Chiral Higher-Spin Double Copy

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Plan

General overview/introductory part ~ 30 min

Review of 2409.19449 ~ 10 min

Key concepts & motivation

Higher-spin theories

Higher-spin theories are massless theories of higher-spin fields. These, include spin-2 field to be identified with graviton

Why interesting: toy models of quantum gravity. Some explicit companions of loop corrections have been carried out confirming the expectations of good UV properties

[Skvortsov, Tran, Tsulaia '18; DP, Sezgin, Skvortsov '19]

It is very difficult to make these fields interact without relaxing some standard assumptions

[Weinberg '64; Coleman, Mandula '67]

Nevertheless, some toy higher-spin models are known. For these one or another standard assumption is, indeed, violated

Chiral higher-spin theories

Chiral higher-spin theory is one of such toy models. It can be regarded as a higher-spin generalisation of self-dual Yang-Mills theory or gravity

[Metsaev '91; DP, Skvortsov '16]

In a related work a supersymmetric extension of self-dual Yang-Mills theory with an arbitrary number of supersymmetries was constructed, thus, showcasing that in a self-dual setup higher-spin fields can, indeed, interact

[Devchand, Ogievetsky '96]

At this point chiral higher-spin theory can be regarded as a closed sector of a parity-even theory, which is yet to be found

It was originally constructed in the light-cone gauge form. In this form it is extremely simple, featuring only cubic vertices

Color-kinematics duality

Color-kinematics duality consists of two statements: the BCJ relations and the double copy

[Bern, Carrasco, Johansson '08,'10]

The BCJ relations highlight a hidden kinematic symmetry of the Yang-Mills theory

The double copy relates the Yang-Mills theory and gravity via a certain squaring procedure

Both these work at the level of amplitudes, which are, moreover, properly preprocessed.

Exception! For self-dual theories in the light-cone gauge both the BCJ relations and the double copy become manifest at the level of action. The cubic form of the action plays a crucial role here

[O'Connell, Monteiro '11]

Our goal

We will study whether the double copy procedure can be applied to chiral higher-spin theories, that is whether chiral higher-spin theories can be squared.

We will also study a more general setup, in which we take products of powers of chiral higher-spin theories, self-dual Yang-Mills theory and self-dual gravity

Motivation

Learn whether double copy applies to higher-spin theories

We will construct new higher-spin theories, thus, extending a not very numerous list of higher-spin theories known.

These will feature stringy spectra, in the sense that fields of each spin appear more than once (in particular, infinite degeneracy as in string theory). Unlike for the typical higher-spin spectrum, extended spectra can potentially accommodate various higher-spin symmetry breaking scenarios

Light-cone deformation procedure and chiral higher-spin theories

Light-cone gauge

Consider the example of Maxwell's theory in 4d. One imposes a gauge

$$A^+ = 0$$

Here

$$x^+ = \frac{1}{\sqrt{2}}(x^3 + x^0), \qquad x^- = \frac{1}{\sqrt{2}}(x^3 - x^0),$$
$$x = \frac{1}{\sqrt{2}}(x^1 - ix^2), \qquad \bar{x} = \frac{1}{\sqrt{2}}(x^1 + ix^2)$$

are the light-cone coordinates.

Light-cone gauge

Equations of motion imply

$$\partial_- A^- + \partial_x A^x + \partial_{\bar{x}} A^{\bar{x}} = 0.$$

Components

$$\Phi^{+1} \equiv A^x, \qquad \Phi^{-1} \equiv A^{\bar{x}}$$

correspond to helicities +1 and -1 respectively. The remaining component is auxiliary

$$A^- = -\frac{\partial^{\bar{x}}}{\partial^+}\Phi^{+1} - \frac{\partial^x}{\partial^+}\Phi^{-1}.$$

The action then reads

$$S = -\int d^4x \partial_m \Phi^{-1} \partial^m \Phi^{+1}.$$

Light-cone gauge

Comment.

