Inverse Bayesian method for centrality determination

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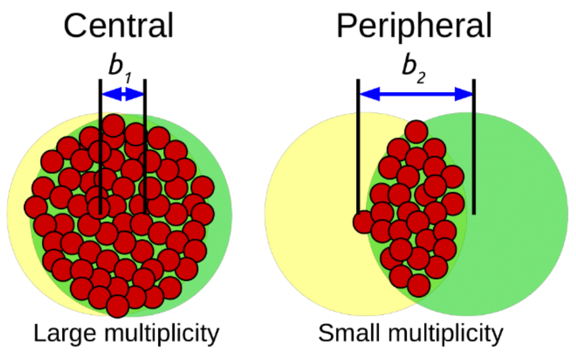
Outline

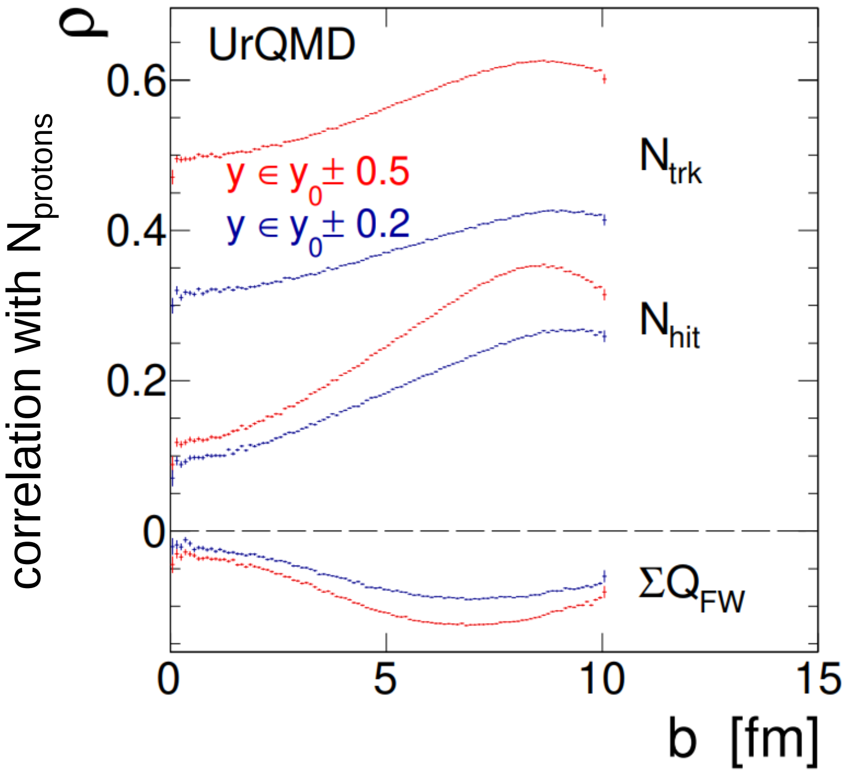
- Introduction
- Inverse Bayesian method
  - Main assumptions
  - Application to experimental data
- Centrality determination at BM@N
  - Corrections for efficiency and pileup
  - 2D fit
- The Bayesian approach in NA61/SHINE
- Summary and outlook

Centrality

- Evolution of matter produced in heavy-ion collisions depend on its initial geometry
- Centrality procedure maps initial geometry parameters with measurable quantities (multiplicity or energy of the spectators)
- **This allows comparison of the future BM@N results with the data from other experiments (STAR BES, NA49/NA61 scans) and theoretical models**

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{A-A}} \int_0^b \frac{d\sigma}{db'} db'$$

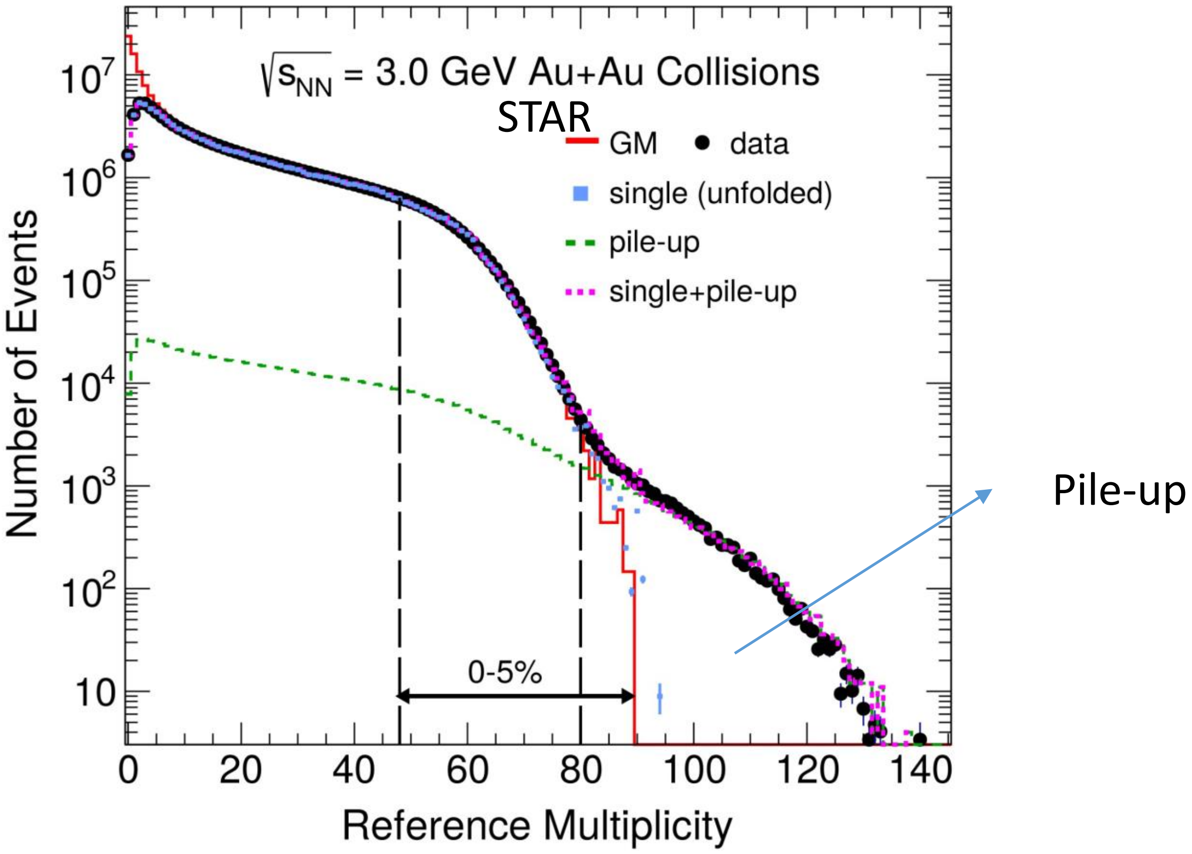




HADES; Phys.Rev.C 102 (2020) 2, 024914

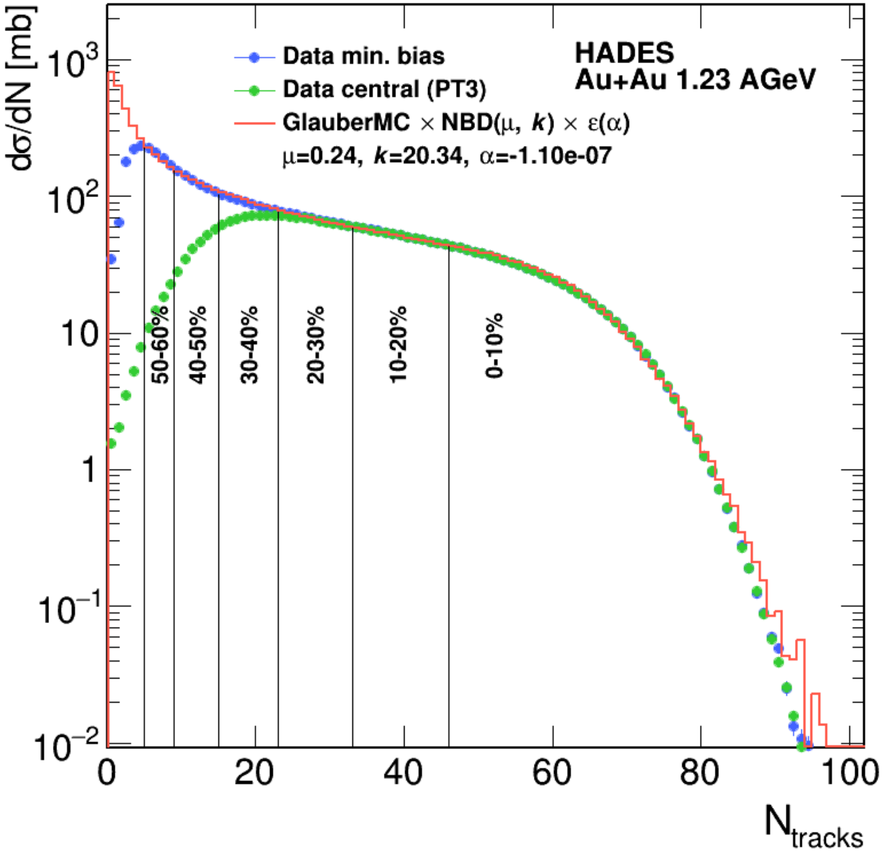
- A number of produced protons is stronger correlated with the number of produced particles (track & RPC+TOF hits) than with the total charge of spectator fragments (FW)
- to suppress self-correlation biases, it is necessary to use spectators fragments for centrality estimation

Centrality determination in the FIX-target experiments



Reference multiplicity distributions (black markers) in the kinematic acceptance within −0.5 < y < 0 and 0.4 < pT < 2.0 GeV/c ,
GM (red histogram), and single and pile-up contributions from unfolding.

https://arxiv.org/abs/2112.00240



The cross section as a function of Ntracks for minimum bias (blue symbols) and central (PT3 trigger, green symbols) data in comparison with a fit using the Glauber MC model (red histogram).

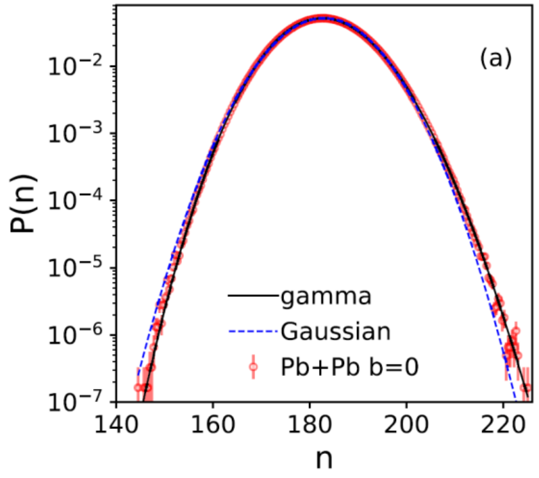
https://arxiv.org/abs/1712.07993

The Bayesian inversion method (Γ-fit): main assumptions

- Relation between multiplicity N_{ch} and impact parameter b is defined by the fluctuation kernel:

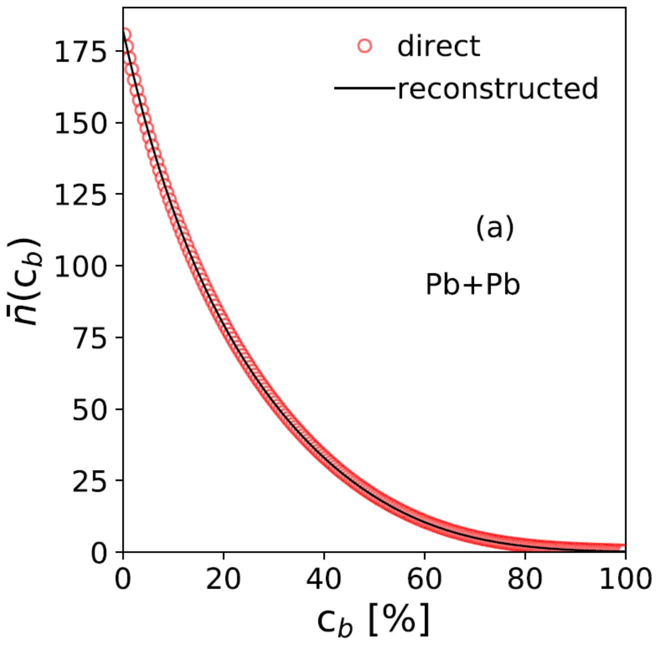
$$P(N_{ch}|c_b) = \frac{1}{\Gamma(k(c_b))\theta^k} N_{ch}^{k(c_b)-1} e^{-n/\theta}$$

$$c_b = \int_0^b P(b')db' \simeq \frac{\pi b^2}{\sigma_{inel}}$$ – centrality based on impact parameter



The results of fitting the multiplicity distribution for a fixed impact parameter TRENTo model, $\sqrt{S_{nn}} = 5.02\ TeV$

R. Rogly, G. Giacalone and J. Y. Ollitrault, Phys.Rev. C98 (2018) no.2, 024902



The dependence of the average value of multiplicity on centrality and the results of its fit

$$\frac{\sigma^2}{\langle N_{ch}\rangle} = \theta \simeq const$$

$$\langle N_{ch}\rangle = N_{knee} \exp\left(\sum_{j=1}^{3} a_j c_b^j\right), \quad k = \frac{\langle N_{ch}\rangle}{\theta}$$

