CFTs and their integrable deformations defined by screeening charges

Based on M. Alfimov, B. Feigin, B. Hoare and A. Litvinov, JHEP**12**(2020)040
and work in progress with B. Feigin

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Bogolibov Laboratory of Theoretical Physics, Dubna, Russia, 13.02.2025

Motivation

- The integrability-preserving deformations of $O(N)$ sigma models are known to admit the dual description in terms of a coupled theory of bosons and Dirac fermions with exponential interactions of the Toda type (Fateev, Onofri, Zamolodchikov'93, Fateev'04, Litvinov, Spodyneiko'18).
- On the other hand, there are known examples of the integrable superstring theories, such as type IIB $\mathrm{AdS}_5 \times \mathrm{S}^5$ (dual to $\mathcal{N} = 4$ SYM) and others, which also have integrable deformations.
- Our strategic goal is to build a similar dual description for the deformed $\mathrm{AdS}_5 \times \mathrm{S}^5$ type IIB superstring (Arutyunov, Frolov et al.) and, possibly, other theories of this type.
- There are three major things to do on this way:
  1. Incorporate the fermionic degrees of freedom into the construction of dual theory.
  2. Adapt the whole construction to describe the sigma models with non-compact target space.
  3. The superstring theory possesses the reparametrization symmetry and requires gauge fixing, which implies inclusion of this symmetry into the dual description.
- In this talk we are going to address the general scheme to build the dual description of the deformed $O(N)$ and $OSp(N|2m)$ sigma models.

The $OSp(N|2m)$ sigma model S-matrix

- The model with such a symmetry has the following rational S-matrix (Saleur, Wehefritz-Kaufmann'01)

$$\check{S}_{i_1 i_2}^{j_2 j_1}(\theta) = \sigma_1(\theta) E_{i_1 i_2}^{j_2 j_1} + \sigma_2(\theta) P_{i_1 i_2}^{j_2 j_1} + \sigma_3(\theta) I_{i_1 i_2}^{j_2 j_1} \ ,$$

  where

$$\sigma_1(\theta) = -\frac{2i\pi}{(N-2m-2)(i\pi-\theta)}\sigma_2(\theta) \ , \quad \sigma_3(\theta) = -\frac{2i\pi}{(N-2m-2)\theta}\sigma_2(\theta) \ .$$

- Besides rational solution, the Yang-Baxter equation

$$\check{R}_{i_1 i_2}^{k_2 k_1}(\mu) \check{R}_{k_1 i_3}^{k_3 j_1}(\mu+\rho) \check{R}_{k_2 k_3}^{j_3 j_2}(\rho) = \check{R}_{i_2 i_3}^{k_3 k_2}(\mu) \check{R}_{i_1 k_3}^{j_3 k_1}(\mu+\rho) \check{R}_{k_1 k_2}^{j_2 j_1}(\rho)$$

  has the trigonometric solution (Bazhanov, Shadrikov'87) with the parameter q.
- Introducing the parametrization

$$q = e^{2i\pi\lambda} \ , \quad \mu = (N-2m-2)\lambda\theta \ ,$$

  we observe that for $\lambda = 0$ it is consistent with the rational limit and in the special point $\lambda = \frac{1}{2}$ the $\check{R}$-matrix demonstrates an interesting behaviour.
- It becomes proportional to the S-matrix, corresponding to the scattering of the free theory consisting of $\frac{N}{2}$ Dirac fermions and m bosonic particles in the case of even N and the same plus one boson in the case of odd N.

Special point of the $OSp(N|2m)$ R-matrix

- The $O(3)$ example with $N = 3$, $m = 0$ at $\lambda = \frac{1}{2}$:

$$\frac{\check{R}^{j_2 j_1}_{i_1 i_2}}{\check{R}^{22}_{22}} = \begin{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{pmatrix} + \mathcal{O}\left(\lambda - \frac{1}{2}\right)$$

- The $OSp(1|2)$ example with $N = 1$, $m = 1$ at $\lambda = \frac{1}{2}$:

$$\frac{\check{R}^{j_2 j_1}_{i_1 i_2}}{\check{R}^{22}_{22}} = \begin{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{pmatrix} + \mathcal{O}\left(\lambda - \frac{1}{2}\right) .$$

The deformed $O(3)$ dual model

- In the work (Fateev, Onofri, Zamolodchikov'93) there was studied the dual description of the sigma model with the metric ($\lambda = \nu + \mathcal{O}(\nu^2)$)

$$ds^2 = \frac{\kappa}{\nu}\left(\frac{dr^2}{(1-r^2)(1-\kappa^2 r^2)} + \frac{1-r^2}{1-\kappa^2 r^2}d\phi^2\right) .$$

In the other limit $\lambda \to \frac{1}{2}$ the special integrable perturbation of the Sine-Liouville theory ($\lambda = \frac{1}{2} - \frac{b^2}{2} + \mathcal{O}(b^4)$)

$$\mathcal{L} = \frac{(\partial_\mu \Phi)^2}{8\pi} + \frac{(\partial_\mu \varphi)^2}{8\pi} - \\ - \frac{m}{4}\left(e^{b\Phi + i\beta\varphi} + e^{b\Phi - i\beta\varphi} + e^{-b\Phi + i\beta\varphi} + e^{-b\Phi - i\beta\varphi}\right) - \\ - \frac{m^2}{32\pi b^2}\left(e^{2b\Phi} - 2 + e^{-2b\Phi}\right) , \quad \beta = \sqrt{1+b^2} .$$

The sigma model coupling constant in the regime $b \to \infty$ is $\nu = \frac{2}{b^2} + \mathcal{O}\left(\frac{1}{b^4}\right)$.

