New Cosmological Solutions of a Nonlocal Gravity Model

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General theory of relativity

- GTR or ETG assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold *M* with metric $(g_{\mu\nu})$ of signature (1,3).
- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
 - sphere S³ (of constant positive sectional curvature).
 - flat space R³ (of curvature equal 0),
 - hyperbolic space III² (of constant negative sectional cutvature).
- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where a(t) is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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$$S = \int \left(\frac{R-2\Lambda}{16 \pi G c^4} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
(2)

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$\Gamma = diag(-\rho \, g_{00}, g_{11}\rho, g_{22}\rho, g_{33}\rho), \tag{3}$$

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Hubble parameter describes the expansion of the Universe

$$H = \frac{a}{a} . \tag{4}$$

- Despite to the great success of GRT, observational discoveries of 20th century imply that they could not be explained by GTR without additional matter.
- Problem of Bing Bang singularity.
- It means that GRT should be modified. There are two approaches:
- (A1) Dark matter and energy

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General theory of relativity

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- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

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Nonlocal modified gravity

- Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function *F*(*R*, □), where □ = ∇_µ∇^µ is d'Alembert operator and ∇_µ denotes the covariant derivative
- Let M be a four-dimensional pseudo-Riemannian manifold with metric (g_{µν}) of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmolo-

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In the case of FRW metric the scalar curvature and d'Alambert operator are given by

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For calculating variation of the action, $\delta S = \frac{1}{16\pi G} \delta S_0 + \delta S_1$, we need the following

Lemma 1. For any two scalar functions G and H hold

$$\begin{split} \int_{M} \mathcal{H}\delta(\sqrt{-g}) \, \mathrm{d}^{4}x &= -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}\delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta \mathcal{H}\sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(\mathcal{R}_{\mu\nu} \mathcal{H} - \mathcal{K}_{\mu\nu} \mathcal{H}\right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x, \\ \int_{M} \mathcal{H}\delta(\mathcal{F}(\Box)\mathcal{G})\sqrt{-g} \, \mathrm{d}^{4}x &= \int_{M} \left(\mathcal{R}_{\mu\nu} - \mathcal{K}_{\mu\nu}\right) \left(\mathcal{G}'\mathcal{F}(\Box)\mathcal{H}\right) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x \\ &+ \sum_{n=1}^{\infty} \frac{\hbar}{2} \sum_{k=0}^{n-1} \int_{M} \mathcal{S}_{\mu\nu} (\Box^{l}\mathcal{H}, \Box^{n-1-l}\mathcal{G}) \delta g^{\mu\nu} \sqrt{-g} \, \mathrm{d}^{4}x. \end{split}$$

where

$$\begin{split} & K_{\mu\nu} = \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ & S_{\mu\nu}(A, B) = g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \end{split}$$

Equations of motion

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$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \tag{5}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

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$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
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 \circledast The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x + \Lambda \int_M g_{\mu\nu} \sqrt{-g} \delta g^{\mu\nu} d^4 x, \qquad (5)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor.

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$$\delta S_{1} = -\frac{1}{2} \int_{M} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \delta g^{\mu\nu} \sqrt{-g} d^{4}x + \int_{M} \left(R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g} d^{4}x.$$
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Equations of motion

- \circledast Let us note that $abla^{\mu} ilde{G}_{\mu
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- EOM are invariant on the replacement of functions G and H in S.