Five fit parameters N_{knee}, θ, a_j

Reconstruction of *b*

- Normalized multiplicity distribution $P(N_{ch})$

$$P(N_{ch}) = \int_0^1 P(N_{ch}|c_b)dc_b$$

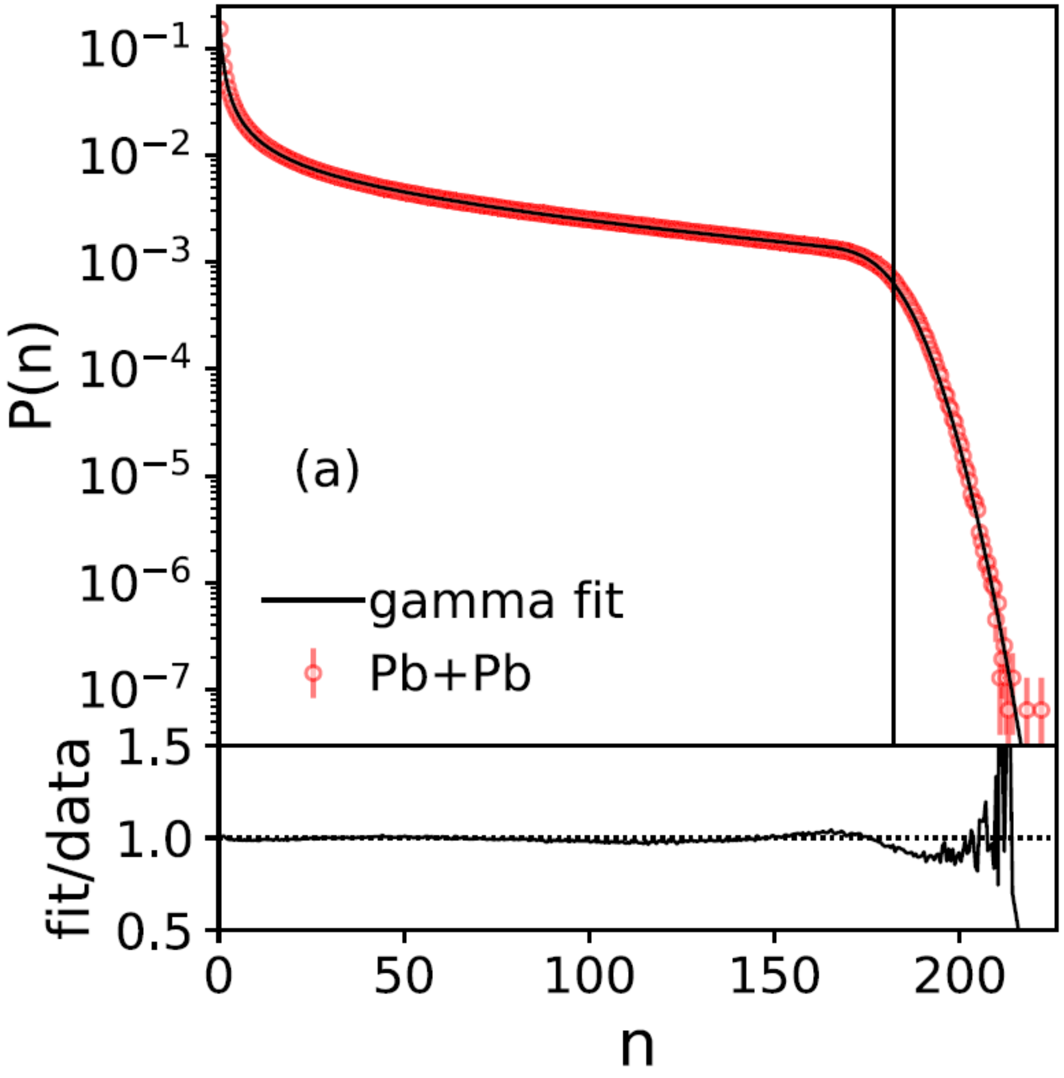
- Find probability of *b* for fixed range of N_{ch} using Bayes' theorem:

$$P(b|n_1 < N_{ch} < n_2) = P(b)\frac{\int_{n_1}^{n_2} P\,(b|N_{ch})dN_{ch}}{\int_{n_1}^{n_2} P\,(N_{ch})dN_{ch}}$$

- **The Bayesian inversion method consists of 2 steps:**

–Fit normalized multiplicity distribution with $P(N_{ch})$

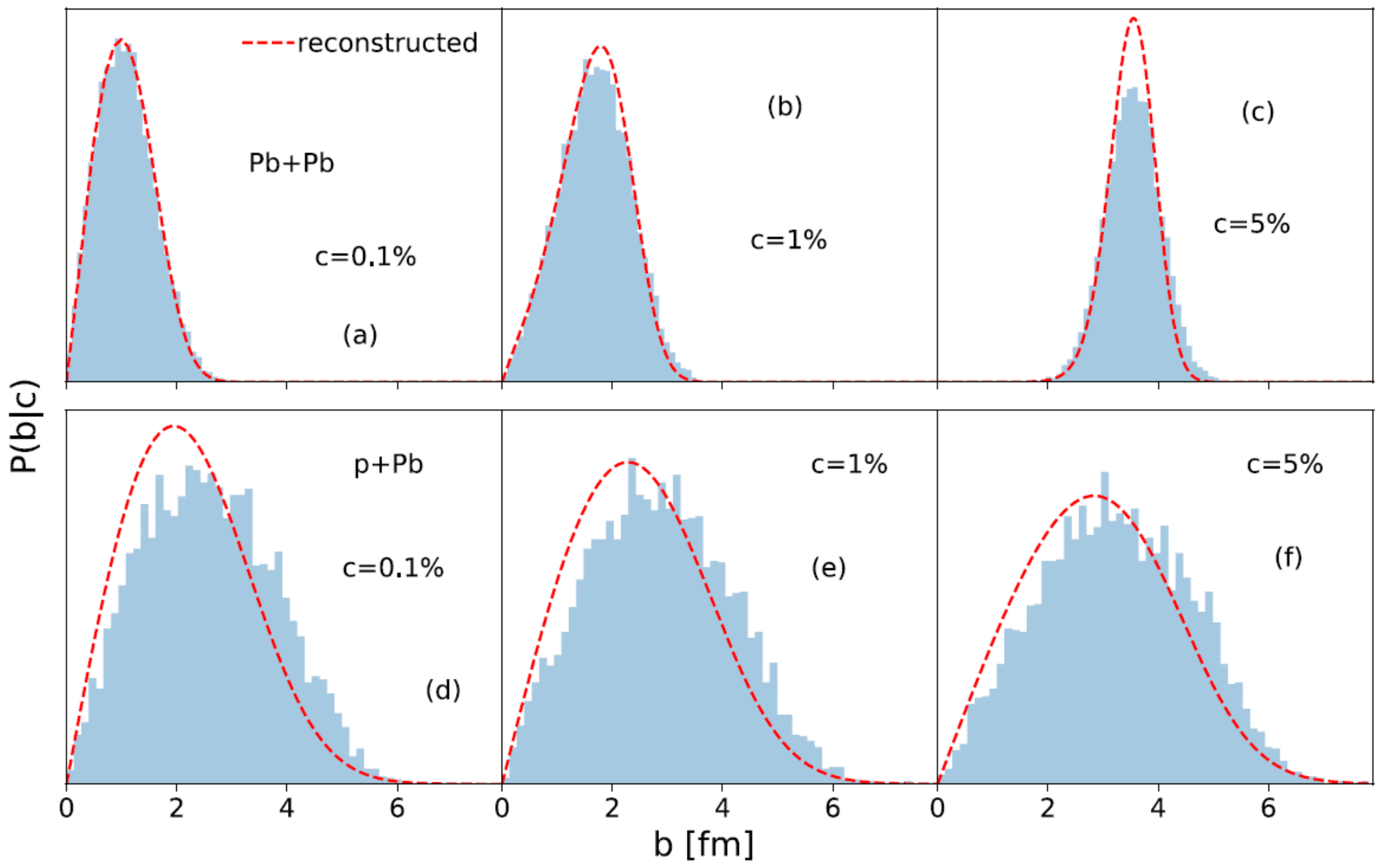
–Construct $P(b|N_{ch})$ using Bayes' theorem with parameters from the fit



TRENTo model, $\sqrt{S_{nn}} = 5.02\ TeV$

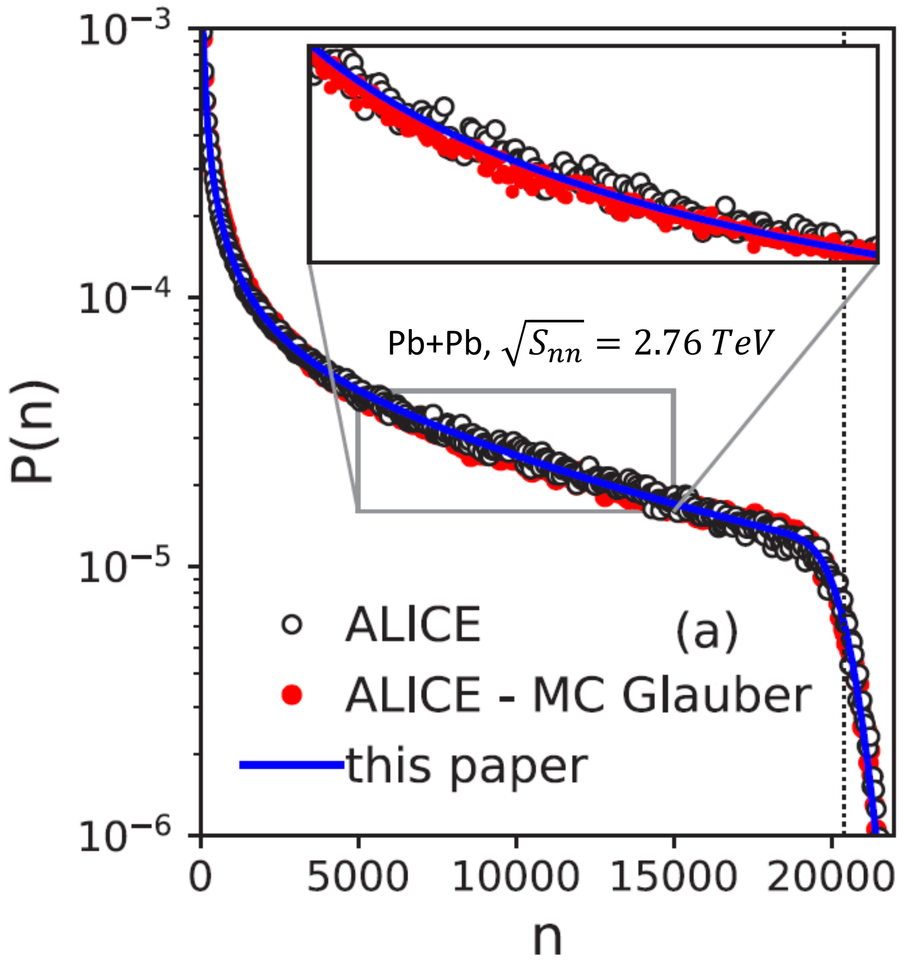
R. Rogly, G. Giacalone and J. Y. Ollitrault, Phys.Rev. C98 (2018) no.2, 024902

Reconstruction of *b*

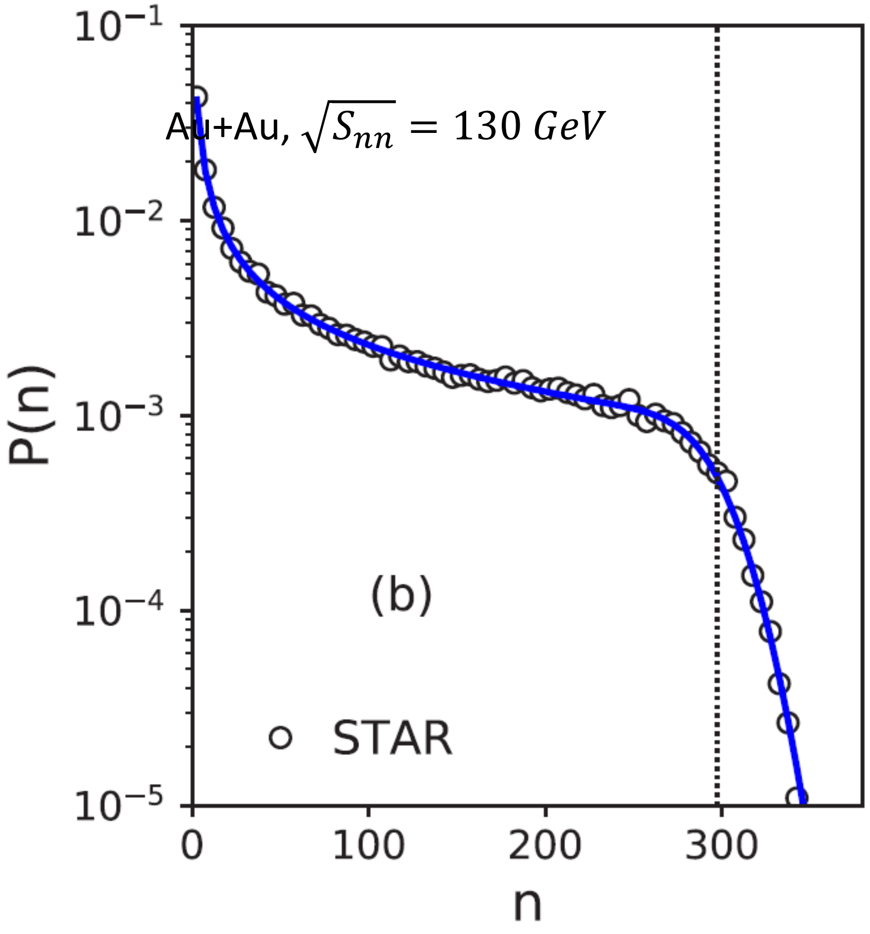


Probability distribution of *b* at fixed centrality returned by the fit (lines) and calculated directly by binning in *b* (shaded area)
good agreement with data from the model

Application to experimental data



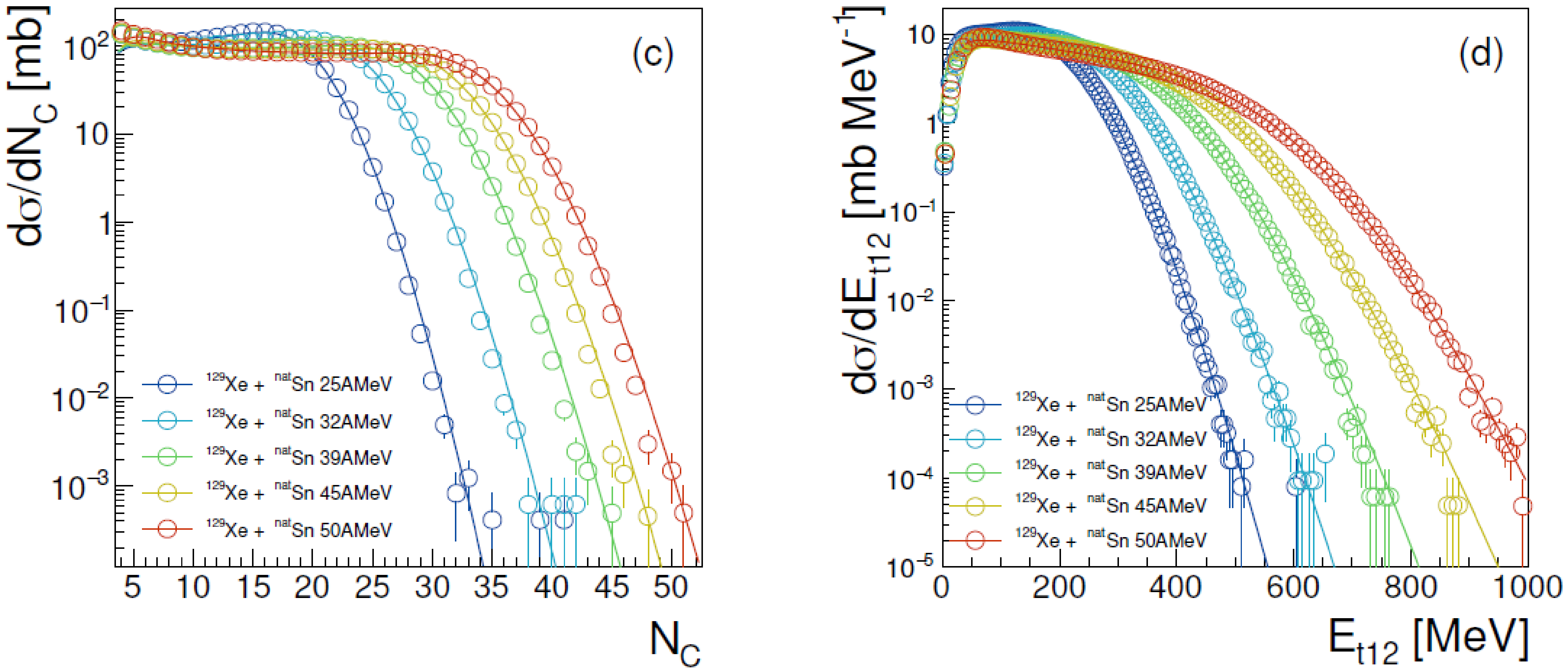
Empty symbols show distribution of the VZERO amplitude



Open symbols denote the multiplicity of negatively charged particles
The vertical line in both panels indicates the position of the knee.
The method was also applied to data from LHC, CMS, and ATLAS experiments.

