New Cosmological Solutions of a Nonlocal Gravity Model

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- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold M with metric $(g_{\mu\nu})$ of signature $(1,3)$.
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
  - sphere S^3 (of constant positive sectional curvature),
  - flat space $\mathbb{R}^3$ (of curvature equal 0),
  - hyperbolic space $\mathbb{H}^3$ (of constant negative sectional curvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where $a(t)$ is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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- GTR is based on Einstein-Hilbert action:

$$S = \int \left(\frac{R - 2\Lambda}{16\,\pi\, G\, c^4} + \mathcal{L}_m \right) \sqrt{-g}\; d^4x$$

where R is scalar curvature, $g = \det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and $\mathcal{L}_m$ is Lagrangian of matter.

- The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, G\, T_{\mu\nu}, \quad c = 1 \tag{2}$$

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

- The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \mathrm{diag}(-\rho\, g_{00}, g_{11}p, g_{22}p, g_{33}p), \tag{3}$$

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- Now, Einstein equation implies Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \qquad H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}.$$

- Hubble parameter describes the expansion of the Universe

$$H = \frac{\dot{a}}{a}\,. \tag{4}$$

- Despite to the great success of GRT, observational discoveries of 20th century imply that they could not be explained by GTR without additional matter.
- Problem of Bing Bang singularity.
- It means that GRT should be modified. There are two approaches:

(A1) Dark matter and energy

(A2) Modification of GTR, i.e. modification of its Lagrangian $\mathcal{L}$

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Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

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- Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $\mathcal{F}(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is d'Alembert operator and ∇_μ denotes the covariant derivative
- Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

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Theorem 2 (EOM) *The equations of motion for system given by* S *are:*

$$\tilde{G}_{\mu\nu} = 0, \tag{7}$$

where

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- If we suppose that the manifold M is endowed with FRW metric, then we have just linearly independent equations (trace and 00-equation):

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$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\, \mathcal{F}(\Box)\, \mathcal{G}(R) \Big) \sqrt{-g}\, d^4x,$$

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1. $\mathcal{H}(R) = R,\ \mathcal{G}(R) = R,$
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- Recently, we have considered classes of nonlocal gravity models with cosmological constant Λ and without matter, given by

$$\text{(M4)} \qquad S = \frac{1}{16\pi G}\int_M \left(R - 2\Lambda + (R - 4\Lambda)\,\mathcal{F}(\Box)(R - 4\Lambda)\right)\sqrt{-g}\,d^4x,$$

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where $P(R)$ and $Q(R)$ are some differentiable functions of R, while $\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n}\Box^{-n}$, $\Box = \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)$ is d'Alembert-Beltrami operator and Λ is cosmologicalconstant.

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- The last equation transforms to

$$\begin{aligned}(G_{\mu\nu} + \Lambda g_{\mu\nu})\left(1 + 2\mathcal{F}(q)PP'\right) + \mathcal{F}(q)g_{\mu\nu}\left(-\frac{1}{2}P^2 + PP'(R - 2\Lambda)\right)\\ -2\mathcal{F}(q)K_{\mu\nu}PP' + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(P,P) = 0. \qquad (12)\end{aligned}$$

- Let now $P = R - 4\Lambda$, then $P\,P' = P = R - 4\Lambda$ and EOM simplify to

$$\begin{aligned}\left(G_{\mu\nu} + \Lambda g_{\mu\nu} + \mathcal{F}(q)\left(G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{R}{2}g_{\mu\nu} - 2K_{\mu\nu}\right)P\right)\\ + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(P,P) = 0. \qquad (13)\end{aligned}$$

- It is evident that EOM are satisfied if

$$\mathcal{F}'(p) = 0, \quad \text{and} \qquad (14)$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + \mathcal{F}(q)\left(G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{R}{2}g_{\mu\nu} - 2K_{\mu\nu}\right)P = 0. \qquad (15)$$

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$$\hat{G}_{\mu\nu} = G_{\mu\nu} + \Lambda g_{\mu\nu} - 8\pi G \hat{T}_{\mu\nu} = 0\,. \qquad (16)$$

- The corresponding Friedmann equations are

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{p}) + \frac{\Lambda}{3}\,, \qquad \frac{\dot{a}^2 + k}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3}\,, \qquad (17)$$

where $\bar{\rho}$ and $\bar{p}$ play a role of the energy density and pressure of the dark side of the universe, respectively.