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Theorem 2 (EOM) The equations of motion for system given by S vare:

$$\tilde{G}_{\mu\nu} = 0, \tag{7}$$

where

$$\begin{split} \tilde{G}_{\mu\nu} &= \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu}, \\ \Omega_{\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} S_{\mu\nu} \big(\Box^l \mathcal{H}(R), \Box^{n-1-l} \mathcal{G}(R) \big), \\ K_{\mu\nu} &= \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ S_{\mu\nu}(A, B) &= g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \\ W &= \mathcal{H}'(R) \mathcal{F}(\Box) \mathcal{G}(R) + \mathcal{G}'(R) \mathcal{F}(\Box) \mathcal{H}(R). \end{split}$$

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If we suppose that the manifold M is endowed with FRW metric, then we have just endowed independent equations (trace and 00-equation):

$$-2\mathcal{HF}(\Box)\mathcal{G} + RW + 3\Box W + \frac{1}{2}\Omega = \frac{R-4\Lambda}{16\pi G}, \quad \Omega = g^{\mu\nu}\Omega_{\mu\nu},$$
$$\frac{1}{2}\mathcal{HF}(\Box)\mathcal{G} + R_{00}W - K_{00}W + \frac{1}{2}\Omega_{00} = -\frac{G_{00}-\Lambda}{16\pi G}.$$

If we take

- $\operatorname{source}(R) := \mathcal{G}(R)$ and
- 0.0 < 0.01 be an eigenfunction of the consequently $\mathcal{F}(C) \mathcal{F}(R) = \mathcal{F}(Q) \mathcal{F}(R)$

$$\begin{aligned} G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) \mathcal{G}^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) \mathcal{G} \mathcal{G}' \qquad (8) \\ + \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(\mathcal{G}, \mathcal{G}) = 0. \end{aligned}$$

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$$\begin{split} -2\mathcal{HF}(\Box)\mathcal{G}+RW+3\Box W+\frac{1}{2}\Omega=\frac{R-4\Lambda}{16\pi G}, \quad \Omega=g^{\mu\nu}\Omega_{\mu\nu},\\ \frac{1}{2}\mathcal{HF}(\Box)\mathcal{G}+R_{00}W-K_{00}W+\frac{1}{2}\Omega_{00}=-\frac{G_{00}-\Lambda}{16\pi G}. \end{split}$$

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$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(\mathbf{R}) = \mathbf{R}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R,$
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(\boldsymbol{R}) = (\boldsymbol{R} + \boldsymbol{R}_0)^m, \, \mathcal{G}(\boldsymbol{R}) = (\boldsymbol{R} + \boldsymbol{R}_0)^m,$

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 Recently, we have considered classes of nonlocal gravity models with cosmological constant ∧ and without matter, given by

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 $G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) R_{\mu\nu} P P' - 2\mathcal{F}(q) K_{\mu\nu} P P' + \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$ (11) The last equation transforms to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu}) \left(1 + 2\mathcal{F}(q)PP'\right) + \mathcal{F}(q)g_{\mu\nu} \left(-\frac{1}{2}P^2 + PP'(R - 2\Lambda)\right) - 2\mathcal{F}(q)K_{\mu\nu}PP' + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(P, P) = 0.$$
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The corresponding Friedmann equations are

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{\rho}) + \frac{\Lambda}{3}, \qquad \frac{\dot{a}^2 + k}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3}, \qquad (17)$$

where $\bar{\rho}$ and $\bar{\rho}$ play a role of the energy density and pressure of the dark side of the universe, respectively.

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$$\bar{\boldsymbol{\rho}}(t) = \bar{\boldsymbol{w}}(t)\,\bar{\boldsymbol{\rho}}(t).\tag{18}$$

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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{\rho}) + \frac{\Lambda}{3}, \qquad \frac{\dot{a}^2 + k}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3}, \qquad (17)$$

where $\bar{\rho}$ and \bar{p} play a role of the energy density and pressure of the dark side of the universe, respectively.

The related equation of state is

$$\bar{\boldsymbol{p}}(t) = \bar{\boldsymbol{w}}(t)\,\bar{\boldsymbol{\rho}}(t). \tag{18}$$

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I. Cosmological solution for $a(t) = A \sqrt{t} e^{\frac{\pi}{4}t}$, k = 0.

