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Holographic RG flows in gauged supergravity

Problems of Modern Mathematical Physics 2025

Statements and results

- Weak gauge/gravity duality: on-shell (super)gravity action corresponds to the generating functional of connected correlators in a QFT;
- Depending on the type of boundary conditions different QFT's arise;
- A proper renormalization procedure is required;
- Poincare (flat) domain walls describe holographic renormalization flow. There exist two possibilities:
  1. flows started by a deformation of the UV CFT;
  2. flows started by a phase with non-zero VEV;
- For a given model (3D gauge sugra) on a phase diagram of RG flows we
  1. identify flows for various boundary conditions;
  2. find exotic RG flows (non-trivial turning points, multivalued beta-function, cycles);
  3. calculate correlation functions.

Weak form of AdS/CFT

Approximate $Z[J(x)]$ by its saddle point

$$e^{-S_{\text{on-shell}}[\phi]}\Big|_{\phi(0,x)=J(x)} = \left\langle \exp \int_{\partial M} d^d x J(x)\mathcal{O}(x) \right\rangle_{\text{QFT}} .$$

Calculate correlation functions by simple variation

$$\langle \mathcal{O}(x) \rangle_J = \frac{\delta S_{\text{on-shell}}}{\delta \phi(x)},$$
$$\langle T_{ij}(x) \rangle_J = \frac{\delta S_{\text{on-shell}}}{\delta g_{ij}(x)}.$$

Subtleties:

1. On-shell action diverges: a renormalization procedure is needed;
2. The answer depends on boundary conditions;
3. Boundary conditions must be imposed at infinity and not on a finite cut-off.

Scalar field in AdS_{d+1}

Choose the metric

$$ds^2 = dr^2 + e^{-2r}dx^2.$$

Near boundary (at $r \to \infty$) behavior:

$$\phi(z,x) = e^{-\Delta_- r}\phi_-(x) + e^{-\Delta_+ r}\phi_+(x),$$

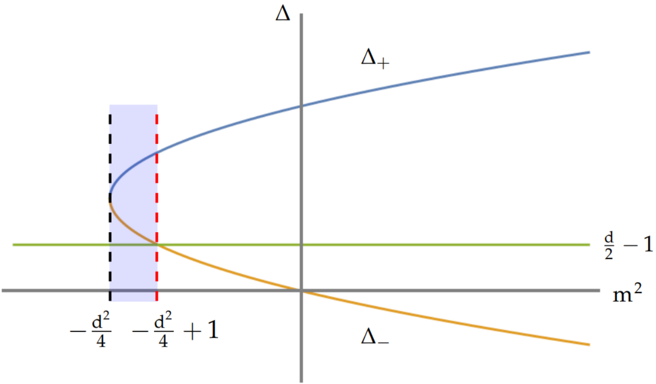
with conformal dimensions

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{m^2 l^2 + \frac{d^2}{4}}.$$

For the normalizable mode $S_{\text{on-shell}}$ is finite and the correspondence reads

$$\begin{aligned} \phi_+(x) &= \left\langle \mathcal{O}_{\Delta_+} \right\rangle_J & &: \text{ correlation function (the VEV);} \\ \phi_-(x) &= J(x) & &: \text{ boundary data (the source).} \end{aligned}$$

Boundary conditions: scaling dimensions



- Equation $\Delta = \Delta(m^2)$ has two branches;
- Unitarity bound for scalars: $\Delta \geqslant \frac{d}{2} - 1$;
- Shaded region: scalar field on AdS describes 2 different theories→ two different quantizations of the scalar field on AdS.

Holographic RG group

(Flat) domain wall solutions describe RG flows that break conformal invariance

$$ds^2 = dr^2 + e^{2A(r)} dx^2.$$

- ✓ Minisuperspace approach: all moduli of the background domain wall metric are frozen except the scale function $A(r)$;
- ✓ Formulate sugra EoM's as first order equations; [Freedman, Gubser, Pilch, Warner (1999)]
- ✓ Use Hamiltonian evolution along r; [Akhmedov(1998), Haro, Skenderis, Solodukhin (2000)]
- ✓ Define boundary conditions in conformally covariant term;
- ✓ Introduce a regulating surface Σ_r at some finite r, renormalize physical quantities (correlators) and then send $r \to \infty$; [Papadimitrou, Skenderis (2004)]
- ✓ Properly subtract counter-terms to preserve SUSY on an RG flow;
- ✓ Calculate correlation functions and central charge using prescriptions.