Derivatives in denominators that have appeared above may seem unattractive. These will show up in interactions as well. These can be easily avoided if one solves

$$\partial_- A^- + \partial_x A^x + \partial_{\bar{x}} A^{\bar{x}} = 0$$

as

$$A^x = \partial^+ \tilde{\Phi}^{+1}, \qquad A^{\bar{x}} = \partial^+ \tilde{\Phi}^{-1}, \qquad A^- = -\partial^{\bar{x}} \tilde{\Phi}^{+1} - \partial^x \tilde{\Phi}^{-1}.$$

We will use the first approach, as it is more standard in the higher-spin literature. In the literature on self-dual lower-spin theories one usually employs the second set of notations.

Light-cone gauge

Extension to free massless higher-spin fields is straightforward. For example,

$$S = -\int d^4x \partial_m \Phi^{-s} \partial^m \Phi^{+s}.$$

Controlling symmetries

Poincare symmetry (its Lorentz part) of

$$S = -\int d^4x \partial_m \Phi^{-s} \partial^m \Phi^{+s}$$

is not at all manifest, though, it is present. By the Noether theorem it entails the existence of the conserved currents and charges associated with each symmetry generator

$$O \quad \to \quad Q[O].$$

Moreover, these commute with the Poisson (Dirac) bracket in the same way as the associated Lie algebra generators commute with the Lie bracket

$$\Big[Q[O_1], Q[O_2]\Big]_{\mathrm{P}} = Q\Big[[O_1, O_2]_{\mathrm{L}}\Big].$$

This is how the symmetry is going to be controlled at non-linear level as well

Light-cone deformation procedure

Light-cone deformation procedure amounts to solving

$$\Big[Q[O_1], Q[O_2]\Big]_{\mathrm{P}} = Q\Big[[O_1, O_2]_{\mathrm{L}}\Big]$$

perturbatively in interactions. In other words, we start from the free theory charges and add non-linear corrections

$$Q[O_i] = Q_2[O_i] + Q_3[O_i] + Q_4[O_i] + \dots$$

Then one solves for non-linear corrections from the requirement of Poincare invariance as stated above.

The action can be recovered via the standard formula

$$S = \int d^4x \Pi \partial_+ \Phi - \int Q[P^-] dx^+ .$$

Light-cone deformation procedure

A simple consideration shows that most of the generators do not receive corrections at non-linear level. Due to that, most of the constraints either trivialise or are easy to solve. In essence, we need to solve only

$$[J, H] = 0, \qquad H \equiv Q[P^-], \qquad J \equiv Q[J^{x-}]$$

and its complex conjugate.

Light-cone deformation procedure

Expanding these in powers of fields we get a series of conditions

$$\begin{aligned} [J_2, H_3] + [J_3, H_2] &= 0, \\ [J_2, H_4] + [J_3, H_3] + [J_4, H_2] &= 0, \\ &\dots \end{aligned}$$

and their complex conjugate.

Cubic vertices

In the higher spin case, the analysis of

$$[J_2, H_3] + [J_3, H_2] = 0,$$
$$[\bar{J}_2, H_3] + [\bar{J}_3, H_2] = 0$$

leads to

$$S_3 = C^{\lambda_1;\lambda_2;\lambda_3} \int d^4x \frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} \Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}, \qquad \lambda_1+\lambda_2+\lambda_3 > 0,$$
$$S_3 = C^{\lambda_1;\lambda_2;\lambda_3} \int d^4x \frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}} \Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}, \qquad \lambda_1+\lambda_2+\lambda_3 < 0,$$
$$S_3 = C^{0;0;0} \int d^4x \Phi^0\Phi^0\Phi^0, \qquad \lambda_1 = \lambda_2 = \lambda_3 = 0,$$

where

$$\bar{\mathbb{P}}_{ij} \equiv \bar{\partial}_i\beta_j - \bar{\partial}_j\beta_i = -\bar{\mathbb{P}}_{ji}, \qquad \mathbb{P}_{ij} \equiv \partial_i\beta_j - \partial_j\beta_i = -\mathbb{P}_{ji},$$
$$\bar{\mathbb{P}} \equiv \bar{\mathbb{P}}_{12} = \bar{\mathbb{P}}_{23} = \bar{\mathbb{P}}_{31}, \qquad \mathbb{P} \equiv \mathbb{P}_{12} = \mathbb{P}_{23} = \mathbb{P}_{31}, \qquad \beta_i \equiv \partial_i^+.$$

