On Carrollian Conformal Field Theory

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Collaborators: Jue Hou, Reiko Liu, Hao-wei Sun and Yu-fan Zheng
Based on 2112.10514, 2301.06011, 2405.04105 and 2406.17451

Galilei & Carroll

The Galilei group could be produced by considering the $c \to \infty$ (non-relativistic) limit of the Poincaré group.
On the contrary, the Carroll group was found by considering the $c \to 0$ (ultra-relativistic) limit. J. Lévy-Leblond (1965); N.D. Sen Gupta (1966)

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Carrollian boosts

$$\vec{x}\,' = \vec{x}, \qquad t' = t - \vec{b} \cdot \vec{x}.$$

With the translations and the rotations among spacial directions, we obtain the Carroll group $Carr(d+1)$.

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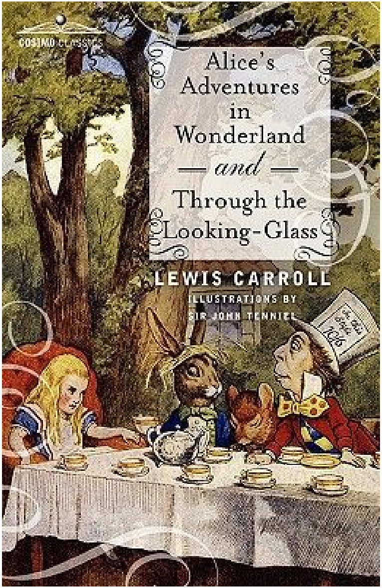
Intuitively, under the Carrollian limit, the lightcones collapse.

"since absence of causality as well as arbitrarinesses in the length of time intervals is especially clear in Alice's adventures (in particular in the Mad Tea-Party) this did not seem out of place to associate Lewis Carroll's name" (Lévy-Leblond (1965))

Carroll's world



The Red Queen effect: running without moving, "ultralocal"

... *The most curious part of the thing was, that the trees and the other things round them never changed their places at all: however fast they went, they never seemed to pass anything.*

A free Carrollian particle is at rest and does not move! C. Duval et.al 1402.0657, E. Bergshoeff et al. 1405.2264

Carrollian physics

The Carrollian boosts appear in the isometry group of plane-gravitational wave. J. M. Souriau (1973);...

The Carrollian limit controls the dynamics of the gravitational field near a spacelike singularity (BKL limit) M. Henneaux (1979);...

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4. Carrollian ModMax:
   with J. Hou and H.W. Sun, 2405.04105

Carrollian conformal algebra (CCA) and H.W. Reps.

One can obtain CCA_d by taking the Carrollian limit of the usual d-dim. conformal algebra.

$$\{D, P^\mu, K^\mu, J^{\mu\nu}\} \longrightarrow \{D, P^\mu, K^\mu, B^i, J^{ij}\},$$

where $\mu = 0, 1, \ldots, d-1,\ \ i, j = 1, \ldots, d-1$. The Carrollian boost generators B^i come from the rotation generators: $J^{i0} \xrightarrow{c \to 0} B^i$.

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$$\begin{aligned}
&[D, P^\mu] = P^\mu, \quad [D, K^\mu] = -K^\mu, \quad [D, B^i] = [D, J^{ij}] = 0, \\
&[J^{ij}, G^k] = \delta^{ik} G^j - \delta^{jk} G^i, \quad G \in \{P, K, B\} \\
&[J^{ij}, P^0] = [J^{ij}, K^0] = 0, \\
&[J^{ij}, J^{kl}] = \delta^{ik} J^{jl} - \delta^{il} J^{jk} + \delta^{jl} J^{ik} - \delta^{jk} J^{il}, \\
&[B^i, P^j] = \delta^{ij} P^0, \quad [B^i, K^j] = \delta^{ij} K^0, \quad [B^i, B^j] = [B^i, P^0] = [B^i, K^0] = 0, \\
&[K^0, P^0] = 0, \quad [K^0, P^i] = -2B^i, \quad [K^i, P^0] = 2B^i, \quad [K^i, P^j] = 2\delta^{ij} D + 2J^{ij}.
\end{aligned}$$

Stabilizer algebra and highest weight representations

The stabilizer algebra g_0 is generated by dilation D, generalized rotations $M = \{J, B\}$ and special conformal transformations (SCTs) K

$$[D, M] = 0, \quad [D, K] \subset K, \ [M, K] \subset K.$$

The commutativity of the dilatation and the rotations implies that the local operators $\mathcal{O}^a$ can be diagonalized simultaneously into the eigenstates of the dilation and the representations of generalized rotations,

$$[D, \mathcal{O}] = \Delta_{\mathcal{O}} \mathcal{O}, \quad [M, \mathcal{O}^a] = M^a_b \mathcal{O}^b.$$

$\Delta_{\mathcal{O}}$: conformal weight

Highest weight repr.: $[K, \mathcal{O}^a] = 0$.
This is often referred to as the primary condition.