- The related equation of state is

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1. Cosmological solution for $a(t) = A\sqrt{t}\, e^{\frac{\Lambda}{4}t^2}$, $k = 0$

- For this solution we have

$$\dot{a}(t) = a(t)\frac{1}{2}\Big(\frac{1}{t} + \Lambda t\Big), \qquad \ddot{a}(t) = a(t)\frac{1}{4}\Big(\Lambda^2 t^2 + 4\Lambda - \frac{1}{t^2}\Big), \tag{19}$$

- and scalar curvature becomes

$$R(t) = 3\Lambda(\Lambda t^2 + 3). \tag{20}$$

- The Hubble parameter

$$H(t) = \frac{1}{2}\Big(\frac{1}{t} + \Lambda\, t\Big). \tag{21}$$

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- From the corresponding equation of state, $\bar{p}(t) = \bar{w}(t)\,\bar{\rho}(t)$, it follows

$$\bar{w} = \frac{t^{-2} - 3\Lambda^2 t^2 - 6\Lambda}{3t^{-2} + 3\Lambda^2 t^2 + 2\Lambda} \to \begin{cases} -1, & t \to \infty, \\ \frac{1}{3}, & t \to 0. \end{cases}$$

- The expressions (28) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model,
- and $\bar{w}(t) \to 1/3$ when $t \to 0$, what corresponds to early times as for the case of radiation.
- From expression for Hubble parameter, (21), follows:
- the first term ($\frac{1}{2t}$) is the same as for the radiation dominance in Einstein's gravity, while the second term ($\frac{\Lambda t}{2}$) can be related to the dark energy generated by cosmological constant Λ.

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Model $\mathcal{H}(R) = \mathcal{G}(R) = R - 4\Lambda$ - old solutions

- At the present cosmic time $t_0 = 13.801 \cdot 10^9$ yr and $\Lambda = 0.98 \cdot 10^{-35}$ s^{-2}, both terms in (21) are of the same order of magnitude.
- Since, the value for the Hubble parameter, and $H(t_0) = 100.2$ km/s/Mpc, is larger than current Planck mission result $H_0 = 67.40 \pm 0.50 km/s/Mpc$, this cosmological solution may be of interest for the early universe with radiation dominance and for far-future accelerated expansion.

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* 2. Cosmological solution for $a(t) = A\, e^{\Lambda t^2}$, $k = 0$

- For this solution we have

$$\dot{a}(t) = a(t)\, 2\Lambda t, \qquad \ddot{a}(t) = a(t)\, 2\Lambda(2\Lambda t^2 + 1) \tag{28}$$

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$$R(t) = 12\Lambda(4\Lambda t^2 + 1). \tag{29}$$

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- According to our solutions $a(t) = A\sqrt{t}e^{\frac{\Lambda}{4}t^2}$ and $a(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$, it follows that effects of the dark radiation ($\sqrt{t}$), the dark matter ($t^{\frac{2}{3}}$) and the dark energy ($e^{\alpha\Lambda t^2}$) at the cosmic scale can be generated by suitable nonlocal gravity models.

Old results.

- The solutions $a_1(t) = A\sqrt{t}e^{\frac{\Lambda}{4}t^2}$ and $a_2(t) = Ae^{\Lambda t^2}$ are not contained in Einstein's gravity with cosmological constant Λ. The solution $a_1(t)$ mimics interference between expansion with radiation $a_1(t)$ and a dark energy $a_2(t)$.
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- Let us consider the scale factor

$$a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}, \tag{38}$$

- and the corresponding eigenvalue problem

$$\Box(R - 4\Lambda) = p(R - 4\Lambda), \tag{39}$$

for some constant p.