3D gauged gravity

$\mathcal{N} = 2$ matter coupled AdS_3 gauged supergravity with scalar sector given by the cosets $SU(1,1)/U(1) = \mathbb{H}^2$ or $SU(2)/U(1) = \mathbb{S}^2$:

$$S_0 = \frac{1}{4}\int d^3x\sqrt{-g}\left(R - \frac{1}{a^2}(\partial\phi)^2 - 4V(\phi)\right) + \mathrm{S}_B + S_{\mathsf{GHY}},$$
$$V(\phi) = -2\cosh^2\phi\left[\left(1-2a^2\right)\cosh^2\phi + 2a^2\right]; \quad \textit{hyperbolic}$$
$$V(\phi) = -2\cos^2\phi\left[\left(1+2a^2\right)\cos^2\phi - 2a^2\right]. \quad \textit{trigonometric}$$

[Deger, Kaya, Sezgin, Sundell (2000)]

Domain wall ansatz $ds^2 = dr^2 + e^{2A(r)}dx^2$ makes equations of motion first order:

$$\frac{dA}{dr} = -\frac{1}{2}W(\phi), \quad \frac{d\phi}{dr} = \frac{1}{a^2}\frac{dW(\phi)}{d\phi};$$

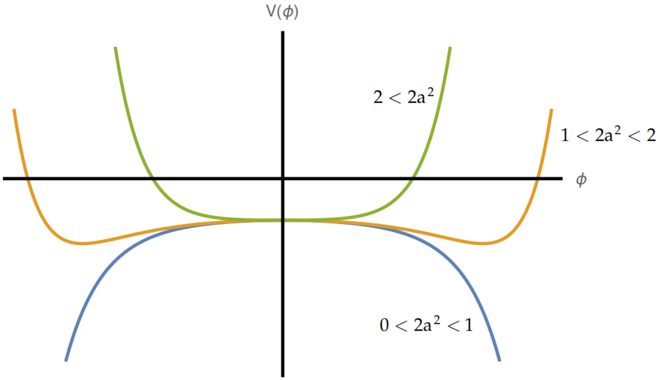
$$V = \frac{a^2}{4}W'^2 - \frac{1}{2}W^2;$$

Here $A(r)$ plays the role of renormalization scale $\longleftrightarrow$ monotonic function of the holographic coordinate r.

Critical points of hyperbolic potential

The potential exhibits different behavior

- $0 < 2a^2 < 1$: one critical point (UV);
- $1 < 2a^2 < 2$: three ctitical points (one UV and two IR);
- $2 < 2a^2$: one critical point (IR).



Near critical point expansion

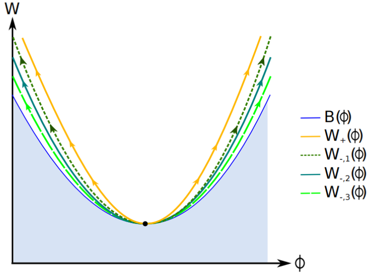
Near a critical point $\phi = \phi_*$ we can expand the potential as

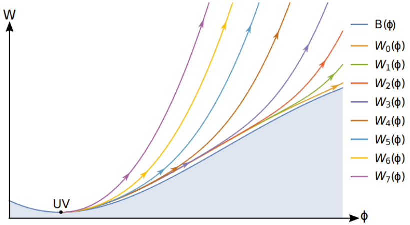
$$V = V_* + \frac{m^2}{2}(\phi - \phi_*)^2 + \mathcal{O}\left(\phi^3\right)$$

and for the two branches of the superpotential we can write

$$W_+(\phi) = \sqrt{-2V_*} + \frac{1}{2a^2}\Delta_+(\phi - \phi_*)^2 + \mathcal{O}\big((\phi - \phi_*)^3\big);$$
$$W_-(\phi) = \sqrt{-2V_*} + \frac{1}{2a^2}\Delta_-(\phi - \phi_*)^2 + C|\phi - \phi_*|^{d/\Delta_-}\left[1 + \mathcal{O}(\phi - \phi_*)\right] + \mathcal{O}\left(C^2\right),$$

where C is an integration constant.