The nontrivial part is the representation of generalized rotation!

Multiplet

For $d \geq 3$ CCFT, the generalized rotation group, CCA rotation group, is the Euclidean group $ISO(d-1)$. It is not semi-simple, and its finite dimensional representations are generally reducible but indecomposable, and can be organized as multiplet representations.

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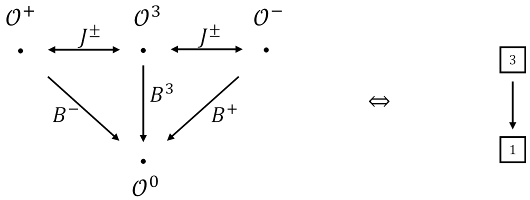
Example: vector representation $\mathcal{O}^{\mu}$ of CCA$_4$

$$[J^{ij}, \mathcal{O}^k] = \delta^{ik}\mathcal{O}^j - \delta^{jk}\mathcal{O}^i, \quad [B^i, \mathcal{O}^j] = \delta^{ij}\mathcal{O}^0, \quad [J^{ij}, \mathcal{O}^0] = [B^i, \mathcal{O}^0] = 0.$$



Here

$$J = -iJ^{12}, \qquad J^{\pm} = \frac{1}{\sqrt{2}}(\mp J^{23} + iJ^{31}), \qquad B^{\pm} = \frac{1}{\sqrt{2}}(iB^1 \pm B^2)$$

Tensor representation

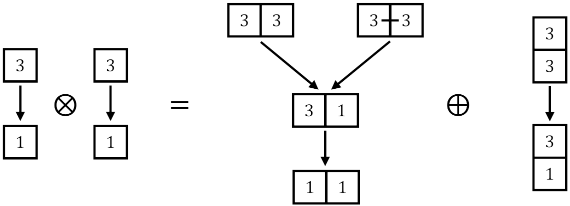


Figure: The rank-2 tensor representation of CCA_4. It is decomposed into a 10-dimensional representation and a 6-dimensional representation

The multiplet representations for $d > 2$ case have much more complicated structures since there is a non-trivial $ISO(d-1)$ part, and lead to net representations rather than just chain-like ones in $\log CFT_2$ or $CCFT_2$.

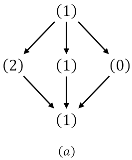


Figure: All the four net representations are legal.

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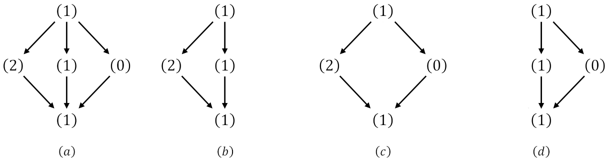

Figure: All the four net representations are legal.

Nevertheless, the finite dimensional representation of the CCA rotations are all multiplet representations with every sub-sector being irreducible representation of $SO(d-1)$, due to a theorem by H. P. Jakobsen (2011).

Notations: the numbers in the bracket indicate the irr. representations w.r.t. $SO(d-1)$, the arrows stand for the actions of the generators B_i.

Chain representations

The possible chain representations must take the following patterns:
rank 2

$$
\begin{aligned}
&(j) \to (j+1), \\
&(j) \to (j), \quad j \neq 0, \\
&(j) \to (j-1).
\end{aligned}
$$

rank 3 or higher

$$
\begin{gathered}
(0) \to (1) \to (0), \\
\cdots \to (j) \to (j+1) \to (j+2) \to \cdots \ , \\
\cdots \to (j) \to (j-1) \to (j-2) \to \cdots \ ,
\end{gathered}
$$

where the patterns works for all possible values of $j \in \{0\} \cup \mathbb{Z}_+/2$.

