Geometric formulation of energy-momentum tensor induced deformations

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Problems of the Modern Mathematical Physics – PMMP'25"@Dubna

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Phys.Rev.Lett. 133 (2024) 11, 111602, ArXiv: 2405.03465[hep-th]

Outline

- Background & Motivations
- Geometric realization of TTbar and Root TTbar
- Duality between two different gravitational theories
- Summary and perspectives

Background & Motivations

To understand nature of Quantum Gravity

Holographic nature of Quantum Gravity proposed by t' Hooft and Susskind

AdS/CFT correspondence, Maldacena 1997

$$AdS_5 \times S^5 \Longleftrightarrow N = 4 \text{ SYM theory}$$

Maldacena

I. AdS5/CFT4

II. AdS4/CFT3 (ABJM)

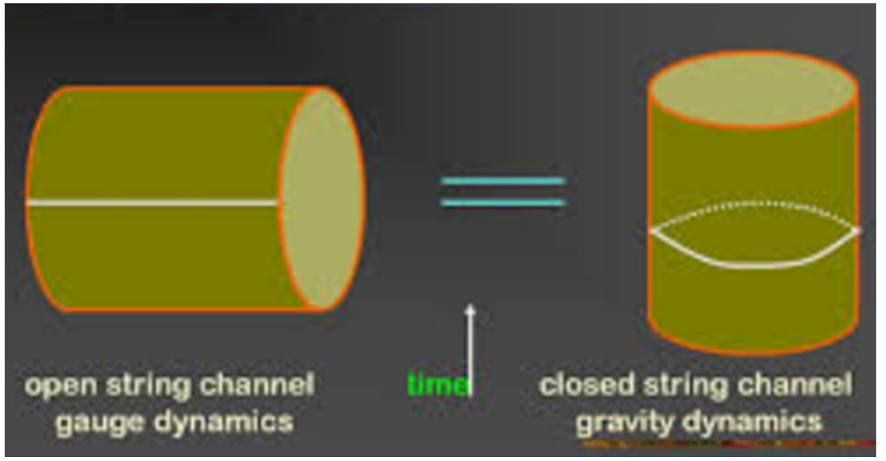
III. AdS3/CFT2

IV. nAdS2/nCFT1, nAdS2/SYK4

V. Non-AdS/CFT (Celestial Holograph)

VI. DS/CFT

VII.….



How to realize the Open-Close duality in target spacetime?
Alternative Way to understand gravity (ew duality)?
Preliminary thought experiment to go beyond holography……