[Kiritsis, Nitti, Silva Pimenta (2018)]

$2a^2 < 1$

One UV critical point $V'(p_1) = 0$ at $\phi = 0$;
Two IR attractors:

- p_4 - saddle;
- p_5 - stable node;

Single W_+ solution (green):

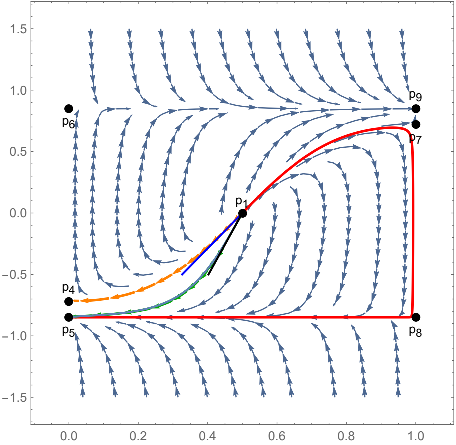
$$\mathbf{D}: \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 2\left(2a^2 - 1\right)\phi_+(x).$$

$$\mathbf{N}: \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

Family of W_- solutions (inside the red contour):

$$\mathbf{D}: \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 0.$$

$$\mathbf{N/M}: \left\langle \mathcal{O}_{\Delta_-} \right\rangle \sim \phi_-(x).$$



Orange: the solution $W_- = -2\cosh^2\phi$

To the right from $Z = p_1$: exotic RG flows → bounce solutions → beta-function is multiple-valued

$1 < 2a^2 < 2$

Three critical points: p_1(UV) and p_2, p_3(IR);
Two IR attractors:

- p_4 - stable node;
- p_5 - saddle;

Single $W_+ = -2\cosh^2\phi$ solution (orange):

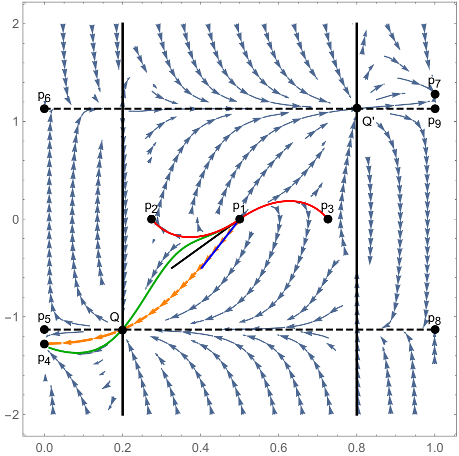
$$\mathbf{D} : \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 2\left(1 - 2\mathrm{a}^2\right)\phi_+(x).$$

$$\mathbf{N} : \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

Family of W_- solutions (green and red):

$$\mathbf{D} : \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 0.$$

$$\mathbf{N/M} : \left\langle \mathcal{O}_{\Delta_-} \right\rangle \sim \phi_-(x).$$



Flows diverge at two $Z = \pm Z_0$ with Q, Q' safe points $\longleftarrow$ metastable pendulum points?