[Bengtsson, Bengtsson, Brink ’83; Bengtsson, Bengtsson, Linden ’87]

Cubic vertices

At this order structure constants C for each triplet of helicities are arbitrary.

Note the (chirality) splitting of the vertices into pieces each depending on P or P-bar alone

$$S_3 = S_3[\mathbb{P}] + S_3[\bar{\mathbb{P}}].$$

The same is true for charges

$$H_3 = H_3[\mathbb{P}] + H_3[\bar{\mathbb{P}}],$$
$$J_3 = J_3[\mathbb{P}] + J_3[\bar{\mathbb{P}}].$$

Cubic coupling constants

The the next order one finds

$$[J_4, H_2] + [J_3, H_3] + [J_2, H_4] = 0,$$
$$[\bar{J}_4, H_2] + [\bar{J}_3, H_3] + [\bar{J}_2, H_4] = 0.$$

It turns out that the first line entails

$$[J_3(\bar{\mathbb{P}}), H_3(\bar{\mathbb{P}})] = 0.$$

This allows one to solve for C with non-negative total helicity

$$C^{\lambda_1,\lambda_2,\lambda_3} = g\frac{l^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}, \qquad \lambda_1+\lambda_2+\lambda_3 \geq 0.$$

Analogously,

$$C^{\lambda_1,\lambda_2,\lambda_3} = \bar{g}\frac{\bar{l}^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}, \qquad \lambda_1+\lambda_2+\lambda_3 \leq 0.$$

[Metsaev '91]

Parity-invariance

For a parity-invariant theory one has to demand

$$l = \bar{l}, \qquad g = \bar{g}.$$

In this case, one can show that the complete consistency conditions

$$[J_4, H_2] + [J_3, H_3] + [J_2, H_4] = 0,$$
$$[\bar{J}_4, H_2] + [\bar{J}_3, H_3] + [\bar{J}_2, H_4] = 0.$$

do not admit a local solution for H4 and J4.

Chiral higher-spin theory

However, one can notice that in the chiral case

$$\bar{l} = 0, \qquad \bar{g} = 0$$

the consistency conditions

$$[J_4, H_2] + [J_3, H_3] + [J_2, H_4] = 0,$$
$$[\bar{J}_4, H_2] + [\bar{J}_3, H_3] + [\bar{J}_2, H_4] = 0.$$

can be easily solved with

$$H_4 = 0, \qquad J_4 = 0.$$

Thus, the light-cone deformation procedure can be terminated at the cubic order.

Chiral higher-spin theory

As a result, one finds the chiral higher-spin theory

$$S_{CHS} = \frac{1}{2}\sum_{\lambda\in\mathbb{Z}}\int d^4x \Phi^{\lambda}\Box\Phi^{-\lambda} + g\sum_{\lambda_i\in\mathbb{Z}} \frac{l^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}\int d^4x \frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}.$$

[DP, Skvortsov '16]

Versions of chiral higher-spin theories

2-derivative colorless

$$S_{PCHS} = \frac{1}{2}\sum_{\lambda\in\mathbb{Z}}\int d^4x\Phi^{\lambda}\Box\Phi^{-\lambda} + g\sum_{\lambda_i\in\mathbb{Z}}\delta_2^{\lambda_1+\lambda_2+\lambda_3}\int d^4x\frac{\bar{\mathbb{P}}^2}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}.$$