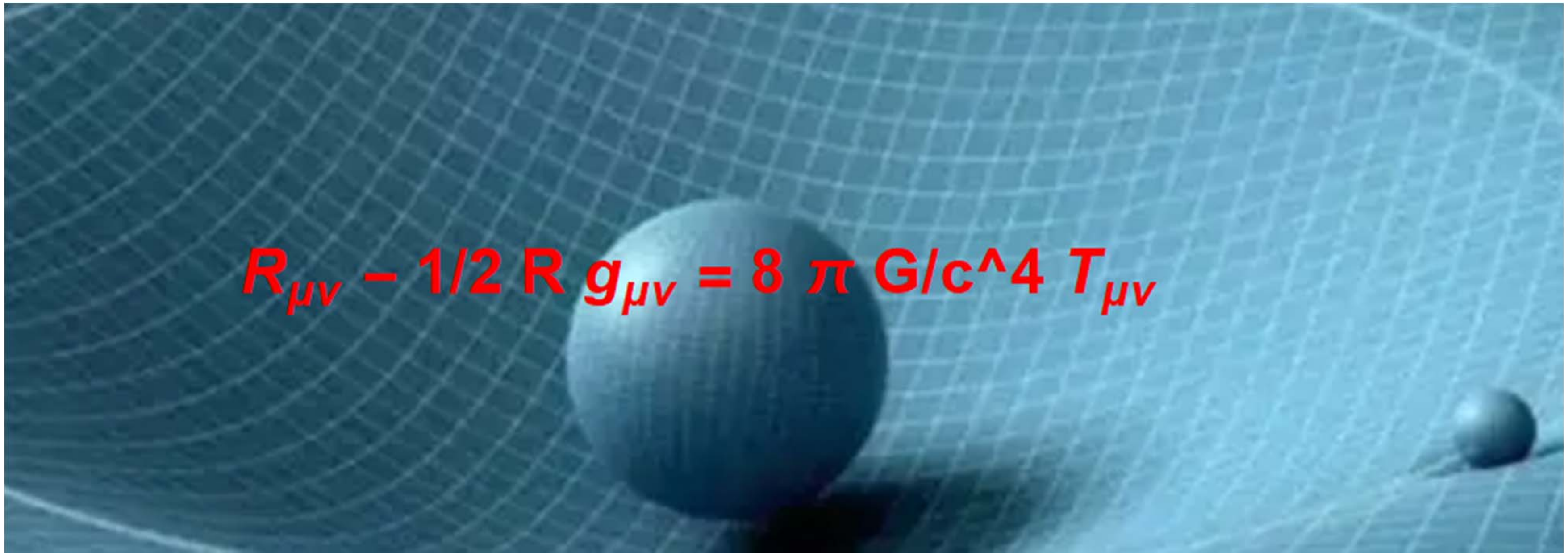
Background & Motivations

To understand nature of Quantum Gravity

Can we Go Beyond the holography?

Possible Candidate: Energy-Momentum Tensor induced deformation

$$S = \frac{g}{2} \int_{T^2} d^2 x \partial_\mu \phi \partial^\mu \phi,$$



$$T^{(0)} = -2\pi g (\partial \phi)^2, \quad \bar{T}^{(0)} = -2\pi g (\bar{\partial} \phi)^2, \quad \Theta^{(0)} = 0,$$

Definition of Ttbar & Root TTbar:

Spacetime dimension is 2, We tend to depict theories by using complex coordinates

$$z = x + it,$$
$$\bar{z} = x - it,$$

R. Conti, J. Romano, and R. Tateo, JHEP 09 (2022) 085; H. Babaei-Aghbolagh, K. Babaei Velni, etc.., Phys. Rev. D 106 no. 8, (2022) 086022;C. Ferko, A. Sfondrini, L. Smith, and G. Tartaglino-Mazzucchelli, Phys. Rev. Lett. 129 no. 20, (2022) 201604

Then in { z, $\bar{z}$ } coordinates,

$$O_{T\bar{T}} = -\det\left(T^{\mu}{}_{\nu}\right) = \frac{1}{2}\left(T^{\mu\nu}T_{\mu\nu} - T^{\mu}{}_{\mu}T^{\nu}{}_{\nu}\right)$$

$$R = \sqrt{\frac{1}{2}T^{\mu\nu}T_{\mu\nu} - \frac{1}{4}T^{\mu}{}_{\mu}T^{\nu}{}_{\nu}},$$

where in 2D CFT, we define

$$\det[T_{\mu\nu}] = -4\left(T\bar{T} - \Theta^2\right),$$

F. A. Smirnov and A. B. Zamolodchikov, Nucl. Phys. B 915 (2017) 363–383, arXiv:1608.05499.

$$T := T_{zz}, \quad \bar{T} := T_{\bar{z}\bar{z}}, \quad \Theta := T_{z\bar{z}}.$$

Deformation of TTbar

$$S^{(\lambda+\delta\lambda)} = S^{(\lambda)} + \delta\lambda \int d^2x \sqrt{\pm\gamma} O^{(\lambda)}_{T\bar{T}},$$

$$\frac{\partial S^{(\lambda)}}{\partial \lambda} = \int d^2x \sqrt{\pm\gamma} O^{(\lambda)}_{T\bar{T}}.$$

$$\frac{\mathrm{d}\mathcal{L}^{\lambda}}{\mathrm{d}\lambda} = \frac{1}{2}\epsilon^{\mu\nu}\epsilon^{\rho\sigma}T^{\lambda}_{\mu\rho}T^{\lambda}_{\nu\sigma},$$

$$\frac{\partial \mathcal{L}_{\gamma}}{\partial \gamma} = R_{\gamma} = \sqrt{\frac{1}{2}T^{\mu\nu}T_{\mu\nu} - \frac{1}{4}T^{\mu}{}_{\mu}T^{\nu}{}_{\nu}},$$