Correlators of singlets

In principle, the 2-pt and 3-pt functions of the operators in CCFT can be determined by using the Ward identities. However, due to complicated structure in representations, it is hard to discuss the most general case. We discussed the correlators of the operators in chain representations carefully. BC, Reiko Liu and Yu-fan Zheng, 2112.10514

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For a singlet in CCFT$_4$, there is

$$\langle \mathcal{O}_1(t_1,\vec{x}_1)\mathcal{O}_2(t_2,\vec{x}_2)\rangle = c_1\frac{1}{r^{\Delta_1+\Delta_2}} + c_2\delta^{(3)}(\vec{x}_{12})\frac{1}{t^{\Delta_1+\Delta_2-3}},$$

- If $c_1\neq 0$, $c_2=0$, the Ward identities of K^i will force $\Delta_1=\Delta_2$, and the resulting 2-pt function coincides with the scalar 2-pt function in CFT$_3$.
- If $c_1=0$, $c_2\neq 0$, it can be understood in an concrete model: the Carrollian free scalar with the action

$$S=\int d^3\vec{x}dt\,\phi\partial_t^2\phi.$$

Close relation between 3D Carrollian CFT and celestial holography!

L. Donnay *et al.*. 2202.04702,2212.12553; Bagchi *et al.*. 2202.08438; ...

Correlators of chain representations: trivial one

In the following discussion on correlators, we focus on the one with only spatial dependence.
Generic structure of 2-pt correlators:

$$\left\langle \mathcal{O}_1^{(m_1,q_1)}(x_1)\mathcal{O}_2^{(m_2,q_2)}(x_2)\right\rangle = f^{\,m_1,m_2}_{\,q_1,q_2}(x_{12})$$

where q_i is the order of the i-th operator in a multiplet.

For the trivial case that $\mathcal{O}_1, \mathcal{O}_2 \in (1) \to (0)$,

Level 3: $f^{\,m_1,m_2}_{\,2,2} = \dfrac{C\, I^{m_1,m_2}_{1,1}}{|\vec{x}_{12}|^{2\Delta}}$,

Level 2: $f^{\,0,m_2}_{\,1,2} = 0$, $f^{\,m_1,0}_{\,2,1} = 0$,

Level 1: $f^{\,0,0}_{\,1,1} = 0$,

$$\mathcal{O}_1 \in \begin{matrix}(1)\\ \downarrow\\ (0)\end{matrix} \qquad \mathcal{O}_2 \in \begin{matrix}(1)\\ \downarrow\\ (0)\end{matrix}$$

with $\Delta_1 = \Delta_2 = \Delta$.

Correlators of chain representations: the simplest nontrivial case

For the simplest nontrivial case,

$$\mathcal{O}_1 \in (1) \to (0), \qquad \mathcal{O}_2 \in (0) \to (1).$$

Level 3: $$f^{\,m_1,0}_{\,2,2} = \frac{C\ t_{12}/|\vec{x}_{12}|\ I^{m_1}_{1,0}}{|\vec{x}_{12}|^{2\Delta}},$$

Level 2: $$f^{\,0,0}_{\,1,2} = \frac{C}{|\vec{x}_{12}|^{2\Delta}}, \qquad f^{\,m_1,m_2}_{\,2,1} = \frac{C\ I^{m_1,m_2}_{1,1}}{|\vec{x}_{12}|^{2\Delta}},$$

Level 1: $$f^{\,0,m_2}_{\,1,1} = 0.$$

$$\mathcal{O}_1 \in \begin{array}{c}(1)\\ \downarrow\\ (0)\end{array} \qquad \mathcal{O}_2 \in \begin{array}{c}(0)\\ \downarrow\\ (1)\end{array}$$

with $\Delta_1 = \Delta_2 = \Delta$.

Here $I^{\,m_1,m_2}_{\,j_1,j_2}$ is the 2-point tensor structure.

Remarks on correlators

- Due to the multiplet structure of the representations, the correlators present multi-level structures. At each level, there are more than one 2-pt coefficients. Even if considering the basis change and renormalization of the operators, not all 2-pt coefficients can be fixed by the Ward identities;

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- As the representations are reducible, there is short of selection rule on the representations. This means that the 2-pt correlators of the operators in different representations could be nonvanishing.
- We explored the 2-pt correlators of net representations and the 3-pt correlators of chain representations. It turns out that the constraints from the Ward identities are quite loose, and we had to compute them case by case.

Constructions of CCFT

The explicit examples are vital to understand various properties of CCFT.

