Reconstruction of inflationary scenarios in EGB gravity

E.O. Pozdeeva

Skobeltsyn Institute of Nuclear Physics
Moscow State University

based on arXiv:2411.16194

International Workshop
Problems of Modern Mathematical Physics (PMMP25)
BLTP, 10-14 Feb 2025

The $R+R^2$ gravity is the earliest inflationary model [1] which is in good agreement with modern observations[2]
The $R+R^2$ and the Higgs-driven inflation [3] belong cosmological attractors models [4] with inflationary parameters

$$n_s \simeq 1 - \frac{2}{N+N_0}, \quad r \simeq \frac{12C_\alpha}{(N+N_0)^2}, \quad C_\alpha, \quad N_0 \ll 60 \quad \text{are constants} \tag{1}$$

the case $C_\alpha = 1$ corresponds to $R+R^2$ gravity [5].

---

[1] A. A. Starobinsky, Phys. Lett. B **91** (1980), 99-102

[2] P. A. R. Ade *et al.* [BICEP and Keck], Phys. Rev. Lett. **127** (2021) no.15, 151301, arXiv:2110.00483 [astro-ph.CO]; Y. Akrami *et al.* [Planck], Astron. Astrophys. **641** (2020), A10, arXiv:1807.06211 [astro-ph.CO].

[3] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659** (2008), 703-706, arXiv:0710.3755 [hep-th]

[4] M. Galante, R. Kallosh, A. Linde and D. Roest, Phys. Rev. Lett. **114** (2015) no.14, 141302, arXiv:1412.3797 [hep-th]

[5] The relation between time derivative and e-folding number derivative is $\frac{d}{dt} = -H\frac{d}{dN}$ ($H = -\frac{dN}{dt}$, $N = -\ln(a/a_0)$, a_0 is any number), the inflation interval is $0 < N < 65$ e-folding numbers

• Early we use the cosmological attractor inflationary parameters to reconstruct scenarios in Einstein-Gauss-Bonnet gravity[6] due to application of standard slow-roll regime.

• In the Einstein–Gauss–Bonnet (EGB) gravity models, the slow-roll approximation has been extended by taking into account the first-order slow-roll parameter $\delta_1 = -2\, H^2\, \xi'/U_0$, which is proportional to the first derivative of the Gauss-Bonnet coupling function ξ with respect to the e-folding number.

• These extensions lead to the question of the accuracy of effective potential reconstruction during the generalization of attractors in EGB gravity.

• We have reconstructed models using the extended slow-roll approximations and compared them with the exact expressions and the standard slow-roll approximation.

[6] E. O. Pozdeeva, Eur. Phys. J. C **80** (2020) no.7, 612, arXiv:2005.10133 [gr-qc]. E. O. Pozdeeva and S. Y. Vernov, Eur. Phys. J. C **81** (2021) no.7, 633, arXiv:2104.04995 [gr-qc]

EGB gravity

The Einstein-Gauss-Bonnet gravity described by the following action:

$$S = \int d^4x\sqrt{-g}\left[U_0 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)\mathcal{G}\right], \qquad (2)$$

where $U_0 > 0$ is a constant, the functions $V(\phi)$ and $\xi(\phi)$ are differentiable ones,
R is the Ricci scalar,
$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss–Bonnet term.
Varying the action 2 with respect to the metric and considering the obtained equations in the spatially flat Friedmann space

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2),$$

We formulate the Einstein equations in terms of e-folding number derivatives:

$$12\,Q\,\left(U_0 + 2\xi'\,Q\right) = Q\Phi + 2V, \tag{3}$$

$$2\,(Q)'\,(U_0 + 2\xi'\,Q) = Q\Phi - 4Q\left(Q\xi'' + \left(\frac{(Q)'}{2} + Q\right)\xi'\right), \tag{4}$$

where $' = \frac{d}{dN}$, $\Phi = \chi^2$, $\chi = \frac{d\phi}{dN}$, $Q = H^2$.
Varying of action (2) with respect to field and working in the spatially flat Friedmann Universe, reformulating obtained equation in terms of e-folding derivative, we get:

$$\frac{Q}{2}\Phi' + \frac{Q'}{2}\Phi - 3\,Q\,\Phi = -V' - 12\,Q\,\xi'\left(-\frac{Q'}{2} + Q\right). \tag{5}$$