Rogly, R., Giacalone, G. & Ollitrault, J. Y. *Phys. Rev. C* 98, 1–9 (2018).

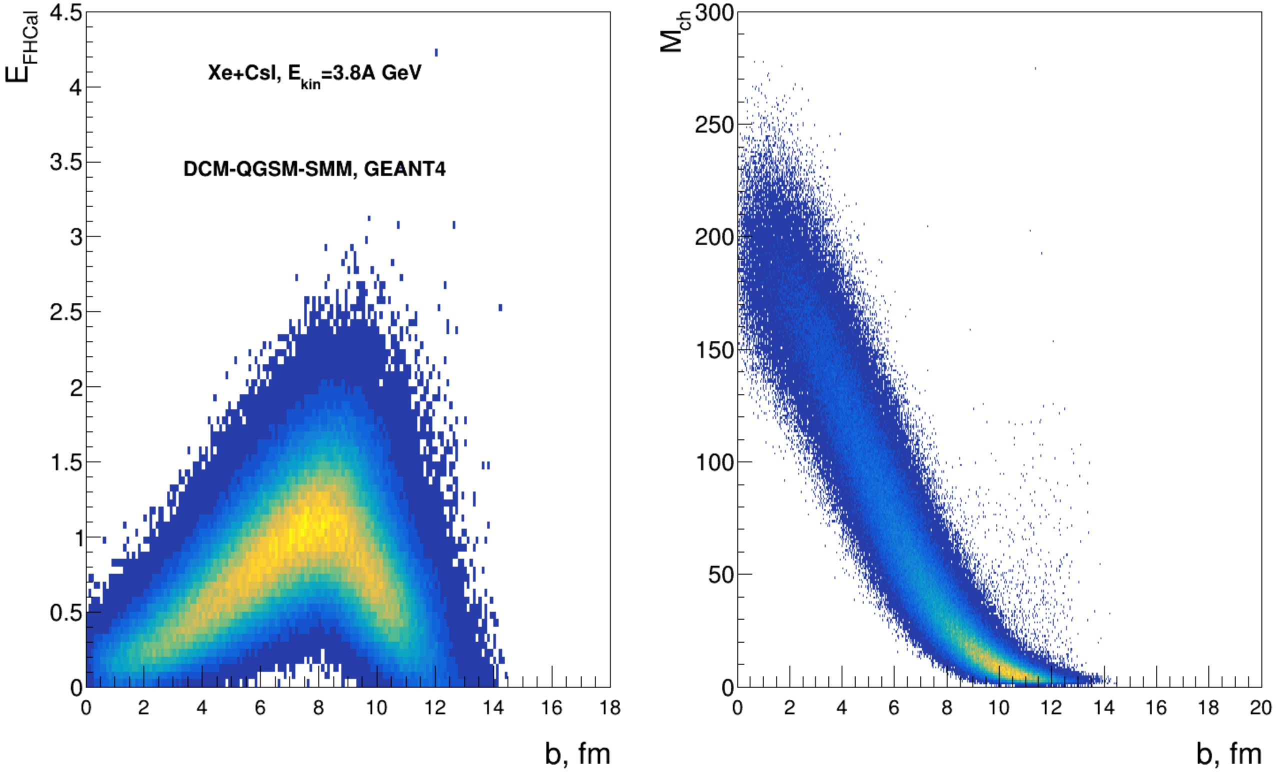
Application to experimental data 2



Results of fits to the multiplicity of charged particles and transvers energy Et12 for the 129Xe+natSn data.
The Bayesian approach was applied to a very wide range of data for different collisions measured with INDRA between 25 MeV/nucleon and 100 MeV/nucleon

Centrality determination in BM@N

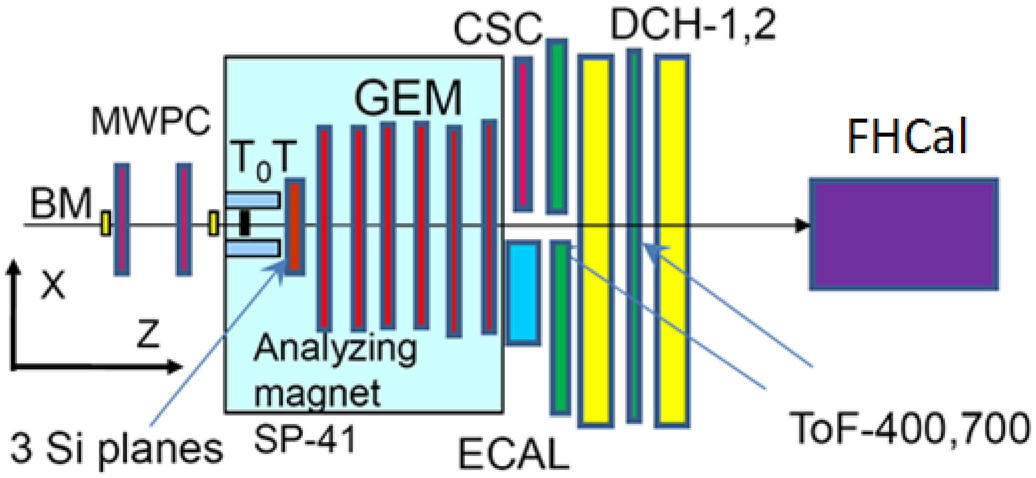
DCM-QGSM-SMM → GEANT4 → Reconstruction → Centrality determination



Relation between impact parameter and track multiplicity

Centrality determination: Multiplicity of produced charged particles in tracking system

Simulated data sets: Xe+Cs, Nev=500k



BM@N setup overview

The Bayesian inversion method (Γ-fit): DCM-QSM-SMM based

- The fluctuation kernel Gamma distr.:

$$P(M \mid c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} M^{k(c_b)-1} e^{-M/\theta}$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$$\theta = \frac{D(M)}{\langle M \rangle}, \quad k = \frac{\langle M \rangle}{\theta}$$

$\langle M \rangle, D(M)$ – average and variance of Multiplicty

$$P(M) = \int_0^1 P(M \mid c_b)dc_b$$

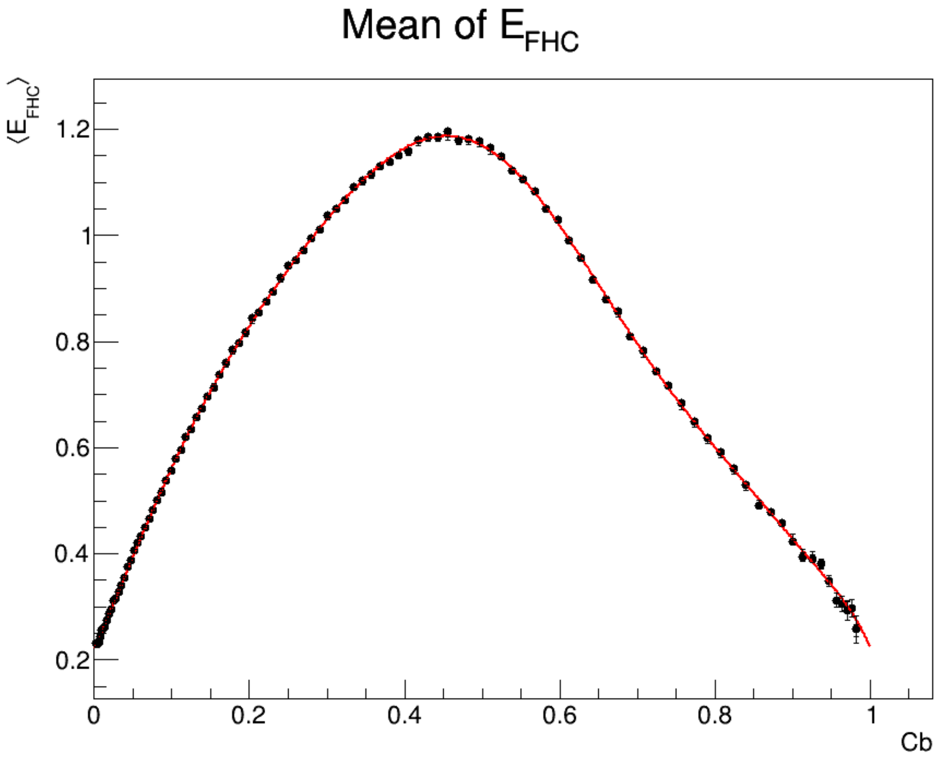
$$\langle M \rangle = m_1 \cdot \langle M' \rangle$$

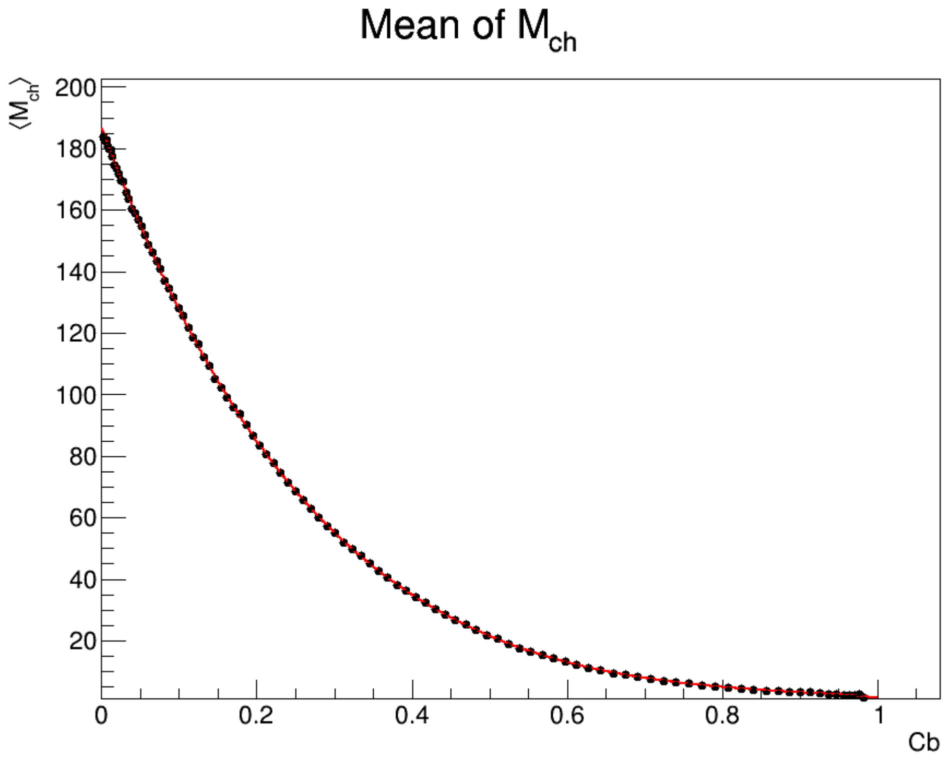
$$D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2 \langle M' \rangle$$

$\langle M'(c_b) \rangle$ – average value and var. of energy/mult.
$D(M'(c_b))$ from the rec. model data

- can be approximated by polynomials and exponential polynomial

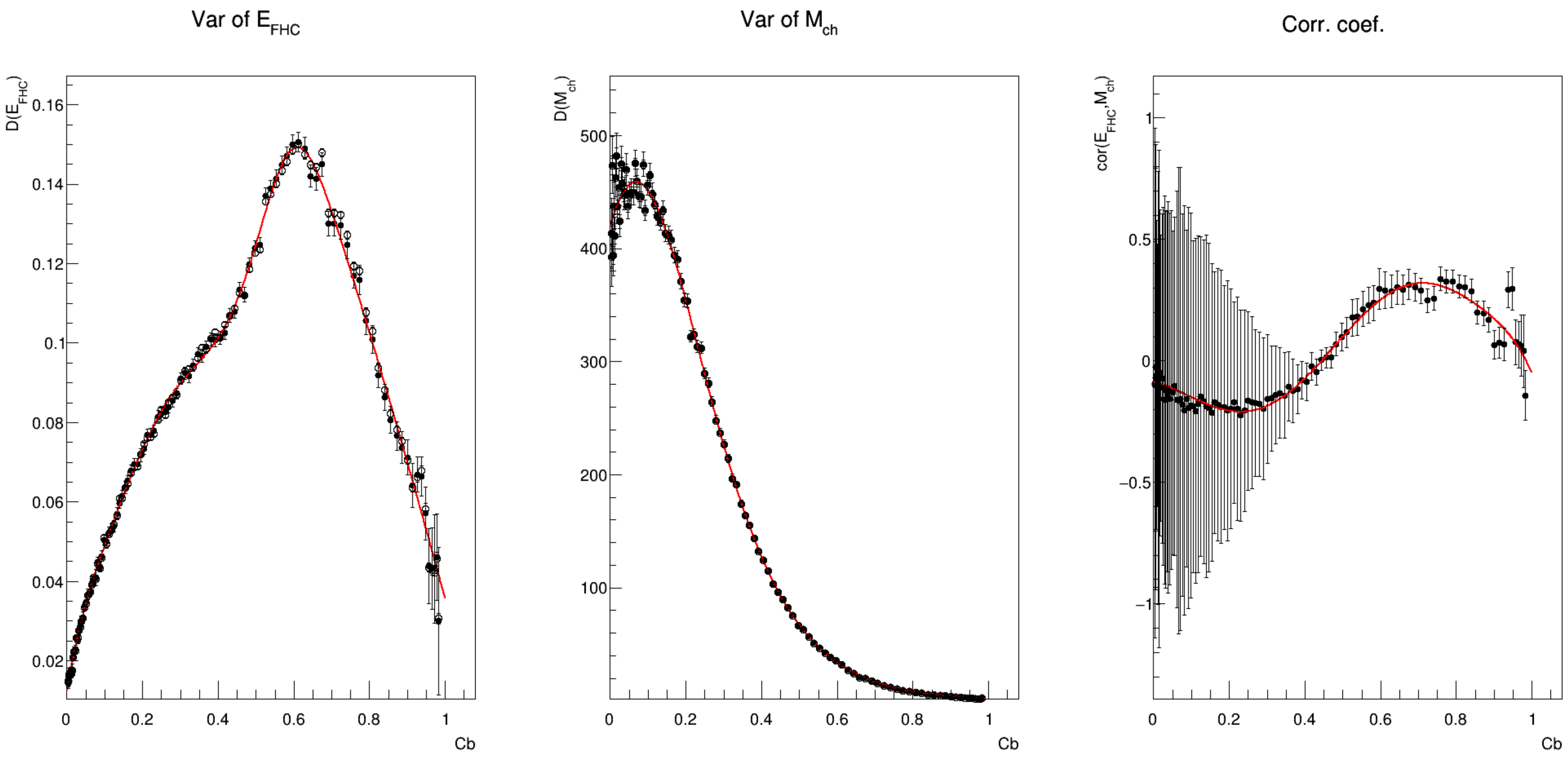
Dependence of the average value of multiplicity and energy on centrality





the distribution of the mean value of observables is well fit by exponential polynomials

Dependence of the variance of multiplicity and energy on centrality



Good fit quality

Probabilistic model of pileup

$M_{pu}(b_1,b_2)=M_1(b_1)+M_2(b_2)$ - pileup as two independent events, with impact parameters b_1,b_2

$$\langle M_{pu}(b_1,b_2)\rangle=\langle M_1(b_1)\rangle+\langle M_2(b_2)\rangle,\quad D(M_{pu}(b_1,b_2))=D(M_1(b_1))+D(M_2(b_2))$$

$$P_{pu}(M_{pu}\,|\,b_1,b_2)=\frac{1}{\Gamma(k_p)\theta_p{}^2}M_{pu}{}^{k_p-1}e^{-M_{pu}/\theta_p}$$