- Using the Coleman-Mandelstam boson-fermion duality (Coleman'75, Mandelstam'75) $(\partial\varphi)^2/(8\pi) \to i\bar{\psi}\gamma^\mu\partial_\mu\psi$, $e^{\pm i\beta\varphi} \to \bar{\psi}(1 \pm \gamma_5)\psi$, we obtain

$$\mathcal{L} = \frac{(\partial_\mu \Phi)^2}{8\pi} + i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{\pi b^2}{2(1+b^2)}(\bar{\psi}\gamma^\mu\psi)^2 - \\ - m\bar{\psi}\psi\cosh(b\Phi) - \frac{m^2}{8\pi b^2}\sinh^2(b\Phi) .$$

Building of the dual model

Guiding principles to look for the dual description (Litvinov, Spodyneiko'18)

1. The theory has to be renormalizable in the certain sense (Friedan'80). In the case of the deformed $O(N)$ and $OSp(N|2m)$ it can be checked by solving the RG flow equation.

2. The dual theory is found as an integrable perturbation from the special "free" point of the S-matrix and is determined by the set of screening charges, which commute with the integrals of motion in the leading order in the mass parameter

$$\left[I_k^{\text{free}}, \int e^{(\boldsymbol{\alpha}_r,\phi)} dz\right] = 0 \,.$$

3. In the case of the deformed $O(3)$ they are $e^{b\Phi+i\beta\varphi}$, $e^{b\Phi-i\beta\varphi}$, $e^{-b\Phi+i\beta\varphi}$ and $e^{-b\Phi-i\beta\varphi}$, where b is some continuous parameter and $\beta=\sqrt{1+b^2}$. Also, for instance, the two operators $e^{b\Phi+i\beta\varphi}$ and $e^{b\Phi-i\beta\varphi}$ define sine-Liouville CFT, therefore the dual description can be understood as an integrable perturbation of this CFT.

4. Our $O(N)$ and $OSP(N|2m)$ models are integrable deformations of some CFT, based on the cosets

$$\frac{\widehat{\mathfrak{so}}(N)_w}{\widehat{\mathfrak{so}}(N-1)_w} \quad \text{and} \quad \frac{\widehat{\mathfrak{osp}}(N|2m)_w}{\widehat{\mathfrak{osp}}(N-1|2m)_w}\,.$$

   respectively.

CFT's defined by screening charges

- Let $\boldsymbol{\varphi}(z) = (\varphi_1(z), \dots, \varphi_N(z))$ be the $N-$component holomorphic bosonic field normalized as
$$\varphi_i(z)\varphi_j(z') = -\delta_{ij}\log(z-z') + \dots \quad \text{at} \quad z \to z',$$
and $\vec{\boldsymbol{\alpha}} = (\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_N)$ be the set of linear independent vectors.
- We define $W_{\vec{\boldsymbol{\alpha}}}$-algebra as a set of currents $W_s(z)$ of integer spins s such that
$$\oint_{\mathcal{C}_z} e^{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi}(\xi))} W_s(z) d\xi = 0\,, \quad r = 1, \dots, N\,.$$
- For generic $\vec{\alpha}$ there is a spin 2 current
$$W_2(z) = -\frac{1}{2}(\partial\boldsymbol{\varphi}(z)\cdot\partial\boldsymbol{\varphi}(z)) + (\boldsymbol{\rho}\cdot\partial^2\boldsymbol{\varphi}(z))\,, \quad \boldsymbol{\rho} = \sum_{r=1}^{N}\left(1 + \frac{(\boldsymbol{\alpha}_r\cdot\boldsymbol{\alpha}_r)}{2}\right)\hat{\boldsymbol{\alpha}}_r\,,$$
and $(\boldsymbol{\alpha}_r \cdot \hat{\boldsymbol{\alpha}}_s) = \delta_{r,s}$. The corresponding central charge is
$$c = N + 12(\boldsymbol{\rho}\cdot\boldsymbol{\rho})\,.$$
- For $N = 1$ we have a current
$$T(\varphi) = -\frac{1}{2}(\partial\varphi)^2 + \left(\frac{1}{\alpha} + \frac{\alpha}{2}\right)\partial^2\varphi\,.$$
The same algebra can be defined through the dual screening charge $\oint e^{\alpha^\vee \varphi} dz$ with $\alpha^\vee = \frac{2}{\alpha}$.

Bosonic and fermionic roots

- Depiction of bosonic roots

$$\bigcirc \text{ – bosonic root: } (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r) = \text{generic}$$

- If the current W_s satisfies commutativity condition it should be of a special form

$$W_s = W_s\Big(T(\varphi_{\|}), \boldsymbol{\varphi}_{\perp}\Big) \,,$$

where

$$\varphi_{\|} \overset{\text{def}}{=} \frac{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi})}{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)^{\frac{1}{2}}}, \quad \boldsymbol{\varphi}_{\perp} \overset{\text{def}}{=} \boldsymbol{\varphi} - \frac{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\varphi})}{(\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)}\boldsymbol{\alpha}_r \,,$$

and $T(\varphi_{\|})$ is given by $W_2(z)$ with $\alpha = (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r)^{\frac{1}{2}}$.

