Bounds on the invariant charges in QED and QCD

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This talk is based on my papers;

1. N.V.K.,Phys.Lett. 103B, 212 (1981).
2. N.V.K.,Nucl.Phys. B192, 497 (1981).

Plus some new results

Outline

1. Introduction
2. QED
3. QCD
4. Applications
5. Conclusions

1.Introduction

In quantum field theory the knowledge of the beta functions and the anomalous dimensions allows to determine the ultraviolet or infrared behavior of the model. For the Green function $G_n(p_1, ..., p_n) = \int exp(ip_k x_k) < 0|T(O_1(x_1)...O_n(x_n)|0 >$

$$(\mu^2 \frac{d}{d\mu^2} + \beta(\alpha) \frac{d}{d\alpha} + \sum_i \gamma_i(\alpha)) G_n = 0 \, .$$

General inequalities on the example of QED

The KL representation for the transverse part of the photon propagator in QED has the form [1]

$$D^{tr}(k^2, \alpha_0, m) = \frac{1}{k^2 + i\epsilon} + \int_0^\infty \frac{\rho(t, \alpha_0, m)}{k^2 - t + i\epsilon} dt \,, \qquad (2)$$

$$\bar{\alpha}(x, y, \alpha) = \alpha_0 k^2 D^{tr}(k^2, \alpha_0, m) \,,$$

where $x = \frac{-k^2}{\mu^2} > 0$, $y = \frac{m^2}{\mu^2}$ and

$$\alpha = \bar{\alpha}(x = 1, y, \alpha) \,.$$

$$\bar{\alpha}(x, y, \alpha) = x \int_0^\infty \frac{\bar{\rho}(t, \alpha, m)}{x + t/\mu^2} dt \,,$$

where $\bar{\rho}(t, \alpha, y) = \alpha_0(\delta(t) + \mu^2 \rho(t\mu^2, \alpha_o, m)) \geq 0.$

General inequalities on the example of QED

The renormalization group equation for the invariant charge has the form [1, 2, 3]

$$x\frac{\partial\bar{\alpha}(x,y,\alpha)}{\partial x} = \psi(\frac{y}{x},\bar{\alpha})\,,$$

where

$$\psi(y,\alpha) = F(x=1,y,\alpha)\,,$$

$$F(x,y,\alpha) = x\frac{\partial\bar{\alpha}(x,y,\alpha)}{\partial x}\,,$$

Using the representation (7) and the definition of the GL function (10,11) one can finc [4, 5]

$$0 \le \psi(y,\alpha) = \int_0^\infty \frac{t\bar{\rho}(t,\alpha,y)}{(1+t)^2}dt \le \int_0^\infty \frac{\bar{\rho}(t,\alpha,y)}{1+t}dt = \alpha$$

The representation for $\frac{\alpha_2(\frac{q^2}{\mu^2},y,\alpha)}{q^2} = \frac{d}{dq^2}\bar{\alpha}(\frac{q^2}{\mu^2},\frac{m^2}{\mu^2},\alpha)$ can be rewritten in the form

$$\frac{\alpha_2(q^2/\mu^2,y,\alpha)}{q^2} = \int_0^\infty \frac{\rho_1(t,y,\alpha)}{(t+q^2)^2}dt\,,$$

General inequalities on the example of QED

where $q^2 = -k^2 \geq 0$ and $\rho_1(t, y, \alpha) = \alpha_0 t \rho(t, \alpha_0, m) \geq 0$. Let us define

$$\frac{\alpha_{2+n}}{(q^2)^{1+n}} \equiv (\frac{d}{dq^2})^n (\frac{\alpha_2}{q^2}) \, .$$

Using the KL representation (13) one can find that

$$\frac{\alpha_{2+n}}{(q^2)^{1+n}} = (-1)^n (n+1)! \int_0^\infty \frac{\rho_1(t, y, \alpha)}{(t+q^2)^{2+n}} dt \, .$$

General inequalities on he example of QED

Since we shall be interested in ultraviolet asymptotics we neglect the mass m, i.e. we put $y = 0$ in our formulae.[2] The renormalization group equation for the invariant charge $\alpha_{2+n}(\frac{q^2}{\mu^2}, \alpha) = \alpha_{2+n}(\frac{q^2}{\mu^2}, y = 0, \alpha)$ has the form

$$q^2 \frac{d\alpha_{2+n}}{dq^2} = \beta(\bar{\alpha}) \frac{d\alpha_{2+n}(\bar{\alpha})}{d\bar{\alpha}} . \tag{16}$$