- The fluctuation of multiplicity can be describe by Gamma distribution

$$\theta_p=\frac{D(M(b_1,b_2))}{\langle M(b_1,b_2)\rangle},\quad k_p=\frac{\langle M(b_1,b_2)\rangle}{\theta_p}$$

- The parameters of Gamma distribution

$P_{pu}(M_{pu})$ – the probability distribution of pileup can be calculated as

$$P_{pu}(M_{pu})=\int_0^{b_{\max}}\int_0^{b_{\max}}P(M_{pu}\,|\,b_1,b_2)P(b_1)P(b_2)db_1db_2=\int_0^{c_{b1}}\int_0^{c_{b2}}P_{pu}(M_{pu}\,|\,c_{b1},c_{b2})dc_{b1}dc_{b2}$$

Corrections for efficiency and pileup

- **Correction for efficiency of normalized multiplicity distribution P(M)**

$$P(M) = \frac{dN}{dM} / N_{ideal}^{ev} = \frac{N_{raw}^{ev}}{N_{ideal}^{ev}} \cdot \frac{1}{N_{raw}^{ev}} \frac{dN_r}{dM} = \frac{1}{K} \cdot Norm.Histogr$$

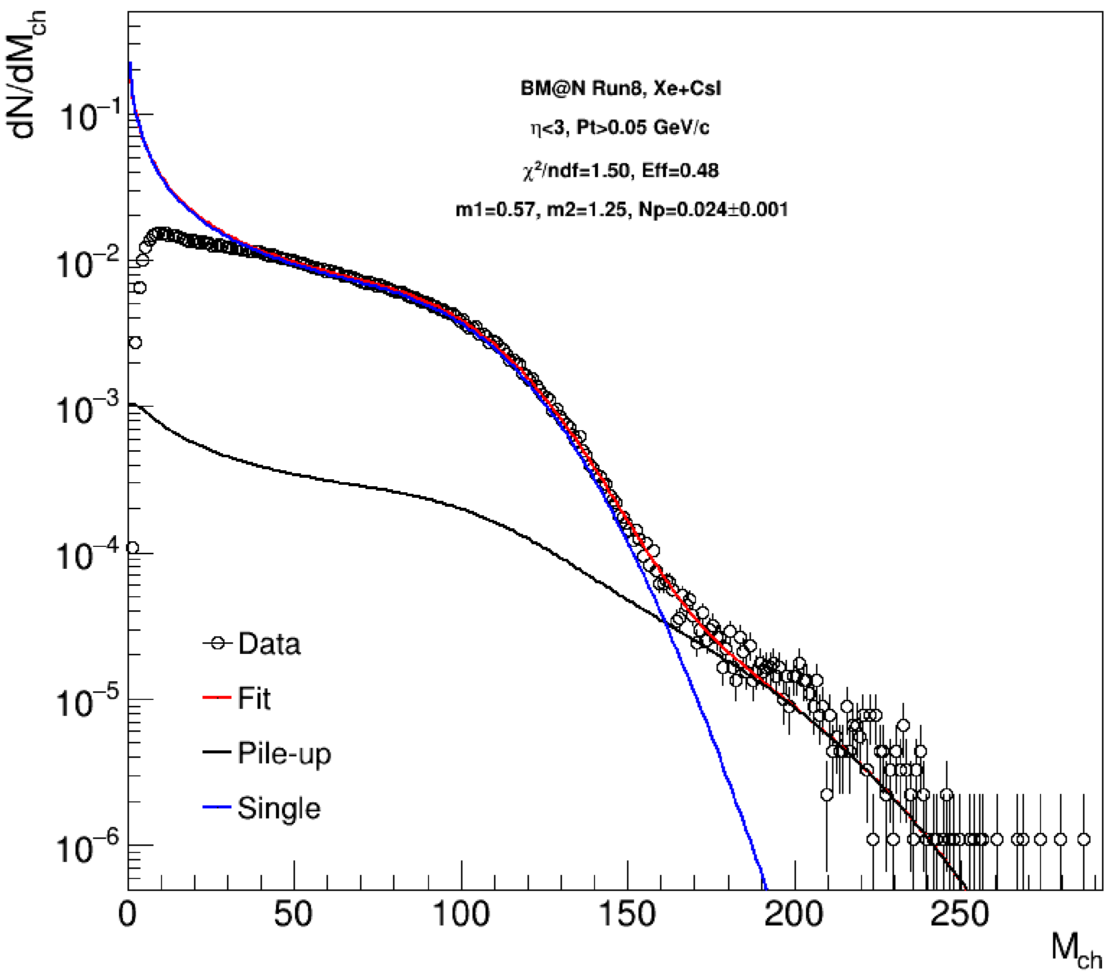
$$Eff = \frac{N_{raw}^{ev}}{N_{ideal}^{ev}} = \frac{1}{K}$$ integral efficiency

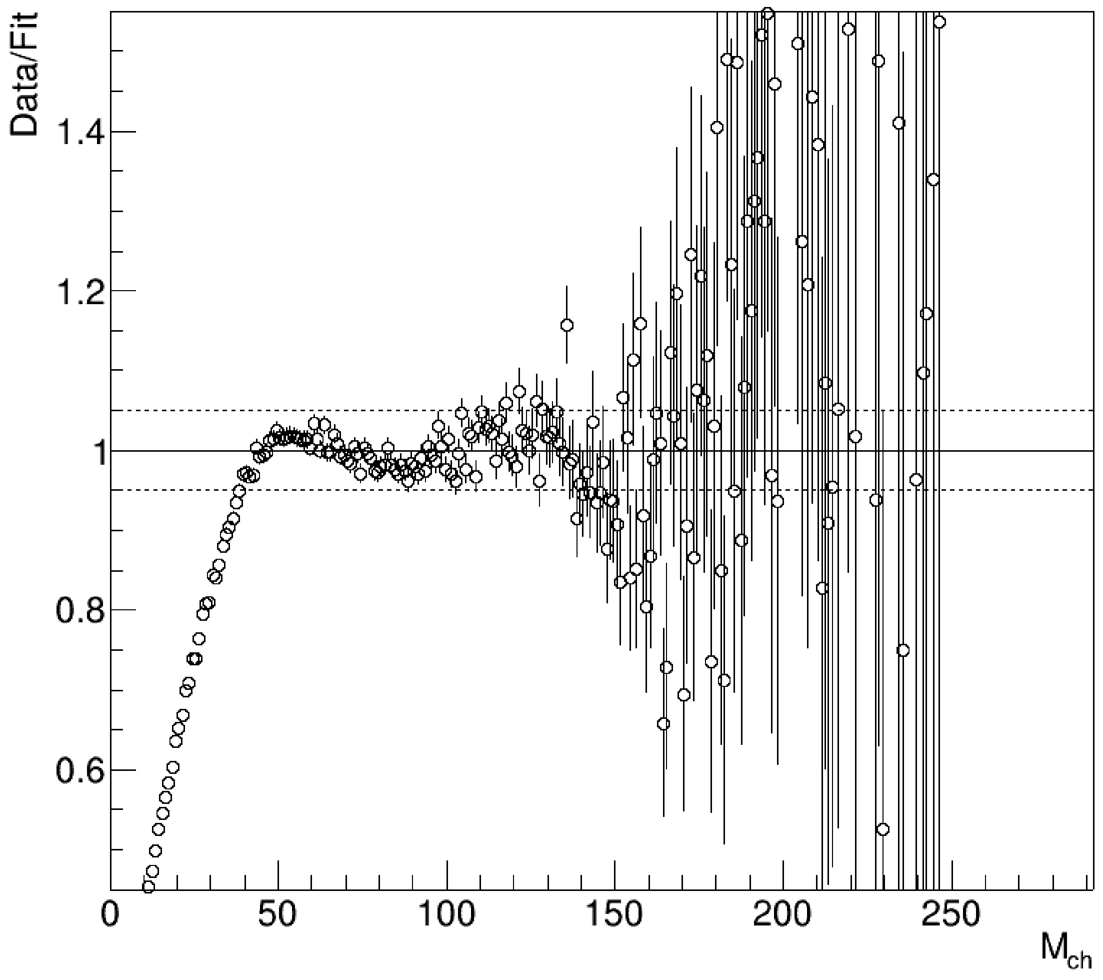
- **Fit function for multiplicity distribution P(M)**

$$F(M) = K \cdot P_{total}(M), \;\; P_{total}(M) = N_p \cdot P_{pu}(M) + (1 - N_p) \cdot P(M)$$

m_1, m_2, K, N_p- fit parameters, $F(M) -$ fit function, corrected for efficiency and pileup

Fit results





Vertex Cuts: CCT2, N_{vtxTr}>1,|$V_{x,y}$ –(0.3,0.14)|<1 cm, |V_z -0.07|< 0.2 cm

Good agreement with fit

Track selection: Nhit>4, η<3, Pt>0.05 GeV/c

The Bayesian inversion method (Γ-fit): 2D fit

- The fluctuation kernel for energy and multiplicity at fixed impact parameter can be describe by 2D Gamma distr.:

$$P(E,M\mid c_b) = G_{2D}(E,M,\langle E\rangle,\langle M\rangle,D(E),D(M),R)$$

$$c_b = \int_0^b P(b')db'$$ – centrality based on impact parameter

$\langle E\rangle, D(E)$ – average value and variance of energy

$\langle M\rangle, D(M)$ – average value and variance of mult.

$R(E,M)$ – Pirson correlation coefficient

$$R(E,M) = \varepsilon_1 \cdot m_1 \cdot R(E',M')\sqrt{\frac{D(E')D(M')}{D(E)D(M)}}$$

$\varepsilon_0, \varepsilon_1, \varepsilon_2, m_1, m_2$ - fit parameters

$\langle E'(c_b)\rangle$, $D(E'(c_b))$ – average value and var. of energy/mult. from the rec. model data

$$\langle E\rangle = \varepsilon_1\langle E'(c_b)\rangle + \varepsilon_0,\quad D(E) = \varepsilon_2 D(E'(c_b))$$

$$\langle M\rangle = m_1\langle M'(c_b)\rangle,\quad D(M) = m_1^2 \cdot D(M') + m_1 \cdot m_2\langle M'\rangle$$

$\langle E'(c_b)\rangle,\quad D(E'(c_b))$ - can be approximated by polynomials

$$\langle E'(c_b)\rangle = \sum_{j=1}^{12} a_j c_b^j,\quad D(E'(c_b)) = \sum_{j=1}^{19} b_j c_b^j$$

$$\langle M'(c_b)\rangle = \sum_{j=1}^{12} a_j c_b^j,\quad D(M'(c_b)) = \sum_{j=1}^{6} b_j c_b^j$$

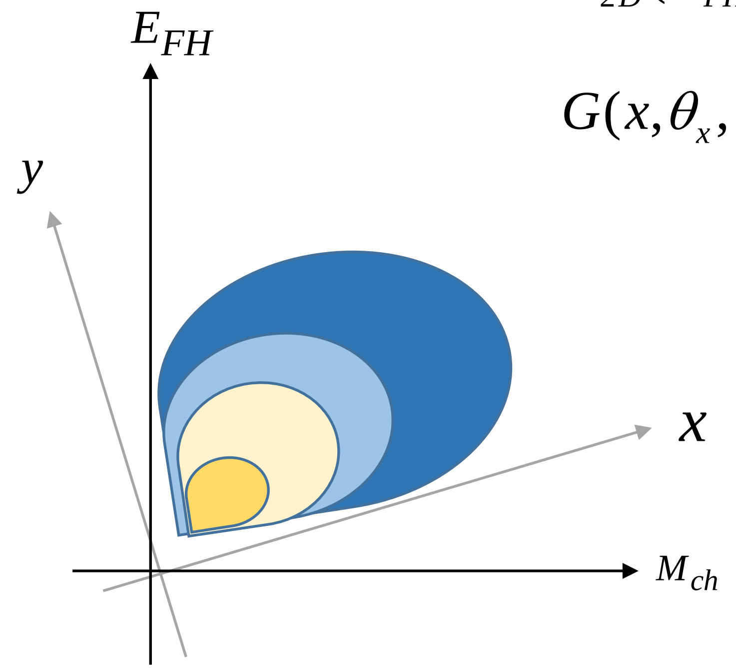
2D Gamma distribution

It is possible to find such a rotation angle of the system that $\mathrm{cov}(x,y)=0$

Then the two-dimensional distribution in the new coordinate system will be

$$G_{2D}(E_{FH},M_{ch},\langle E\rangle,\langle M\rangle,D(E),D(M),R)=G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)$$

$$G(x,\theta_x,k_x)\cdot G(y,\theta_y,k_y)=\frac{(x)^{k_x(c_b)-1}e^{-x/\theta_x}}{\Gamma(k_x(c_b))\theta_x^2}\cdot\frac{(y)^{k_y(c_b)-1}e^{-y/\theta_y}}{\Gamma(k_y(c_b))\theta_y^2}$$



$$\theta_x=\frac{D(x)}{\langle x\rangle},\quad k_x=\frac{\langle x\rangle^2}{D(x)},\theta_y=\frac{D(y)}{\langle y\rangle},\quad k_y=\frac{\langle y\rangle^2}{D(y)}$$

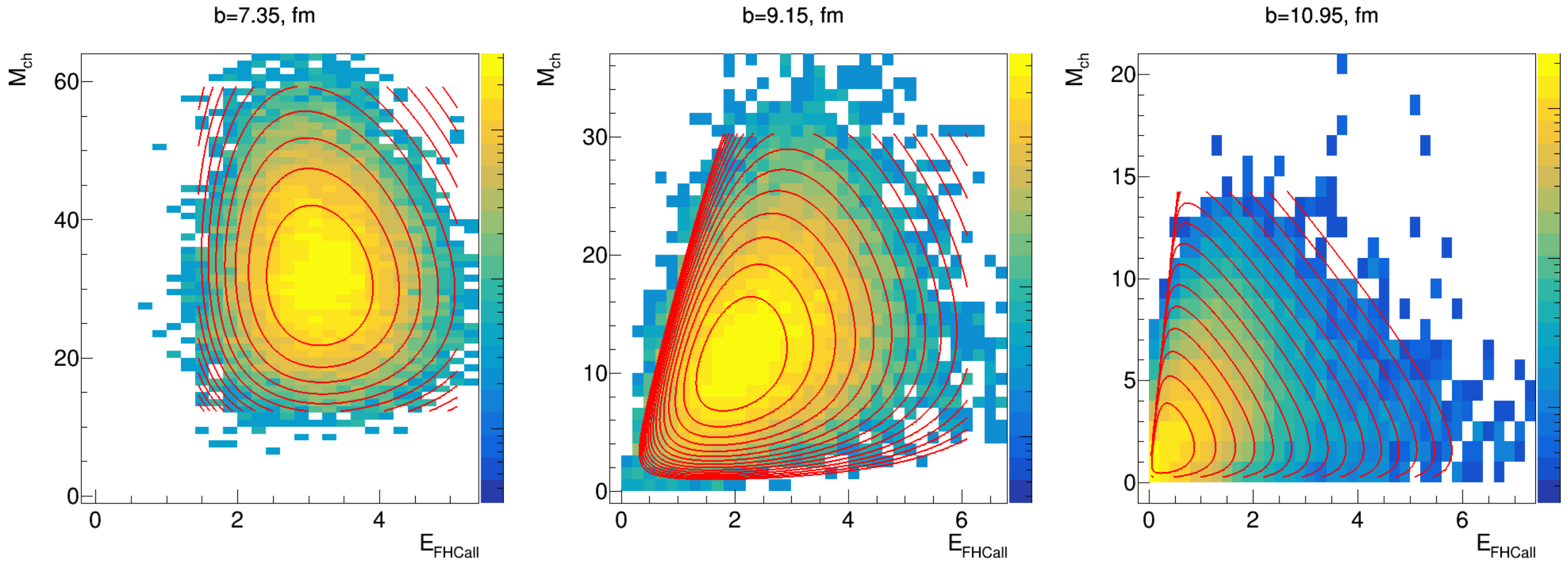
$$\alpha=\arctan\left(\frac{2\sqrt{D(E)D(M)}R(E,M)}{D(E)-D(M)}\right)$$

mean value and variance in the new coordinate system

$$\langle x\rangle=\cos(\alpha)\langle E\rangle+\sin(\alpha)\langle M\rangle \qquad D(x)=D(E)\cos(\alpha)^2+R(E,M)\sqrt{D(E)D(M)}\sin(2\alpha)+D(M)\sin(\alpha)^2$$

$$\langle y\rangle=-\sin(\alpha)\langle E\rangle+\cos(\alpha)\langle M\rangle \qquad D(y)=D(E)\sin(\alpha)^2-R(E,M)\sqrt{D(E)D(M)}\sin(2\alpha)+D(M)\cos(\alpha)^2$$