The first slow-roll parameters

$$\epsilon_1 \quad = \quad \frac{1}{2}\frac{d\ln(Q)}{dN}, \;\; \delta_1 = -\frac{2\,Q}{U_0}\frac{d\xi}{dN}, \tag{6}$$

were early introduced in terms of time derivatives [7]
In the slow-roll regime ($\ddot{\phi} \ll 3H\dot{\phi}$ or equivalently
$\ddot{\phi}\phi' = \frac{Q}{2}\Phi' + \frac{Q'}{2}\Phi \ll 3\,Q\,\Phi$) the equation (5) can be reduced to

$$3\,Q\,\Phi = V' + 12\,Q^2\,\xi'\,(1-\epsilon_1) \tag{7}$$

leading to

$$\Phi_{1,2} = \frac{V'_{1,2} + 12\,Q^2\,\xi'\,(1-\epsilon)}{3Q} \tag{8}$$

for the extended slow-roll approximations and

$$\Phi_{sl} = \frac{V'_{sl} + 12\,Q^2\,\xi'}{3Q} \tag{9}$$

for the standard slow-roll approximation ($\frac{Q'}{2} \ll Q$ or $\epsilon_1 \ll 1$).

[7] Z. K. Guo and D. J. Schwarz, Phys. Rev. D **81** (2010), 123520, arXiv:1001.1897 [hep-th]
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) no.6, 063519, arXiv:1512.04768 [hep-ph]

The first Friedmann equation can be presented in the form:

$$6\,U_0\,H^2\,(1-\delta_1) = \frac{\dot{\phi}^2}{2} + V. \tag{10}$$

Supposition $\delta_1 \ll 1$ and $\dot{\phi}^2/2 \ll 6U_0H^2$ leads to the standard slow-roll approximation:

$$V \approx 6\,U_0\,Q. \tag{11}$$

The slow-roll parameter δ_1 grows during inflation and can becomes equal to one before end of inflation and can be taken into account considering slow-roll approximation of (3):

$$6U_0\,Q(1-\delta_1) \approx V. \tag{12}$$

From here we get the potential approximations (16) and (15) using relation

$$(1-\delta_1) \approx \frac{1}{1+\delta_1}. \tag{13}$$

Potentials

Thus, the standard slow-roll approximation supposes:

$$V_{sl} = 6\,U_0\,Q. \tag{14}$$

The expressions of the potential for the expanded slow-roll approximations are :

$$V_1 \approx \frac{6\,U_0\,Q}{1+\delta_1}, \tag{15}$$

$$V_2 \approx 6\,U_0\,(1-\delta_1)\,Q. \tag{16}$$

The obtained evolution equations allows to get expression for $\Phi = \chi^2$ using (4)

$$\Phi_{exact} = \frac{2\, U_0\, Q' + 6\, Q\, Q'\, \xi' + 4\, Q^2\, (\xi'' + \xi')}{Q} \tag{17}$$

and for the potential V substituting (17) to (3)

$$V_{exact} = -\left(3\, \xi'\, Q + U_0\right)\, Q' + 2\, \left(5\, \xi' - \xi''\right) Q^2 + 6\, U_0\, Q. \tag{18}$$

Substituting (16) to Φ_2 from (8) and using the definition of the slow-roll parameter δ_1 in terms of e-folding number derivatives (6), we get (17), thus $\Phi_2 = \Phi_{exact}$.