Using the definition (14) of the α_{2+n} and the renormalization group equation (16) we find that

$$\alpha_{2+n+1}(\bar{\alpha}) = -(1+n)\alpha_{2+n}(\bar{\alpha}) + \frac{d\alpha_{2+n}(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) \tag{17}$$

As a consequence of (17 we find that

$$\alpha_3(\bar{\alpha}) = -\alpha_2(\bar{\alpha}) + \frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) , \tag{18}$$

$$\alpha_4(\bar{\alpha}) = -2\alpha_3(\bar{\alpha}) + \frac{d\alpha_3(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) . \tag{19}$$

Using the KL representation (13) for $\frac{\alpha_{2+n}}{(q^2)^{1+n}}$ and nonegativity $\rho_1(t) \geq 0$ of spectral density we find the inequality

$$0 \leq (-1)^{n+1} \alpha_{2+n+1}(\bar{\alpha}) \leq (-1)^n (n+2) \alpha_{2+n}(\bar{\alpha}) . \tag{20}$$

As a consequence of the formula (17) the inequality (21) takes the form

$$(1+n)(-1)^{n+1} \alpha_{2+n}(\bar{\alpha}) \leq (-1)^{n+1} \frac{d\alpha_{2+n}(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) \leq \alpha_{2+n}(\bar{\alpha})(-1)^n . \tag{21}$$

For $n = 0$ and $n = 1$ the inequality (21) takes the form

$$-\alpha_2(\bar{\alpha}) \leq -\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) \leq \alpha_2(\bar{\alpha}) , \tag{22}$$

$$2\alpha_3(\bar{\alpha}) \leq \frac{d\alpha_3(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) \leq -\alpha_3(\bar{\alpha}) . \tag{23}$$

As a consequence of the relation (18) the inequality (23) takes the form

$$-2\alpha_2(\bar{\alpha}) + 3\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha}) \leq \frac{d}{d\bar{\alpha}} [\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}} \beta(\bar{\alpha})] \beta(\bar{\alpha}) \leq \alpha_2(\bar{\alpha}) . \tag{24}$$

Inequalities for GL function in QED

For $n = 0$ and $n = 1$ the inequality (21) takes the form

$$-\alpha_2(\bar{\alpha}) \leq -\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}}\beta(\bar{\alpha}) \leq \alpha_2(\bar{\alpha}) \,,$$

$$2\alpha_3(\bar{\alpha}) \leq \frac{d\alpha_3(\bar{\alpha})}{d\bar{\alpha}}\beta(\bar{\alpha}) \leq -\alpha_3(\bar{\alpha}) \,.$$

As a consequence of the relation (18) the inequality (23) takes the form

$$-2\alpha_2(\bar{\alpha}) + 3\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}}\beta(\bar{\alpha}) \leq \frac{d}{d\bar{\alpha}}[\frac{d\alpha_2(\bar{\alpha})}{d\bar{\alpha}}\beta(\bar{\alpha})]\beta(\bar{\alpha}) \leq \alpha_2(\bar{\alpha}) \,.$$

In QED $\alpha_2(\bar{\alpha}) = \psi(\bar{\alpha})$ and $\beta(\bar{\alpha}) = \psi(\bar{\alpha})$ where $\psi(\bar{\alpha})$ is the GL function. As a consequence of the inequalities (23) and (24) we find that

$$|\frac{d\psi(\bar{\alpha})}{d\bar{\alpha}}| \leq 1 \,, \tag{25}$$

$$-2 + 3\frac{d\psi(\bar{\alpha})}{d\bar{\alpha}} \leq \frac{d}{d\bar{\alpha}}[\frac{d\psi(\bar{\alpha})}{d\bar{\alpha}}\psi(\bar{\psi})] \leq 1 \,. \tag{26}$$

Note that for the regularization related with the GL function (MOM scheme) we define α as $\alpha = \bar{\alpha}(1, \alpha)$ with $\bar{\alpha}$ defined by the equation (8) to be proportional to the transverse part

GL in QED

of the photon propagator. In general renormalization scheme the renormalization condition (9) has the form

$$\bar{\alpha}(1,\alpha) \equiv c(\alpha) = \alpha + \sum_{k=2}^{\infty} c_k \alpha^k \,. \tag{27}$$