1-derivative with color

$$S_{CCHS} = \frac{1}{2}\sum_{\lambda\in\mathbb{Z}}\int d^4x\Phi^{a|\lambda}\Box\Phi^{a|-\lambda} + gf^{abc}\sum_{\lambda_i\in\mathbb{Z}}\delta_1^{\lambda_1+\lambda_2+\lambda_3}\int d^4x\frac{\bar{\mathbb{P}}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\Phi^{a|\lambda_1}\Phi^{b|\lambda_2}\Phi^{c|\lambda_3}.$$

Matrix-valued color theory

$$S_{MCHS} = \frac{1}{2}\sum_{\lambda\in\mathbb{Z}}\int d^4x\ \mathrm{Tr}\big[\Phi^{\lambda}\Box\Phi^{-\lambda}\big] + g\sum_{\lambda_i\in\mathbb{Z}}\frac{l^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}\int d^4x\frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\mathrm{Tr}\big[\Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}\big].$$

Some recent developments

Chiral higher-spin theories were covariantised, unfolded, deformed to AdS

[Krasnov, Skvortsov ’21; Sharapov, Skvortsov, Sukhanov, Van Dongen ’22; Didenko ’22]

1-loop corrections in flat space were found to vanish

[Skvortsov, Tran, Tsulaia ‘18]

Holographic dual in flat space was proposed

[DP ’22]

Extension to 6d suggested

[Basile ’24]

Twistor reformulations were suggested

[Krasnov, Skvortsov, Tran, Adamo, Herfray. … ’21-’22]

Some classical solutions were found

[Tran ’25]

Color-kinematics duality

Original color-kinematics duality

4-pt amplitudes in YM theory can be presented in the form

$$A_4^{YM} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

$$c_s \equiv f^{a_1 a_2}{}_e f^{a_3 a_4 e}, \qquad c_t \equiv \dots, \qquad c_u \equiv \dots.$$

This requires to split quartic vertices!

Color Jacobi identity

$$c_s \pm c_t \pm c_u = 0.$$

Original color-kinematics duality

BCJ relations state that there is a kinematic counterpart of the Jacobi identity

$$n_s \pm n_t \pm n_u = 0.$$

Double copy states that 4-pt gravity amplitude can be found as

$$A_4^{GR} = \frac{n_s n_s}{s} + \frac{n_t n_t}{t} + \frac{n_u n_u}{u}.$$

Schematically,

$$GR = YM^2$$

Extends to n points, integrands of loop amplitudes, other theories.

One does not have to deal with an infinite series of vertices in GR!

[Bern, Carrasco, Johansson '08,'10]

Self-dual color-kinematics duality

Self-dual Yang-Mills theory in the light-cone gauge can be written as

$$S_{SDYM} = \int d^4x(\Phi^{a|-1}\Box\Phi^{a|+1} + gf^{abc}\frac{\bar{\mathbb{P}}\beta_1}{\beta_2\beta_3}\Phi^{a|-1}\Phi^{b|+1}\Phi^{c|+1}).$$

$$ds^2 = 2dx^+dx^- + 2dxd\bar{x}.$$

$$\bar{\mathbb{P}} \equiv \bar{\partial}_1\beta_2 - \bar{\partial}_2\beta_1, \qquad \beta_i = \partial_i^+$$

For self-dual gravity one has

$$S_{SDGR} = \int d^4x(\Phi^{-2}\Box\Phi^{+2} + \kappa\frac{\bar{\mathbb{P}}^2\beta_1^2}{\beta_2^2\beta_3^2}\Phi^{-2}\Phi^{+2}\Phi^{+2}).$$

Action is already cubic, no splitting of higher vertices is needed!