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$$\text{2D Carrollian conformal group} \simeq \text{BMS}_3$$

In 2D, a few field theories with BMS symmetry have been constructed and studied:

BMS free scalar theory P.x. Hao *et al.*. 2111.04701,

BMS free fermion theory Z.f. Yu & BC. 2211.06926; P.x. Hao *et al.* 2211.06927; A. Banerjee *et al.* 2211.11639

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We propose a novel off-shell way to construct Carrollian (conformal) field theories, starting from the Bargmann field theories. We have successfully reproduced all Carrollian field theories in the literatures. BC, *et al.*, 2301.06011

Quantum aspects of CCFT

Some essential questions: quantizations, vacuum, state-operator correspondence, ...

Quantization: Path-integral BC *et al.* 2301.06011, 2302.05975;Canonical quantization on massive scalar J. de Boer *et al.* 2307.06827; J. Cotler *et al.* 2407.11971

We studied the quantization of Carrollian conformal scalar theories in 2D and 3D BC *et al.* 2406.17451

Q1: induced vacuum or highest-weight (h.w.) vacuum?

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induced vacuum: can define unitary Hilbert space

Q2: state-operator correspondence: No!

ModMax theory

ModMax: Modified Maxwell (ModMax)

$$\mathcal{L}_\gamma(\mathcal{S},\mathcal{P}) = -\frac{1}{2}\cosh\gamma\mathcal{S} + \frac{1}{2}\sinh\gamma\sqrt{\mathcal{S}^2+\mathcal{P}^2}, \quad \gamma \in \mathbf{R},$$

with

$$\mathcal{S} \equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu}, \qquad \mathcal{P} \equiv \frac{1}{2}F_{\mu\nu}(*F)^{\mu\nu}.$$

1. It is maximally symmetric nonlinear extensions of Maxwell: conformal invariant and EM invariant $(\mathbf{E}+i\mathbf{B}) \to e^{-i\theta}(\mathbf{E}+i\mathbf{B})$. I. Bandos *et al.* 2007.09092; B. Kosyakov 2007.13878

2. ModMax can be generated from the Maxwell theory by the $\sqrt{T\bar{T}}$ flow perturbatively in $d=4$ in the sense that H. Babaei-Aghbolagh *et al.* 2202.11156; C. Ferko *et al.* 2203.01085

$$\frac{\partial \mathcal{L}_\gamma^{\text{ModMax}}}{\partial\gamma} = O^\gamma_{\sqrt{T^2}}, \qquad \mathcal{L}_\gamma^{\text{ModMax}} = \mathcal{L}^{\text{Maxwell}} + \int O^\gamma_{\sqrt{T^2}}\, d\gamma,$$

where

$$O^\gamma_{\sqrt{T^2}} = \sqrt{\frac{1}{d}\left(T^\mu{}_\nu T^\nu{}_\mu - r T^\mu{}_\mu T^\nu{}_\nu\right)}.$$

Carrollian ModMax BC, J.Hou and H.W. Sun, 2405.04105

$$\mathcal{L}_\gamma^{\text{CMM}}(\mathcal{S},\mathcal{P}) = \frac{1}{4}\left(e^\gamma \mathcal{S} \mp e^{-\gamma}\frac{\mathcal{P}^2}{\mathcal{S}}\right),$$

with

$$\mathcal{S} \equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu} = m^{\mu\rho}\gamma^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}, \quad \mathcal{P} \equiv \frac{1}{2}F_{\mu\nu}\widetilde{F}^{\mu\nu}.$$

Note: The theory is defined on Carrollian geometry, and the Hodge dual should be defined carefully.

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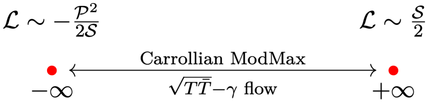
1. It is Carrollian SO(2) EM duality invariant as well as conformal invariant.
2. The Carrollian ModMax theories in the family deform among themselves under $\sqrt{T\bar{\bar{T}}}$ flow, except two end-points $\gamma \to \pm\infty$, where the flow is non-invertible.



Summary

1. We tried to study the higher dimensional ($d \geq 3$) Carrollian conformal invariant theories in a systematic way. As the conformal algebra is not semi-simple, the finite dimensional h.w.r. present some novel features: multiplet structure, staggered module, chain-like and even net-like representations.

- We classified all the chain representations
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3. We studied the quantization of Carrollian conformal scalar theories in 2D and 3D, and discussed the physical implications of different vacua.