Solvability, integrability, non-locality

$$T^{\lambda}_{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S^{\lambda}}{\delta g^{\mu\nu}} = 2\frac{\partial \mathcal{L}^{\lambda}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}^{\lambda}.$$

Example:

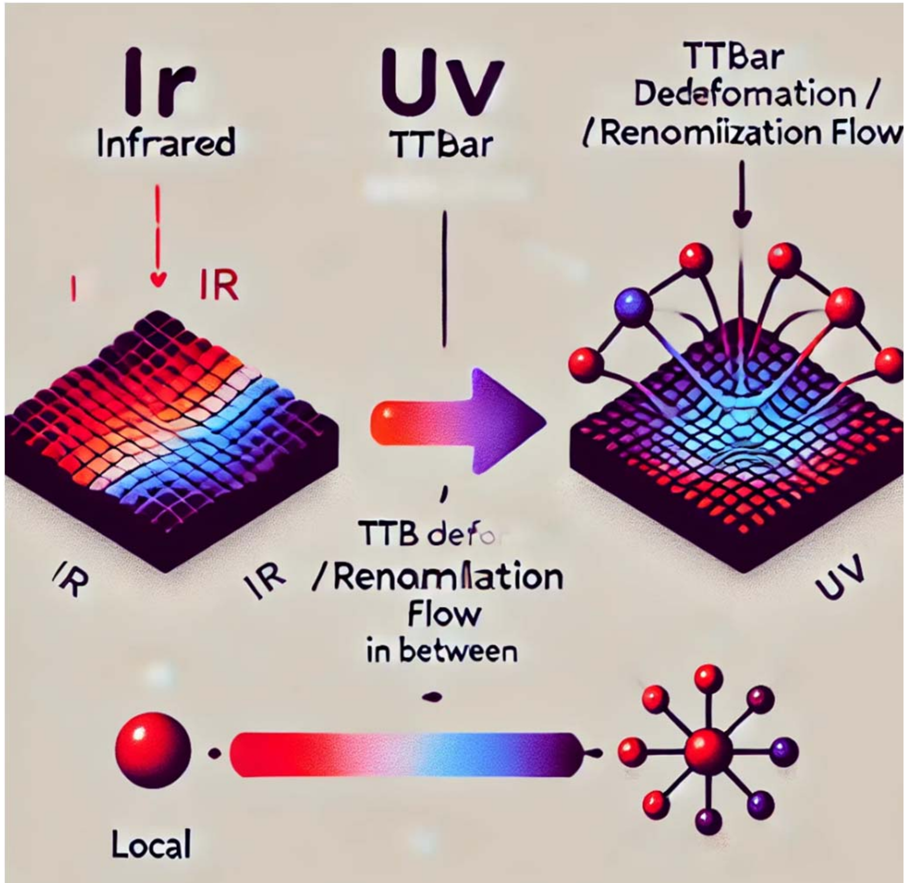
TTbar From free boson to Nambu-Goto

$$\mathcal{L}^0 = \partial\phi\bar{\partial}\phi$$

Before the deformation: (Free boson)

$$\mathcal{L}^\lambda = \frac{1}{2\lambda}\sqrt{4\lambda\partial\phi\bar{\partial}\phi + 1} - \frac{1}{2\lambda}$$

After the deformation: (Nambu-Goto - Noncritical String theory)