Self-dual color-kinematics duality

We can identify

$$\sqrt{c} \equiv f^{abc}, \qquad \sqrt{n} \equiv \frac{\bar{\mathbb{P}}\beta_1}{\beta_2\beta_3}.$$

Then cubic vertices have the following schematic form

$$V_3^{SDYM} = \sqrt{c}\sqrt{n}, \qquad V_3^{SDGR} = \sqrt{n}\sqrt{n}.$$

Double copy becomes manifest at the level of action (cubic vertices)

BCJ relations follow from the fact that $\sqrt{n}$ satisfy the Jacobi identity. These are the structure constants of the algebra of area-preserving diffeomorphisms (2d symplectic transformations)

[Monteiro, O'Connell '11]

More natural double copy

There is another, more natural version of lower-spin double copy. Schematically, it works as

$$YM^2 = GR + KR + dilaton.$$

For self-dual theories, we start by writing the SDYM theory as

$$S_{SDYM} = \int d^4x(\Phi^{a|-1}\Box\Phi^{a|+1} + \frac{g}{3}f^{abc}\sum_{\lambda=\pm1}\frac{\bar{\mathbb{P}}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}(\delta_1^{\lambda_1}\delta_1^{\lambda_2}\delta_{-1}^{\lambda_3} + \delta_1^{\lambda_1}\delta_{-1}^{\lambda_2}\delta_1^{\lambda_3} + \delta_{-1}^{\lambda_1}\delta_1^{\lambda_2}\delta_1^{\lambda_3})\Phi^{a|\lambda_1}\Phi^{b|\lambda_2}\Phi^{c|\lambda_3}).$$

Upon squaring we get

$$S_{SDGR+KR+D} = \int d^4x(\Phi^{-1,-1}\Box\Phi^{+1,+1} + \Phi^{-1,+1}\Box\Phi^{+1,-1}$$
$$+\frac{\kappa}{9}\sum_{\lambda,\mu=\pm1}\frac{\bar{\mathbb{P}}^2}{\beta_1^{\lambda_1+\mu_1}\beta_2^{\lambda_2+\mu_2}\beta_3^{\lambda_3+\mu_3}}(\delta_1^{\lambda_1}\delta_1^{\lambda_2}\delta_{-1}^{\lambda_3} + \delta_1^{\lambda_1}\delta_{-1}^{\lambda_2}\delta_1^{\lambda_3} + \delta_{-1}^{\lambda_1}\delta_1^{\lambda_2}\delta_1^{\lambda_3})$$
$$(\delta_1^{\mu_1}\delta_1^{\mu_2}\delta_{-1}^{\mu_3} + \delta_1^{\mu_1}\delta_{-1}^{\mu_2}\delta_1^{\mu_3} + \delta_{-1}^{\mu_1}\delta_1^{\mu_2}\delta_1^{\mu_3})\Phi^{\lambda_1,\mu_1}\Phi^{\lambda_2,\mu_2}\Phi^{\lambda_3,\mu_3}).$$

More natural double copy

Helicity is the sum of two field labels.

Here

$$\begin{aligned} \text{GR}: &\qquad \Phi^{-1,-1} \oplus \Phi^{1,1}, \\ \text{dilaton}: &\qquad \Phi^{-1,+1} + \Phi^{1,-1}, \\ \text{Kalb-Ramond}: &\qquad i\Phi^{-1,+1} - i\Phi^{1,-1}. \end{aligned}$$

<u>What is natural</u>: at the little group level double copy boils down to the tensor square of representations

Gravity forms a closed subsector of GR+dilaton+KR, thus, we can get back to the original double copy by dropping dilaton and the Kalb-Ramond field.

Alternative BCJ pattern
and
chiral higher-spin theories

Alternative BCJ pattern

It turns out that for all chiral (cubic vertices only, depend on P-bar, but not on P) theories the following BCJ' pattern holds

$$V_3 = \sqrt{n} f,$$

where f are structure constants of some Lie algebra, the rest is the kinematic factor of the YM vertex.