4. We investigated the duality symmetry group of Carrollian (nonlinear) electrodynamics and proposed a family of Carrollian ModMax theories, which are Carrollian EM and conformal invariant.

Thanks for your attention!

Carroll group as kinematical group

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Poincaré group, Galilei group, AdS/dS isometry group,
Newton-Hooke group, Carroll group

Different relativity groups are related by Inönü-Wigner contractions. Actually all these groups can be obtained by certain contraction of AdS and dS groups.

Backup: Carrollian particle

To study the motion of a free Carrollian particle, we may start from the massive particle moving in AdS/dS spacetime and then take the Carrollian limit. In the end, we find the action

$$S_C = \int d\tau(-E\dot{t} + \dot{\vec{x}} \cdot \vec{p} - \frac{e}{2}(E^2 - M^2))$$

which is invariant under the Carrollian transformation

$$t' = t - \vec{b} \cdot R\vec{x} + a_t, \qquad \vec{x}\,' = R\vec{x} + \vec{a},$$
$$\vec{p}\,' = R\vec{p} - \vec{b}E, \qquad E' = E.$$

The free Carrollian particle is at rest and does not move! C. Duval et al 1402.0657, E. Bergshoeff *et al.* 1405.2264

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Symmetries: E. Bergshoeff *et al.* 1405.2264
1. The free Carrollian particle has infinite dimensional symmetry.
2. For the massless one, the symmetries get enhanced to Carrollian conformal symmetry.

However, for two-particle system, there is non-trivial dynamics!

In flat holography, most of studies has been focused on 3D flat spacetime, whose asym. symm. group is BMS_3.

$$\mathrm{BMS}_3 \simeq \text{2D Carrollian group} \simeq \text{2D Galilean group} \tag{0.1}$$

We have been studying 2D Galilean/Carrollian analytic conformal

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1. Multiplet structure
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3. Harmonic analysis of GCA: GCPW
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Q: how about higher dimensional Carrollian conformal field theories (CCFT)?

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A Bargmann manifold has three ingredients, $(\mathcal{B}, G, \xi)$, where $\mathcal{B}$ is a $(d+1)$-dimensional manifold with metric G of Lorentz signature and a vertical vector ξ, a nowhere vanishing null vector.

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$$\mathcal{B} = \mathbb{R} \times \mathbb{R}^{d-1} \times \mathbb{R}, \qquad G = 2dudv + \delta_{ij}dx^i dx^j, \qquad \xi = \partial_u,$$

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where u, v are the lightcone coordinates. The Bargmann group is the isometries of the flat Bargmann structure, which is a subgroup of Poincaré group

$$Barg(d,1) = ISO(d,1)/\{J^{0}{}_{d+1}, 1/\sqrt{2}\left(J^{i}{}_{0} - J^{i}{}_{d+1}\right)\}$$

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that keep the null vector ξ invariant. The Bargmann generators are $\{P_\alpha, J^i{}_j, B_i^{\mathcal{B}}\}$, where $B_i^{\mathcal{B}}$ is the Bargmann boost. The actions on point $(u, \vec{x}, v)$ in the manifold are shown in the following Table

generator	vector field	finite transformation
p_α	∂_α	$x^\alpha + x_0^\alpha$
$m^i{}_j$	$x^i\partial_j - x_j\partial^i$	$(u, \mathbf{M}\vec{x}, v)$
$b_i^{\mathcal{B}}$	$v\partial_i - x_i\partial_u$	$\left(u - \vec{\nu}\cdot\vec{x} - \frac{1}{2}\vec{\nu}^2 v, \vec{x} + \vec{\nu}v, v\right)$

Carrollian symmetry from Bargmann symmetry

Restricting the Bargmann group on the null hyper-surface $v = 0$, we can immediately see the Bargmann structure reduce to Carrollian structure $(\mathcal{C}, g, \xi)$ with the coordinates $(t = u, \vec{x})$, the degenerated metric

$$g_{\mu\nu} = G_{\mu\nu} = \delta^i_\mu \delta^j_\nu \delta_{ij}$$

while ξ^μ being the timelike vector, and the Carroll group is subgroup of Bargmann group $Carr(d) = Barg(d, 1)/\{P_v\}$.

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This motivates us to construct Carrollian field theories by restricting Bargmann field theories on the null hyper-surface.

However, trivially restricting Bargmann fields with configuration $\Phi(u, \vec{x}, v) = \Phi(u, \vec{x})\delta(v)$ on $v = 0$ causes many difficulty from the Dirac delta function. The trick is to introduce an uniformly distributed function over small interval of v.