Cavaglia, Negro, Szecsenyi, Tateo '16

Geometric Realization of TTbar and Root TTbar in 2D

2D TTbar

Massive Gravity

A. J. Tolley, 1911.06142

$$S_\lambda = S_{\rm grav}[g_{\mu\nu}, \gamma_{\mu\nu}] + S_0[g_{\mu\nu}, \psi].$$

$$g_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu \,, \quad \gamma_{\mu\nu} = \delta_{ab} f^a_\mu f^b_\nu \,,$$

$$S_{\rm grav}[e^a_\mu, f^a_\mu] = \frac{1}{2\pi^2\lambda} \int d^2x \, \epsilon^{\mu\nu} \epsilon_{ab} (e^a_\mu - f^a_\mu)(e^b_\nu - f^b_\nu)$$

EOM of f^a_μ

$$T^\mu_a \equiv \frac{2\pi}{\det(f^a_\mu)} \frac{\delta S_\lambda[e, f, \psi]}{\delta f^a_\mu} = -\frac{2}{\pi\lambda \det(f^a_\mu)} \epsilon^{\mu\nu} \epsilon_{ab} (e^b_\nu - f^b_\nu)$$

V.S.

EOM of e^a_μ

$$\frac{1}{\pi^2\lambda} \epsilon^{\mu\nu} \epsilon_{ab} (e^{*b}_{\ \nu} - f^b_\nu) + \frac{\delta S_0[e, \psi]}{\delta e^{*a}_{\ \mu}} = 0,$$

$T^a_{0\mu}$

on-shell value of $S_{\rm grav}$

$$\det f T^\mu{}_a = \frac{\delta S_0[\varphi]}{\delta e^a_\mu}\Big|_{e=e_*} = \det e_* T_0{}^\mu{}_a(\varphi, e_*)$$

$$\frac{dS_\lambda}{d\lambda} = -\frac{1}{4} \int d^2x \det(T^a_\mu)$$

Same as TTbar flow eq.

Massive Gravity

1D TTbar

$$S\left[e_\mu, v^\mu, \phi\right] = S_{\text{grav}}\left[e_\mu, v^\mu\right] + S_0\left[e_\mu, \phi\right]$$

$$S_0 = \int dt e_t\left(\frac{1}{e_t} p\partial_t\phi - H_0(\phi, p)\right)$$

$$S_{\text{grav}}\left[e_\mu, v^\mu\right] = \frac{1}{\lambda}\int dt e_t B\left(e_t v^t\right)$$

$$v^T = \frac{dT}{dt}, \quad e_T = 1, \quad e_t = \frac{dT}{dt}$$

$$\frac{dT}{dt}B'\left(\frac{dT}{dt}\right) + B\left(\frac{dT}{dt}\right) - \lambda H_0 = 0.$$

One H deformation

SH, Zhuoyu Xian, Phys.Rev.D 106 (2022) 4, 046002
2104.03852

$$f(H) = \frac{1-\sqrt{1-8H\lambda}}{4\lambda},$$

1D TTbar deformed Hamiltonian

D. J. Gross, J. Krutho, A. Rolph and E. Shaghoulian,1912.06132

$$B(x) = \frac{(x-1)^2}{8x^2}.$$

Unify 2D TTbar and Root TTbar deformation

$$S_{\gamma,\lambda}[\phi, e^a_\mu, f^a_\mu] = S_0[\phi, e^a_\mu] + S_{\text{grav}}[e^a_\mu, f^a_\mu],$$

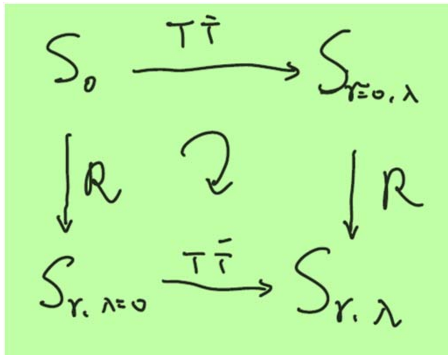
$$g_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu \,, \quad \gamma_{\mu\nu} = \delta_{ab} f^a_\mu f^b_\nu \,,$$