[DP '17]

Alternative BCJ pattern

This structure follows solely from Lorentz invariance (no underlying string constructions need to be involved, as in standard proves of CK duality)

Can be phrased as: every chiral theory is a self-dual Yang-Mills theory. Parallels Cartan's approach to GR and frame-like approach to massless higher spins.

This structure entails a list of properties, in particular, theories that have it are integrable

[DP '17; Monteiro '22]

Equivalence of BCJ patterns

Let us look again at lower-spin self-dual theories

The original BCJ relation tells us that the YM vertex with the color part removed

$$V_3^{SDYM} = \sqrt{n}\sqrt{c}, \qquad V_3^{SDGR} = \sqrt{n}\sqrt{n}$$

satisfies the Jacobi identity.

The alternative BCJ relation tells us that the GR vertex divided by the kinematic factor of the YM vertex

$$V_3^{SDYM} = \sqrt{n}\sqrt{c}, \qquad V_3^{SDGR} = \sqrt{n}\sqrt{n}$$

satisfies the Jacobi identity.

These two statements are equivalent once double copy is taken into account.

Alternative BCJ pattern

BCJ' also holds for chiral higher-spin theories

$$V_3^{CHS} = \sqrt{n} f^{CHS}.$$

The associated f corresponds to an algebra defined by a 2d Moyal bracket.

What about chiral higher-spin double copy?

Thus, BCJ arguably applies to chiral higher-spin theories.

What about the double copy, the second ingredient of the color-kinematics duality?

In other words, is there a chiral higher-spin theory with cubic vertices of the form

$$\sqrt{n} f^{CHS} \sqrt{n} f^{CHS} \quad \text{or} \quad \sqrt{n} f^{CHS} f^{CHS}$$

or some other form? The lower-spin double copy does not seem to suggest any precise pattern for the vertex to expect.

Chiral higher-spin double copy

Our setup

1) We consider a theory with a spectrum

$$\Phi^{\lambda,\mu}, \qquad \lambda, \mu \in \mathbb{Z}, \qquad h = \lambda + \mu.$$

2) A theory is chiral, so it has only cubic vertices of one chirality. These are required to factorise

$$V_3^{CHS^2} = C^{\lambda_1,\lambda_2,\lambda_3} \frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} \tilde{C}^{\mu_1,\mu_2,\mu_3} \frac{\bar{\mathbb{P}}^{\mu_1+\mu_2+\mu_3}}{\beta_1^{\mu_1}\beta_2^{\mu_2}\beta_3^{\mu_3}}.$$

3) We use the light-cone deformation procedure to impose Lorentz invariance and fix the coupling constants.

Imposing Lorentz invariance

In practice, we only need to impose Lorentz invariance at order g^2, where cubic vertices enter quadratically.

In other words, we solve

$$[J_3(\bar{\mathbb{P}}), H_3(\bar{\mathbb{P}})] = 0.$$

The solution is

$$C^{\lambda_1,\lambda_2,\lambda_3} = \frac{l^{\sum \lambda - 1 - a}}{(\sum \lambda - 1 - a)!}, \qquad \tilde{C}^{\mu_1,\mu_2,\mu_3} = \frac{m^{\sum \mu + a}}{(\sum \mu + a)!}$$

where a is an arbitrary parameter.

Redundancy

An arbitrary parameter a is inessential. Indeed,

$$\Phi^{\lambda,\mu} \to \Phi^{\lambda-x,\mu+x}$$

preserves helicity and can be regarded as a change of a variable that labels spectrum degeneration. We can fix it in any convenient way.

Chiral higher-spin double copy

Eventually, we find the double copy of the chiral higher-spin theory with the following cubic vertex

$$V_3^{CHS^2} = g \frac{l^{\sum \lambda - 1}}{(\sum \lambda - 1)!} \frac{m^{\sum \mu}}{(\sum \mu)!} \frac{\bar{\mathbb{P}}^{\sum \lambda + \sum \mu}}{\beta_1^{\lambda_1 + \mu_1} \beta_2^{\lambda_2 + \mu_2} \beta_3^{\lambda_3 + \mu_3}} .$$

Non-triviality

One may wonder whether this result is trivial in one way or another.