The fluctuation of energy and multiplicity at fixed impact parameter

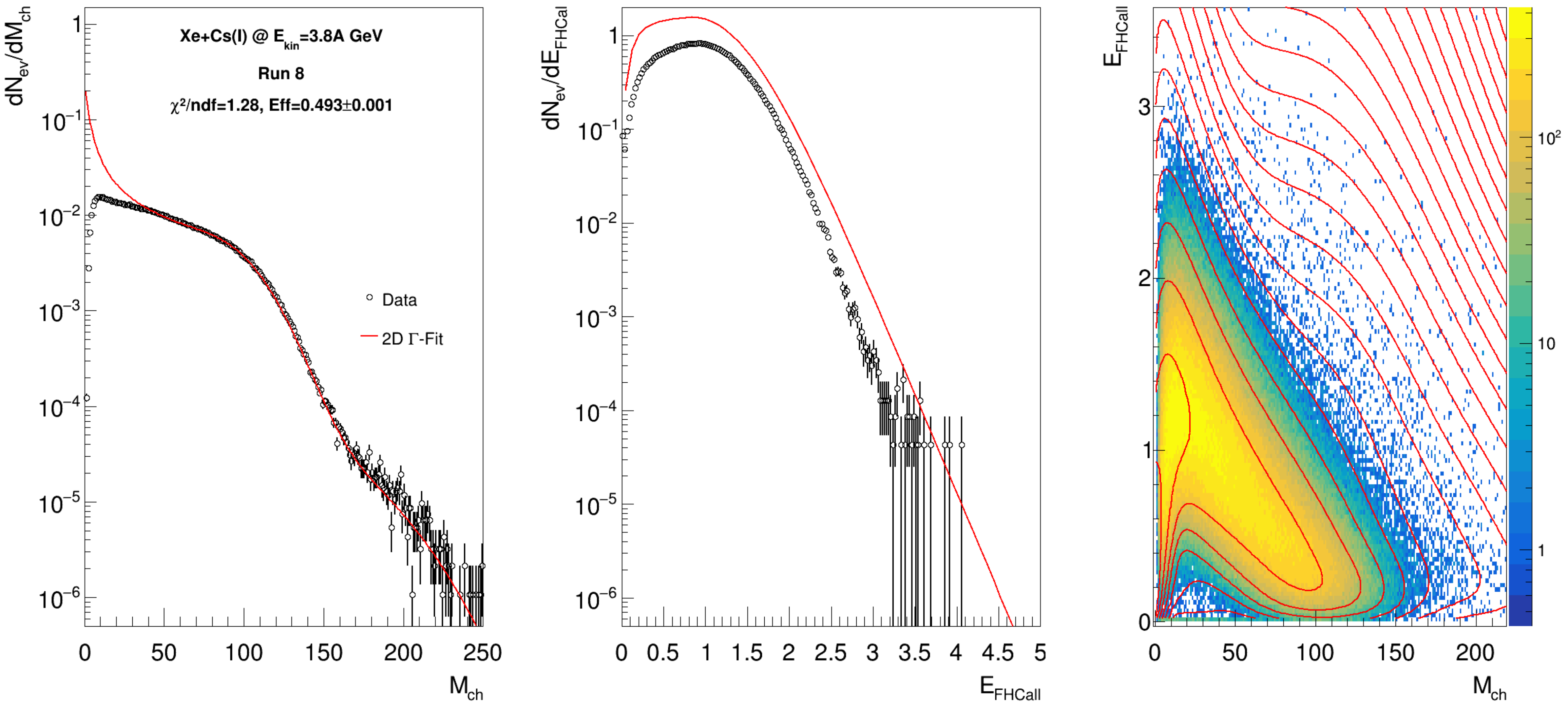


The distribution of energy and multiplicity at a fixed impact parameter is well described by the gamma distribution

- Find probability of *b* for fixed range of E and M using Bayes' theorem:

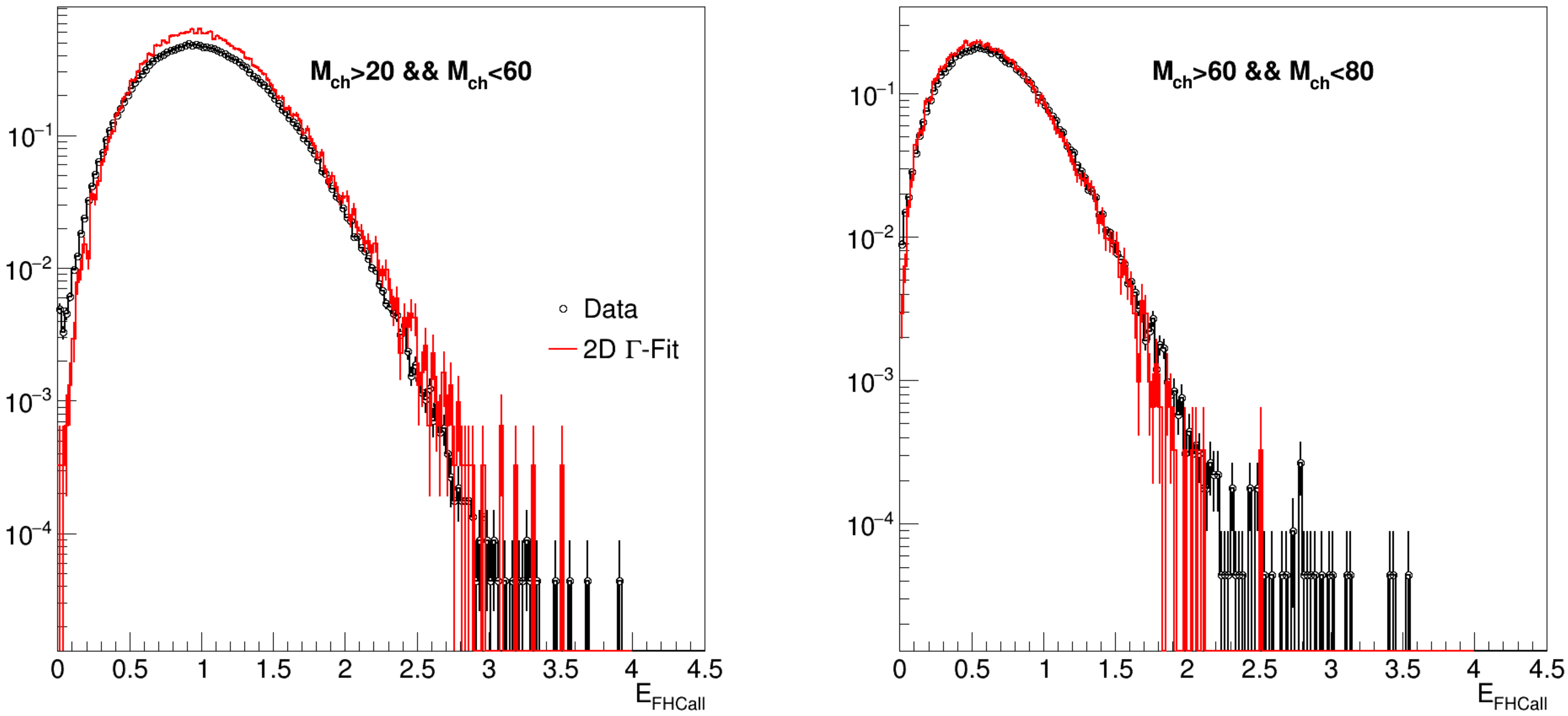
$$P(b \mid E_1 < E < E_2, M_1 < M < M_2) = P(b) \frac{\int_{E_1}^{E_2} \int_{M_1}^{M_2} P(E, M \mid c_b) dM dE}{\int_{E_1}^{E_2} \int_{M_1}^{M_2} \int_{0}^{1} P(E, M \mid c_b) dM dE dc_b}$$

2D fit results



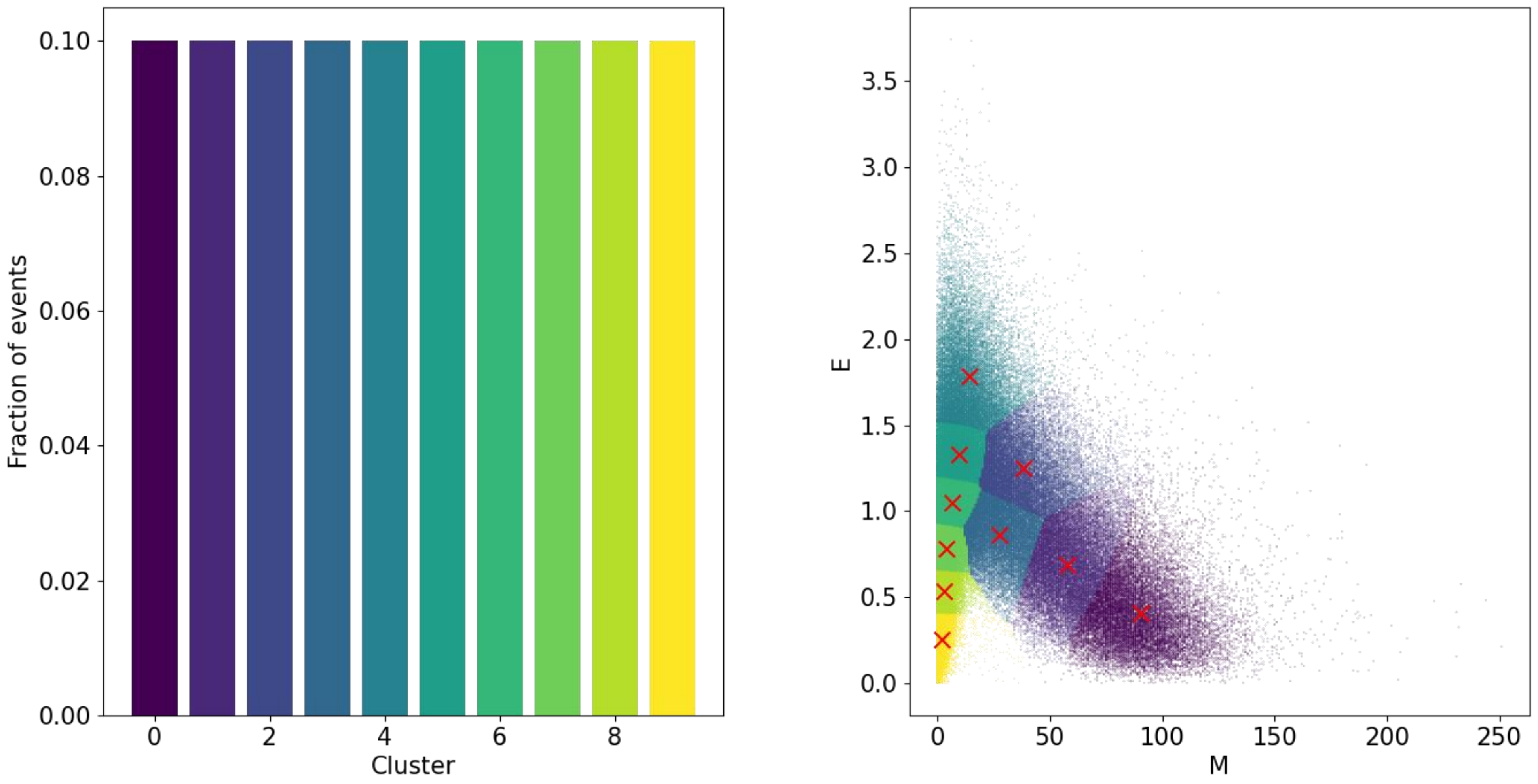
Good agreement between fit and data.

Energy distribution



Good agreement between fit and data.