- Solving the eigenvalue problem (39) we found that it is satisfied in the following two cases:
  1. $\gamma = 1, p = 2\lambda^2, \Lambda = 3\lambda^2, k \in \{0, -1, 1\}$
  2. $\gamma = \frac{1}{2}, \Lambda = \frac{3}{4}\lambda^2, k = 0.$

- Let us consider the scale factor

$$a(t) = (\alpha \cos \lambda t + \beta \sin \lambda t)^{\gamma}, \tag{40}$$

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- We found that this nonlocal gravity model has the following new cosmological soulutions.

(i1) for $\Lambda \geq 0$, and scaling factors of the form

$$a_3(t) = \alpha e^{\sqrt{\frac{1}{3}\Lambda}\, t} + \beta e^{-\sqrt{\frac{1}{3}\Lambda}\, t}, \tag{42}$$

$$a_4(t) = \left(\alpha e^{\sqrt{\frac{2}{3}\Lambda}\, t} + \beta e^{-\sqrt{\frac{2}{3}\Lambda}\, t}\right)^{\frac{1}{2}}, \tag{43}$$

(i2) for $\Lambda \leq 0$, and the trigonometric scaling factors of the form

$$a_5(t) = \alpha \cos\sqrt{-\frac{1}{3}\Lambda}\, t + \beta \sin\sqrt{-\frac{1}{3}\Lambda}, \tag{44}$$

$$a_6(t) = A\left(\alpha \cos\sqrt{\frac{2}{3}\Lambda}\, t + \beta \sin\sqrt{\frac{2}{3}\Lambda}\, t\right)^{\frac{1}{2}}. \tag{45}$$

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- 3. Cosmological solution for $a(t) = \alpha e^{\sqrt{\frac{\Lambda}{3}}\,t} + \beta e^{-\sqrt{\frac{\Lambda}{3}}\,t}$

- For this solution we have

$$\dot{a}(t) = \sqrt{\frac{\Lambda}{3}}\Big(\alpha e^{\sqrt{\frac{\Lambda}{3}}\,t} - \beta e^{-\sqrt{\frac{\Lambda}{3}}\,t}\Big), \qquad \ddot{a}(t) = \frac{\Lambda}{3}\,a(t), \tag{46}$$

$$R(t) = 4\Lambda + (6k - 8\Lambda\alpha\beta)\,a(t)^{-2}, \tag{47}$$

$$H(t) = \sqrt{\frac{\Lambda}{3}}\Big(1 - 2\beta e^{-\sqrt{\frac{\Lambda}{3}}\,t} a(t)^{-1}\Big), \tag{48}$$

$$R_{00} = -\Lambda, \qquad G_{00} = \Lambda + (3k - 4\Lambda\alpha\beta)\,a(t)^{-2}. \tag{49}$$

- The corresponding eigenvalue problem has the solutions,

$$\Box(R - 4\Lambda) = \frac{2}{3}\Lambda(R - 4\Lambda), \quad \mathcal{F}(\Box)(R - 4\Lambda) = \mathcal{F}(\frac{2}{3}\Lambda)(R - 4\Lambda). \tag{50}$$

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- Equations of motion are satisfied in the following 3 nontrivial cases:
  - 3.1 $\alpha\beta = \frac{3k}{4\Lambda}$,
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  - 3.3 $\alpha\beta = -\frac{k}{4\Lambda}$, $\mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}$, $\mathcal{F}'(\frac{2}{3}\Lambda) = 0$.

Case 3.1 $\alpha\beta = 0$, $R(t) = 4\Lambda$.

- 3.1.1 For $k = 0$ we have $a(t) \sim e^{\pm\sqrt{\frac{\Lambda}{3}}\, t}$,
- 3.1.2 $\Lambda > 0, k = +1$, gives $\quad a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}}\, t$
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Case 3.2 $\alpha = 0$ or $\beta = 0$ and $R(t) = 6ka(t)^{-2} + 4\Lambda$,

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  - 3.1 $\alpha\beta = \frac{3k}{4\Lambda}$,
  - 3.2 $\alpha\beta = 0$, $\mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}$, $\mathcal{F}'(\frac{2}{3}\Lambda) = \frac{1}{24\Lambda^2}$, $k \neq 0$,
  - 3.3 $\alpha\beta = -\frac{k}{4\Lambda}$, $\mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}$, $\mathcal{F}'(\frac{2}{3}\Lambda) = 0$.