For this solution we have

$$\ddot{a}(t) = a(t)\frac{1}{2}\left(\frac{1}{t} + \Lambda t\right), \qquad \ddot{a}(t) = a(t)\frac{1}{4}\left(\Lambda^2 t^2 + 4\Lambda - \frac{1}{t^2}\right),$$
 (19)

and scalar curvature becomes

$$R(t) = 3\Lambda(\Lambda t^2 + 3). \tag{20}$$

The Hubble parameter

$$H(t) = \frac{1}{2} (\frac{1}{t} + \Lambda t).$$
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$$(22)$$

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$$\mathcal{F}(\Box) \left(R - 4\Lambda \right) = \mathcal{F}(-3\Lambda) \left(R - 4\Lambda \right).$$
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EOM are satisfied under conditions

$$\mathcal{F}(-3\Lambda) = -\frac{1}{10\Lambda}, \qquad \mathcal{F}'(-3\Lambda) = 0, \quad \Lambda \neq 0, \tag{25}$$

which are satisfied by nonlocal operator

$$\mathcal{F}(\Box) = \frac{\Box}{30\Lambda^2} \exp\left(\frac{\Box}{3\Lambda} + 1\right).$$
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Friedman equations imply

$$\bar{\rho}(t) = \frac{3t^{-2} + 3\Lambda^2 t^2 + 2\Lambda}{32\pi G}, \quad \bar{\rho}(t) = \frac{t^{-2} - 3\Lambda^2 t^2 - 6\Lambda}{32\pi G}$$
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• From the corresponding equation of state, $\bar{p}(t) = \bar{w}(t) \bar{\rho}(t)$, it follows

$$\bar{w} = \frac{t^{-2} - 3\Lambda^2 t^2 - 6\Lambda}{3t^{-2} + 3\Lambda^2 t^2 + 2\Lambda} \to \begin{cases} -1, & t \to \infty, \\ \frac{1}{3}, & t \to 0. \end{cases}$$

- The expressions (28) implies that w
 (t) → −1 when t → ∞, what corresponds to an analog of ∧ dark energy dominance in the standard cosmological model.
- and w
 w(t) → 1/3 when t → 0, what corresponds to early times as for the case of radiation.
- From expression for Hubble parameter, (21), follows:
- the first term (¹/_{2l}) is the same as for the radiation dominance in Einstein's gravity, while the second term (^Λ/₂) can be related to the dark energy generated by cosmological constant Λ.

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- At the present cosmic time t₀ = 13.801 · 10⁹ yr and Λ = 0.98 · 10⁻³⁵ s⁻², both terms in (21) are of the same order of magnitude.
- Since, the value for the Hubble parameter, and *H*(*t*₀) = 100.2 km/s/Mpc, is larger than current Planck mission result *H*₀ = 67.40 ± 0.50 km/s/Mpc, this cosmological solution may be of interest for the early universe with radiation dominance and for far-future accelerated expansion.

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- \circledast 2. Cosmological solution for $a(t) = A e^{\Lambda \Gamma}$, k = 0
- For this solution we have

$$\dot{a}(t) = a(t) 2\Lambda t, \qquad \ddot{a}(t) = a(t) 2\Lambda (2\Lambda t^2 + 1)$$
 (28)

and scalar curvature becomes

$$R(t) = 12\Lambda(4\Lambda t^2 + 1). \tag{29}$$

The Hubble parameter

$$H(t) = 2\Lambda t. \tag{30}$$

There is useful equality

$$\Box(R-4\Lambda) = -12\Lambda(R-4\Lambda), \tag{31}$$

which implies

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- 8 2. Cosmological solution for a(t) = A e^{At²}, k = 0
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, $k = 0$