$$y_1 = \operatorname{tr}(e^{-1} f) = f^a_\mu e^\mu_a,$$
$$y_2 = \operatorname{tr}[(e^{-1} f)^2] = f^a_\mu e^\mu_b f^b_\nu e^\nu_a.$$

$$S_{\text{grav}}[e^a_\mu, f^a_\mu] = \frac{1}{2\lambda} \int d^2x \det e$$
$$\times \left(2 + y_1^2 - y_2 - 2y_1 \cosh \frac{\gamma}{2} + 2\sqrt{2y_2 - y_1^2} \sinh \frac{\gamma}{2} \right)$$

$$\det e \, (T^{[0]})^\mu{}_\nu \equiv \frac{\delta S_0}{\delta e^a_\mu} e^a_\nu = -\frac{\delta S_{\text{grav}}}{\delta e^a_\mu} e^a_\nu,$$

$$\frac{\partial S_{\gamma,\lambda}}{\partial \lambda} = -\int d^2x \det f \det(T^\nu{}_\mu),$$
$$\frac{\partial S_{\gamma,\lambda}}{\partial \gamma} = \int d^2x \det f \sqrt{\frac{1}{2} T^\mu_\nu T^\nu_\mu - \frac{1}{4}(T^\nu_\nu)^2}.$$



H. Babaei-Aghbolagh, **SH**, Tommaso Morone, Hao Ouyang, Roberto Tateo, Phys.Rev.Lett. 133 (2024) 11, 111602

Unify 2D TTbar and Root TTbar deformation

$$S_{\gamma,\lambda}[\phi, e^a_\mu, f^a_\mu] = S_0[\phi, e^a_\mu] + S_{\text{grav}}[e^a_\mu, f^a_\mu],$$

$$g_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu \ , \quad \gamma_{\mu\nu} = \delta_{ab} f^a_\mu f^b_\nu \ ,$$

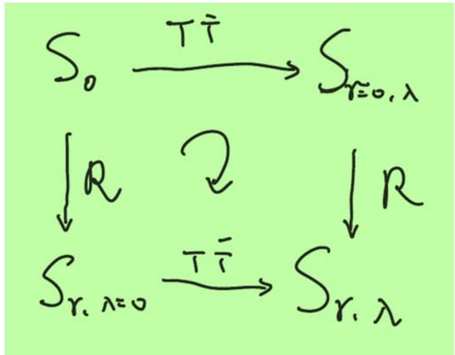
$$e^{-1} f = U \, \text{diag}(\alpha_1, \ldots, \alpha_d) \, U^{-1}$$

$$S_{\text{grav}}[e^a_\mu, f^a_\mu] = \frac{1}{\lambda} \int d^2x \det e (\alpha_1 - e^{\frac{\gamma}{2}})(\alpha_2 - e^{-\frac{\gamma}{2}}).$$

$$\det e \, (T^{[0]})^\mu{}_\nu \equiv \frac{\delta S_0}{\delta e^a_\mu} e^a_\nu = -\frac{\delta S_{\text{grav}}}{\delta e^a_\mu} e^a_\nu,$$

$$\frac{\partial S_{\gamma,\lambda}}{\partial \lambda} = -\int d^2x \det f \det(T^\nu{}_\mu),$$

$$\frac{\partial S_{\gamma,\lambda}}{\partial \gamma} = \int d^2x \det f \sqrt{\frac{1}{2} T^\mu_\nu T^\nu_\mu - \frac{1}{4} (T^\nu_\nu)^2}.$$



H. Babaei-Aghbolagh, **SH**, Tommaso Morone, Hao Ouyang,Roberto Tateo, Phys.Rev.Lett. 133 (2024) 11, 111602

Example for free Scalar

$$S_0[\phi, e^a_\mu] = \int d^2x \det e \left(\frac{1}{2}\eta^{ab} e^\mu_a e^\nu_b \partial_\mu\phi\partial_\nu\phi\right).$$

$$S_{\text{grav}}[e^a_\mu, f^a_\mu] = \frac{1}{2\lambda}\int d^2x \det e$$
$$\times\left(2 + y_1^2 - y_2 - 2y_1\cosh\frac{\gamma}{2} + 2\sqrt{2y_2 - y_1^2}\sinh\frac{\gamma}{2}\right)$$