The expressions for the inflationary parameters: the tensor-to-scalar ratio, the spectral index of scalar perturbation and the amplitude of scalar perturbation were early formulated in EGB gravity [8]:

$$r \approx 8|2\epsilon_1 - \delta_1| \tag{19}$$

$$n_s \approx 1 - 2\epsilon_1 + \frac{d\ln(r)}{dN} \tag{20}$$

$$A_s \approx \frac{Q}{\pi^2\, U_0\, r}. \tag{21}$$

[8] J.-c. Hwang and H. Noh,Phys. Rev. D (2005) [arXiv:gr-qc/0412126 [gr-qc]]; J.-c. Hwang and H. Noh, Phys. Rev. D (2005) [arXiv:gr-qc/0412126 [gr-qc]]; Z. K. Guo and D. J. Schwarz, Phys. Rev. D (2010) [arXiv:1001.1897 [hep-th]], C. van de Bruck and C. Longden, Phys. Rev. D (2016) [arXiv:1512.04768 [hep-ph]]

We start our reconstruction from (19) using (1) to get equation

$$8|2\epsilon_1 - \delta_1| = \frac{12C_\alpha}{(N+N_0)^2} \tag{22}$$

with the following form of solution

$$Q = Q_0 \exp\left(-\frac{3}{2}\frac{C_\beta}{(N+N_0)}\right), \ \xi = \frac{\xi_0\, Q_0}{Q}. \tag{23}$$

where C_β, N_0, Q_0, ξ_0 are model constants.
We substitute these expressions for Q and ξ into the formulas for the first slow-roll parameters (6):

$$\epsilon_1 = \frac{3}{4}\frac{C_\beta}{(N+N_0)^2}, \ \delta_1 \quad = \quad \frac{3\,\xi_0\, Q_0}{U_0}\frac{C_\beta}{(N+N_0)^2}. \tag{24}$$

To get exit from inflation at $N=0$ ($\epsilon_1 = 1$) we put $C_\beta = \frac{4\,N_0^2}{3}$.
We obtain following expressions for the inflationary parameters: the tensor-to-scalar ratio, the spectral index of scalar perturbation and the amplitude of scalar perturbation:

$$r \approx 8|2\epsilon_1 - \delta_1| \approx \frac{16\,N_0^2}{(N+N_0)^2}\left|1 - \frac{4\,\xi_0\,Q_0}{U_0}\right|, \quad (25)$$

$$n_s \approx 1 - 2\epsilon_1 + \frac{d\ln(r)}{dN} \approx 1 - \frac{2}{N+N_0}\left(1 + \frac{N_0}{N+N_0}\right), \quad (26)$$

$$A_s \approx \frac{Q}{\pi^2\,U_0\,r} \approx \frac{Q_0(N+N_0)^2\exp\left(-\frac{2\,N_0^2}{(N+N_0)}\right)}{16\pi^2\,U_0\,N_0^2\left|1 - \frac{4\,\xi_0\,Q_0}{U_0}\right|}. \quad (27)$$

We derive the expressions for the potential of the exponential model under consideration using approximations and the exact formula:

$$V_{sl} = 6U_0 Q_0 \exp\left(-\frac{2{N_0}^2}{N+N_0}\right), \tag{28}$$

$$V_1 = V_{sl}\left(1+\frac{4Q_0\,\xi_0\,{N_0}^2}{U_0\,(N+N_0)^2}\right)^{-1}, \tag{29}$$

$$V_2 = V_{sl}\left(1-\frac{4\,Q_0\,\xi_0\,{N_0}^2}{U_0\,(N+N_0)^2}\right), \tag{30}$$

$$V_{exact} = V_{sl}\left(1-\frac{N_0^2}{3\,(N+N_0)^2}-\frac{2Q_0\xi_0{N_0}^2(5\,(N+N_0)^2-N_0^2+2\,(N+N_0))}{3\,U_0\,(N+N_0)^4}\right)$$