Case 3.1 $\alpha\beta = 0$, $R(t) = 4\Lambda$.

- 3.1.1 For $k = 0$ we have $a(t) \sim e^{\pm\sqrt{\frac{\Lambda}{3}}\,t}$,
- 3.1.2 $\Lambda > 0, k = +1$, gives $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}}\, t$
- 3.1.3 $\Lambda > 0, k = -1$, gives $a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}}\, t$

Case 3.2 $\alpha = 0$ or $\beta = 0$ and $R(t) = 6ka(t)^{-2} + 4\Lambda$,

- 3.2.1 For $\alpha = 0$ we have $a(t) = \beta e^{-\sqrt{\frac{\Lambda}{3}}\,t}$,
- 3.2.2 For $\beta = 0$ we have $a(t) = \alpha e^{\sqrt{\frac{\Lambda}{3}}\,t}$.

Case 3.3 $R(t) = 4\Lambda + 8ka(t)^{-2}$,

- 3.3.1 For $k = 1$ we have $a(t) = \frac{1}{\sqrt{\Lambda}} \sinh(\varphi + \sqrt{\frac{\Lambda}{3}} t)$,
- 3.2.2 For $k = -1$ we have $a(t) = \frac{1}{\sqrt{\Lambda}} \cosh(\varphi + \sqrt{\frac{\Lambda}{3}} t)$.

- Effective density and pressure are given by:

$$\bar{\rho} = \frac{3}{8\pi G}(k - \frac{4}{3}\Lambda\alpha\beta)a(t)^{-2}, \quad \bar{p} = -\frac{1}{8\pi G}(k - \frac{4}{3}\Lambda\alpha\beta)\,a(t)^{-2}. \qquad (51)$$

- For $k \neq \frac{4}{3}\Lambda\alpha\beta$ the corresponding $\bar{w}$ parameter is $\bar{w} = -\frac{1}{3}$.

- 4. Cosmological solution for $a(t) = \left(\alpha e^{2\sqrt{\frac{\Lambda}{3}}\,t} + \beta e^{-2\sqrt{\frac{\Lambda}{3}}\,t}\right)^{\frac{1}{2}}$,

- From the related eigenvalue problem follows: $k = 0$ and $R = 4\Lambda$.
- The EOM yield the condition

$$\alpha\,\beta = 0. \qquad (52)$$

Hence, we only have solutions with $a(t) \sim e^{\pm\sqrt{\frac{\Lambda}{3}}\,t}$, which we already have in in the Einstein theory of gravity.

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5. Cosmological solution for $a(t) = \alpha \cos \sqrt{-\frac{\Lambda}{3}}\, t + \beta \sin \sqrt{-\frac{\Lambda}{3}}\, t$

- In this case we have

$$\dot{a}(t) = \sqrt{-\frac{\Lambda}{3}}\Big(\beta \cos \sqrt{-\frac{\Lambda}{3}}\, t - \alpha \sin \sqrt{-\frac{\Lambda}{3}}\, t\Big), \quad \ddot{a}(t) = \frac{\Lambda}{3} a(t), \qquad (53)$$

$$R(t) = 4\Lambda + 6\Big(k - (\alpha^2 + \beta^2)\frac{\Lambda}{3} a(t)^{-2}\Big), \qquad (54)$$

$$H(t) = \sqrt{-\frac{\Lambda}{3}}\Big(\beta \cos \sqrt{-\frac{\Lambda}{3}}\, t - \alpha \sin \sqrt{-\frac{\Lambda}{3}}\, t\Big) a(t)^{-1}, \qquad (55)$$

$$R_{00} = -\Lambda, \; G_{00} = 3\Big(k - \frac{\Lambda}{3}\Big(\beta \cos \sqrt{-\frac{\Lambda}{3}}\, t - \alpha \sin \sqrt{-\frac{\Lambda}{3}}\, t\Big)^2\Big) a(t)^{-2}.$$