Bargmann scalar field theories

The building blocks of Bargmann field theories are geometric invariants $G^{\alpha\beta}$ and ξ^α. For a free scalar Φ, the Bargman invariant action could be

$$S_E^{\mathcal{B}} = \frac{1}{2}\int d^{d+1}x\, \xi^\alpha\xi^\beta\partial_\alpha\Phi\partial_\beta\Phi, \qquad S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, G^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi.$$

The subscript M is for magnetic sector and E for electric sector, corresponding to magnetic/electric Carrollian field theories. M. Henneaux and P. Salgado-Rebolledo, 2109.06708

Electric sector

$$S_E^{\mathcal{B}} = \frac{1}{2}\int d^{d+1}x\, \xi^\alpha \xi^\beta \partial_\alpha \Phi \partial_\beta \Phi.$$

Expand Φ near $v = 0$, we have

$$\Phi(u, \vec{x}, v) = \phi(u, \vec{x}) + v\pi(u, \vec{x}) + \mathcal{O}(v^2).$$

Inserting this in the action, and noticing $\xi^\alpha = (1, \vec{0}, 0)$, we have

$$S_E^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, \partial_u \Phi \partial_u \Phi = -\frac{1}{2}\int d^{d+1}x\, \partial_u \phi \partial_u \phi + 2v\partial_u \pi \partial_u \phi + \mathcal{O}(v^2),$$

and thus we have the Carrollian action

$$S_E^{\mathcal{C}} = \lim_{\epsilon \to 0} S_{E,\epsilon}^{\mathcal{B}} = -\frac{1}{2}\int d^d x\, \partial_0 \phi \partial_0 \phi.$$

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$$S_E^{\mathcal{C}} = \lim_{\epsilon \to 0} S_{E,\epsilon}^{\mathcal{B}} = -\frac{1}{2}\int d^d x\, \partial_0 \phi \partial_0 \phi.$$

Actually, it is not only Carrollian invariant, but even Carrollian conformal invariant. From

$$\langle \phi(x)\phi(y)\rangle = \frac{i|t|}{2}\delta^{(d-1)}(\vec{x}),$$

we see that ϕ is a primary operator when $d > 2$.

Magnetic sector

$$S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\; G^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi.$$

Insert the expansion of Φ, we get:

$$S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, 2\partial_u\Phi\partial_v\Phi + \partial_i\Phi\partial_i\Phi = -\frac{1}{2}\int d^{d+1}x\, 2\pi\partial_u\phi + \partial_i\phi\partial_i\phi + \mathcal{O}(v).$$

Thus we reproduce the action of magnetic Carrollian scalar theory M. Henneaux and P. Salgado-Rebolledo, 2109.06708.

$$S_M^{\mathcal{C}} = -\frac{1}{2}\int d^d x\; 2\pi\partial_0\phi + \partial_i\phi\partial_i\phi$$

The fundamental fields in this theory are ϕ and π.

The above action is Carrollian conformal invariant as well. The scalar ϕ is still a primary fields, and the field π appears as part of staggered module of ϕ's conformal family.

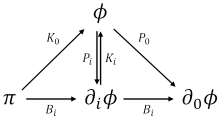

Figure: The staggered structure of fields ϕ, $\partial_\mu\phi$ and π.

$$\langle \phi(\vec{x}_1, t_1)\phi(\vec{x}_2, t_2)\rangle = 0$$

$$\langle \phi(\vec{x}_1, t_1)\pi(\vec{x}_2, t_2)\rangle = -\frac{i\ sign(t)}{2(1-\alpha^2)}\delta^{(d-1)}(\vec{x})$$

$$\langle \pi(\vec{x}_1, t_1)\pi(\vec{x}_2, t_2)\rangle = \frac{i|t|}{2(1-\alpha^2)}\vec{\partial}^2\delta^{(d-1)}(\vec{x})$$

where $t = t_1 - t_2$ and $\vec{x} = \vec{x}_1 - \vec{x}_2$, and α is a parameter determined by quantization. They indeed satisfy the Ward identities of CCA.

The similar construction can be applied to other field theories: BC, H.W. Sun and Y.F. Zheng, work in progress

1. Carrollian p-form field theories, including electromagnetic theory
2. Carrollian Yang-Mills theory
3. Carrollian scalar QED
4. ······