Chiral effective potential in Wess-Zumino model at 3 loops

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Setup: massless $\mathcal{N}$=1 WZ model

Buchbinder, Petrov et al '94
Buchbinder, Kuzenko'98

- Action:

Kahler potential — Chiral potential with complex coupling constant

$$\mathcal{S}[\Phi,\bar\Phi] = \int d^8z\ \Phi\bar\Phi + \frac{\lambda}{3!}\int d^6z\ \Phi^3 + \frac{\bar\lambda}{3!}\int d^6\bar z\ \bar\Phi^3$$

contains quartic and Yukawa interactions

Super-measure

$$d^8z = d^4x d^2\theta d^2\bar\theta$$
$$d^6z = d^4x d^2\theta$$
$$d^6\bar z = d^4x d^2\bar\theta$$

Quantum corrections can supplement:

- Kahlerian part (from the first loop)
- Auxiliary field part
- Chiral part**

$$\Gamma[\Phi,\bar\Phi] = \sum_{L=1}^{\infty} \hbar^L \Gamma^{(L)}[\Phi,\bar\Phi]$$

$$\mathbf{K} = \sum_L \hbar^L \mathbf{K}^{(L)}(\Phi\bar\Phi) \qquad \mathbf{A} = \sum_L \hbar^L \mathbf{A}^{(L)}(D\Phi,\bar D\bar\Phi)$$

Non-renormalization theorem:
all the loop corrections to the effective action in such a theories are expressed by integrals over the whole superspace, but not over its chiral subspace.

$$\mathbf{W} = \sum_L \hbar^L \mathbf{W}^{(L)}(\Phi)$$

Setup: massless $\mathcal{N}$=1 WZ model

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- Action:

Kahler potential

Chiral potential
with complex coupling constant

$$\mathcal{S}[\Phi, \bar{\Phi}] = \int d^8z\ \Phi\bar{\Phi} + \frac{\lambda}{3!}\int d^6z\ \Phi^3 + \frac{\bar{\lambda}}{3!}\int d^6\bar{z}\ \bar{\Phi}^3$$

Super-measure

$$d^8z = d^4x d^2\theta d^2\bar{\theta}$$
$$d^6z = d^4x d^2\theta$$
$$d^6\bar{z} = d^4x d^2\bar{\theta}$$

Anyway, there is a loophole:

$$D^2\bar{D}^2D^2 = 16D^2\partial^2$$

$$\int d^8z\ u(\Phi)\left(-\frac{D^2}{4\partial^2}\right)v(\Phi) = \int d^6z\ u(\Phi)v(\Phi) \quad (*)$$

Possible only for massless diagrams

Therefore, one can obtain finite (three-point functions do not diverge) corrections to the chiral effective potential

$$\mathbf{W} = \sum_L \hbar^L \mathbf{W}^{(L)}(\Phi)$$

this finite terms are non-local in coordinate space

Buchbinder, Petrov et al '94

Effective $\mathcal{N}=1$ superpotential

Effective action after Legendre transformation of generating functional is

$$\exp\left(\frac{i}{\hbar}\Gamma[\Phi,\bar{\Phi}]\right) = \int \mathcal{D}\phi\mathcal{D}\bar{\phi}\ \exp\frac{i}{\hbar}\mathcal{S}[\Phi+\sqrt{\hbar}\phi,\bar{\Phi}+\sqrt{\hbar}\bar{\phi}]-\left(\sqrt{\hbar}\int d^6z\ \phi(z)\frac{\delta\bar{\Gamma}}{\delta\Phi(z)}+h.c.\right)$$

$$\mathcal{S}^{(2)}[\phi,\bar{\phi}] = \int d^8z\ \phi\bar{\phi} + \left(\frac{1}{2}\int d^6z\ \lambda\Phi\phi^2 + h.c.\right) \qquad \Gamma[\Phi,\bar{\Phi}] = \mathcal{S}[\Phi,\bar{\Phi}] + \bar{\Gamma}[\Phi,\bar{\Phi}] \qquad \Gamma[\Phi,\bar{\Phi}] = \sum_{L=1}^{\infty}\hbar^L\Gamma^{(L)}[\Phi,\bar{\Phi}]$$