- The corresponding eigenvalue problem has the same solution as in the previous case (50),

$$\Box(R - 4\Lambda) = \frac{2}{3}\Lambda(R - 4\Lambda), \quad \mathcal{F}(\Box)(R - 4\Lambda) = \mathcal{F}(\frac{2}{3}\Lambda)(R - 4\Lambda). \qquad (56)$$

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Case 5.1 In this case we have $R(t) = 4\Lambda$.

Case 5.2 In this case, we have $R(t) = 8\, k\, a(t)^{-2} + 4\Lambda$,

5.2.1 For $k = 1$ we can transform scale factor $a(t) = \alpha \cos \sqrt{-\frac{\Lambda}{3}}\, t + \beta \sin \sqrt{-\frac{\Lambda}{3}}\, t$ into form:

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$$\rho = \frac{3k - \Lambda(\alpha^2 + \beta^2)}{8\pi G\, a(t)^2}, \qquad \bar{p} = \frac{\Lambda(\alpha^2 + \beta^2) - 3k}{24\pi G\, a(t)^2}. \qquad (57)$$

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- In this case, $k = 0$ and $R = 4\Lambda$.
- From the EOM follows

$$\alpha^2 + \beta^2 = 0. \tag{58}$$

Hence, there are no nontrivial solutions of the form

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⊛ **On new cosmological solutions.** In the previous considerations, related to the finding of new cosmological solutions of nonlocal gravity model, in a class of possible scale factors of the form

$$a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$$

we found four new solutions when $\gamma = 1$ and no nontrivial solutions if $\gamma \neq 1$. The new solutions are:

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$$G_{\mu\nu} = 8\,\pi G\, T_{\mu\nu}, \quad \Box\varphi = V'(\varphi). \tag{61}$$

⊛ Now, we obtain

$$8\pi G\rho = \frac{1}{2}\dot\varphi^2 + V(\varphi), \qquad 8\pi G p = \frac{1}{2}\dot\varphi^2 - V(\varphi). \tag{62}$$

Therefore we have

$$8\pi G(\rho + p) = \dot\varphi^2 \qquad 4\pi G(\rho - p) = V(\varphi). \tag{63}$$

⊛ Let us start with the action

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\, R\, d^4x + \frac{1}{8\pi G}\int \sqrt{-g}(-\frac{1}{2}\,\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi))d^4x. \tag{59}$$

⊛ By variation of the previous action with respect to metric $g^{\mu\nu}$ we obtain

$$\frac{1}{16\pi G}G_{\mu\nu} + \frac{1}{8\pi G}\Big(\frac{1}{4}g_{\mu\nu}\nabla^\rho\varphi\nabla_\rho\varphi + \frac{1}{2}g_{\mu\nu}V(\varphi) - \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi\Big) = 0. \tag{60}$$

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- In the case of cosmological solution for $a(t) = A\sqrt{t}\, e^{\frac{\Lambda}{4}t^2}$, $k = 0$
- Corresponding effective density and pressure for this solution are:

$$\rho = \frac{\Lambda t^2\left(3\Lambda t^2 + 2\right) + 3}{32\pi G t^2}, \qquad p = \frac{1 - 3\Lambda t^2\left(\Lambda t^2 + 2\right)}{32\pi G t^2}. \qquad (64)$$