The renormalizarion group equations for the invariant charge $\bar{\alpha}(\frac{k^2}{\mu^2},\alpha)$ in arbitrary renormalization scheme has the form

$$(\mu^2 \frac{d}{d\mu^2} + \beta(\alpha)\frac{d}{d\alpha})\bar{\alpha}(x,\alpha)\,, \tag{28}$$

$$x\frac{\bar{\alpha}'(x,\alpha)}{d\alpha} = \beta(\bar{\alpha}')\,. \tag{29}$$

$$\bar{\alpha}'(1,\alpha) = \alpha\,. \tag{30}$$

The solution of the renormalization group equation (28) has the form

$$\bar{\alpha}(x,\alpha) = \bar{\alpha}(1,\bar{\alpha}'(x,\alpha) = c(\bar{\alpha}'(x,\alpha))\,. \tag{31}$$

The principal difference between the GL scheme and arbitrary scheme is that in the GL scheme due to the renormalization condition $c(\alpha) = 1$ the knowledge of the GL function allows to restore completely the dependence of the photon propagator on the k^2 while in arbitrary renormalization scheme we have to know two functions $c(\alpha)$ and $\beta(\alpha)$.

Bounds in QCD

For the propagator of the gauge invariant local operator $O(x)$ the KL representation has the form[6]

$$F(-p^2) =\equiv i \int \exp(ipx) < 0|T(O(x)O(0)| > d^4x =$$

$$\sum_{k=0}^{N-1} c_k (p^2)^k + \int_0^\infty \rho(t)[\frac{1}{t - p^2 - i\epsilon} - \ subtractions\ at\ p^2 = -\mu^2] \tag{46}$$

and

$$\rho(t) \geq 0.$$

Due to the asymptotic freedom of the QCD the ultraviolet asymptotics of the propagator $F(-p^2)$ coincides up to logarithms with the corresponding asymptotics for the propagator with free operators $O(x)$ and as a consequence the number of subtractions in representation (46) is equal to $N = d_O - 1$, where d_O is the dimension of the operator $O(x)$. The function $\Phi(x)$

$$\Phi(x) \equiv x \frac{(-1)^N}{N!} \frac{\partial^N F(x)}{\partial x^N} = x \int_0^\infty \frac{\rho(t)}{(t+x)^{N+1})}, \tag{47}$$

is ultraviolet finite for free operators. For the case with multiplicatively transformed operatotr $O(x) \to Z_O O(x)$ the function $\Phi(x)$ transforms as $\Phi(x) \to Z_O^2 \Phi(x)$ and the renormalization group equation for the (47) reads

$$[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} + 2\gamma_O(\alpha_s)]\Phi(x) = 0\,. \tag{48}$$

Here $\beta(\alpha_s)$ is the QCD β-function and $\gamma_O(\alpha_s)$ is the anomalous dimension of the operator $O(x)$. For simplicity we consider QCD with massless quarks. In QCD with the f quarks the β-function in two-loop approximation is [?]

$$\beta(\alpha_s) = -\beta_2 \alpha_s^2 - \beta_3 \alpha_s^3 \tag{49}$$

$$\beta_2 = \frac{11 - \frac{2f}{2}}{4\pi}$$

$$\beta_3 = \frac{102 - \frac{38f}{3}}{16\pi^2}$$

QCD bounds

In the derivation of the bound on the β-function in QCD we shall use the equation [?]

$$T^{\mu}_{\mu} = \frac{1}{2}\frac{\beta(\alpha_s)}{\alpha_s}(F^a_{\mu\nu})^2 \,. \tag{52}$$

for the trace of energy-momentum tensor in QCD with massless quarks. As a consequence of the relation (52) one can find [?] that the anomalous dimension of the operator $\frac{1}{4}\frac{\beta(\alpha_s)}{\alpha_s}(F^a_{\mu\nu})^2$ is equal to zero. For the two-point function

$$F(-p^2) \equiv i\int \exp(ipx) < 0|T(T^{\mu}_{\mu}(x)T^{\mu}_{\mu}(0)| > d^4x$$

the KL representation has the form

$$F(x) = x^2\int_0^{\infty}\rho(t)[\frac{1}{t+x-i\epsilon} - \frac{1}{t+\mu^2-i\epsilon}] + c_{\mu}\,, \tag{53}$$

$$\rho(t) \geq 0\;,$$

where c_{μ} is an arbitrary subtraction constant. The dimensionless quantity

$$\Phi(x) \;=\; -x\frac{d}{dx}(\frac{F(x)}{x^2}) \;=\; x\int_0^{\infty}\frac{\rho(t)dt}{(t+x)^2} \tag{54}$$

is renormalization group invariant and the renormalization group equation for $\Phi(x)$ reads

$$[\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}]\Phi(x) \;=\; 0\,. \tag{55}$$