On shell solution of e^a_μ

$$e^{*a}_{\ \mu} = \frac{1}{2}e^{\frac{\pm\gamma}{2}}\left(\frac{1}{\sqrt{1-4\lambda e^{\pm\gamma}X}} + 1\right) f^a_\mu$$
$$\mp\left(\frac{\sinh\frac{\gamma}{2} \pm 2\lambda e^{\pm\frac{\gamma}{2}}X}{X\sqrt{1-4\lambda e^{\pm\gamma}X}} + \frac{\sinh\frac{\gamma}{2}}{2X}\right)\eta^{ab} f^\nu_b \partial_\nu\phi\partial_\mu\phi,$$

EOM of e^a_μ

$$\det e\,(T^{[0]})^\mu{}_\nu \equiv \frac{\delta S_0}{\delta e^a_\mu} e^a_\nu = -\frac{\delta S_{\text{grav}}}{\delta e^a_\mu} e^a_\nu,$$

Non-Critical NG String: $S_{\gamma,\lambda}[\phi, e^{*a}_{\ \nu}, f^a_\nu] = \int d^2x \frac{1-\sqrt{1-4e^{\pm\gamma}\lambda X}}{2\lambda},$

$$X = \frac{1}{2}\eta^{ab} f^\mu_a f^\nu_b \partial_\mu\phi\partial_\nu\phi.$$

C. Ferko, A. Sfondrini, L. Smith, and G. Tartaglino-Mazzucchelli, PhysRevLett.129.201604

Geometric realization of TTbar & Root TTbar In Higher D

TTbar & Root TTbar in Higher D

$$S_{\rm grav}[e^a_\mu, f^a_\mu] = \int d^d x \det e B(e^{-1}f),$$

$$S_{\gamma,\lambda}[\phi, e^a_\mu, f^a_\mu] = S_0[\phi, e^a_\mu] + S_{\rm grav}[e^a_\mu, f^a_\mu],$$

$$T \equiv \frac{1}{\det f}\frac{\delta S_{\rm grav}}{\delta f} f = \sum_{n=1}^{d} \frac{n}{\det(e^{-1}f)}(e^{-1}f)^n \partial_{y_n} B,$$

Engenvalue approach

$$T = (\prod_{k=1}^{d} \alpha_k)^{-1} U \,\mathrm{diag}(\alpha_1 \partial_{\alpha_1} B, ..., \alpha_d \partial_{\alpha_d} B)\, U^{-1},$$

$$S_{\rm grav}[e^a_\mu, f^a_\mu] = \frac{1}{\lambda}\int d^2 x \det e (\alpha_1 - e^{\frac{\gamma}{2}})(\alpha_2 - e^{-\frac{\gamma}{2}}).$$

Generalized ansatz

$$B = \frac{1}{\lambda^{\Sigma-1}} \prod_{k=1}^{d} (\alpha_k^{p_k} - \beta_k^{p_k})^{1/p_k},$$

TTbar-like flow:

$$\frac{\partial S_{\rm grav}}{\partial \lambda} = -(\Sigma - 1)\int d^d x \det f \left(\prod_{k=1}^{d} \tau_k^{1/p_k}\right)^{\frac{1}{\Sigma-1}}.$$

Root TTbar-like flow:

$$\sum_{k=1}^{d} v^k \frac{\partial S_{\beta,\lambda}}{\partial \log \beta_k} = -\int d^d x \det f \left(\sum_{k=1}^{d} v^k \tau_k\right),$$