Clusterization with k means for centrality classes



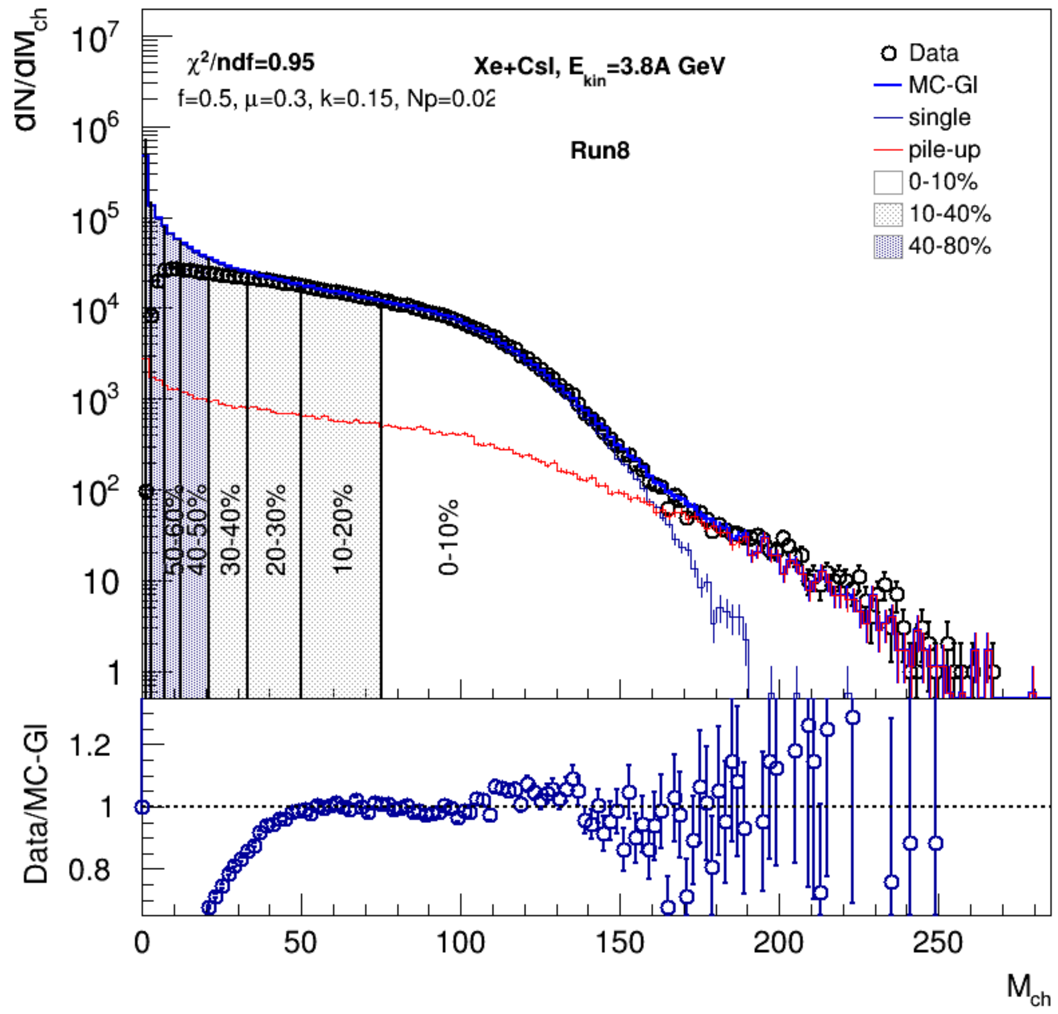
the bivariate fit distribution was divided into 10 centrality classes

MC-Glauber based centrality framework

MC Glauber data → Evaluate N_a: $N_a = fN_{part}+(1-f)N_{coll}$ → Call $NBD(\mu,k) \times N_a \times \varepsilon(\alpha)$ → Build multiplicity fitting function

Input multiplicity distribution

Build multiplicity fitting function, Input multiplicity distribution → Evaluate χ^2 → Minimize χ^2 to find α, μ, k



NBD – negative binomial distribution

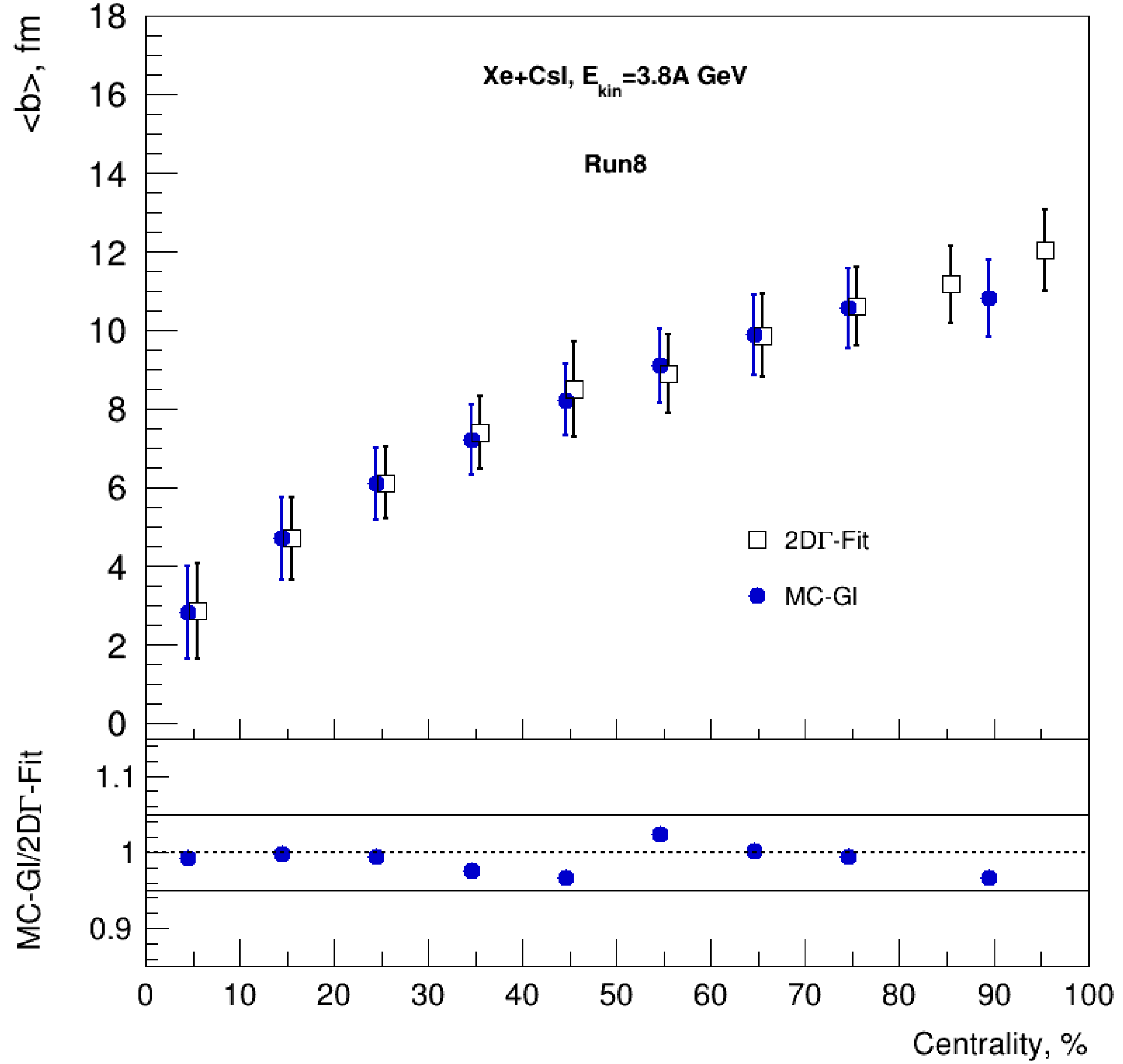
Parameters of the fit:

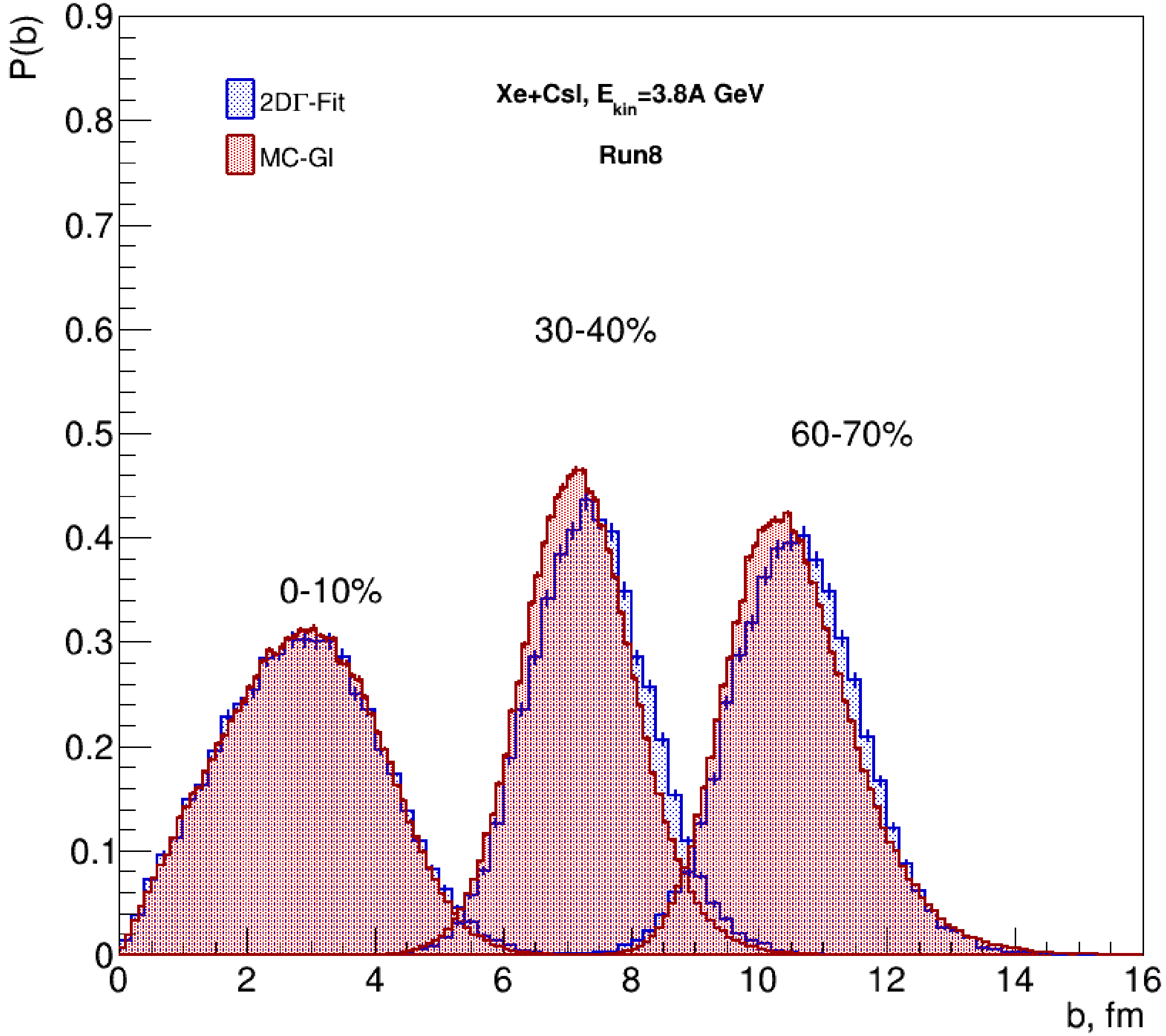
- **α** – coefficient in efficiency function
- **μ** – mean multiplicity value
- **k** – width of the multiplicity distribution, can be connected to the fluctuations

Implamentation for MPD: https://github.com/FlowNICA/CentralityFramework

P. Parfenov, et al., Particles. 2021; 4(2):275-287

Comparison with MC-Glauber fit





There is agreement within 5%.

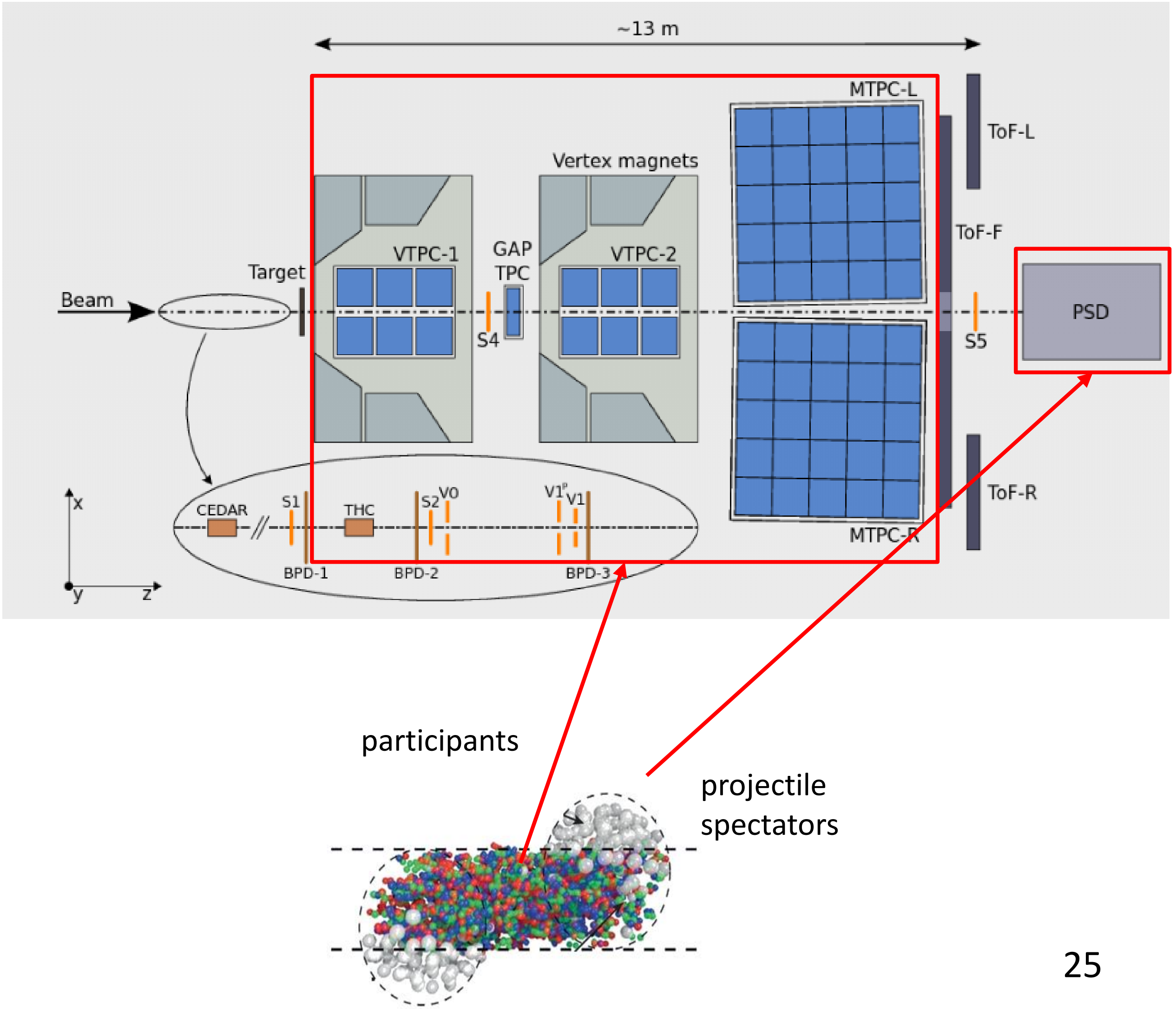
NA61/SHINE experimental setup

Data samples:

- Pb-Pb @ p_{beam} = 13A GeV/c
- data from 2016 physics run
- DCM-QGSM-SMM x Geant4
  M.Baznat et al. PPNL 17 (2020) 3, 303