In the derivation of the bound on β function we we shall use the renormalization condition

$$\Phi(x=\mu^2,\mu,\alpha_s) = (\frac{1}{2}\frac{\beta(\alpha_s)}{\alpha_s})^2\frac{1}{8\pi^2}\,. \tag{56}$$

Two-loop approximation

$$\beta(\alpha_s) = -\beta_2\alpha_s^2 - \beta_3\alpha_s^3$$

$$\beta_2 = \frac{11 - \frac{2f}{2}}{4\pi}$$

$$\beta_3 = \frac{102 - \frac{38f}{3}}{16\pi^2}$$

QCD bounds

As a consequence of the normalization condition (55) and the inequality (25) for $n = 0$ we find that

$$|\alpha_s \frac{d}{d\alpha_s}(\frac{\beta(\alpha_s)}{\alpha_s})| \leq \frac{1}{2}. \quad (57)$$

For the two-loop approximation (49) with $f = 3$ quarks the inequality (57) is not valid for $\alpha_s \geq 0.46$. The use of the inequality (29) for $n = 1$ leads to the inequality

$$-2(\frac{\beta}{\alpha_s})^2 + 4(\frac{\beta^2}{\alpha_s})(\frac{\beta}{\alpha_s})' \leq \beta[\frac{2\beta^2}{\alpha_s}(\frac{\beta}{\alpha_s})']' \leq (\frac{\beta}{\alpha_s})^2 \quad (58)$$

The inequality (58 can be rewritten as

$$-1 + 2\alpha_s(\frac{\beta}{\alpha_s})' \leq \beta\alpha_s(\frac{\beta}{\alpha_s})'' + \alpha_s^2((\frac{\beta}{\alpha_s})')^2 + \alpha_s\beta'(\frac{\beta}{\alpha_s})' \leq 1/2. \quad (59)$$

For two-loop approximation with $f = 3$ quarks the inequality (58) is valid only at $\alpha_s \leq 0.32$ while for $f = 0$(gluedynamics) the inequality (58) is valid only at $\alpha_s \leq 0.25$ So we conclude that for $f = 3$ the perturbation theory does not work at $\alpha_s \geq 0.32$

Implications

QED with N identical fermions

GL function in 4 loops

A.L.Kataev, s.A.Gorishny, S.A.Larin

5 loops K.G.Chetyrkin et al.

QED with N identical fermions

As is well known the β-function in renormalizable field theory with single effective charge does not depend on the renormalization scheme in two-loop approximation. For QED with N identical fermions in the one-loop approximation the GL function is

$$\psi(\alpha, N) = N\frac{\alpha^2}{3\pi}\,. \tag{32}$$

In two-loop approximation the GL function has the form

$$\psi(\alpha, N) = N[\frac{\alpha^2}{3\pi} + \frac{\alpha^3}{4\pi^2}]\,. \tag{33}$$

For one-loop approximation (32) the inequality (26) is valid up to $\frac{\alpha_{cr}}{\pi} = \frac{1.23}{N}$. For $\frac{\alpha_{cr}}{\pi} = \frac{1.23}{N}$ the ratio of two loop approximatiom to one loop approximation for GL function is equal to $\frac{0.92}{N}$ and for $N \geq 10$ it is less than 10 percent. One can show that higher order corrections to GL function qualitatively don't change our conclusions. Really the GL function up to four loops is [6]

$$\psi(\alpha, N) = N[\frac{\alpha^2}{3\pi} + \frac{\alpha^3}{4\pi^2}]4(\frac{\alpha}{4\pi})^4(-46N + (\frac{64}{3}\zeta(3) - \frac{184}{9})N^2) +$$

$$4(\frac{\alpha}{4\pi})^5(-46N + (104 + \frac{512}{3}\zeta(3) - \frac{1280}{3}\zeta(5))N^2 + (128 - \frac{256}{3}\zeta(3))N^3)\,. \tag{34}$$

QED with N identical fermions

For $\frac{\alpha_{cr}}{\pi} = \frac{1.23}{N}$ the higher order contribution is less than 15 percent.