- Depiction of fermionic roots

$$\otimes \text{ – fermionic root: } (\boldsymbol{\alpha}_r \cdot \boldsymbol{\alpha}_r) = -1$$

- In the coordinates defined above it corresponds to the complex fermion. The communant of the corresponding screening charge $\oint e^{-i\varphi_{\|}(z)}dz$ consists of all $w_s = \psi^{+}\partial^{s-1}\psi$, $s = 2, 3, \ldots$
- Among these currents only w_2 and w_3 are independent. Therefore

$$W_s = W_s\Big(w_2(\varphi_{\|}), w_3(\varphi_{\|}), \boldsymbol{\varphi}_{\perp}\Big).$$

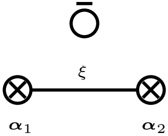
Properties of the systems with bosonic/fermionic roots

- **Bosonic root duality:** the bosonic roots always appear in pairs

$$\boldsymbol{\alpha} \quad \text{and} \quad \boldsymbol{\alpha}^{\vee} = \frac{2\boldsymbol{\alpha}}{(\boldsymbol{\alpha}\cdot\boldsymbol{\alpha})} \ .$$

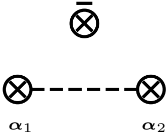
- **Dressed/sigma-model bosonic screening:** $(\boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) = \xi$ is arbitrary

$$\mathcal{S}_B = \oint (\boldsymbol{\alpha}_1 \cdot \partial\boldsymbol{\varphi}) e^{(\boldsymbol{\beta}_{12}\cdot\boldsymbol{\varphi})} dz, \quad \text{where} \quad \boldsymbol{\beta}_{12} = \frac{2(\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2)}{(\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2)^2}$$



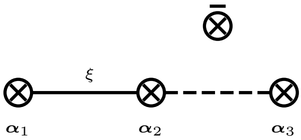
- **Dressed/sigma-model fermionic screening:** $(\boldsymbol{\alpha}_1 \cdot \boldsymbol{\alpha}_2) = -1$

$$\mathcal{S}_F = \oint (\boldsymbol{\alpha}_1 \cdot \partial\boldsymbol{\varphi}) e^{(\boldsymbol{\beta}_{12}\cdot\boldsymbol{\varphi})} dz, \quad \text{where} \quad \boldsymbol{\beta}_{12} = \nu\boldsymbol{\alpha}_1 - (1+\nu)\boldsymbol{\alpha}_2$$



Dressed/sigma-model fermionic screening

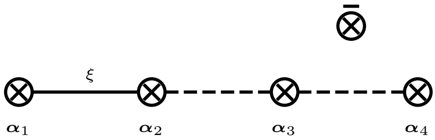
- The parameter ν cannot be fixed if only the two roots α_1 and α_2 are present.
- One way to fix the parameter ν is to embed in larger diagram. For example, consider the diagram



Then the parameter ν in the vector $\boldsymbol{\beta}_{23}$ is fixed from the condition

$$(\boldsymbol{\beta}_{23} \cdot \boldsymbol{\alpha}_1) = -1 \quad \Longrightarrow \quad \nu = -\frac{1}{\xi}.$$

- Another case also important for us is



Then the parameter ν in the vector $\boldsymbol{\beta}_{34}$ is fixed from the condition

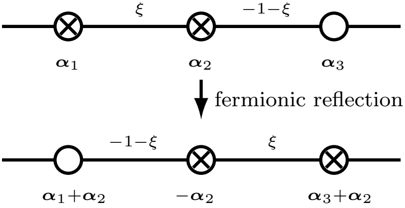
$$(\boldsymbol{\beta}_{34} \cdot \boldsymbol{\alpha}_2) = 1 - \xi \quad \Longrightarrow \quad \nu = \xi - 1.$$

Fermionic reflection

- There is another transformation, which involves given screening and neighbouring ones (Litvinov, Spodyneiko'16).
- This transormation is based on the Coulomb integral identities (Baseilhac, Fateev'99).
- If we have a CFT, defined by a set of screenings $\mathcal{S}_j = \oint e^{(\boldsymbol{\alpha}_j, \varphi(z))} dz$, then the same CFT is defined by a set of screenings $\tilde{\mathcal{S}}_j = \oint e^{(\tilde{\boldsymbol{\alpha}}_j, \varphi(z))} dz$ with

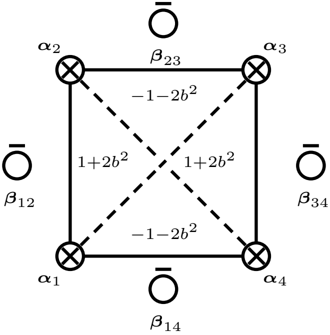
$$\tilde{\boldsymbol{\alpha}}_j = \begin{cases} -\boldsymbol{\alpha}_j & \text{if} \quad j = r\,, \\ \boldsymbol{\alpha}_j + \boldsymbol{\alpha}_r & \text{if} \quad (\boldsymbol{\alpha}_j, \boldsymbol{\alpha}_r) \neq 0\,, \\ \boldsymbol{\alpha}_j & \text{otherwise} \end{cases}$$

  for the fermionic reflection with respect to the screening $\boldsymbol{\alpha}_r$.
- This operation can be illustrated with an example