Subsystems

- Multiplicity: TPCs
- Spectators energy: PSD



Fit results for NA61

- the fluctuation kernel:

$$P(E\,|\,c_b) = \frac{1}{\Gamma(k(c_b))\theta^2} E^{k(c_b)-1} e^{-E/\theta}$$

where

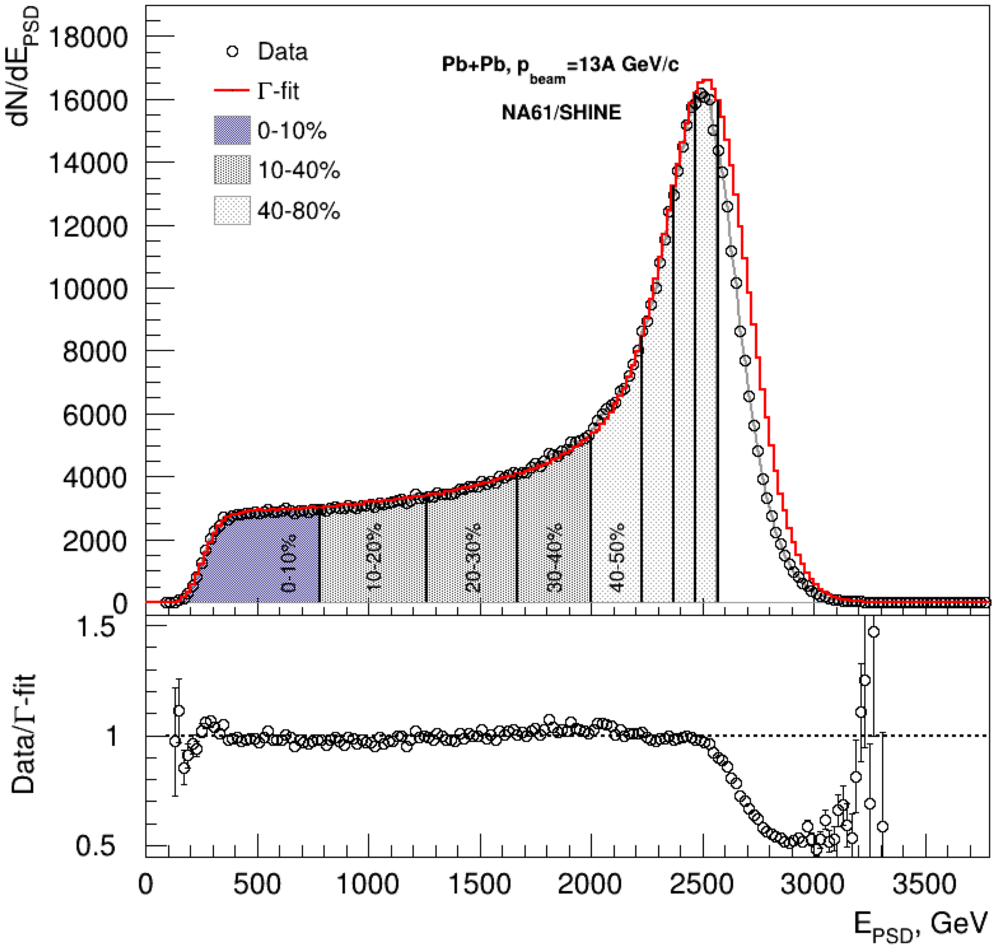
$$\theta = \frac{D(E)}{\langle E \rangle}, \quad k = \frac{\langle E \rangle}{\theta}$$

$\langle E \rangle, \quad D(E)$ – average value and variance of energy

$$\langle E \rangle = \mu_1 \langle E'(c_b) \rangle + \lambda_1, \quad D(E) = \mu_2 D(E'(c_b))$$

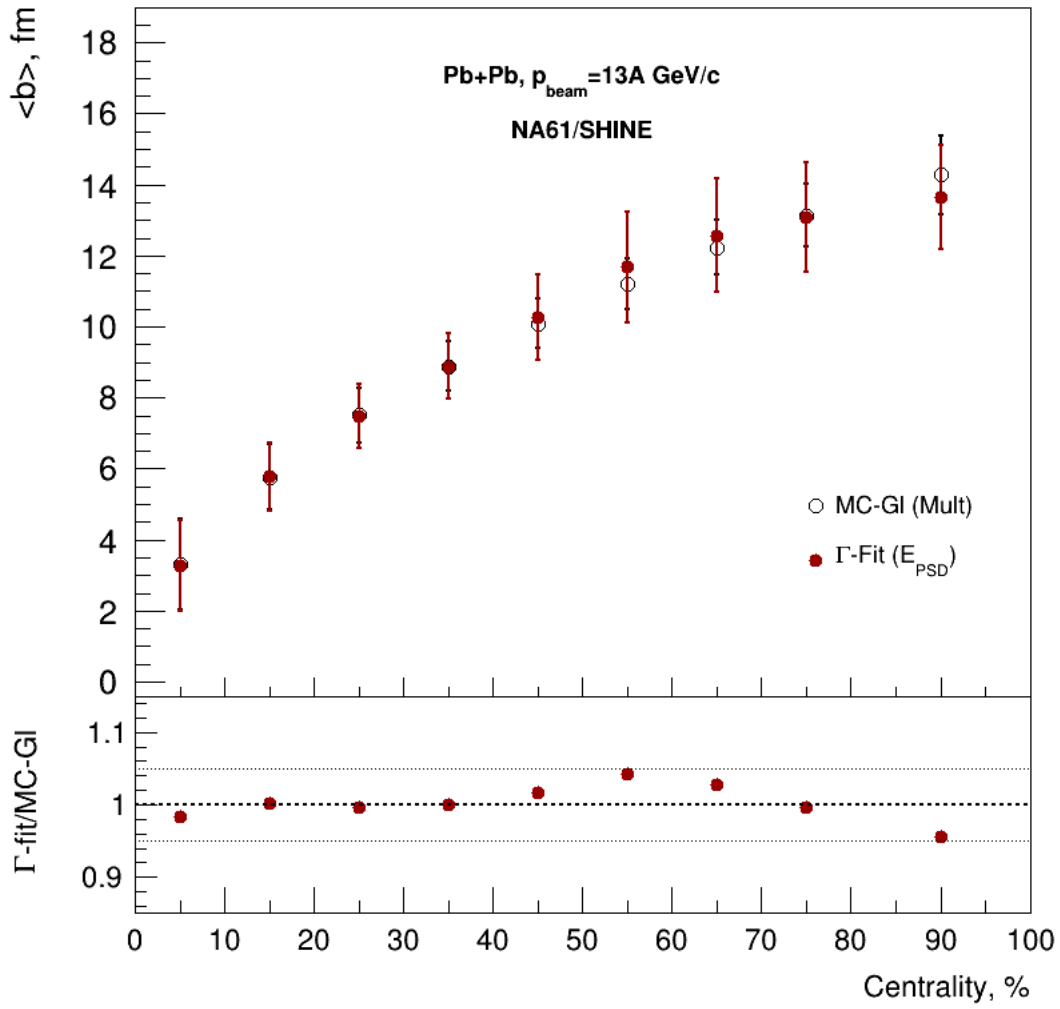
$\langle E'(c_b) \rangle, \quad D(E'(c_b))$ – average value and variance of energy from the rec. model data

Three fit parameters μ_1, μ_2, λ_1

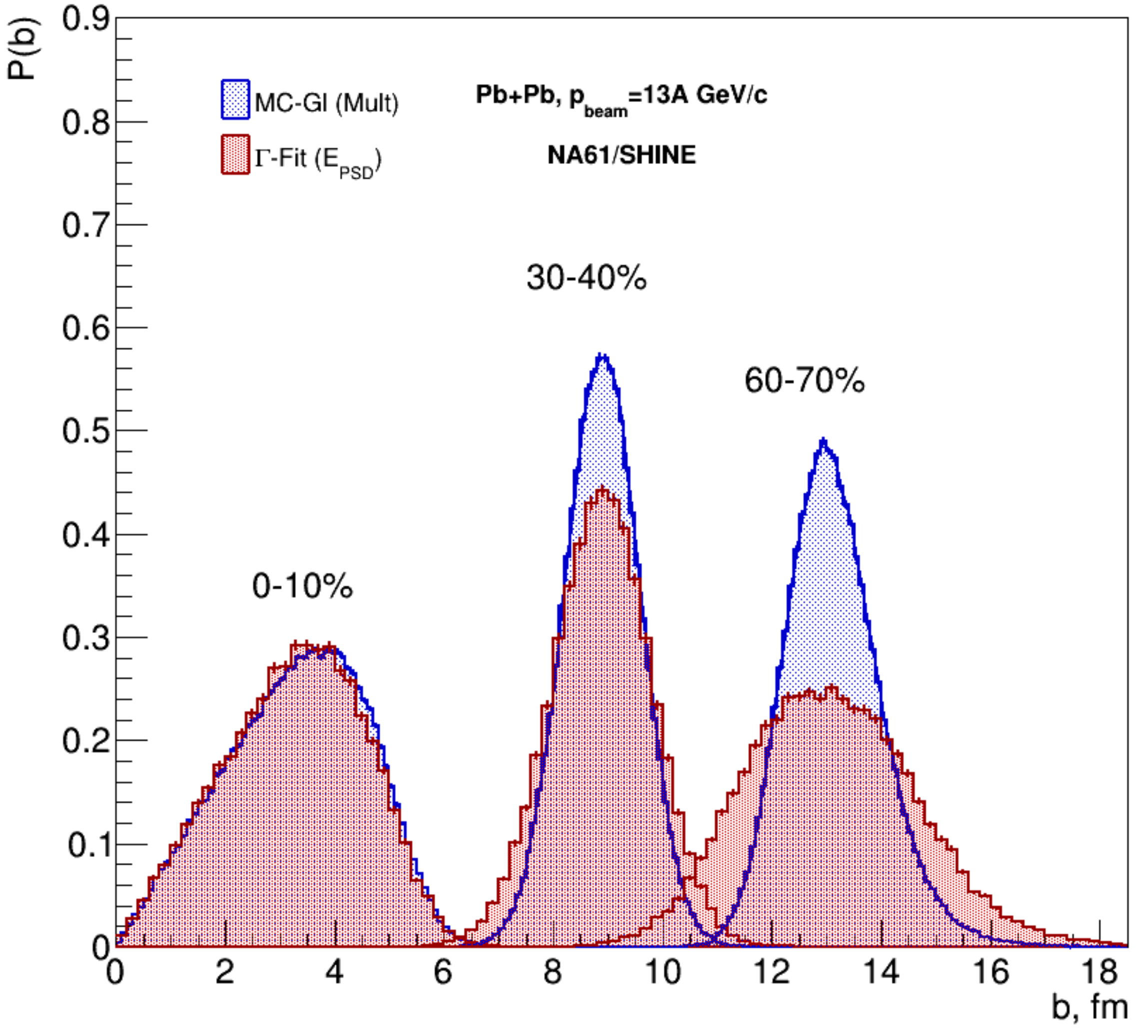


The method reproduces the energy distribution well.
The difference in the peripheral region is due to the trigger efficiency

Comparison with MC-Glauber fit



Good agreement between fit and data.



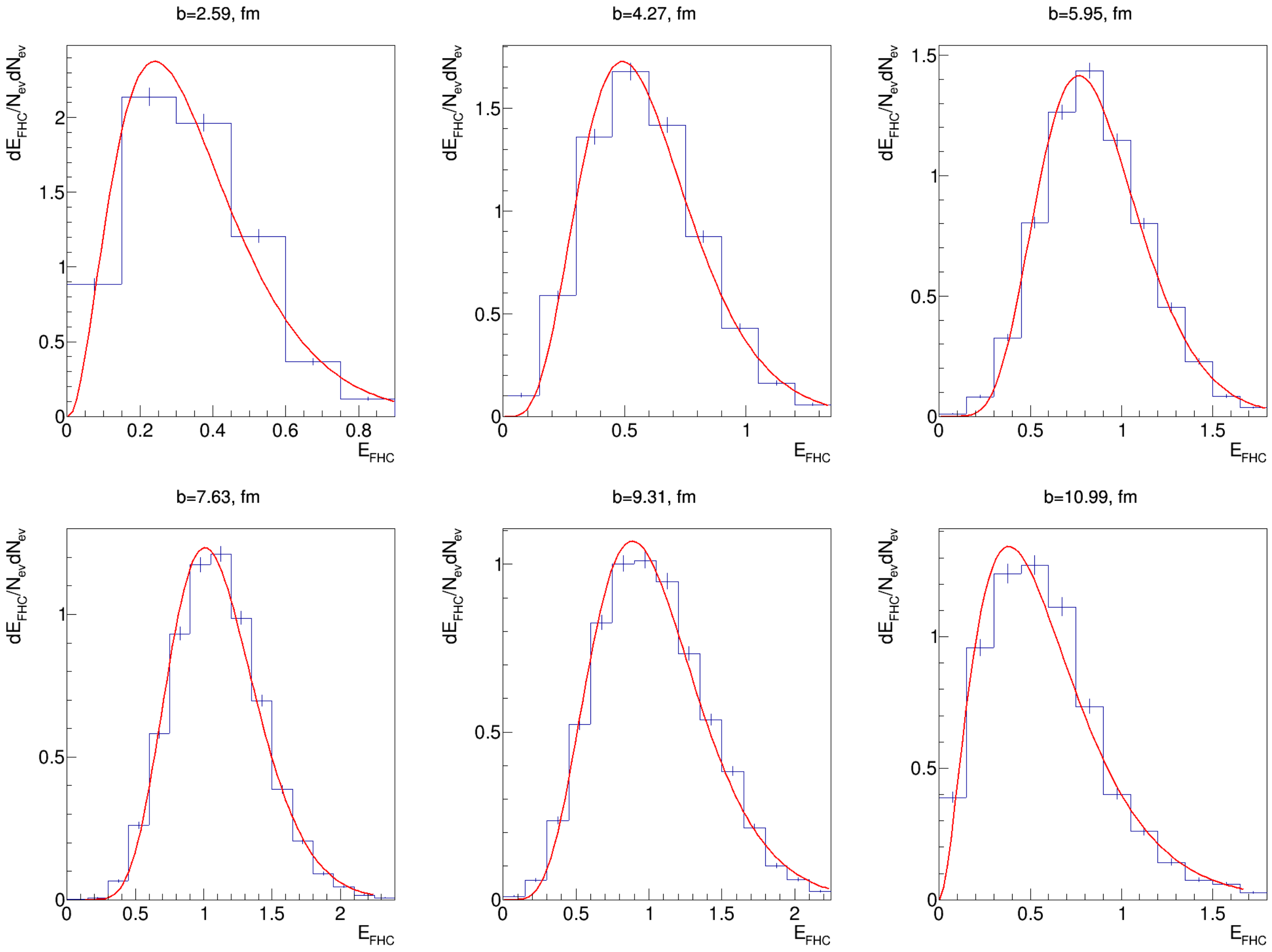
There is agreement within 5%.