For this solution we have

$$\dot{a}(t) = a(t) 2\Lambda t, \qquad \ddot{a}(t) = a(t) 2\Lambda (2\Lambda t^2 + 1)$$
 (28)

and scalar curvature becomes

$$R(t) = 12\Lambda(4\Lambda t^2 + 1).$$
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• The Hubble parameter

$$H(t) = 2\Lambda t. \tag{30}$$

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Friedman equations give

$$\bar{\rho}(t) = \frac{\Lambda(12\Lambda t^2 - 1)}{8\pi G}, \quad \bar{\rho}(t) = -\frac{3\Lambda(4\Lambda t^2 + 1)}{8\pi G}.$$
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- The solutions $a_1(t) = A\sqrt{t}e^{\frac{\alpha}{2}t^{\alpha}}$ and $a_2(t) = Ae^{At^{\alpha}}$ are not contained in Einstein's gravity with cosmological constant Λ . The solution $a_1(t)$ minics interference between expansion with radiation $a_1(t)$ and a dark energy $a_2(t)$.
- The solution a₂(t) is a nonsingular bounce one and an even function of cosmic time. An exact cosmological solution of the type a(t) = Ae^{αΛt²}, where α ∈ ℝ, appears also at least in the following two models: (1) P(R) = Q(R) = R, and (2) P(R) = Q(R) = √R - 2Λ.
- The nonlocal analytic operator $\mathcal{F}(\Box)$ that takes into account both solutions $a_1(t)$ and $a_2(t)$ have the form $\mathcal{F}(\Box) = a_{\Lambda}^{u} \exp(bu^3 + cu^2 + du)$, where a, b, c, d, are constants and $u = \Box / \Lambda$ is dimensionless operator.
- According to our solutions a(t) = A√1e^{A/t} and a(t) = At⁸e^{A/t}, it follows that effects of the dark radiation (√1), the dark matter (t⁸) and the dark energy (e^{aAt²}) at the cosmic scale can be generated by suitable nonlocal gravity models.

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Let us consider the scale factor

$$a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}, \tag{38}$$

22

and the corresponding eigenvalue problem

$$\Box(R-4\Lambda) = \rho(R-4\Lambda), \tag{39}$$

for some constant *p*.

Solving the eigenvalue problem (39) we found that it is satisfied in the following two cases:

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$$\gamma = 1, p = 2\lambda^2, \Lambda = 3\lambda^2, k \in \{0, -1, 1\}$$

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(ii) for $\Lambda \ge 0$, and scaling factors of the form

$$a_{i}(t) = \left(a_{i}(t)^{2} f^{i}(t) g_{i}(t)^{2} f^{i}(t) g_{i}(t)^{2} f^{i}(t) g_{i}(t)^{2} f^{i}(t)^{2} f^$$

(i2) for $\Lambda \leq 0$, and the trigonometric scaling factors of the form

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It has solutions in the following two cases:

• $\gamma = 1, q = -2\lambda^2, \Lambda = -3\lambda^2, k \in \{0, -1, 1\}$ • $\gamma = \frac{1}{2}, \Lambda = -\frac{3}{4}\lambda^2, k = 0.$

We found that this nonlocal gravity model has the following new cosmological soulutions.

(ii) for A ≥ 0, and scaling factors of the form.

12) for $\Lambda < 0$, and the trigonometric scaling factors of the form

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It has solutions in the following two cases:

We found that this nonlocal gravity model has the following new cosmological soulutions.

(i1) for $\Lambda \geq 0,$ and scaling factors of the form

$$a_{3}(t) = \alpha \, e^{\sqrt{\frac{1}{3}} \wedge t} + \beta e^{-\sqrt{\frac{1}{3}} \wedge t}, \qquad (42)$$

23

$$a_4(t) = \left(\alpha e^{\sqrt{\frac{2}{3}}\Lambda t} + \beta e^{-\sqrt{\frac{2}{3}}\Lambda t}\right)^{\frac{1}{2}},\tag{43}$$

(i2) for $\Lambda \leq 0$, and the trigonometric scaling factors of the form

$$_{5}(t) = \alpha \cos \sqrt{-\frac{1}{3}\Lambda} t + \beta \sin \sqrt{-\frac{1}{3}\Lambda},$$
 (44)

$$a_{6}(t) = A \left(\alpha \cos \sqrt{\frac{2}{3}} \wedge t + \beta \sin \sqrt{\frac{2}{3}} \wedge t \right)^{\frac{1}{2}}.$$
 (45)