Deformed $O(3)$ sigma model

- We want to check whether the metric is consistent with the screening charges corresponding to the η- and λ-deformed $O(3)$ sigma model (Fateev et al.'93).
- Let us recall that the theory in question may be determined by the following set of fermionic screenings

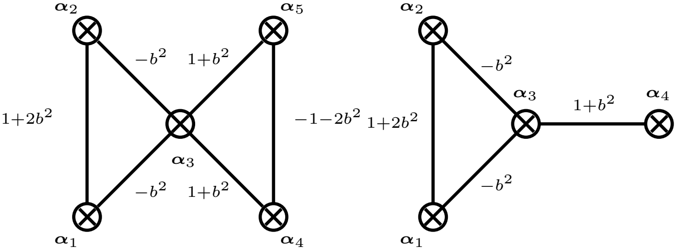


- By utilizing Cartesian coordinates as in (Litvinov, Spodyneiko'18) we can parametrize the fermionic screening lengths as follows

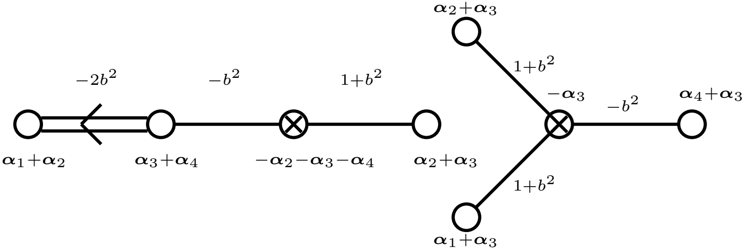
$$\boldsymbol{\alpha}_1 = bE_1 + i\beta e_1 \ , \quad \boldsymbol{\alpha}_2 = bE_1 - i\beta e_1 \ ,$$
$$\boldsymbol{\alpha}_3 = -bE_1 + i\beta e_1 \ , \quad \boldsymbol{\alpha}_4 = -bE_1 - i\beta e_1 \ .$$

Deformed $O(5)$ sigma model

- Screening picture and corresponding underlying CFT $\frac{\widehat{\mathfrak{so}}(5)_{-b^2-3}}{\widehat{\mathfrak{so}}(4)_{-b^2-3}}$ with the central charge $c = 4 + \frac{30}{b^2} - \frac{12}{1+b^2}$ lead to the following diagrams

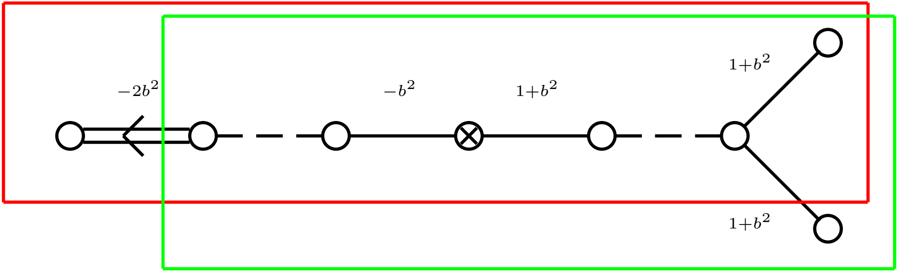


- Different applications of fermionic reflections lead to

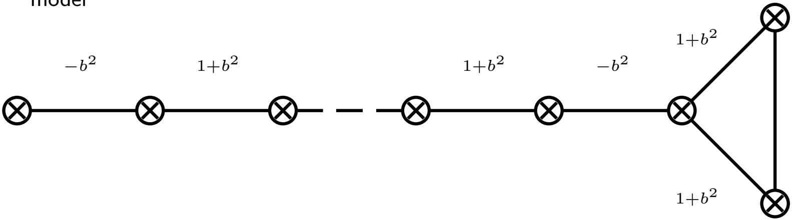


Deformed $O(N)$ model

- Therefore, these two representations above can be encoded in the following picture consisting of N screenings



- Application of fermionic reflections in both cases leads to the CFT, integrable deformation of which leads to the set of screenings describing the $O(N)$ sigma model



The Yang-Baxter deformation of the $OSp(N|2m)$ sigma model

- The action for the Yang-Baxter deformed model is (Klimcik'02,Delduc'13)

$$\mathcal{S}_\eta = \int d^2x\, \mathcal{L}_\eta = -\frac{\eta}{2\nu} \int d^2x\ \mathrm{STr}[J_+ P \frac{1}{1-\eta \mathcal{R}_g P} J_-] \,,$$

where $J_\pm = g^{-1}\partial_\pm g$ takes values in the Grassmann envelope of the Lie superalgebra $\mathfrak{osp}(N|2m;\mathbb{R})$, η is the deformation parameter and ν is the sigma model coupling.

- The operator $\mathcal{R}_g$ is defined in terms of an operator $\mathcal{R} : \mathfrak{g} \to \mathfrak{g}$ through

$$\mathcal{R}_g = \mathrm{Ad}_g^{-1}\, \mathcal{R}\, \mathrm{Ad}_g \,,$$

with $\mathcal{R}$ an antisymmetric solution of the (non-split) modified classical Yang-Baxter equation.