We substitute the potentials to corresponding expressions for $\Phi = (\phi')^2$ and get:

$$\Phi_{sl} = \frac{4\, {N_0}^2\, (U_0 - 2\, Q_0\, \xi_0)}{(N + N_0)^2}, \tag{31}$$

$$\Phi_1 = \frac{4\, U_0^2\, N_0^2 \left(1 + \frac{4\xi_0\, Q_0 (N+N_0)}{U_0 (N+N_0)^2 + 4\xi_0\, Q_0\, N_0^2}\right)}{U_0 (N + N_0)^2 + 4\xi_0\, Q_0\, N_0^2} + \frac{8\, Q_0 \xi_0\, N_0^2 \left(\frac{N_0^2}{(N+N_0)^2} - 1\right)}{(N + N_0)^2}, \tag{32}$$

$$\Phi_2 = \frac{4\, N_0^2}{(N + N_0)^2} \left(U_0 - 2\, Q_0 \xi_0 + \frac{4\, Q_0\, \xi_0}{(N + N_0)} - \frac{2\, Q_0 \xi_0\, N_0^2}{(N + N_0)^2}\right), \tag{33}$$

$$\Phi_{exact} = \Phi_2 \tag{34}$$

The field ϕ can be expressed analytically for the exponential model. To avoid complications during subsequent numerical calculations, it is better to integrate $\sqrt{\Phi_{exact}}$ after selecting the model parameters.
The expression Φ_1 is very long and does not lead to an analytical expression for $\phi_1(N)$.
In the standard slow-roll approximation, the expression for the field's dependence on the e-folding number during inflation has a simple form

$$\phi_{sl} = 2\, N_0\, \sqrt{U_0 - 2\, Q_0\, \xi_0}\, \ln\left(N + N_0\right) + c_{sl}, \tag{35}$$

where c_{sl} is constant, which should be chosen in order to compare the behavior of ϕ and ϕ_{sl} in numerical analysis.

Choice of model parameters, numerical estimation

To analyze the behavior of $\phi(N)$ we should choose the model parameters, from eq. (25) we get:

$$N_0^2 \left| 1 - \frac{4\xi_0\, Q_0}{U_0} \right| = \frac{3}{4}\, C_\alpha, \tag{36}$$

where C_α is a constant . At the choice (36), the expression for the scalar perturbation amplitude at the beginning of inflation $(N = N_b)$ is

$$A_s|_{N=N_b} = \frac{Q_0\,(N_b + N_0)^2\, \exp\left(-\frac{2\,N_0^2}{N_b+N_0}\right)}{12\pi^2\, U_0^2\, C_\alpha}, \tag{37}$$

from where we can determine the value of Q_0:

$$Q_0 = \frac{12\pi^2\, U_0^2\, C_\alpha \cdot (A_s|_{N=N_b}) \cdot \exp\left(\frac{2\,N_0^2}{N_b+N_0}\right)}{(N_b + N_0)^2}. \tag{38}$$

Fixing N_0^2, C_α and Q_0 we automatically fix ξ_0, such as

$$\left| 1 - \frac{4\xi_0\, Q_0}{U_0} \right| = \frac{3}{4}\,\frac{C_\alpha}{N_0^2}. \tag{39}$$

The expression in the module brackets can be positive or negative:

- if $\frac{4\xi_0 Q_0}{U_0} < 1$, then $\xi_0 = \xi_{01} = \frac{U_0}{4Q_0}\left(1 - \frac{3}{4}\frac{C_\alpha}{N_0^2}\right)$ from where
  1. if $\frac{3}{4}\,C_\alpha < N_0^2$, then ξ_0 is positive,
  2. if $N_0^2 < \frac{3}{4}\,C_\alpha$, then ξ_0 is negative;

  which is possible for sufficiently large positive ξ_0, then
- if $1 < \frac{4\xi_0 Q_0}{U_0}$, when $\xi_0 = \xi_{02} = \frac{U_0}{4\,Q_0}\left(\frac{3}{4}\frac{C_\alpha}{N_0^2} + 1\right)$, which corresponds to positive ξ_0.