For example, can it be reduced to the direct sum of chiral higher-spin theories upon field redefinitions?

Answer: No

BCJ' Lie algebra

The Lie algebra associated with the BCJ' relations is as follows.

First, we define a generating function for «gauge» parameters

$$\tilde{E}(x,z,w) \equiv \sum_{\lambda,\mu=-\infty}^{\infty} \tilde{E}^{\lambda_i,\mu_i}(x) z^{\check{\lambda}} w^{\mu}, \qquad \check{\lambda}_i \equiv \lambda_i - 1.$$

Then, the Lie bracket is

$$[\tilde{E}_1(x,z,w), \tilde{E}_2(x,z,w)] = \sinh\left(\left[\frac{l}{z} + \frac{m}{w}\right](\bar{\partial}_1 \partial_2^+ - \partial_1^+ \bar{\partial}_2)\right) \tilde{E}_1(x,z,w) \tilde{E}_2(x,z,w).$$

BCJ' Lie algebra

For comparison, for the single copy we have

$$[\tilde{E}_2(x,z), \tilde{E}_3(x,z)] = \sinh\left(\frac{l}{z}(\bar{\partial}_2\partial_3^+ - \partial_2^+\bar{\partial}_3)\right)\tilde{E}_2(x,z)\tilde{E}_3(x,z).$$

In both cases the space-time part is just the Moyal bracket. It is also decorated with commuting variables z and w that count helicities.

Chiral higher-spin double copy: schematic structure

1) Double copy works in the following form

$$V_3^{CHS} = \sqrt{n} f^{CHS} \quad \rightarrow \quad V_3^{CHS^2} = \sqrt{n} f^{CHS} f^{CHS}.$$

2) BCJ' does hold, that is

$$f^{CHS} f^{CHS}$$

satisfies the Jacobi identity.

Chiral higher-spin theory
times
self-dual Yang-Mills theory

CHS x SDYM

One can consider products of power of different theories in a similar way. For example, for CHSxSDYM case one finds

$$C^{\lambda_1,\mu_1;\lambda_2,\mu_2;\lambda_3,\mu_3} = g\frac{l^{\sum\lambda}}{(\sum\lambda)!}(\delta_1^{\mu_1}\delta_1^{\mu_2}\delta_{-1}^{\mu_3} + \delta_1^{\mu_1}\delta_{-1}^{\mu_2}\delta_1^{\mu_3} + \delta_{-1}^{\mu_1}\delta_1^{\mu_2}\delta_1^{\mu_3}).$$

After a field redefinition

$$\Psi^{\lambda+1,+} \equiv \Phi^{\lambda,1}, \qquad \Psi^{\lambda-1,-} \equiv \Phi^{\lambda,-1}$$

the cubic vertex acquires the form

$$V_3 = 3g\sum_{h_i\in\mathbb{Z}}\frac{l^{h_1+h_2+h_3-1}}{(h_1+h_2+h_3-1)!}\int d^4x\frac{\bar{\mathbb{P}}^{h_1+h_2+h_3}}{\beta_1^{h_1}\beta_2^{h_2}\beta_3^{h_3}}\Psi^{h_1,+}\Psi^{h_2,+}\Psi^{h_3,-}.$$

EOM's for Psi+ are the same as in chiral higher-spin theory. At the same time, Psi- can be regarded as fields, propagating on a background formed by Psi+. This structure is reminiscent of SDYM in Chalmers-Siegel form.