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- \circledast 3. Cosmological solution for $a(t) = \alpha e^{\sqrt{\frac{\alpha}{3}}t} + \beta e^{-\sqrt{\frac{\alpha}{3}}t}$
- For this solution we have

$$\dot{a}(t) = \sqrt{\frac{\Lambda}{3}} \left(\alpha e^{\sqrt{\frac{\Lambda}{3}} t} - \beta e^{-\sqrt{\frac{\Lambda}{3}} t} \right), \qquad \ddot{a}(t) = \frac{\Lambda}{3} a(t), \tag{46}$$

$$R(t) = 4\Lambda + (6k - 8\Lambda\alpha\beta) a(t)^{-2}, \qquad (47)$$

$$H(t) = \sqrt{\frac{\Lambda}{3}} \left(1 - 2\beta e^{-\sqrt{\frac{\Lambda}{3}}t} a(t)^{-1} \right), \tag{48}$$

$$R_{00} = -\Lambda, \quad G_{00} = \Lambda + (3k - 4\Lambda\alpha\beta) a(t)^{-2}.$$
 (49)

The corresponding eigenvalue problem has the solutions

$$\Box(R-4\Lambda) = \frac{2}{3}\Lambda(R-4\Lambda), \quad \mathcal{F}(\Box)(R-4\Lambda) = \mathcal{F}(\frac{2}{3}\Lambda)(R-4\Lambda).$$
(50)

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- \circledast 3. Cosmological solution for $a(t) = \alpha e^{\sqrt{\frac{\Lambda}{3}}t} + \beta e^{-\sqrt{\frac{\Lambda}{3}}t}$
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24

$$R(t) = 4\Lambda + (6k - 8\Lambda\alpha\beta) a(t)^{-2}, \qquad (47)$$

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• The corresponding eigenvalue problem has the solutions,

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Equations of motion are satisfied in the following 3 north 3.1 $\alpha\beta = \frac{3k}{4\Lambda}$, 3.2 $\alpha\beta = 0, \mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}, \mathcal{F}'(\frac{2}{3}\Lambda) = \frac{1}{24\Lambda^2}, k \neq 0$, 3.3 $\alpha\beta = -\frac{k}{4\Lambda}, \mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}, \mathcal{F}'(\frac{2}{3}\Lambda) = 0$.

Case 3.1 $\alpha\beta = 0$, $R(t) = 4\Lambda$.

3.1.1 For k = 0 we have $a(t) \sim e^{\pm \sqrt{\frac{3}{2}t}}$, 3.1.2 $\Lambda > 0, k = +1$, gives $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}t}$ 3.1.3 $\Lambda > 0, k = -1$, gives $a(t) = \sqrt{\frac{3}{\Lambda}} \sinh \sqrt{\frac{\Lambda}{3}t}$

Case 3.2 $\alpha = 0$ or $\beta = 0$ and $R(t) = 6ka(t)^{-2} + 4\Lambda$, 3.2.1 For $\alpha = 0$ we have $a(t) = \beta e^{-\sqrt{\frac{2}{3}}t}$, 3.2.2 For $\beta = 0$ we have $a(t) = \alpha e^{\sqrt{\frac{2}{3}}t}$. Case 3.3 $R(t) = 4\Lambda + 8ka(t)^{-2}$, 3.3.1 For k = 1 we have $a(t) = \frac{1}{\sqrt{\lambda}} \sinh(\varphi + \sqrt{\frac{2}{3}}t)$.

3.2.2 For k = -1 we have a(t) = -1

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3.3 $\alpha\beta = -\frac{k}{4\Lambda}, \mathcal{F}(\frac{2}{3}\Lambda) = \frac{1}{12\Lambda}, \mathcal{F}'(\frac{2}{3}\Lambda) = 0$.