- In terms of coordinates on the target superspace

$$\mathcal{L}_\eta = (G_{MN}(z) + B_{MN}(z))\, \partial_+ z^N \partial_- z^M \,, \quad z^M = (x^\mu, \psi^\alpha) \,,$$

where $G_{MN} = (-1)^{MN} G_{NM}$ and $B_{MN} = -(-1)^{MN} B_{NM}$.

$OSp(N|2m)$ action from $O(N+2m)$ action

- Although the general form of this trick is known to us, for conciseness let us consider the case $N = 2n+1$ and $m = 1$. The simplest way to write the deformed $O(2n+1)/O(2n)$ action is to use "stereographic" coordinates

$$ds^2 = \sum_{k=1}^{n} \frac{\kappa_k}{\nu} \frac{dz_k d\bar{z}_k}{(1+z_k\bar{z}_k)^2 \left(1 - \kappa_k^2 \left(\frac{1-z_k\bar{z}_k}{1+z_k\bar{z}_k}\right)^2\right)} \, ,.$$

where

$$\kappa_k = \kappa \prod_{j=1}^{k-1} \left(\frac{1-z_j\bar{z}_j}{1+z_j\bar{z}_j}\right)^2 \, , \quad k = 1, \ldots, n \, .$$

- The transition to different deformations $OSp(N|2)$ action from the $O(N+2)$ is made by the substitution for some z_k

$$z_k \to \frac{\psi}{\sqrt{2}} = \frac{\psi^1 + i\psi^2}{\sqrt{2}} \, , \quad \bar{z}_k \to \frac{\bar{\psi}}{\sqrt{2}} = \frac{\psi^1 - i\psi^2}{\sqrt{2}} \, .$$

  Further we concentrate on the case $k = 2$.

- Also we go back to the "spherical" parametrization of the coordinates z_j

$$z_j = \sqrt{2\frac{1-r_j}{1+r_j}} e^{i\phi_j} \, .$$

The deformed $OSp(5|2)$ sigma model action

- Let us now turn to the specific case $OSp(5|2)$. The deformed sigma model is parametrised by four bosons, ϕ_1, ϕ_2, r_1 and r_2, and a symplectic fermion, ψ^a, where $a = 1, 2$.

- The Lagrangian following from the previous slide is

$$
\begin{aligned}
\mathcal{L}^{(i)}_\kappa = & \frac{\kappa}{\nu(1-\kappa^2 r_1^2)}\left[\frac{\partial_+ r_1 \partial_- r_1}{1-r_1^2} + (1-r_1^2)\partial_+\phi_1\partial_-\phi_1 + \right. \\
& \left. + i\kappa r_1(\partial_+ r_1 \partial_-\phi_1 - \partial_+\phi_1\partial_- r_1)\right] + \frac{\kappa r_1^2(1-\kappa^2 r_1^4 r_2^2 + (1+\kappa^2 r_1^4 r_2^2)\psi\cdot\psi)}{\nu(1-\kappa^2 r_1^4 r_2^2)^2} \times \\
& \times \left[\frac{\partial_+ r_2 \partial_- r_2}{1-r_2^2} + (1-r_2^2)\partial_+\phi_2\partial_-\phi_2 + i\kappa r_1^2 r_2(1+\psi\cdot\psi)(\partial_+ r_2\partial_-\phi_2 - \partial_+\phi_2\partial_- r_2)\right] - \\
& - \frac{\kappa r_1^2(1-\kappa^2 r_1^4 + \frac{1}{2}(1+\kappa^2 r_1^4)\psi\cdot\psi)}{\nu(1-\kappa^2 r_1^4)^2}\left[\partial_+\psi\cdot\partial_-\psi - i\kappa r_1^2(1+\tfrac{1}{2}\psi\cdot\psi)\partial_+\psi\wedge\partial_-\psi\right] ,
\end{aligned}
$$

  where we have introduced the following contractions of the symplectic fermion

$$
\chi\cdot\chi' = \epsilon_{ab}\chi^a\chi'^b , \quad \chi\wedge\chi' = \delta_{ab}\chi^a\chi'^b .
$$

- The sigma model with action above is 1-loop renormalizable, however, at higher loops it receives corrections.

UV limit of the deformed $OSp(5|2)$ sigma model

- We are interested in the expansion around the UV fixed point, that is $\kappa = 1$. The specific limit we consider (Litvinov'18) is given by first setting

$$r_1 = \exp(-\epsilon e^{-2x_1}) \ , \quad r_2 = \tanh x_2 \ , \quad \psi^a = 2\epsilon^{\frac{1}{2}}\theta^a \ , \quad \kappa = 1 - \frac{\epsilon^2}{2} \ ,$$

  and subsequently expanding around $\epsilon = 0$.