$2a^2 = 1$

One UV critical point at $\phi = 0$, saturates the BF bound;
Two IR attractors:

- $p_4 = p_5$ - stable node;
- q_2 - saddle;

Single $W_+ = -2\cosh^2\phi$ solution (orange):

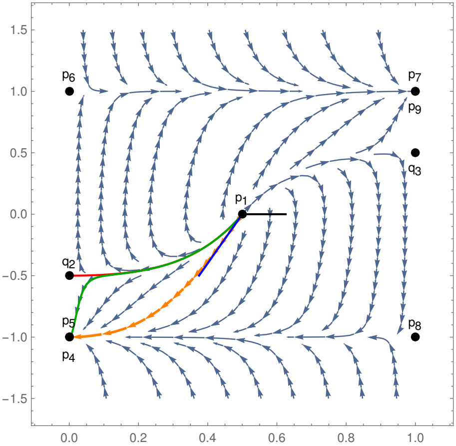
$$\mathbf{D}: \left\langle \mathcal{O}_{\Delta_+} \right\rangle \sim \phi_0(x).$$

$$\mathbf{N}: \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

Family of W_- solutions (log-type) (green and red):

$$\mathbf{D}: \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 0.$$

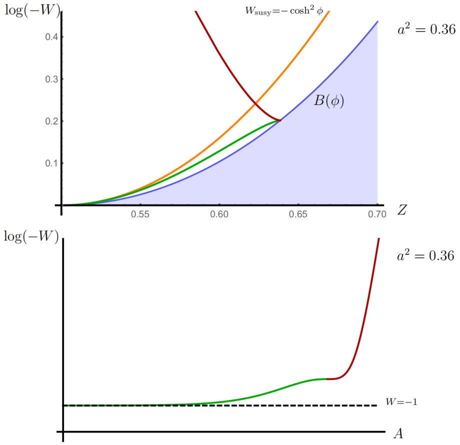
$$\mathbf{N/M}: \left\langle \mathcal{O}_{\Delta_-} \right\rangle \sim \tilde{\phi}_0(x).$$

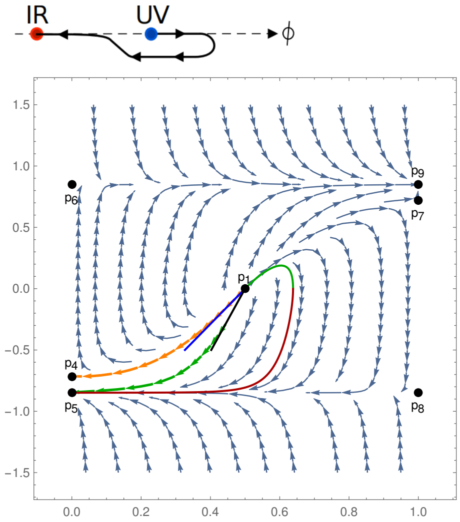


a^2 is a bifurcation parameter, at $2a^2 = 1$ - transcritical bifurcation $\longleftarrow$ phase transition in QFT?

[Bose, Ghosh (2019)]

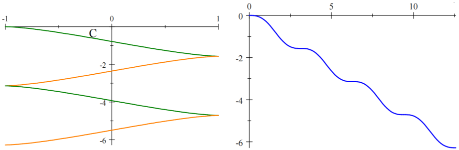
Exotic RG flows: bounces



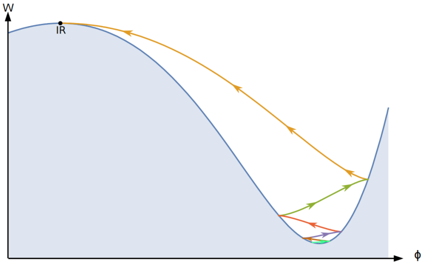


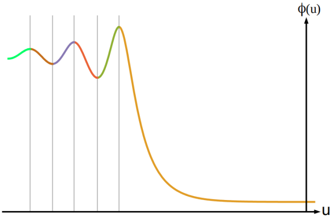
Exotic RG flows: bounces

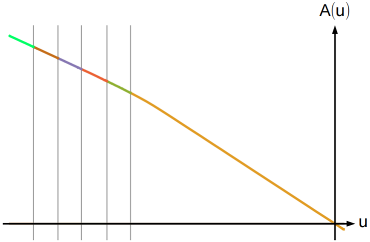
- Cascade solutions $\leftrightarrow$ UV violates the BF bound
- The holographic C-function still monotonic, but multivalued.