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Case 3.2 $\alpha = 0$ or $\beta = 0$ and $R(t) = 6ka(t)^{-2} + 4A$ 3.2.1 For $\alpha = 0$ we have $a(t) = \beta e^{-\sqrt{\frac{\Lambda}{3}}t}$, 3.2.2 For $\beta = 0$ we have $a(t) = \alpha e^{\sqrt{\frac{\Lambda}{3}}t}$.

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Equations of motion are satisfied in the following 3 nontrivial cases:

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$$\bar{\rho} = \frac{3}{8\pi G} (k - \frac{4}{3} \Lambda \alpha \beta) a(t)^{-2}, \quad \bar{\rho} = -\frac{1}{8\pi G} (k - \frac{4}{3} \Lambda \alpha \beta) a(t)^{-2}.$$
(51)

- \circledast For $k
 eqrac{4}{3}\Lambdalphaeta$ the corresponding $ar{w}$ parameter is $ar{w}=-rac{1}{3}$
- 3 4. Cosmological solution for $a(t) = \left(\alpha e^{2\sqrt{\frac{5}{3}t}} + \beta e^{-2\sqrt{\frac{5}{3}t}}\right)^{\frac{3}{2}}$
- From the related eigenvalue problem follows: k = 0 and R = 4Λ.
- The EOM yield the condition

$$\alpha \beta = 0. \tag{52}$$

$$\bar{\rho} = \frac{3}{8\pi G} (k - \frac{4}{3} \Lambda \alpha \beta) a(t)^{-2}, \quad \bar{\rho} = -\frac{1}{8\pi G} (k - \frac{4}{3} \Lambda \alpha \beta) a(t)^{-2}.$$
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- The EOM yield the condition

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- S. Cosmological solution for $a(t) = \alpha \cos \sqrt{-\frac{h}{3}} t + \beta \sin \sqrt{-\frac{h}{3}} t$
- In this case we have

$$\dot{a}(t) = \sqrt{-\frac{\Lambda}{3}} \left(\beta \cos \sqrt{-\frac{\Lambda}{3}} t - \alpha \sin \sqrt{-\frac{\Lambda}{3}} t\right), \quad \ddot{a}(t) = \frac{\Lambda}{3} a(t), \quad (53)$$

$$R(t) = 4\Lambda + 6 \left(k - (\alpha^2 + \beta^2) \frac{\Lambda}{3} a(t)^{-2}\right), \quad (54)$$

$$H(t) = \sqrt{-\frac{\Lambda}{3}} \left(\beta \cos \sqrt{-\frac{\Lambda}{3}} t - \alpha \sin \sqrt{-\frac{\Lambda}{3}} t\right) a(t)^{-1}, \quad (55)$$

$$R_{00} = -\Lambda, \quad G_{00} = 3 \left(k - \frac{\Lambda}{2} \left(\beta \cos \sqrt{-\frac{\Lambda}{2}} t - \alpha \sin \sqrt{-\frac{\Lambda}{2}} t\right)^2\right) a(t)^{-2}.$$

 The corresponding eigenvalue problem has the same solution as in the previous case (50),

$$\Box(R-4\Lambda) = \frac{2}{3}\Lambda(R-4\Lambda), \quad \mathcal{F}(\Box)(R-4\Lambda) = \mathcal{F}(\frac{2}{3}\Lambda)(R-4\Lambda).$$
(56)

$$\circledast$$
 5. Cosmological solution for $a(t) = \alpha \cos \sqrt{-\frac{\Lambda}{3}} t + \beta \sin \sqrt{-\frac{\Lambda}{3}} t$