Now we start numerical estimations. We define the constant coupling U_0 as $U_0 = M_{Pl}^2/2$. For convenience, we put $N_0 = 1$. For simplicity, we follow to estimation for tensor-to-scalar ratio r corresponding to $R + R^2$ inflationary model using $C_\alpha = 1$.
Using the observations of the cosmic microwaves background

$$A_s = (2.10 \pm 0.03) \times 10^{-9}, n_s = 0.9654 \pm 0.0040, r < 0.028$$

[9], we introduce values of the model parameters. We estimate the starting point of inflation, $N = N_b$, by substituting the spectral index value $n_s = 0.9654$ into equation (26):

$$N_b \approx 57.787. \tag{40}$$

We apply expression for Q_0 (38) by substituting $A_s = 2.1 \cdot 10^{-9}$, and obtain:

$$Q_0 \approx 1.8861 \cdot 10^{-12}\pi^2. \tag{41}$$

We calculate $\xi_0 = \xi_{01}$ assuming that the expression in the module brackets is positive:

$$\xi_0 \approx 1.6788 \cdot 10^{10}/\pi^2. \tag{42}$$

For obtained model parameters, we get $r \approx 0.0035$ at the beginning of inflation.

[9] Planck 2018; BICEP 2021; Galloni 2022

Early we introduce the effective potential for EGB gravity [10]. The effective potential V_{eff} for it's equivalent presentation can play the role of potential for studying stability [11]. Here, we consider an equivalent presentation of the effective potential $\tilde{V}_{eff}$:

$$\tilde{V}_{eff} = -V_{eff}^{-1}, \quad \text{where} \quad V_{eff} = -U_0^2/V + \xi/3. \tag{43}$$

Here we should note, that the considering set of the model parameters is mostly toy and used to clear present the behavior of effective potential $\tilde{V}_{eff}$ which is analog to potential in the General Relativity models.

[10] E. O. Pozdeeva, M. Sami, A. V. Toporensky and S. Y. Vernov, Phys. Rev. D (2019) [arXiv:1905.05085 [gr-qc]].

[11] S. Vernov and E. Pozdeeva, Universe (2021) [arXiv:2104.11111 [gr-qc]].

All approximations and exact behavior of $\tilde{V}_{eff}$ have good agreements during inflation and start deviate near $N = 5$. This deviation is mostly interesting after inflation, because after inflation only equivalent exact effective potential has potential well. The all approximations and exact expression of $\tilde{V}_{eff}$ are closed to zero after $N \approx -0.8$. The corresponding behaviors of the equivalent effective potential are presented in Fig.1. The exact $\tilde{V}_{eff}$ has minimum at $N_m \approx -0.4463$ and equals to $\tilde{V}_{eff}(N_m) \approx -4.4571 \cdot 10^{-12}$ (see the Fig.1). The equivalent effective potential $\tilde{V}_{eff} \approx 0$ if $N \approx -0.2392$ and $N \approx -0.7777$.

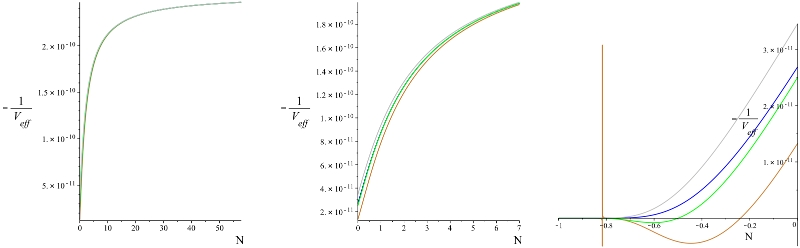

Figure: The behavior of $\tilde{V}_{eff}$ during inflation for slow-roll approximations (the gray line corresponds to the standard slow-roll approximation, the blue line to the approximation (15), the green line to the approximation (16)) and the exact considerations (orange line) at the following values of the parameters: $N_0 = 1$, $N_b = 57.787$, $\xi_0 = 1.6788 \cdot 10^{10}/\pi^2$, $Q_0 \approx 1.8861 \cdot 10^{-12}\pi^2$, $U_0 = M_{Pl}^2/2$, $M_{Pl} = 1$.