[Curtright, Jin, Zachos (2011)]





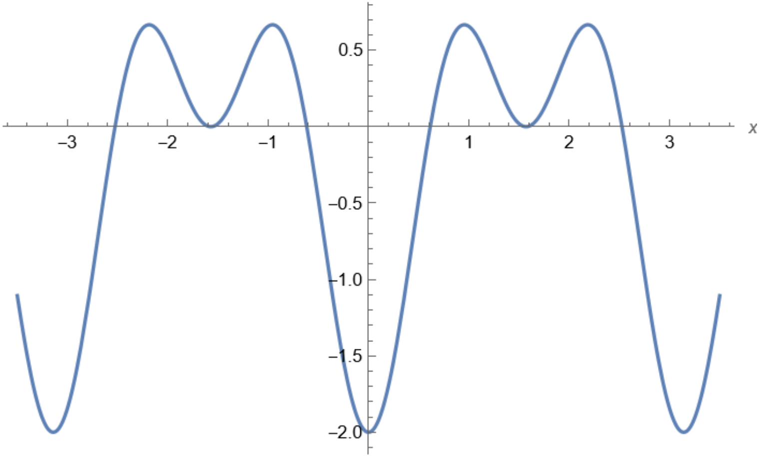


[Kiritsis, Nitti, Silva Pimenta (2018)]

Critical points of trigonometric potential

The potential exhibits seven critical points on the period for all $a > 0$

- Three AdS critical points (IR);
- Four dS ctitical points (IR);
- Two Minkowski critical points (UV).



First order supersymmetric flows

Set of IR critical points at $\phi = \pi n,\ n \in \mathbb{Z}$;
Set of UV attractors at $\phi = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$;
Family of exact supersymmetric Deger solutions

$$A(r) = -\frac{1}{4a^2} \ln[e^{-8ma^2 r} + 1] + \mathrm{c_A}, \quad \mathrm{c_A} \in \mathbb{R};$$

$$\phi(r) = \pm \arctan[e^{4ma^2 r}] + \pi n, \quad n \in \mathbb{Z}.$$

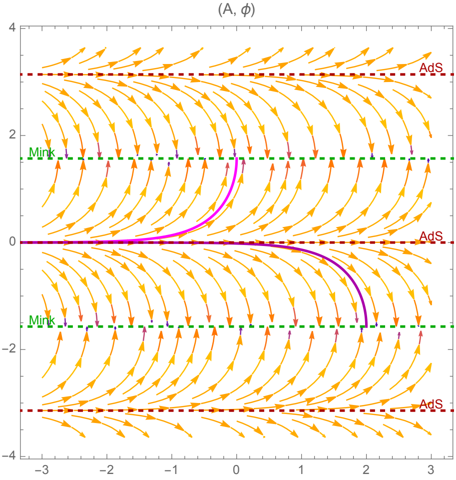
Set of $W_+ = -2\cos^2\phi$ solutions (orange):

$$\mathbf{D} : \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 2\mathrm{a}^2 \phi_+(x).$$

$$\mathbf{N} : \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

For $a = 1$ $\Delta_+ = 4 \longleftarrow T\bar{T}$ deformation and LST

[Giveon, Itzhaki, Kutasov (2020)]



Non-sypersymmetric flows

Set of IR critical points at $\phi = \pi n$, $n \in \mathbb{Z}$;
Set of UV attractors at $\phi = \frac{\pi}{2} + \pi n$, $n \in \mathbb{Z}$;
Set of IR critical points at $\cos^2 \phi = \frac{a^2}{2a^2+1}$;
Set of supersymmetric W_+ solutions (orange):