In this case we have

$$\dot{a}(t) = \sqrt{-\frac{\Lambda}{3}} \left(\beta \cos \sqrt{-\frac{\Lambda}{3}} t - \alpha \sin \sqrt{-\frac{\Lambda}{3}} t\right), \quad \ddot{a}(t) = \frac{\Lambda}{3} a(t), \quad (53)$$

$$R(t) = 4\Lambda + 6\left(k - \left(\alpha^2 + \beta^2\right)\frac{\Lambda}{3}a(t)^{-2}\right),\tag{54}$$

$$H(t) = \sqrt{-\frac{\Lambda}{3}} \left(\beta \cos \sqrt{-\frac{\Lambda}{3}} t - \alpha \sin \sqrt{-\frac{\Lambda}{3}} t\right) a(t)^{-1}, \tag{55}$$

$$R_{00} = -\Lambda, \ G_{00} = 3\left(k - \frac{\Lambda}{3}\left(\beta\cos\sqrt{-\frac{\Lambda}{3}}\ t - \alpha\sin\sqrt{-\frac{\Lambda}{3}}\ t\right)^2\right)a(t)^{-2}.$$

• The corresponding eigenvalue problem has the same solution as in the previous case (50),

$$\Box(R-4\Lambda) = \frac{2}{3}\Lambda(R-4\Lambda), \quad \mathcal{F}(\Box)(R-4\Lambda) = \mathcal{F}(\frac{2}{3}\Lambda)(R-4\Lambda).$$
(56)

* 5. Cosmological solution for
$$a(t) = \alpha \cos \sqrt{-\frac{\Lambda}{3}} t + \beta \sin \sqrt{-\frac{\Lambda}{3}} t$$

In this case we have

$$\dot{a}(t) = \sqrt{-\frac{\Lambda}{3}} \left(\beta \cos \sqrt{-\frac{\Lambda}{3}} t - \alpha \sin \sqrt{-\frac{\Lambda}{3}} t\right), \quad \ddot{a}(t) = \frac{\Lambda}{3} a(t), \quad (53)$$

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- Equations of motion are satisfied in the following two nontrivial cases
 5.1 α² + β² = ^{3k}/_Λ,
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- Case 5.1 In this case we have $R(t) = 4\Lambda$.
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Effective density and pressure are given by:

$$\rho = \frac{3k - h(\alpha^2 + \beta^2)}{8\pi G a(t)^2}, \qquad \bar{\rho} = \frac{h(\alpha^2 + \beta^2) - 3k}{24\pi G a(t)^2}.$$
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- Osmological solution for $a(t) = \left(\alpha \cos \sqrt{-\frac{\hbar}{3}} t + \beta \sin \sqrt{-\frac{\hbar}{3}} t\right)^2$
- In this case, k = 0 and $R = 4\Lambda$.
- From the EOM follows

$$\alpha^2 + \beta^2 = 0. \tag{58}$$

Hence, there are no nontrivial solutions of the form

$$a(t) = \left(\alpha \cos \sqrt{-\frac{\Lambda}{3}} t + \beta \sin \sqrt{-\frac{\Lambda}{3}} t\right)^{\frac{1}{2}}.$$

On new cosmological solutions. In the previous considerations, related to the finding of new cosmological solutions of nonlocal gravity model, in a class of possible scale factors of the form

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Recall that in the de Sitter (anti-de Sitter) case, analogous solutions are:

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$$\text{ (*) } = Ae^{\pm\sqrt{\frac{\Lambda}{3}}t}, \quad R(t) = \frac{6k}{A^2}e^{\pm 2\sqrt{\frac{\Lambda}{3}}t} + 4\Lambda, \quad k = -1, +1, \quad \Lambda > 0,$$

$$\text{ (*) } = \frac{1}{\sqrt{\Lambda}}\cosh(\sqrt{\frac{\Lambda}{3}}t), \quad R(t) = \frac{8k\Lambda}{\cosh^2\sqrt{\frac{\Lambda}{3}}t} + 4\Lambda, \quad k = -1, \quad \Lambda > 0,$$

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- Change of topology. If we compare solutions of de Sitter (anti-de Sitter) and new nonlocal solutions, we can note that for the same cosmological constant Λ, there are analogous scale factors with the same time dependence, but with different curvature constant k.
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Conclusions.