Superpropagator:

effective mass

$$\begin{pmatrix} -\lambda\Phi & \frac{1}{4}\bar{D}^2 \\ \frac{1}{4}D^2 & -\lambda\bar{\Phi} \end{pmatrix}\begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix} = -\begin{pmatrix} \delta_+ & 0 \\ 0 & \delta_- \end{pmatrix}$$

$$\Gamma[\Phi,\bar{\Phi}] = \int d^8z\ \left(\mathbf{K}(\Phi,\ \bar{\Phi}) + \mathbf{A}(D\Phi,\bar{D}\bar{\Phi})\right) + \left(\int d^6z\ \mathbf{W}(\Phi) + c.c.\right) \qquad \partial_a\Phi = \partial_a\bar{\Phi} = 0$$

Kahler potential — Auxiliary field potential — Chiral potential

Buchbinder, Petrov et al '94

Effective $\mathcal{N}$=1 chiral superpotential

- Improved Feynman super-rules:

$$\mathcal{G}_{ab} = \frac{1}{16}\mathbf{D}\bigoplus G_{ab} \qquad (\partial^2 - \frac{1}{4}\bar{\lambda}\bar{\Phi}D^2 - \frac{1}{4}\lambda\Phi\bar{D}^2)\mathcal{G}(z_1, z_2) = \delta^8(z_1 - z_2)$$

- Real superfield propagator:

$$\mathcal{G}(z_1, z_2) = \frac{1}{16}\begin{pmatrix} 0 & -\frac{\bar{D}_1^2 D_2^2}{\partial_1^2}\delta^8(z_1 - z_2) \\ -\frac{D_1^2 \bar{D}_2^2}{\partial_1^2}\delta^8(z_1 - z_2) & -\frac{D_1^2 D_2^2}{\partial_1^2}\left(\lambda\Phi(z_1)\frac{\bar{D}_1^2}{4\partial_1^2}\delta^8(z_1 - z_2)\right) \end{pmatrix}$$

Two-loop diagram example:

$$\Gamma^{(2)}(\Phi) = \frac{\bar{\lambda}^2}{3! \times 2}\int d^6\bar{z}_1 d^6\bar{z}_2\ (\mathcal{G}_{--}(z_1, z_2))^3 \qquad \Phi(z_1)\frac{\bar{D}_1^2}{\partial_1^2}\delta^8(z_1 - z_2) = \int d^8z_3\ \Phi(z_3)\delta^8(z_3 - z_1)\frac{\bar{D}_3^2}{\partial_2^2}\delta^8(z_3 - z_2)$$

This integral can be formulated in terms of standard Feynman technique

Buchbinder, Kuzenko et al '93
Buchbinder, Kuzenko et al '94

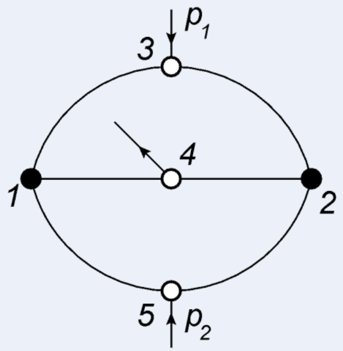
Fixing rules

We can derive rules for specific diagrams contributing to the chiral superpotential

1. As initial interaction is cubic, the correction should be proportional to cubic potential, i.e. every diagram must contain three second derivatives of initial chiral potential.
2. To satisfy the relation (*) the resulting integral up to last integration must contain number of squares of chiral covariant derivatives by one more than antichiral ones
3. Restriction: $2L + 1 - n_2 - n_3 = 0$

Only possible supergraph restricted by the rules at the two-loop level looks like

$$\lim_{p_1, p_2 \to 0}$$

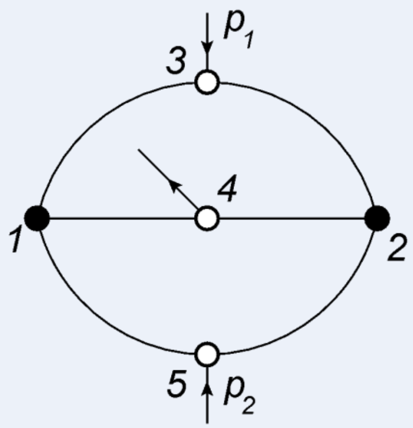


Two-loop contribution

The first contribution to this potential is given by the two-loop diagram

$$\Gamma^{(2)}(\Phi) = \frac{\bar{\lambda}^2}{3! \times 2} \int d^6\bar{z}_1 d^6\bar{z}_2 \ (\mathcal{G}_{--}(z_1, z_2))^3$$