- Introducing the complex fields

$$X_1 = x_1 - i\phi_1 \ , \qquad X_2 = x_2 - i\phi_2 \ , \qquad \Theta = \theta^1 - i\theta^2 \ ,$$

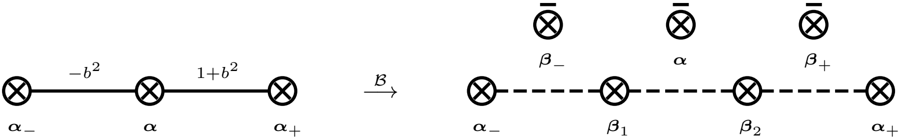
  we find the following expansion

$$\begin{aligned} \mathcal{L}^{(i)}_{\kappa\sim 1} = \frac{1}{\nu}\big(&\partial_+ X_1 \partial_- X_1^* + \partial_+ X_2 \partial_- X_2^* + ie^{2x_1}(1 - ie^{2x_1}\Theta\Theta^*)\partial_+\Theta\partial_-\Theta^*\big) - \\ - \frac{\epsilon}{\nu}\big(&e^{2x_1}\partial_+ X_1 \partial_- X_1^* + e^{-2x_1+2x_2}(1 + 2ie^{2x_1}\Theta\Theta^*)\partial_+ X_2 \partial_- X_2^* \\ &+ e^{-2x_1-2x_2}(1 + 2ie^{2x_1}\Theta\Theta^*)\partial_+ X_2^* \partial_- X_2 + \\ &+ \tfrac{i}{4}e^{4x_1}(1 - 2ie^{2x_1}\Theta\Theta^*)\partial_+\Theta\partial_-\Theta^*\big) + \mathcal{O}(\epsilon^2) \ , \end{aligned}$$

  up to total derivatives.

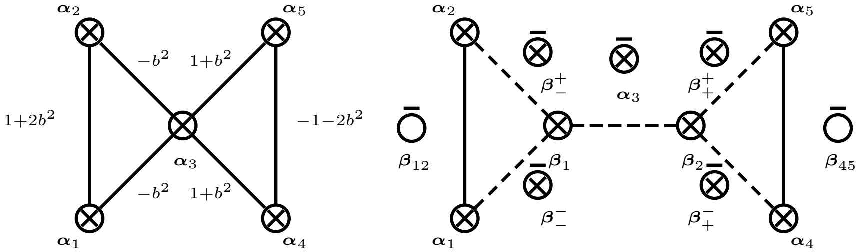
Blow-up transformation

- Now we describe transformation $\mathcal{B}$ of the root system, we call it *blow-up*, which acts as
$$O(N) \to OSP(N|2)\ ,$$
or more generally as
$$OSP(N|2m) \to OSP(N|2m+2)\ .$$
It can be applied to both conformal diagram and its affine counterpart.
- It acts on any root except $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, $\boldsymbol{\alpha}_{2n}$ and $\boldsymbol{\alpha}_{2n+1}$ and produces two fermionic roots out of one. On fermionic root $\boldsymbol{\alpha}$ it acts as follows
$$\boldsymbol{\alpha} = -b\boldsymbol{E} + i\beta\boldsymbol{e} \xrightarrow{\mathcal{B}} \{\boldsymbol{\beta}_1, \boldsymbol{\beta}_2\} = \left\{-\frac{1}{b}\boldsymbol{E} + \frac{i\beta}{b}\boldsymbol{\epsilon}, \frac{ib}{\beta}\boldsymbol{\epsilon} - \frac{i}{\beta}\boldsymbol{e}\right\}\ ,$$
where $\boldsymbol{\epsilon}$ is a new basis vector.
- Altogether this can be shown as follows



Screening charges for the deformed $OSp(5|2)$ sigma model

- Consider the simplest case of $OSP(5|2)$ affine diagram. According to our rule it is obtained from $O(5)$ diagram by blowing up the root $\boldsymbol{\alpha}_3$

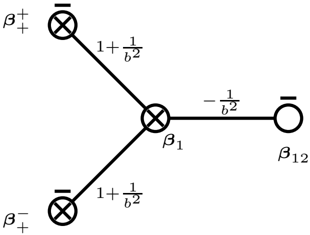


- The vectors $\boldsymbol{\alpha}_r$ can be parameterized as follows ($\beta = \sqrt{1+b^2}$)

$$\boldsymbol{\alpha}_1 = b\boldsymbol{E}_1 + i\beta\boldsymbol{e}_1 \ , \quad \boldsymbol{\alpha}_2 = b\boldsymbol{E}_1 - i\beta\boldsymbol{e}_1 \ , \quad \boldsymbol{\alpha}_3 = -b\boldsymbol{E}_1 + i\beta\boldsymbol{e}_2 \ ,$$
$$\boldsymbol{\alpha}_4 = b\boldsymbol{E}_2 - i\beta\boldsymbol{e}_2 \ , \quad \boldsymbol{\alpha}_5 = -b\boldsymbol{E}_2 - i\beta\boldsymbol{e}_2 \ ,$$
$$\boldsymbol{\beta}_1 = -\frac{1}{b}\boldsymbol{E}_1 + \frac{i\beta}{b}\boldsymbol{\epsilon} \ , \quad \boldsymbol{\beta}_2 = \frac{ib}{\beta}\boldsymbol{\epsilon} - \frac{i}{\beta}\boldsymbol{e}_2 \ , \quad \boldsymbol{\beta}_-^\pm = \pm\frac{i}{\beta}\boldsymbol{e}_1 - \frac{ib}{\beta}\boldsymbol{\epsilon} \ ,$$
$$\boldsymbol{\beta}_+^\pm = \pm\frac{1}{b}\boldsymbol{E}_2 - \frac{i\beta}{b}\boldsymbol{\epsilon} \ , \quad \boldsymbol{\beta}_{12} = \frac{1}{b}\boldsymbol{E}_1 \ , \quad \boldsymbol{\beta}_{45} = \frac{i}{\beta}\boldsymbol{e}_2 \ .$$