According to common lore we can trust one-loop approximation provided two-loop approximation is much smaller one-loop approximation. So we can think that one-loop approximation is correct for $N \geq 10$ and $\alpha = \alpha_{cr}$. In other words we find that one-loop approximation contradicts to the inequality (26) for $\frac{\alpha_{cr}}{\pi} = \frac{1.23}{N}$ and for $N \geq 10$ we can trust the perturbation theory. Probably we can interpret this result as an indication in favour of instability of vacuum in QED with $N \geq 10$ identical fermions. In perturbation theory ($L \geq 2$-loop contribution to the GL function for $N \gg 1$ is proportional to $N^{L-1}(\frac{\alpha}{\pi})^{1+L}$. For $\alpha_{cr} \sim \frac{1}{N}$ it is equal to $O(\frac{1}{N^2}$ and it is much smaller one-loop contribution which is equal to $\frac{0.41}{N}$. So higher order contributions are much smaller one-loop contribution for $N \gg 1$.

Bound on new particles contribution to the SM

As is well known in the SM based on the gauge group $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ the GL function for $U_Y(1)$ subgroup in one-loop approximation in ultraviolet region $p^2 \gg M_Z^2$ is

$$\psi(\alpha_1) = N_{tot} \frac{\alpha^2}{3\pi}\,, \tag{34}$$

where $N_{tot} = 5.125 + \Delta N$ Here 5.125 is the contribution from quarks, leptons, Higgs isodoublet and $\Delta N \geq 0$ is the contribution from new particles beyond the SM. We shall assume that the SM with the gauge group $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ and with possible new particles (isosinglets, isodoublets,...) is valid up to $M_{cr} = \frac{M_{PL}}{10}$. [2] Moreover we assume that for the extended SM and in particular for the gauge group $U_Y(1)$ the perturbation theory is valid for energies up to M_{cr}. As a consequence of the inequality (25) we find that

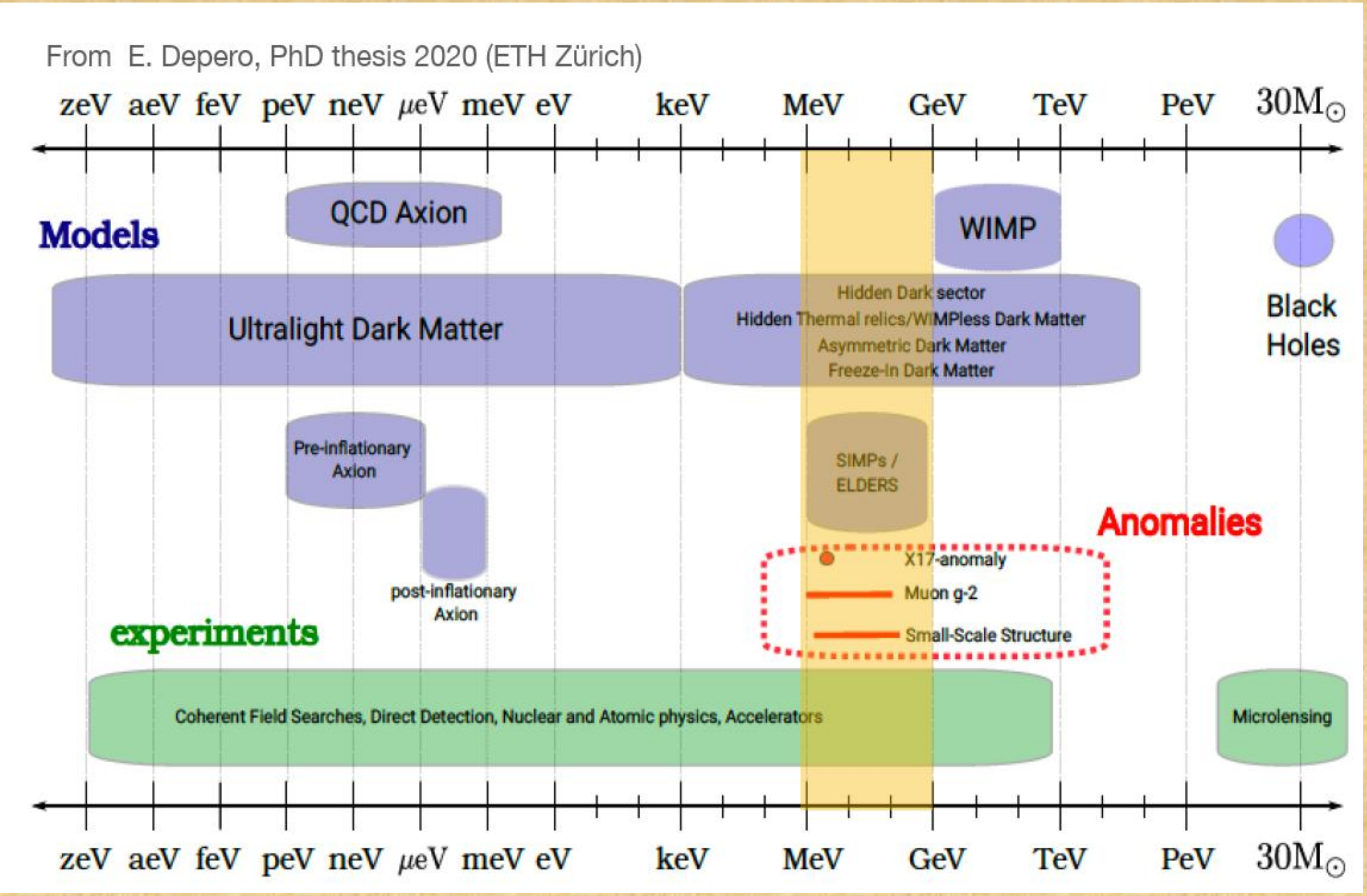
$$\bar{\alpha}_1(\frac{E^2}{M_Z^2}, \alpha_1) \leq \frac{3\pi}{\sqrt{6}N_{tot}}\,. \tag{35}$$