- Substituting the previous expressions into (63) we obtain

$$\dot\varphi^2 = \frac{1}{t^2} - \Lambda,$$

$$\varphi = \pm t\sqrt{\frac{1}{t^2} - \Lambda} \pm \frac{t\sqrt{\frac{1}{t^2} - \Lambda}\,\operatorname{arccot}\left(\sqrt{\Lambda t^2 - 1}\right)}{\sqrt{\Lambda t^2 - 1}} + C, \qquad (65)$$

$$V(\varphi) = \Lambda + \frac{3\Lambda^2 t^2}{4} + \frac{1}{4t^2}.$$

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Some relevant references

- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, *Some cosmological solutions of a new nonlocal gravity model*, Symmetry 12, 917 (2020), arXiv:2006.16041 [gr-qc].
- I.Dimitrijevic, B.Dragovich, Z.Rakic, and J.Stankovic, New Cosmological Solutions of a Nonlocal Gravity Model. *Symmetry* (2022) Volume 14 (1), 3.
- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *Nonlocal de Sitter gravity and its exact cosmological solutions*, Journal of High Energy Physics 2022 (12), 1-28.
- I. Dimitrijevic, B. Dragovich, A. S. Koshelev, Z. Rakic, J. Stankovic, *Cosmological solutions of a nonlocal square root gravity*, Phys. Lett. B 797 (2019) 134848, arXiv:1906.07560 [gr-qc].
- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *Nonlocal de Sitter $\sqrt{dS}$ gravity model and its applications*, to appear in Russian Journal of Mathematical Physics, (2025).
- S. Nojiri, S.D. Odintsov, V. K. Oikonomou, *Modified Gravity Theories on a Nutshell: inflation, bounce, and late-time evolution*, Phys. Rep. **692** (2017), 1–104.
- A. S. Koshelev, S. Yu. Vernov, *On bouncing solutions in non-local gravity*, Phys. Part. Nuclei **43**, 666–668 (2012) [arXiv:1202.1289v1 [hep-th]].
- I. Dimitrijevic, B. Dragovich, Z. Rakic, J. Stankovic, *On the Schwarzschild-de Sitter metric of nonlocal de Sitter gravity*, Filomat, 2023, Vol. 37 (25), 8641-8650.
- I. Dimitrijevic, B. Dragovich, J. Grujic, A.S. Koshelev, Z. Rakic, *Cosmology of modified gravity with a nonlocal $f(R)$,* Filomat 33 (2019) 1163–1178, arXiv:1509.04254[hep=th].
- T. Biswas, T. Koivisto, A. Mazumdar, *Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity*, JCAP **1011** (2010) 008 [arXiv:1005.0590v2 [hep-th]].

THANK YOU FOR

YOUR ATTENTION !!!

Non-trivial Christoffel symbols of Friedman – Robertson – Walker metric

$$\Gamma^1_{01} = \frac{\dot a}{a} \qquad \Gamma^2_{02} = \frac{\dot a}{a} \qquad \Gamma^3_{03} = \frac{\dot a}{a}$$

$$\Gamma^0_{11} = \frac{a\,\dot a}{1 - k\,r^2} \qquad \Gamma^1_{11} = \frac{k\,r}{1 - k\,r^2} \qquad \Gamma^2_{12} = \frac{1}{r}$$

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Non-trivial components of curvature tensor

$$R_{0110} = \frac{a\,\ddot{a}}{1 - k\,r^2} \qquad R_{1221} = -\frac{r^2\,a^2\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

$$R_{0220} = r^2\,a\,\ddot{a} \qquad R_{1331} = -\frac{r^2\,a^2\,\sin^2\theta\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

$$R_{0330} = r^2\,a\,\ddot{a}\,\sin^2\theta \qquad R_{2332} = -r^4\,a^2\,\sin^2\theta\,(\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & u\,g_{11} & 0 & 0 \\ 0 & 0 & u\,g_{22} & 0 \\ 0 & 0 & 0 & u\,g_{33} \end{pmatrix}, \qquad u = \frac{a\,\ddot{a} + 2\,(\dot{a}^2 + k)}{a^2}$$

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Scalar curvature

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3\,(\dot{a}^2+k)}{a^2} & 0 & 0 & 0 \\ 0 & -v\,g_{11} & 0 & 0 \\ 0 & 0 & -v\,g_{22} & 0 \\ 0 & 0 & 0 & -v\,g_{33} \end{pmatrix}, \qquad v = \frac{2\,a\,\ddot{a} + \dot{a}^2 + k}{a^2}$$

FRW metric EOM EOM-2

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