$$\Gamma^{(2),h}(\Phi) = \lambda \frac{|\lambda|^4}{3! \times 2} \int \prod_{i=1}^{5} d^8 z_i \Phi(z_3)\Phi(z_4)\Phi(z_5) \left\{ \frac{1}{\partial_1^2}\delta_{1,3} \frac{D_2^2\bar{D}_3^2}{16\partial_2^2}\delta_{3,2} \quad \frac{1}{16\partial_2^2}\delta_{2,4}\frac{D_1^2\bar{D}_4^2}{16\partial_1^2}\delta_{1,4}\frac{D_1^2\bar{D}_5^2}{16\partial_1^2}\delta_{1,5}\frac{D_2^2}{4\partial_2^2}\delta_{2,5} \right\}$$



...after D-algebra routine:

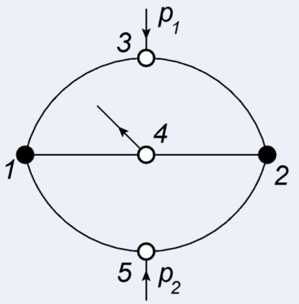
$$\lim_{p_1,p_2\to 0} I_2(p_1,p_2) = \int \frac{d^4q_1}{(4\pi)^4}\frac{d^4q_2}{(4\pi)^4}\frac{q_1^2p_1^2 + q_2^2p_2^2 - 2(q_1\cdot q_2)(p_1\cdot p_2)}{q_1^2q_2^2(q_1+q_2)^2(q_1-p_1)^2(q_2-p_2)^2(q_1+q_2-p_1-p_2)^2}$$

Integral easily can be evaluated by IBP and uniqueness relation

Buchbinder, Kuzenko'94
Buchbinder, Petrov et al '94

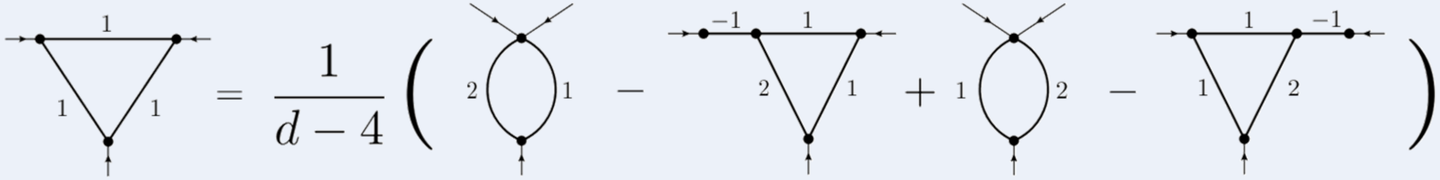
Two-loop contribution

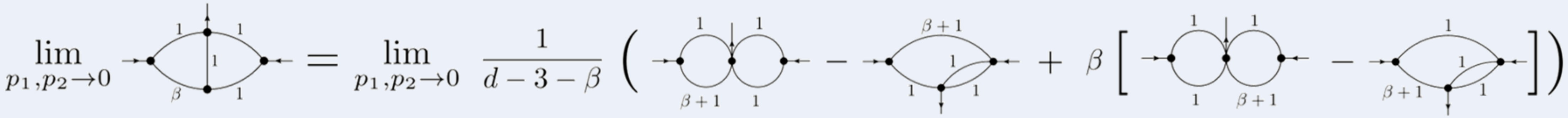
$$\lim_{p_1,p_2\to 0} I_2(p_1,p_2) = \int \frac{d^4q_1}{(4\pi)^4}\frac{d^4q_2}{(4\pi)^4}\frac{q_1^2p_1^2+q_2^2p_2^2-2(q_1\cdot q_2)(p_1\cdot p_2)}{q_1^2q_2^2(q_1+q_2)^2(q_1-p_1)^2(q_2-p_2)^2(q_1+q_2-p_1-p_2)^2}$$



Chetyrkin, Tkachev'81

IBP relation





Integral easily can be evaluated by IBP and uniqueness relation

Kazakov'84

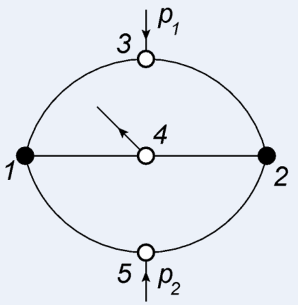
$$\mathbf{W}^{(2)}(\Phi) = \hbar^2\frac{|\lambda|^4}{(4\pi)^4}\frac{1}{2}\zeta(3)\ \lambda\Phi^3$$

The result is finite (no need to renormalise) and given as integral over the half-superspace!
Also it is not holomorphic by coupling constant!