In the SM $\bar{\alpha}_1(M_Z^2) = \frac{\bar{\alpha}_{em}(M_Z^2)}{\cos^2(\theta_W)}$ and for $\bar{\alpha}_{em}(M_Z) = \frac{1}{128}$, $\sin^2(\theta_W) = 0.2245$ [?] we find that $\bar{\alpha}_1(M_Z^2) = 0.010$. The solution of the renormalization group equation with the GL function (34) leads to $\bar{\alpha}_1(M_{cr}^2) = \frac{\bar{\alpha}_1(M_Z^2)}{(1-\bar{\alpha}_1(M_Z^2)\frac{N_{tot}}{3\pi}\ln(\frac{M_{cr}^2}{M_Z^2})}$ As a consequence of the inequality (35) we find that

$$\Delta N \leq 7.3\,. \tag{36}$$

Here the parameter $\Delta N = \sum_k Y_k^2 c_k \frac{\ln(\frac{M_{cr}^2}{M_k^2})}{\ln(\frac{M_{cr}^2}{M_Z^2})}$ is the contributions of new particles with masses M_k and hypercharges Y_k.[3] The inequality (36) allows to restrict possible additional particles in the SM extension. For instance the inequality (36) excludes the existence of the fermion with the hypercharge $Y = 3$ and the mass $O(10)$ TeV and also it excludes the existence of 3 additional vectorlike generations with the masses $O(10)$ TeV.