- Four new exact cosmological solutions are obtained,
- Effective energy density and effective pressure are computed for all new solutions,
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$$S = \frac{1}{16\pi G} \int \sqrt{-g} R d^4 x + \frac{1}{8\pi G} \int \sqrt{-g} (-\frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi)) d^4 x.$$
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$$\frac{1}{16\pi G}G_{\mu\nu} + \frac{1}{8\pi G} \left(\frac{1}{4}g_{\mu\nu}\nabla^{\rho}\varphi\nabla_{\rho}\varphi + \frac{1}{2}g_{\mu\nu}V(\varphi) - \frac{1}{2}\nabla_{\mu}\varphi\nabla_{\nu}\varphi\right) = 0.$$
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 \circledast Variation over arphi yields $\Box arphi = V'(\phi)$. The corresponding EOM are:

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u}, \quad \Box \varphi = V'(\varphi).$$
 (61)

Now, we obtain

$$8\pi G\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad 8\pi G\rho = \frac{1}{2}\dot{\varphi}^2 - V(\varphi).$$
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- Sorresponding effective density and pressure for this solution are:

$$\rho = \frac{\Lambda t^2 \left(3\Lambda t^2 + 2\right) + 3}{32\pi G t^2}, \qquad p = \frac{1 - 3\Lambda t^2 \left(\Lambda t^2 + 2\right)}{32\pi G t^2}.$$
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Substituting the previous expressions into (63) we obtain

$$\begin{split} \dot{\varphi}^2 &= \frac{1}{t^2} - \Lambda, \\ \varphi &= \pm l \sqrt{\frac{1}{t^2} - \Lambda} \pm \frac{l \sqrt{\frac{1}{t^2} - \Lambda} \arccos\left(\sqrt{\Lambda t^2 - 1}\right)}{\sqrt{\Lambda t^2 - 1}} + C, \quad (65) \\ V(\varphi) &= \Lambda + \frac{3\Lambda^2 t^2}{4} + \frac{1}{4t^2}. \end{split}$$

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THANK YOU FOR

YOUR ATTENTION !!!

Zoran Rakić New Cosmological Solutions of a Nonlocal Gravity Model

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Non-trivial Christoffel symbols of Friedman – Robertson – Walker metric

$$\Gamma_{01}^{1} = \frac{\dot{a}}{a} \qquad \Gamma_{02}^{2} = \frac{\dot{a}}{a} \qquad \Gamma_{03}^{3} = \frac{\dot{a}}{a}$$

$$\Gamma_{11}^{0} = \frac{a\dot{a}}{1 - kr^{2}} \qquad \Gamma_{11}^{1} = \frac{kr}{1 - kr^{2}} \qquad \Gamma_{12}^{2} = \frac{1}{r}$$

$$\Gamma_{13}^{3} = \frac{1}{r}$$

$$\Gamma_{22}^{0} = r^{2}a\dot{a} \qquad \Gamma_{22}^{1} = r(kr^{2} - 1) \qquad \Gamma_{23}^{3} = \cot\theta$$

$$\Gamma_{33}^{0} = r^{2}a\dot{a}\sin^{2}\theta \qquad \Gamma_{33}^{1} = r(kr^{2} - 1)\sin^{2}\theta \qquad \Gamma_{33}^{2} = -\sin\theta\cos\theta$$

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Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(\dot{a}^2 + k)}{a^2} & 0 & 0 & 0\\ 0 & -v g_{11} & 0 & 0\\ 0 & 0 & -v g_{22} & 0\\ 0 & 0 & 0 & -v g_{33} \end{pmatrix}, \qquad v = \frac{2 \, a \ddot{a} + \dot{a}^2 + k}{a^2}$$

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