Buchbinder, Kuzenko'98
Buchbinder, Petrov et al '94

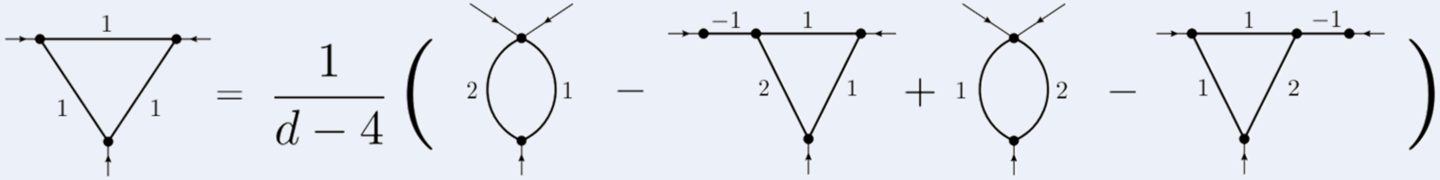
Two-loop contribution

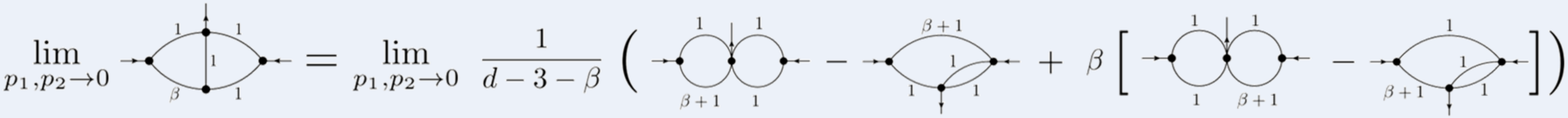
$$\lim_{p_1,p_2\to 0} I_2(p_1,p_2) = \int \frac{d^4q_1}{(4\pi)^4}\frac{d^4q_2}{(4\pi)^4}\frac{q_1^2p_1^2+q_2^2p_2^2-2(q_1\cdot q_2)(p_1\cdot p_2)}{q_1^2q_2^2(q_1+q_2)^2(q_1-p_1)^2(q_2-p_2)^2(q_1+q_2-p_1-p_2)^2}$$



Chetyrkin, Tkachev'81

IBP relation





Integral easily can be evaluated by IBP and uniqueness relation

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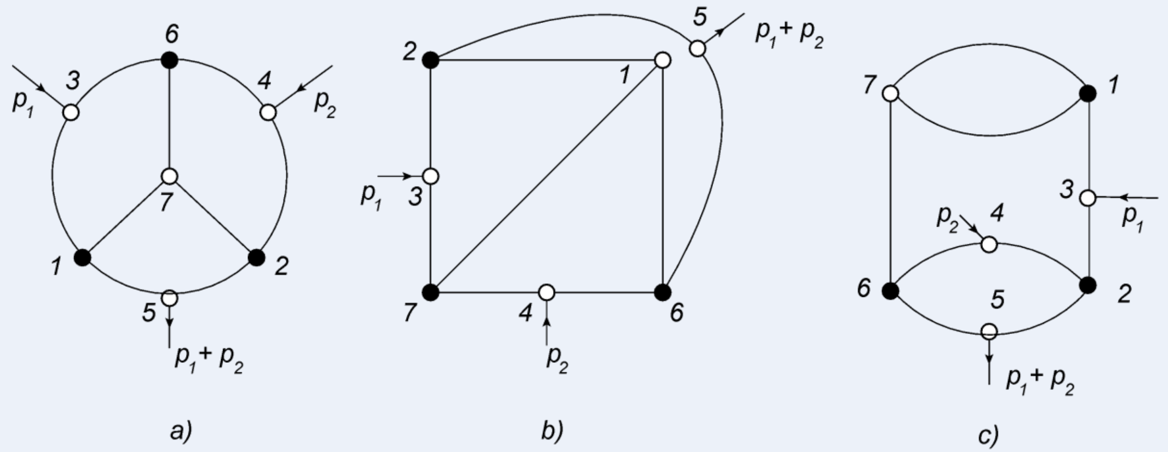
$$\mathbf{W}^{(2)}(\Phi) = \hbar^2\frac{|\lambda|^4}{(4\pi)^4}\frac{1}{2}\zeta(3)\ \lambda\Phi^3$$