Bounds on dark matter models

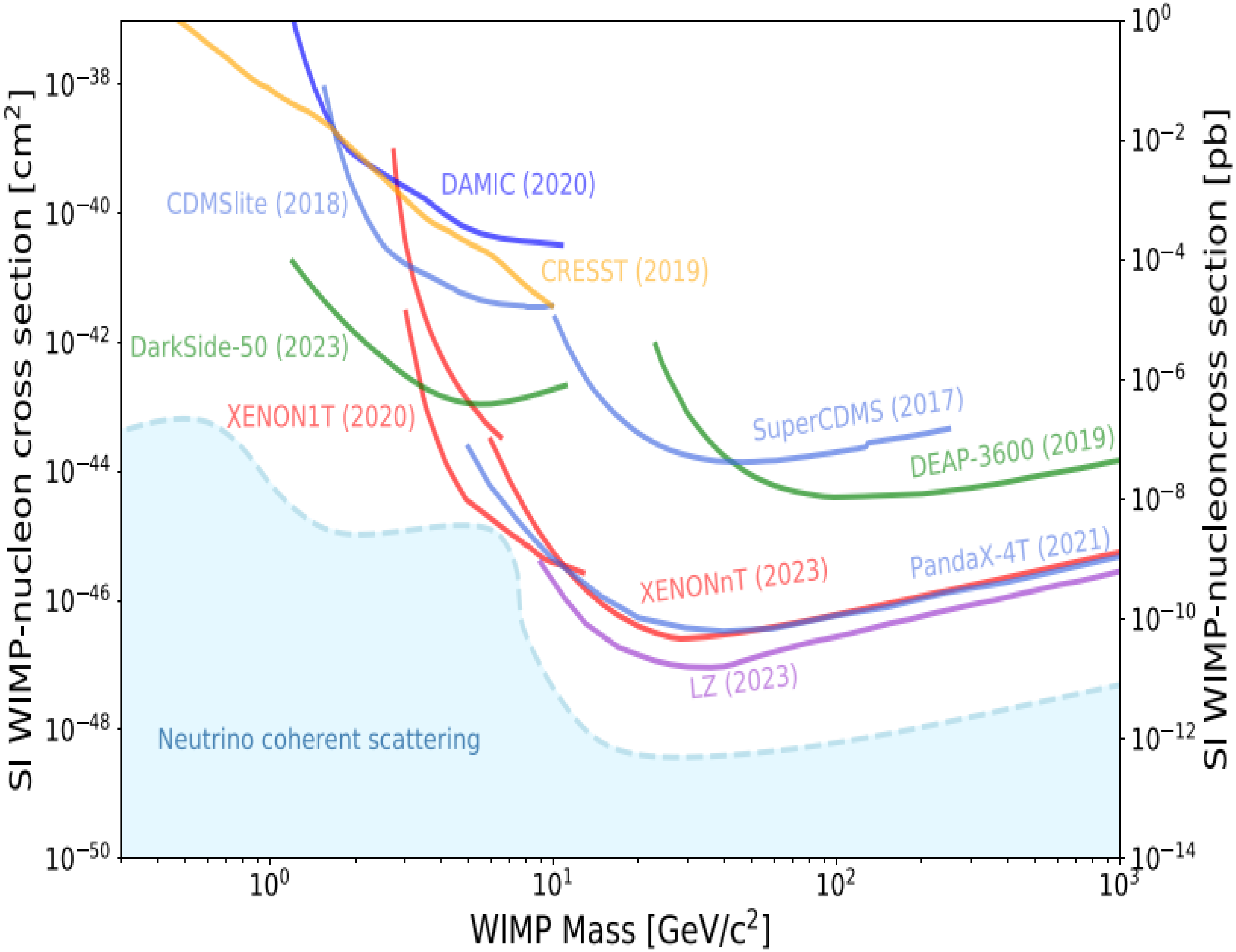


Dark matter models

Three main assumptions

1. Dark matter model is renormalizable model
2. Experimental bound on elastic DM nucleon cross section is valid
3. At the early stage of the Universe DM was in equilibrium wih the SM matter and at some time it decouples. As a consequence the annihilation cross section is predicted to be around O(1) pb

Elastic DM nucleon cross sections bounds . Bounds from underground experiments. Particle data



WIMP

The most popular mass interval from LHC point of view between O(1) GeV and O(1) TeV -> WIMP = weakly interacting massive particles

Also mass interval between O(1) MeV and O(1) GeV is popular for fixed target experiments like NA64, BELLE, SHIP, …

So called light dark matter

Typical models

At LHC bounds depend on particular model. There are a lot of models.

Simplified renormalized models:

A. Models with vector mediator
B. Models wth scalar mediator

Dark Matter: scalar, fermion, Majorana, vector spin 1.

Here as an example we consider models with vector mediator and fermion dark matter

Bounds for vector (B-L) model

In this section we apply obtained inequalities for constraining the models with dark matter. Consider as an example the model with additional vector $B - L$ Z'-boson which plays the role of messenger between our world and the dark matter world. We assume that the dark matter is described by the fermion field ψ_D with a mass m_D and coupling constant g_D with Z' boson. The interaction of Z' boson with quarks and leptons of the SM and dark matter ψ_D has the form

$$L_{int} = g_{B-L}\Big(\sum_{quarks} \frac{1}{3}\bar{q}\gamma^{\mu}q - \sum_{leptons} \bar{l}\gamma^{\mu}l \Big) Z'_{\mu} + g_D\bar{\psi}\gamma^{\mu}\psi Z'_{\mu} \,. \tag{37}$$