Generalisations

Multiple copies and products of different factors

Analogously, one can construct many other theories. For example,

$$C^{\lambda_1,\mu_1,\rho_1;\lambda_2,\mu_2,\rho_2;\lambda_3,\mu_3,\rho_3} = g\frac{l^{\sum\lambda-1}}{(\sum\lambda-1)!}\frac{m^{\sum\mu}}{(\sum\mu)!}\frac{r^{\sum\rho}}{(\sum\rho)!},$$

$$C^{\lambda_1,\mu_1;\lambda_2,\mu_2;\lambda_3,\mu_3} = g\delta_2^{\sum\lambda}\delta_0^{\sum\mu},$$

$$C^{\lambda_1,\mu_1;\lambda_2,\mu_2;\lambda_3,\mu_3} = g\frac{l^{\sum\lambda-1}}{(\sum\lambda-1)!}\delta_0^{\sum\mu},$$

$$C^{\lambda_1,\mu_1,a;\lambda_2,\mu_2,b;\lambda_3,\mu_3,c} = gf^{abc}\delta_1^{\sum\lambda}\delta_0^{\sum\mu},$$

$$C^{\lambda_1,\mu_1,a;\lambda_2,\mu_2,b;\lambda_3,\mu_3,c} = gf^{abc}\delta_0^{\sum\lambda}(\delta_1^{\mu_1}\delta_1^{\mu_2}\delta_{-1}^{\mu_3}+\delta_1^{\mu_1}\delta_{-1}^{\mu_2}\delta_1^{\mu_3}+\delta_{-1}^{\mu_1}\delta_1^{\mu_2}\delta_1^{\mu_3}).$$

Does the double copy work as planned?

The original double copy naturally applies to single copy theories with internal symmetries. Let us, thus, consider chiral higher-spin theory with color

$$C^{\lambda_1,a_1;\lambda_2,a_2;\lambda_3,a_3} = f^{a_1a_2a_3}\delta_1^{\sum\lambda}.$$

By removing the color factor and squaring the rest, we get

$$C^{\lambda_1,\mu_1;\lambda_2,\mu_2;\lambda_3,\mu_3} = \delta_1^{\sum\lambda}\delta_1^{\sum\mu} \sim \delta_2^{\sum\lambda}\delta_0^{\sum\mu},$$

which is identically one of the theories that we constructed previously. Thus, in the given case, the double copy works literally as one can expect from the lower-spin case.

Note also that the resulting double copy theory features a closed sector involving fields with mu=0 only. It gives a 2-derivative chiral higher-spin theory without internal symmetries.

This sector is analogous to the gravitational sector in GR+KR+dilaton.

Other cases

For other multiple copies and products the original double copy does not quite suggest any pattern. We find that the resulting vertex is a bit more complex than just the product of vertices of the two single-copy theories.

Conclusions

Results

1) We constructed a number of Lorentz-invariant chiral higher-spin theories, motivated by the lower-spin double copy procedure.
2) In the case for which the original double copy procedure suggest a natural double copy theory, it proves to work exactly as expected.
3) We have many more examples, for which vertices do not factorise into vertices of single-copy theories identically, though, modifications are minor.
4) Thus, one can argue that the double copy does work for higher-spins. Our results substantially extend the existing duality web.
5) In particular, we constructed the chiral higher-spin double copy of the following schematic form

$$\text{spectrum}: \qquad \Phi^{\lambda,\mu} \quad \mu, \lambda \in \mathbb{Z},$$
$$\text{vertex:} \qquad V_3 = \sqrt{n} f^{CHS} f^{CHS}.$$

Outlook

1) Double copy CHS has fields of each helicity appearing infinitely many times. This makes this theory somewhat reminiscent of string theory in the tensionless limit. Theories with such an extended spectra are also more suitable for higher-spin symmetry breaking.

[Vasilev '12; Vasiliev '18; Skvortsov, Tran, Tsulaia '20; Didenko, Korybut '23]

2) We used very special requirements on the spectrum and on the vertices. It would be nice to solve for theories in which field of each helicity appear more than once systematically. In fact, some other theories of this type are known

[Devchand, Ogievetsky '96]

It may turn out that a complete landscape of higher-spin theories is far richer than currently known.

Thank you!