Seiberg'93

<u>We observe violation of Seiberg's holomorphy principle</u>:
superpotential should be holomorphic in the fields and in the coupling constants but here it is not

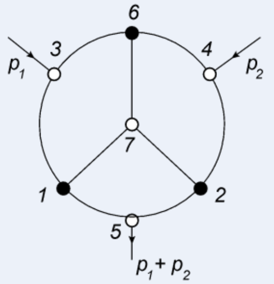
Three-loop contribution

There are only three possible triple-loop diagrams contributing to CESP

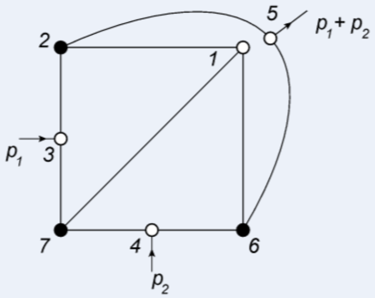


Two are finite and one is divergent due to presence of two-point integral (counterterms are needed)

Finite three-loop contribution



a)



b)

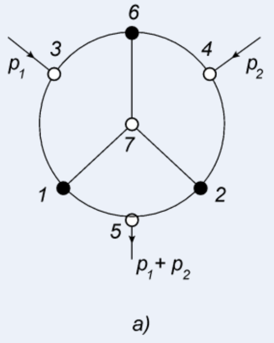
$$\Gamma^{(3a),h}(\Phi) = \lambda\frac{|\lambda|^6}{3!\times 8}\int \prod_{i=1}^{7} d^8 z_i\ \Phi(z_3)\Phi(z_4)\Phi(z_5)$$
$$\Big\{ \frac{1}{\partial_1^2}\delta_{1,3}\frac{\bar{D}_3^2 D_6^2}{16\partial_6^2}\delta_{3,6}\frac{D_6^2\bar{D}_5^2}{16\partial_6^2}\delta_{6,5}\frac{D_2^2}{4\partial_2^2}\delta_{5,2}$$
$$\frac{1}{\partial_2^2}\delta_{2,4}\frac{\bar{D}_4^2 D_1^2}{16\partial_1^2}\delta_{4,1}\frac{1}{\partial_1^2}$$
$$\delta_{1,7}\frac{\bar{D}_7^2 D_6^2}{16\partial_6^2}\delta_{7,6}\frac{\bar{D}_7^2 D_2^2}{16\partial_2^2}\delta_{7,2}\Big\}$$

$$\Gamma^{(3b),h}(\Phi) = \lambda\frac{|\lambda|^6}{3!\times 8}\int \prod_{i=1}^{7} d^8 z_i\ \Phi(z_3)\Phi(z_4)\Phi(z_5)$$
$$\Big\{ \frac{1}{\partial_2^2}\delta_{2,3}\frac{\bar{D}_3^2 D_7^2}{16\partial_7^2}\delta_{3,7}\frac{\bar{D}_7^2 D_4^2}{16\partial_7^2}\delta_{7,4}$$
$$\frac{1}{\partial_6^2}\delta_{4,6}\frac{D_6^2}{4\partial_6^2}\delta_{6,5}\frac{\bar{D}_5^2 D_2^2}{16\partial_2^2}$$
$$\delta_{5,2}\frac{D_2^2\bar{D}_1^2}{16\partial_2^2}\delta_{2,1}\frac{\bar{D}_1^2 D_6^2}{16\partial_6^2}\delta_{1,6}\frac{1}{\partial_7^2}\delta_{1,7}\Big\}$$

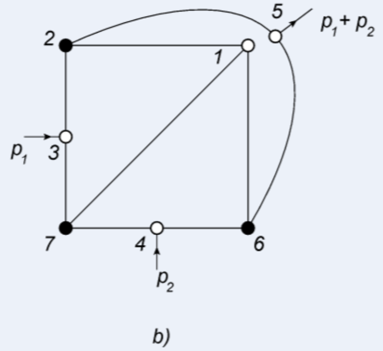
…D-algebra routine…

(`SusyMath.m` and other packages)