Underground experiments

Experimental bound on nucleon dark matter cross section for vector mediator and fermion dark mater

$$\sigma_{el} = \frac{16\pi\alpha_{B-L}\alpha_D m_p^2}{m_{Z'}^4},$$

$$\sigma_{el} \leq 10^{-9}\kappa_{el}\ pb$$

Bounds for vector (B-L) model

$$\sigma_{an} = \frac{16\pi c_{an}\alpha_{B-L}\alpha_D m_{DM}^2}{(m_{Z'}^2 - 4m_{DM}^2)^2} = \frac{16\pi c_{an}\alpha_{B-L}\alpha_D k_{DM}}{m_{Z'}^2} = \tag{41}$$

where $c_{an} = 9$ and $k_{DM}^{-1} = \frac{(m_{Z'}^2 - 4m_{DM}^2)^2}{m_{Z'}^2 m_{DM}^2}$. For often used relation $m_{Z'} = 3m_{DM}$ we find $k_{DM} = 9/25$. We also assume that there is no fine tuning between $m_{Z'}$ and m_{DM}, namely we assume that $|4m_{DM} - m_{Z'}| \geq 0.2m_{DM}$. This assumption means that $k_{DM} \leq 7$. From the requirement that the annihilation cross section reproduces the observable dark matter density the value of the annihilation cross section has to be equal to [?]

$$< \sigma_{an} v_{rel} >= 2.6\kappa_{an} \cdot 10^{-9}\ GeV^{-2}\,, \tag{42}$$

where $\kappa_{an} = O(1)$. As a consequence of the formulae (39, 40, 41, 44) one can find that

$$\frac{\sigma_{el}}{\sigma_{an}} = \frac{\kappa_{el}}{\kappa_{an}} \frac{m_p^2}{m_{Z'}^2} \cdot \frac{1}{c_{an}k} \leq \frac{\kappa_{el}}{\kappa_{an}} \cdot 10^{-9}\,, \tag{43}$$

$$16\pi\alpha_D\alpha_{B-L} \geq 2.6(\frac{1}{c_{an}k})^2 \cdot \frac{\kappa_{el}^2}{\kappa_{an}}\,. \tag{44}$$

Bounds for (B-L) vector model

We shall assume that for $(B-L)$ model the perturbation theory is valid for the scales up to $M_{cr} = \frac{M_{PL}}{10} \leq M_X \leq M_{Z'} = O(10^4\ GeV)$. As a consequence of this assumption we find that in one-loop approximation[5]

$$16\pi\alpha_{B-L}\alpha_D = 16\pi\alpha_{B-L}(M_{Z'})\alpha_D(M_{Z'}) \leq 0.027 \tag{45}$$

The bound (45) contradicts to the bound (44) for the uncertainties $\frac{\kappa^2_{el}}{\kappa_{an}} \geq 0.11$ at $m_{Z'} = 3m_{DM}$ For $c_{DM} \geq O(1)$ we don't have contradiction between the bounds (45) and (44). The bound (45) was derived from an experimental bound (39) on the elastic DM nucleon cross-section and also the assumption that at the early Universe the DM was in the thermodynamic equilibrium with the SM matter has been used. So we have found that the bound (44) at $m_{Z'} = 3m_{DM}$ leads to two large value of $16\pi\alpha_D\alpha_{B-L}$ that contradicts to the bound (45) derived in the assumption that the perturbation theory for $(B-L$ model is valid for the scales up to $10^{18}\ GeV$.

5.Conclusions

1. We have derived rigorous bounds on the renormalization group functions in QFT(QED, QCD,..). The cornstone of the derivation is the fact that in many models the invariant charges are proportional to the propagators. Plus nonegativity of the spectral density in KL representation.

2. In many cases the knowledge of such bounds allows to restrict the parameters of such models with DM. In Particular, dark matter model with (B-L) vector messenger.

THE END

Dark photon model generalization with an additional vector massive field (N.V.K., Phys.Lett. B854(2024)138747)

Direct underground experiments lead to very strong bounds on DM models. In particular, strong bounds arise for dark photon model on mixing parameter ε. The main idea is that ε parameter depends on the square of momentum transfer q^2, i.e. $\varepsilon(q^2)$ and for $\varepsilon(q^2) = cq^2$ at small q^2 direct elastic cross section is suppressed. Two possible realizations of this idea

1. Nonlocal field theory – SM and dark sector are described by renormalizable field theory but the interaction between them

Is described by nonlocal field theory

2. The introduction of additional vector field allows realize this idea.

Suppose we have additional Z' boson interacting only with the SM fields, for instance Z' interacting with (B-L) current of the SM

Introduction

Implications from underground and accelerator experiments for different

DM models are contained in recent review:

M.Lindner et al., arXiv:2403.15860

A lot of models at the level of exclusion