Checked by 4D ModMax, ModMax Born-Infeld-like, Higher D Nambu-Goto

H. Babaei-Aghbolagh, **SH**, Tommaso Morone, Hao Ouyang,Roberto Tateo, Phys.Rev.Lett. 133 (2024) 11, 111602

Dynamical gravity TTbar & Root TTbar in Higher Dim

$$S[h,g,\Gamma,\phi] = \frac{1}{2\kappa}\int d^dx\sqrt{\det h}h^{\mu\nu}R_{\mu\nu}(\Gamma) + \int d^dx\sqrt{\det g}B(g^{-1}h) + S_0[g,\phi],$$

Integrate over g

Integrate over h

$$S[h,\Gamma,\phi] = \frac{1}{2\kappa}\int d^dx\sqrt{\det h}h^{\mu\nu}R_{\mu\nu}(\Gamma) + S_{\text{deformed}}[h,\phi],$$

Duality

$$S[g,\Gamma,\phi] = \int d^dx\sqrt{\det g}\mathcal{L}(g^{-1}R) + S_0[g,\phi],$$

$$\mathcal{L} = \left[B(g^{-1}h) - \frac{1}{d-2}g^{\mu\alpha}h_{\alpha\nu}\frac{\partial B}{\partial(g^{\mu\beta}h_{\beta\nu})}\right]\Bigg|_{h=h^*(g)}$$

TTbar-like flow:

$$\frac{\partial\mathcal{L}(\rho)}{\partial\lambda} = -(\Sigma-1)\kappa^{-\frac{\Sigma}{\Sigma-1}}\left(\prod_{i=1}^{d}\alpha_i\right) \times \left(\prod_{k=1}^{d}(\alpha_k^{-2}\rho_k - \frac{1}{2}\sum_{j=1}^{d}\alpha_j^{-2}\rho_j)^{1/p_k}\right)^{\frac{1}{\Sigma-1}}$$

Root TTbar-like flow:

$$\frac{\partial\mathcal{L}(\rho)}{\partial\rho_k} = \frac{1}{2\kappa\alpha_k^2}\prod_{i=1}^{d}\alpha_i.$$

H. Babaei-Aghbolagh, **SH**, Tommaso Morone, Hao Ouyang, Roberto Tateo, Phys.Rev.Lett. 133 (2024) 11, 111602

TTbar & Root TTbar in Higher D with dynamic gravity

$$S[h,g,\Gamma,\phi] = \frac{1}{2\kappa}\int d^dx\sqrt{\det h}\,h^{\mu\nu}R_{\mu\nu}(\Gamma) + \int d^dx\sqrt{\det g}\,B(g^{-1}h) + S_0[g,\phi],$$

Integrate over g

Integrate over h

Duality

$$S[h,\Gamma,\phi] = \frac{1}{2\kappa}\int d^dx\sqrt{\det h}\,h^{\mu\nu}R_{\mu\nu}(\Gamma) + S_{\text{deformed}}[h,\phi],$$

$$S[g,\Gamma,\phi] = \int d^dx\sqrt{\det g}\,\mathcal{L}(g^{-1}R) + S_0[g,\phi],$$

$$\mathcal{L} = \left[B(g^{-1}h) - \frac{1}{d-2}g^{\mu\alpha}h_{\alpha\nu}\frac{\partial B}{\partial(g^{\mu\beta}h_{\beta\nu})}\right]\Bigg|_{h=h^*(g)}$$

Deformed theory couple with Einstein gravity

Undeformed theory couple with Ricci based gravity

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Summary

- **Propose a prescription to unify the TTbar & Root TTbar deformation in two and higher dimension.**
- **Offer a duality between field theory and gravitational theory.**
- **Possible realization of open string and close string duality in Target spacetime.**
- **Holography & Quantum aspect of deformed theory,**
- **By celestial holograph to construct 4D UV complete gravity.**

SH, Pujian Mao, Xincheng Mao, Phys.Rev.D 110 (2024) 8, L081901;
SH, Pujian Mao, Xincheng Mao, Phys.Rev.D 107 (2023) 10, L101901

Thanks