Finite three-loop contribution



a)



b)

$$I_3^{(a)}(p_1,p_2) = \lim_{p_{1,2}\to 0} \int \prod_{i=1}^{3} \frac{d^4 q_i}{(4\pi)^4} \frac{(a_1 p_1^2 + a_2 p_2^2 + a_3 (p_1 \cdot p_2))}{q_1^2 (q_1+p_1)^2 (p_1+q_2)^2 (q_2+p_1+p_2)^2} \frac{1}{(q_2+p_1+p_2)(q_1-q_2)^2 q_3^2 (q_1-q_3)^2 (q_3-q_2)^2}$$

$$a_1 = q_1^2 \left(q_2 - q_3\right)^2$$

$$a_2 = q_2^2 \left(q_1 - q_3\right)^2$$

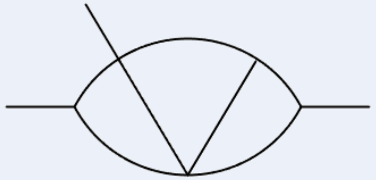
$$a_3 = -2\left(q_3^2 (q_1 \cdot q_2) - q_2^2 (q_3 \cdot q_1) + q_1^2 (q_2 \cdot q_3)\right)$$

$$I_3^{(b)}(p_1,p_2) = \lim_{p_1,p_2\to 0} \int \prod_{i=1}^{3} \frac{d^4 q_i}{(4\pi)^4} \frac{(b_1 p_1^2 + b_2 p_2^2 + b_3 (p_1 \cdot p_2))}{(q_1-p_1)^2 q_1^2 q_2^2 (q_2+p_2)^2 (q_3+p_2)} \frac{1}{(q_3-p_1)^2 (q_3-q_1)^2 (q_3-q_2)^2 (q_1-q_2)^2}$$

$$b_1 = q_2^2 \left(q_1 - q_3\right)^2$$

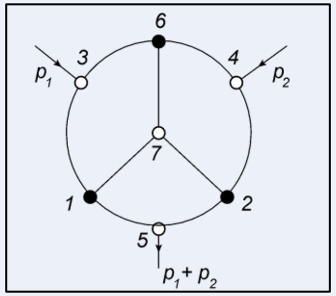
$$b_2 = q_1^2 \left(q_2 - q_3\right)^2$$

$$b_3 = -2\left(q_1^2 \left(q_2 - q_3\right)^2 + q_3^2 \left(q_1 - q_2\right)^2 - q_2^2 \left(q_1 - q_3\right)^2\right)$$

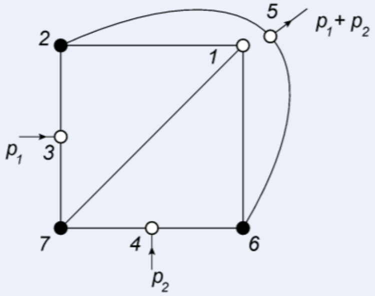


All these integrals reduces to a simple topology

Finite three-loop contribution

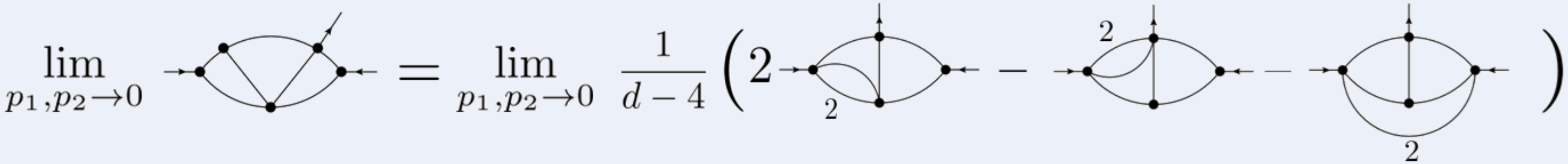


a)



b)

Integral easily can be evaluated by IBP and uniqueness relation



D-algebra and evaluation of master-integral

$$\Gamma^{(3a),h}[\Phi] = |\lambda|^6 \frac{5}{12}\zeta(5) \int d^6z\ \lambda\Phi^3(z)$$