Using the values of the model parameters, we obtain:

$$\Phi = (\phi')^2 \approx \frac{1.75\,(N+1.5469)\,(N+0.73880)}{(N+1)^4}, \tag{44}$$

$$\phi = 0.5\arctan\left(\frac{0.37796(N-1.7202\cdot 10^{-9})}{S_q}\right) \tag{45}$$

$$+ \quad 1.3229\ln((N+1.1429+S_q)\cdot 10^9) - \frac{1.3229\,S_q}{(N+1)} - c_0, \tag{46}$$

where $S_q = \sqrt{(N+1.5468)\cdot(N+0.73886)}$, c_0 is a constant of integration. The choice of integration constant c_0 is not unique. Near $N \approx -0.738$ the field becomes complex and we choose $c_0 \approx 25.431$ to get $\phi|_{N\approx -0.738} = 0$.

We calculate the integration constant included to the slow-roll approximation of the field $c_{sl} = \phi - (2\,N_0\sqrt{U_0 - 2\,Q_0\,\xi_0}\ln(N+N_0))$ at the point $N = N_b$ and get: $c_{sl} \approx 1.75$. The choice of the integration constant allows us fix the same values of fields at the beginning of inflation. We solve the differential equation $\frac{d\phi}{dN} = \sqrt{\Phi}$ numerically assuming $\phi(N = N_b) = 7.145$ for all types of approximations and the exact solution. In Fig. 2, we can see the graphical behavior of the field during inflation for all slow-roll approximations and the exact behavior.

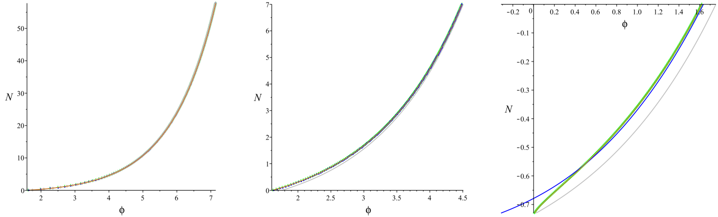

Figure: The dependence of e-folding number form fields $N(\phi)$. The gray line corresponds to the standard slow-roll approximation, the blue line to the approximation (15), the green line to the approximation (16) and the exact behavior at the following values of the parameters: $c_{sl} = 1.7556$, $N_0 = 1$, $N_b = 57.787$, $\xi_0 = 1.6569 \cdot 10^{10}/\pi^2$, $Q_0 \approx 1.8861 \cdot 10^{-12}\pi^2$, $U_0 = M_{Pl}^2/2$, $M_{Pl} = 1$. In the left picture, all lines merge into one. In the central picture, the lines of extended approximations and exact behavior merge into one line again, the gray line has a small deviation from another lines near 5 e-folds before end of inflation. In the right, picture the first extended approximation deviates from the second extended approximation and the exact behavior after end of inflation

In Fig. 3, the dependence of the equivalent effective potential $\tilde{V}_{eff}$ on the field for all type of approximation and exact case are presented.

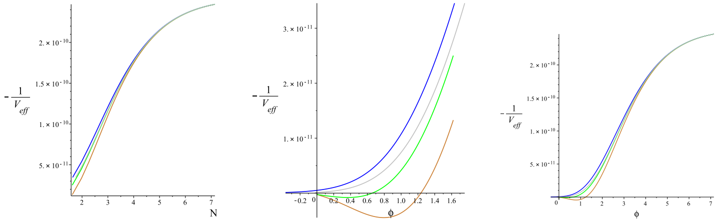

Figure: The graphical behavior of effective potential $\tilde{V}_{eff} = -V_{eff}^{-1}$ during inflation for slow-roll approximations (the gray line corresponds to the standard slow-roll approximation, the blue line to the approximation (15), the green line to the approximation (16)) and the exact considerations (orange line) at the following values of the parameters: $c_{sl} = 1.7556$, $N_0 = 1$, $N_b = 57.787$, $\xi_0 = 1.6569 \cdot 10^{10}/\pi^2$, $Q_0 \approx 1.8861 \cdot 10^{-12}\pi^2$, $U_0 = M_{Pl}^2/2$, $M_{Pl} = 1$. The left picture corresponds to evolution during inflation, the middle picture - to evolution after inflation, the right picture is the join evolution.