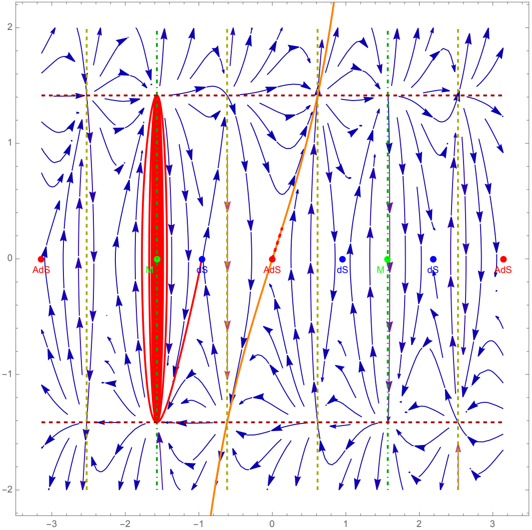
$$\mathbf{D} : \left\langle \mathcal{O}_{\Delta_+} \right\rangle = 2\mathrm{a}^2 \phi_+(x).$$

$$\mathbf{N} : \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

Set of W_+ solutions (settle on closed cycle):

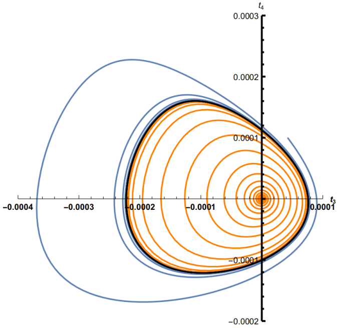
$$\mathbf{D} : \left\langle \mathcal{O}_{\Delta_+} \right\rangle = \sqrt{9 + 8a^2} \phi_+(x).$$

$$\mathbf{N} : \left\langle \mathcal{O}_{\Delta_-} \right\rangle = 0.$$

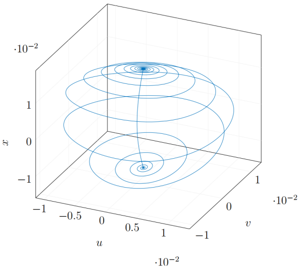


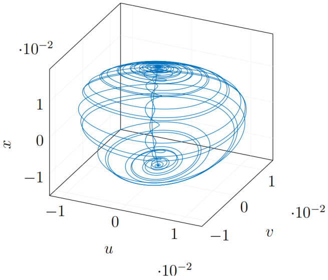
Exotic RG flows: Cycles

- RG flows in $O(N)^2$ and $O(N_1) \times O(N_2)$ tensor models with rational N, N_1, N_2
- N, N_1, N_2 become Hopf bifurcation parameters
- RG cycle as a homoclinic orbit
- Little variations lead to chaotic behaviour
- Price - unitarity



[Jepsen, Klebanov, Popov(2021)]





[Bosschaert, Jepsen, Popov (2022)]

Future work and open questions

- Bouncing RG flows might
  1. signal of additional structure present in the theory: projection of RG flows triggered by multitrace operators, redefefintion of the coupling; [Kiritsis, Nitti, Silva Pimenta (2018)]
  2. be an actual physical property. [Curtright, Jin, Zachos (2011), Bray, Moore (1987)]

- Consider the dS-AdS transitions in case of periodic potential. [Deger, Kaya, Sezgin, Sundell (2000)]

- Gravitational instantons and the vacuum decay - special solutions of marginal operator RG flow. [de Haro, Papadimitrou, Petkou (2007)]

- Apply the analysis to gauged supergravities with known M-theory origin: $\mathcal{N}=4$ SYM $\rightarrow$ $\mathcal{N}=1$ Leigh-Strassler CFT. [Khavaev, Pilch (1998)]

Thanks for attention!

Boundary conditions: correlators

The branches are realized by different boundary conditions [Minces, Rivelles (1999)]

$$\delta S = \int_M \text{EoM's} + \int_{\Sigma_r} \pi_\phi \delta\phi + \delta S_B.$$

1. Dirichlet:

$$\delta\phi|_{\Sigma_r} = 0;$$

$$S^D_{\text{on-shell}} \sim \int d^d x d^d y \frac{\phi_-(x)\phi_-(y)}{|x-y|^{2\Delta_+}}, \quad \phi_-(x) = \lim_{r\to\infty} e^{\Delta_- r}\phi(r,x);$$

$$\left\langle \mathcal{O}_{\Delta_+} \right\rangle^{\text{ren}} = \frac{\delta S_D[\phi_-]}{\delta\phi_-(x)} = \hat{\pi}_+(x). \longleftarrow \text{source for Neumann}$$