Summary and outlook

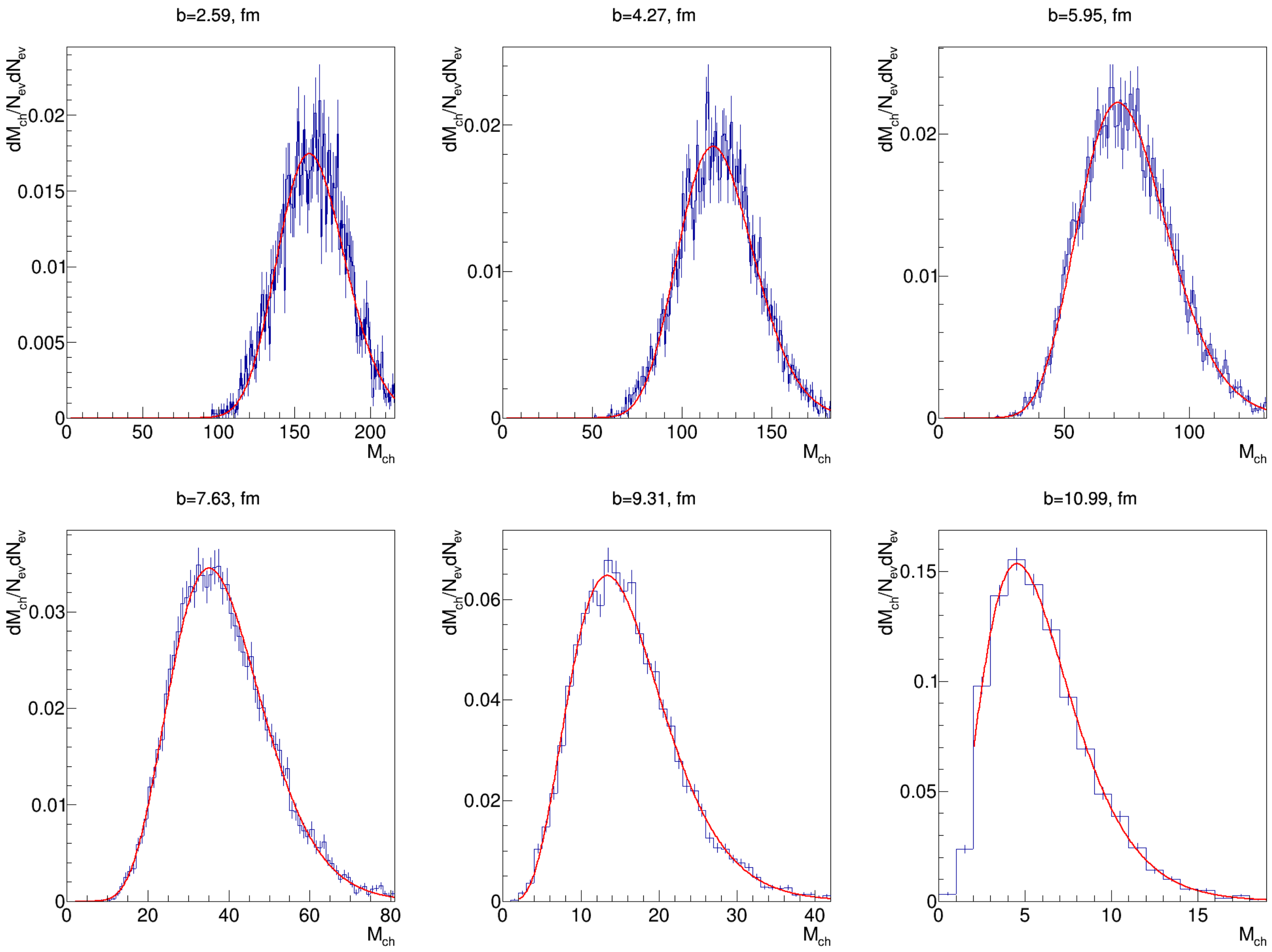
- The new approach can be applied to a wide range of energies and may become a non-dependent method for the estimation of centrality
- A new approach for efficiency and pileup correction was developed
- A new approach for centrality determination with energy of spectators and multiplicity of charged particles was proposed
- The proposed method was applied to the data from BM@N experiment
    - results are consistent with the conventional MC-Glauber based approach
- It is planned to create a two-dimensional method based on a signal from a hodoscope and energy from the FHCal

Thank you for your attention!

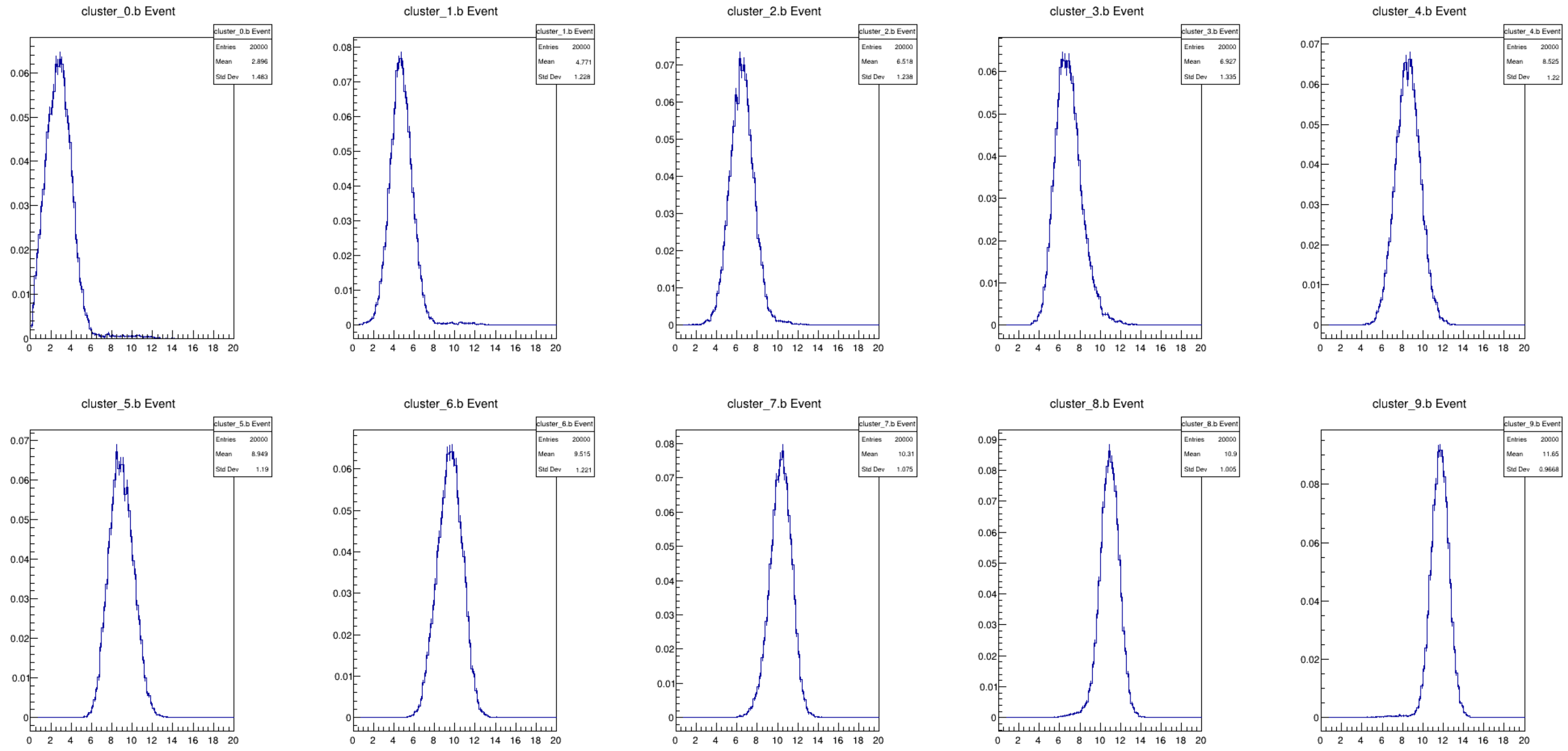
Energy distr. fit



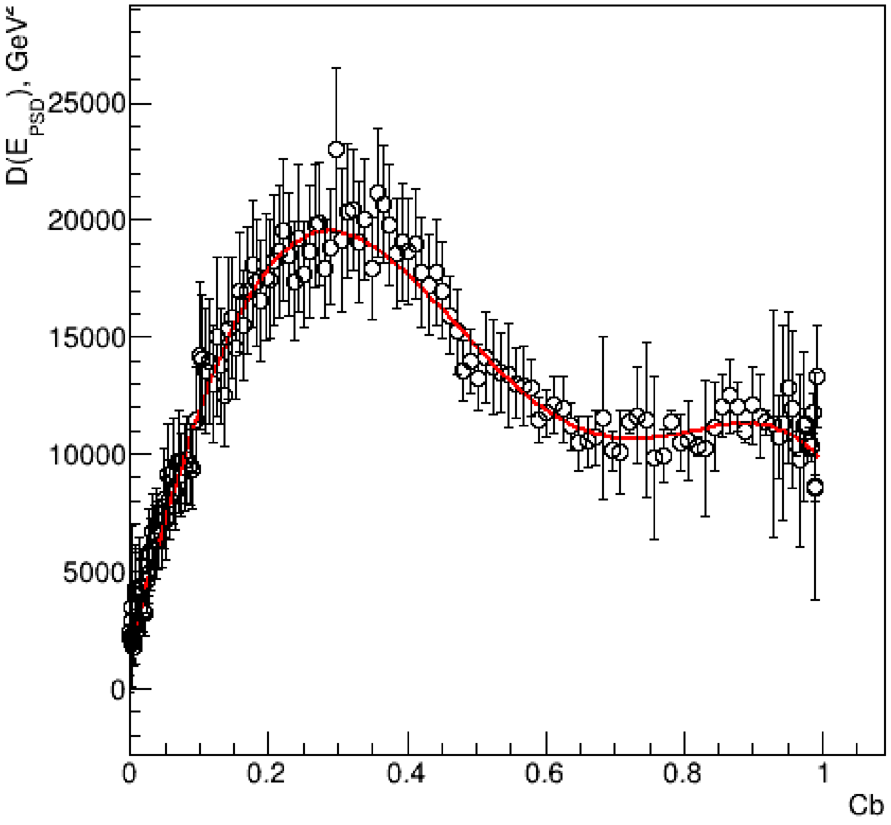
Mult distr. fit

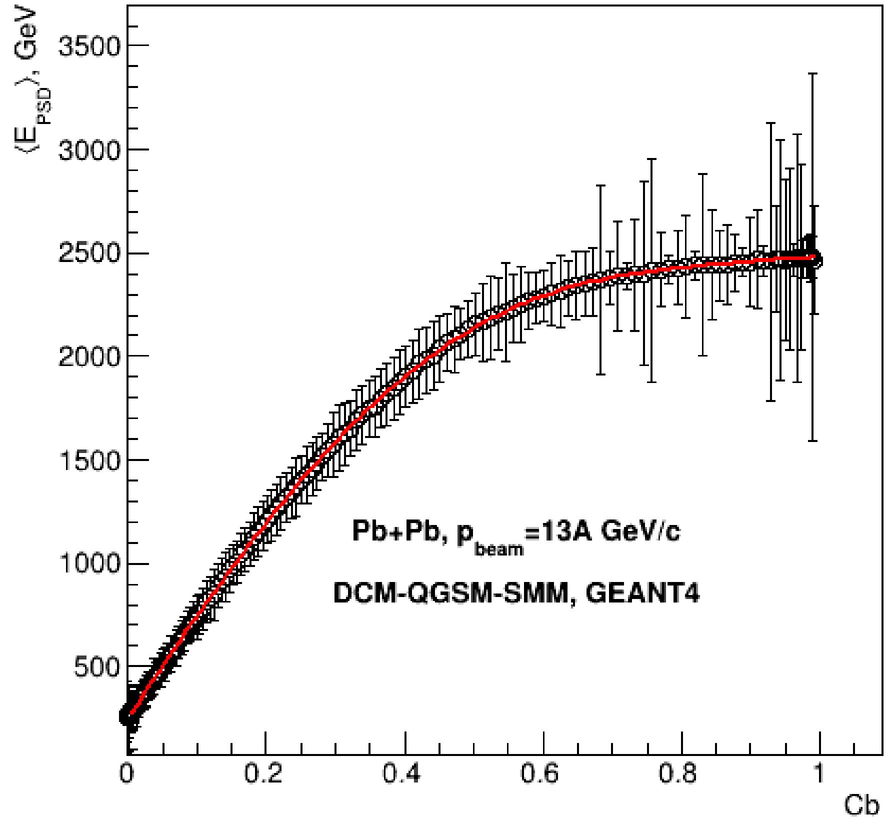


Impact parameter distribution for centrality classes



Dependence of the average value and variance of energy on centrality





The average value and dispersion of energy from the DCM-QGSM-SMM model are well described by polynomials

Reconstruction of *b*

- Normalized energy distribution P(E)

$$P(E)=\int_0^1 P(E\,|\,c_b)\,dc_b$$

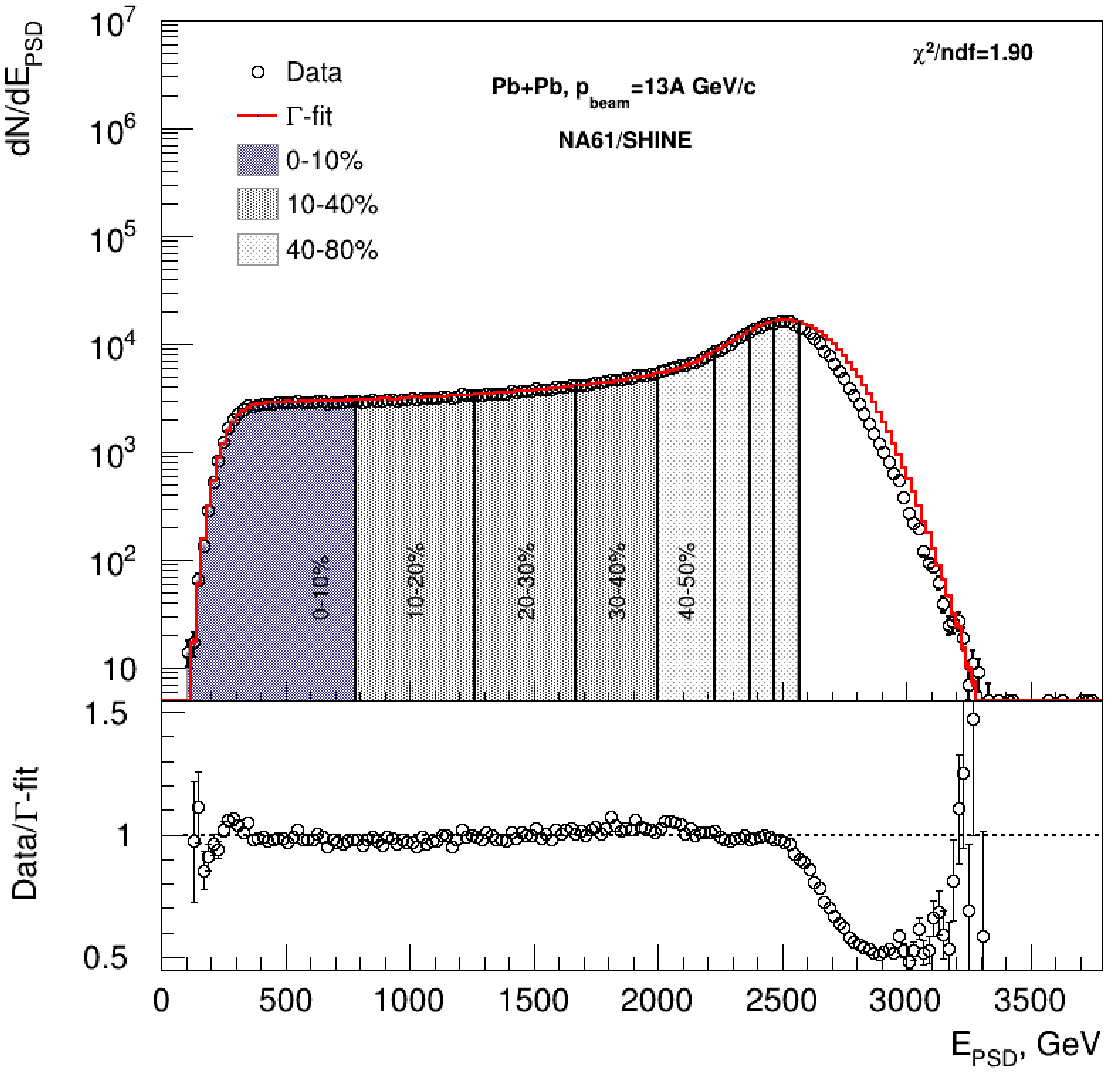
- Find probability of *b* for fixed range of E using Bayes' theorem:

$$P(b\,|\,E_1<E<E_2)=P(b)\frac{\int_{E_1}^{E_2}P(b\,|\,E)\,dE}{\int_{E_1}^{E_2}P(E)\,dE}$$

- **The Bayesian inversion method consists of 2 steps:**

–Fit normalized energy distribution with P(E)

–Construct P(*b*|E) using Bayes' theorem with parameters from the fit



Good agreement between fit and data in wide energy range