The exact dependencies of equivalent effective potential from e-folding number $\tilde{V}_{eff}(N)$ and field $\tilde{V}_{eff}(\phi)$ have potential well after inflation, but approximations of of equivalent effective potential don't have such potential well. At the same time all approximations rather accurate reproduce the exact behavior up to $N \approx 5$. So, we suppose that for the considering model the application of slow-roll approximation and it's extended version is reasonable for description of inflation. And it is reasonable to apply the exact behavior simulation to better understanding processes after inflation.

The exit from inflation is determined by the equality of the first slow-roll parameter to one $\epsilon_1 = 1$. Within the framework of the considering model, the exit from inflation takes place if $N = 0$, regardless of approximations types. If we consider the dependence of the first slow-roll parameter on the field, then the value of field at which the exit from inflation takes place is related with type of considering approximation. The behavior of the first slow-roll parameter on the field is presented in Fig. 4.

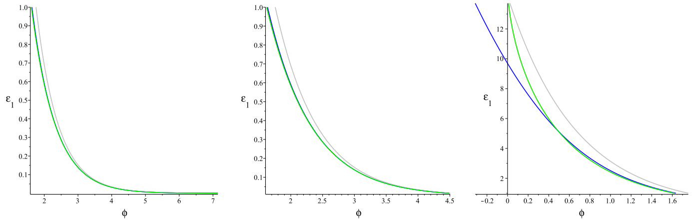

Figure: The behavior $\epsilon_1(\phi)$ (the gray line corresponds to the standard slow-roll approximation, the blue line – to the approximation (15), the green line – to the approximation (16)) and the exact behavior at the following values of the parameters: $c_{sl} = 1.7556$, $N_0 = 1$, $N_b = 57.787$, $\xi_0 = 1.6569 \cdot 10^{10}/\pi^2$, $Q_0 \approx 1.8861 \cdot 10^{-12}\pi^2$, $U_0 = M_{Pl}^2/2$, $M_{Pl} = 1$. The left picture corresponds to the interval $N = 0..N_b$, the central picture corresponds to the interval $N = 0..7$ and the right picture corresponds to the interval $N = -0.73..0$ In some regions the lines of extended slow-roll approximations coincide with exact behavior.

• The standard and the extended slow-roll approximations were considered and verified using the exact solution for the exponential model of EGB gravity.

• The extended slow-roll approximation with $V \sim (1-\delta_1)$ allows us to reconstruct the dependence of exact field on the e-folding numbers. However, the extended slow-roll approximation with $V \sim (1+\delta_1)^{-1}$ does not lead to an analytical dependence of the field on the e-folding numbers.

• All potentials reconstructed in this study preserve the exponential nature of the potential obtained within the standard slow-roll approximation framework. The first terms of all potentials coincide with V_{sl}.

• The standard slow-roll approximation reproduces $\phi(N)$ of the exact model for N between $N \approx 58$ and $N \approx 5$ up to an integration constant. After that, the deviation slowly increases before the end of inflation.

• The substitution of the standard slow-roll approximated $\phi(N)$ instead of the exact consideration to the models depending from field can change the total number of inflationary e-folding number in numerical tests. Despite this the field $\phi(N)$ approximated by standard slow-roll can be used to generate new EGB inflation models due to its simplicity in reasonable values of e-folding number.

• Both the standard and the extended slow-roll approximations reproduce exact effective potential up to $N \approx 5$ e-folding number with high accuracy.

• The effective potential obtained from the standard slow-roll approximation has bigger deviation from exact than obtained from the extended slow-roll approximations. We suppose that in models with difficult exact analytical considerations, the extended approximation with $V \sim (1 - \delta_1)$ can be applied with higher accuracy to describe inflation.

• The obtained results can be applied for generation of EGB gravity models. We hope that the results can be useful for better understanding of post-inflation evolution and for connection of early and later time processes.