2. Neumann:

$$\delta\pi_\phi|_{\Sigma_r} = 0;$$

$$S^N_{\text{on-shell}} \sim \int d^d x d^d y \frac{\hat{\pi}_+(x)\hat{\pi}_+(y)}{|x-y|^{2\Delta_-}}, \quad \hat{\pi}_+(x) = \lim_{r\to\infty} e^{\Delta_+ r}\pi_\phi(r,x);$$

$$\left\langle \mathcal{O}_{\Delta_-} \right\rangle^{\text{ren}} = \frac{\delta S_N[\hat{\pi}_+]}{\delta\hat{\pi}_+(x)} = \phi_-(x). \longleftarrow \text{source for Dirichlet}$$

3. For mixed b.c. $(\delta\pi_\phi + f''(\phi)\delta\phi)|_{\Sigma_r} = 0$ the correlation functions are the same as for Neumann case.

Dynamical system approach

Rewrite the first order equations as the autonomous system:

$$Z = \left(1 + e^{\phi}\right)^{-1}, \quad X = \frac{\dot{\phi}(r)}{\dot{A}(r)}.$$

[Aref'eva, Golubtsova, Polikastro (2018)]

Supergravity (scalar) equations for the domain wall anzats become

$$\frac{dZ}{dA} = XZ(Z-1);$$
$$\frac{dX}{dA} = \left(\frac{X^2}{a^2} - 2\right)\left(X + \frac{a^2}{2}\frac{V'(\phi)}{V(\phi)}\right).$$

Near the critical point $(Z_0, X_0) \equiv (X_0^1, X_0^2)$:

$$\frac{d}{dA} X^i = \mathcal{M}^i_j \left(X^j - X^j_0\right),$$
$$X^i = X^i_0 + e^{\lambda_1 A} u^i_1 + e^{\lambda_2 A} u^i_2.$$

The eigenvalues (λ_1, λ_2) correspond to $(-\Delta_+, -\Delta_-)$.

General procedure

- ✔ Solve the equation

$$4V = a^2 W'^2 - 2W^2;$$

- ✔ Solve 1st order equations

$$\frac{dA}{dr} = -\frac{1}{2} W(\phi), \quad \frac{d\phi}{dr} = \frac{1}{a^2} \frac{dW(\phi)}{d\phi};$$

- ✔ Calculate renormalized effective action and correlations functions

$$S_{\text{eff,ren}} = \int_{\Sigma_r} d^d x \sqrt{\gamma} \big(W(\phi) - U(\phi)\big);$$

[Papadimitrou, Skenderis (2004)]

- ✗ The procedure stops at the first step: only one solution is known

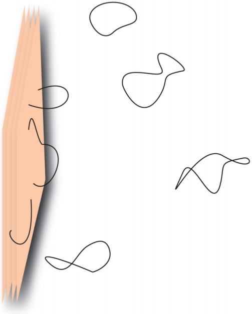
$$W = -2\cosh^2\phi.$$

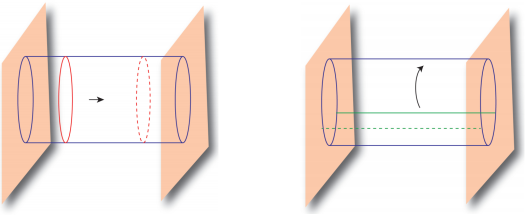
[Deger, Kaya, Sezgin, Sundell (2000)]

Gauge/gravity duality

$$\int\limits_{\Phi(z=0,x)} D\Phi e^{-S_{\text{bulk}}[\Phi]} = \int D\varphi e^{-S[\varphi]+\int\limits_{\partial M} d^d x J(x) O(x)}$$

A well understood example: AdS/CFT correspondence. [Maldacena (1997)]





Closed string tree level ⟷ open string loop

N Dp-branes can be described equivalently by open or closed strings:

- Closed, strong $g_s N \gg 1$: a supergravity background, AdS near the brane;
- Open, weak $g_s N \ll 1$: a gauge theory on the brane.