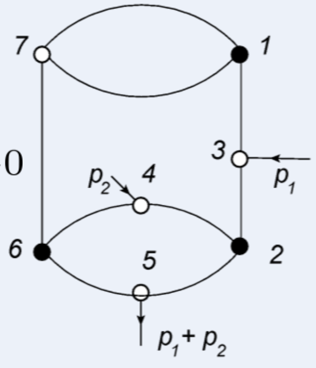
Distinguished series of diagrams (if there were no non-planar diagrams and divergent graphs, one could use the apparatus of superfishnet models)

D-algebra and evaluation of master-integral

$$\Gamma^{(3b),h}[\Phi] = |\lambda|^6 \frac{5}{12}\zeta(5) \int d^6z\ \lambda\Phi^3(z)$$

Divergent three-loop contribution

$$\lim_{p_1, p_2 \to 0}$$

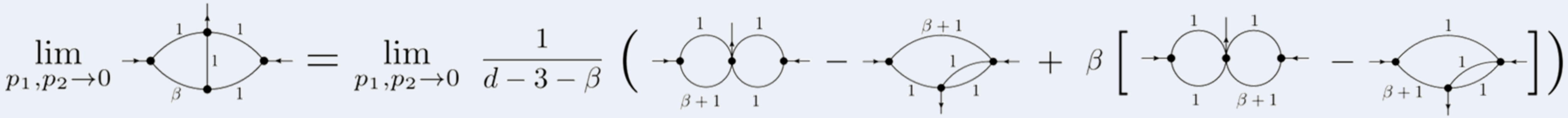


c)

$$\Gamma^{(3c),h}(\Phi) = \lambda \frac{|\lambda|^6}{3! \times 8} \int \prod_{i=1}^{7} d^8 z_i \; \Phi(z_3)\Phi(z_4)\Phi(z_5)$$
$$\Big\{ \frac{1}{\partial_1^2} \delta_{1,3} \frac{\bar{D}_3^2 D_2^2}{16\partial_2^2} \delta_{3,2} \frac{\bar{D}_4^2 D_2^2}{16\partial_2^2} \delta_{2,4} \frac{D_6^2}{4\partial_6^2} \delta_{4,6}$$
$$\frac{\bar{D}_5^2 D_6^2}{16\partial_6^2} \delta_{6,5} \frac{1}{\partial_2^2}$$
$$\delta_{5,2} \frac{1}{\partial_6^2} \delta_{6,7} \frac{\bar{D}_7^2 D_1^2}{16\partial_1^2} \delta_{1,7} \frac{\bar{D}_7^2 D_2^1}{16\partial_1^2} \delta_{7,1} \Big\}$$

$$\mathbf{K}_{1,div} = \frac{|\lambda|^2}{2\epsilon} \Phi\bar{\Phi}$$

$$\Gamma^{(3c),h}[\Phi] = \left(-\frac{3}{8\epsilon}\zeta(3) + O(\epsilon^0) \right) |\lambda|^6 \int d^6 z \; \lambda\Phi^3.$$



Final contribution

$$\Gamma^{(3c),h}[\Phi] = -|\lambda|^6 \left(\frac{3\zeta(3)}{8\epsilon} + \frac{3}{2}\zeta(3) + \frac{9}{16}\zeta(4) \right) \int d^6 z \; \lambda\Phi^3(z)$$

Final result and prospects

Result of three-loop contribution:

$$\bar{\Gamma}^{(3)}_{chiral}[\Phi] = \hbar^3 \int d^6z \, \mathbf{W}^{(3)}_{div}(\Phi) + \mathbf{W}^{(3)}_{fin}(\Phi),$$ total contribution

$$\mathbf{W}^{(3)}_{div} = -\frac{3\lambda|\lambda|^6}{8(4\pi)^8\epsilon}\zeta(3)\Phi^3(z)$$ divergent contribution

$$\mathbf{W}^{(3)}_{fin} = -\frac{\lambda|\lambda|^6}{(4\pi)^8}\left(\frac{3}{2}\zeta(3) + \frac{9}{16}\zeta(4) - \frac{5}{6}\zeta(5)\right)\Phi^3(z)$$ finite contribution

Kade, Staudacher'24

- Four-loop calculations?
- Does $\mathcal{N}$=2 case contain such a chiral corrections?
- General $\mathcal{N}$=1 chiral supersymmetric model three-loop?
- $\mathcal{N}$=1 beta deformed SYM (superfishnet model)/other SYM models?

Thank you for attention!