Metric for the deformed $OSp(5|2)$ sigma model

- By taking the dual screenings we obtain the following system, which includes the dressed screenings



- By choosing $z = x^1 - ix^2$ ($\bar{z} = x^1 + ix^2$) and then conducting Wick rotation $x^2 = ix^0$, we obtain the action in Minkowski signature

$$
\begin{aligned}
\mathcal{L} = \frac{1}{8\pi}\left(\sum_{i=1}^{2}(\partial_+\Phi_i)(\partial_-\Phi_i) + \sum_{j=1}^{3}(\partial_+\phi_j)(\partial_-\phi_j)\right) + \\
+ \Lambda_1 e^{-\frac{i\beta}{b}\phi_3}\left(\partial_+(b\Phi_2 + i\beta\phi_2)\,\partial_-(b\Phi_2 - i\beta\phi_2)\,e^{-\frac{\Phi_2}{b}} + \right. \\
\left. +\partial_+(b\Phi_2 - i\beta\phi_2)\,\partial_-(b\Phi_2 + i\beta\phi_2)\,e^{\frac{\Phi_2}{b}}\right) + \Lambda_2 e^{-\frac{\Phi_1}{b} + \frac{i\beta}{b}\phi_3} + \\
+ \Lambda_3\partial_+(b\Phi_1 + i\beta\phi_1)\,\partial_-(b\Phi_1 - i\beta\phi_1)\,e^{\frac{\Phi_1}{b}} + \frac{\pi b^2}{\beta^2}\Lambda_1\Lambda_2 e^{\frac{\Phi_1}{b}} \times \\
\times\left(\partial_+(b\Phi_2 + i\beta\phi_2)\,\partial_-(b\Phi_2 - i\beta\phi_2)\,e^{-\frac{\Phi_2}{b}} + \partial_+(b\Phi_2 - i\beta\phi_2)\,\partial_-(b\Phi_2 + i\beta\phi_2)\,e^{\frac{\Phi_2}{b}}\right) + \dots ,
\end{aligned}
$$

Restoring the deformed $OSp(5|2)$ sigma model in the UV limit

- Then we fermionize the ϕ_3 field

$$\mathcal{L} = \frac{1}{8\pi}\left(\sum_{i=1}^{2}(\partial_+\Phi_i)(\partial_-\Phi_i) + \sum_{j=1}^{2}(\partial_+\phi_j)(\partial_-\phi_j)\right) + 2i\Psi_1^\dagger\partial_-\Psi_1 + 2i\Psi_2^\dagger\partial_+\Psi_2 +$$

$$+\frac{2\pi}{\beta^2}\Psi_1^\dagger\Psi_2^\dagger\Psi_2\Psi_1 - i\Lambda_1\Psi_1^\dagger\Psi_2 e^{-\frac{i\beta}{b}\phi_3}\Big(\partial_+\left(b\Phi_2 + i\beta\phi_2\right)\partial_-\left(b\Phi_2 - i\beta\phi_2\right)e^{-\frac{\Phi_2}{b}} +$$

$$+\partial_+\left(b\Phi_2 - i\beta\phi_2\right)\partial_-\left(b\Phi_2 + i\beta\phi_2\right)e^{\frac{\Phi_2}{b}}\Big) - i\Lambda_2\Psi_1\Psi_2^\dagger e^{-\frac{\Phi_1}{b}} +$$

$$+\Lambda_3\partial_+\left(b\Phi_1 + i\beta\phi_1\right)\partial_-\left(b\Phi_1 - i\beta\phi_1\right)e^{\frac{\Phi_1}{b}} + \frac{\pi b^2}{\beta^2}\Lambda_1\Lambda_2 e^{\frac{\Phi_1}{b}}\times$$

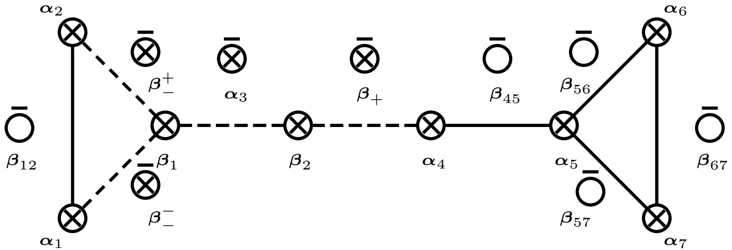
$$\times\Big(\partial_+\left(b\Phi_2 + i\beta\phi_2\right)\partial_-\left(b\Phi_2 - i\beta\phi_2\right)e^{-\frac{\Phi_2}{b}} +$$

$$+\partial_+\left(b\Phi_2 - i\beta\phi_2\right)\partial_-\left(b\Phi_2 + i\beta\phi_2\right)e^{\frac{\Phi_2}{b}}\Big) + \ldots\,,$$

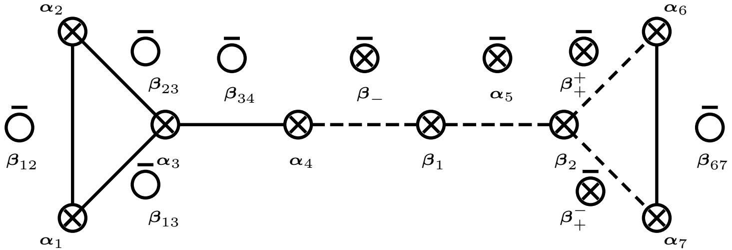
- This after the integrations over the Ψ_1 and $\Psi_2^\dagger$ upon identifying $\Phi_{1,2} = 2bx_{2,1}$, $\phi_{1,2} = 2b\varphi_{2,1}$ and $\Psi_1^\dagger = b\Theta^*$, $\Psi_2 = b\Theta$ together with taking the limit $b \to \infty$ and adjusting properly the coefficients $\Lambda_{1,2,3}$ ($\alpha' = \frac{2}{b^2}$) we obtain dividing by 4 the UV limit originating from the screening picture.

Deformed $OSp(7|2)$ sigma model

- There exist two integrable deformations of $OSp(7|2)$ sigma models, first of them is described by

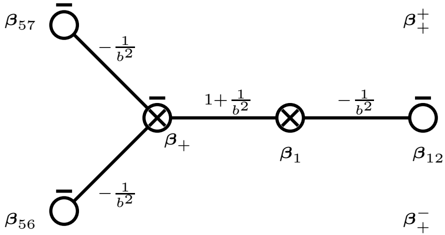


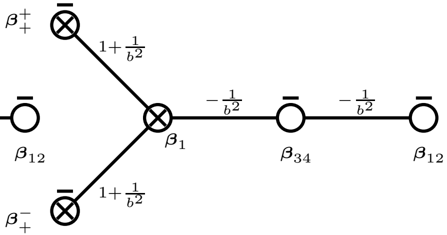
- The second one is described by the screenings



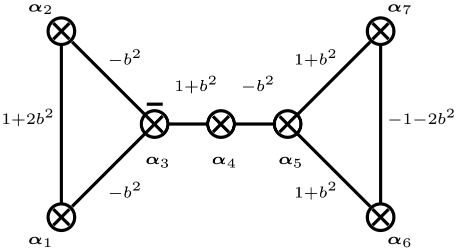
Metric and $b \to 0$ limit for the $OSp(7|2)$ sigma model

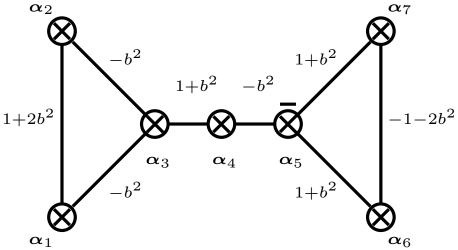
- Metric of the both deformations of $OSp(7|2)$ sigma model





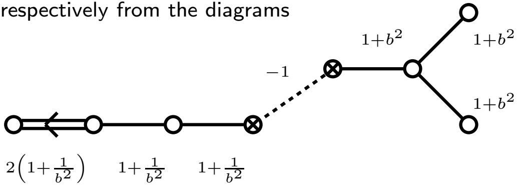
- Respectively in the $b \to 0$ limit we obtain the following screening charges

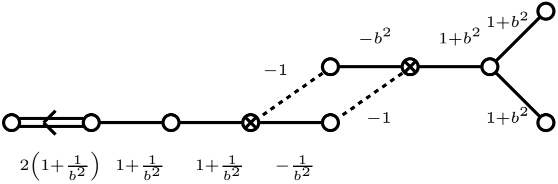




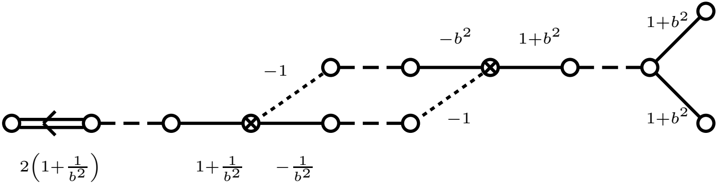
Set of screenings for general $OSp(N|2m)$ sigma model

- In the case of $OSp(7|2)$ and $OSp(7|4)$ we are able to obtain the underlying CFT respectively from the diagrams





- Based on the information above, we can put forward the hypothesis for the structure of the screening for general N and m



Conclusions

Results obtained:

- Presented a systematic way to generate the screening charges picture for deformed $O(N)$ sigma models.
- The system of screening charges, which determines the integrable structure of the $OSp(N|2)$ sigma model, was built.
- By using it we demonstrated how to restore the sigma model action in the deep UV in the cases of $OSp(5|2)$ and $OSp(7|2)$.
- Utilized our system of screenings to write the dual model with the Toda type interactions in the cases of $OSp(7|2)$ and similar ones.
- Put forward a hypothesis on the method to build the set of screening charges for general deformed $OSp(N|2m)$ sigma model.

Future goals:

- Find the system of screening charges for a wider class of integrable sigma models.
- The next interesting step would be to try to adapt the dual description for the sigma models with the non-compact target space (Basso, Zhong'18).
- Include reparametrization invariance into the dual description.

Thanks for your attention!