Partially massless fields and supersymmetry

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Outlook

1. Poincare algebra
2. de Sitter algebra
3. Supermultiplets

Unitary representations $d = 4$

$$C_2 = P^2 = m^2, \qquad C_4 = W^2$$

- massive $m^2 > 0$, $G_{little} = SO(3)$
  - $C_4 = m^2 s(s+1)$, helicities: $0(\pm\frac{1}{2}) \cdots \pm s$
- massless $m^2 = 0$, $G_{little} = E_2$
  - $C_4 = 0$, helicities: $\pm\lambda$
  - $C_4 = \mu^2$, helicities: $0(\pm\frac{1}{2}) \dots \infty$
- tachyonic $m^2 < 0$, $G_{little} = SO(1,2)$
  - $C_4 = 0$, helicity 0
  - $C_4 = m^2 s(s+1)$, helicities: $\pm s \dots \infty$

Note: massless infinite spin representation can be obtained from the massive one in the limit

$$m^2 \to 0, \qquad s \to \infty, \qquad ms = \mu = const$$

Gauge invariant formalism

- Lagrangian

$$\mathcal{L}_0 = \sum_{k=0}^{\infty} [D\Phi^k D\Phi^k + a_k \Phi^{k+1} D\Phi^k + b_k \Phi^k \Phi^k]$$

$$[a_k] \sim m, \qquad [b_k] \sim m^2$$

- Gauge transformations

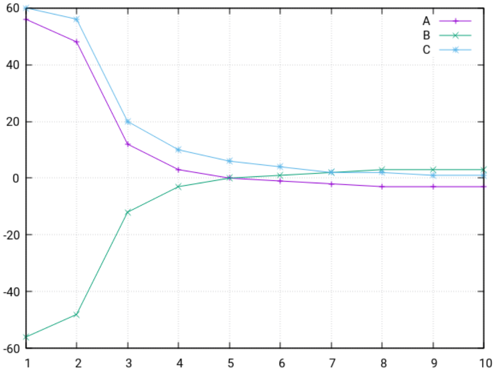
$$\delta\Phi^k = D\xi^k + \alpha_k \xi^{k+1} + \beta_k \xi^{k-1}$$

- General solution

$$a_k{}^2 = \frac{k}{(k-2)} a_2{}^2 - \frac{(k+1)}{(k-1)} b_2, \qquad b_k = \frac{b_2}{k(k^2-1)}$$

- For arbitrary a_2, b_2 we have some representation!

Illustration



A massive finite spin

B massless infinite spin

C tachyonic infinite spin

From Minkowski space to $(A)dS_4$

- The same approach works for $(A)dS_4$:

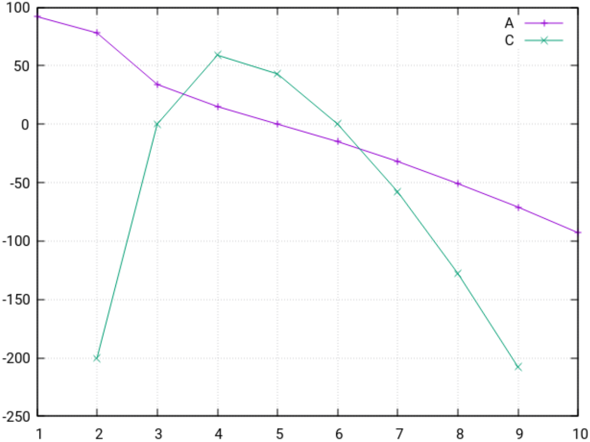
$$[D, D] \sim \Lambda, \qquad g^{\mu\nu} \neq \eta^{\mu\nu}, \qquad Dg^{\mu\nu} = 0$$

- What is mass in AdS?
- General solution

$$a_k{}^2 = \frac{k}{(k-2)} a_2{}^2 - \frac{(k+1)}{(k-1)} b_2 - 4k(k+1)\Lambda$$

- No infinite spin representations for $\Lambda > 0$!

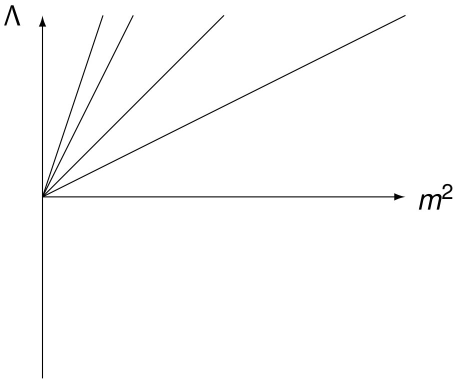
Illustration



A massive finite spin

B partially massless (and massless!)

Unitary forbidden region $m^2 < s(s-1)\Lambda$



Note: for spin 3/2 gravitino unitary forbidden region $m^2 < \Lambda$

Applications

- Partially massless (bi)gravity
  - There exist cubic vertices for PM_2 self-interaction and its interaction with massless gravity
  - All attempts to go beyond the cubic level failed
  - There are several ”no-go” results
- Conformal gravity in $(A)dS$
  - At the quadratic level on $(A)dS_4$ background conformal spin 2 is equivalent to combination of massless and partially massless spin 2 fields
  - Similar relations for all higher spin conformal fields
- Extensions of Vasiliev theory?
  - There exist candidates on the role of infinite dimensional algebra that could describe a theory with massless and partially massless fields

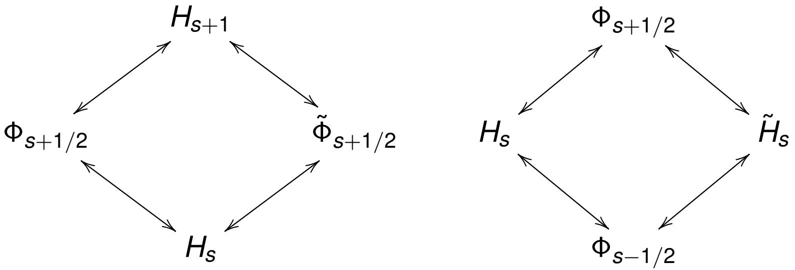
From CFT_3 to AdS_4

Garcia-Saenz, Hinterbichler, Rosen (2018)
"Supersymmetric partially massless fields and non-unitary superconformal representations"

- The new class of non-unitary irreducible representations of three dimensional superconformal group were constructed
- By AdS_4/CFT_3 correspondence this implies the existence of supermultiplets constructed out of partially massless fields
- Partially massless spin 2 supermultiplet was constructed as the first explicit example

From massive to partially massless

Buchbinder, Khabarov, Snegirev, Z (2019)



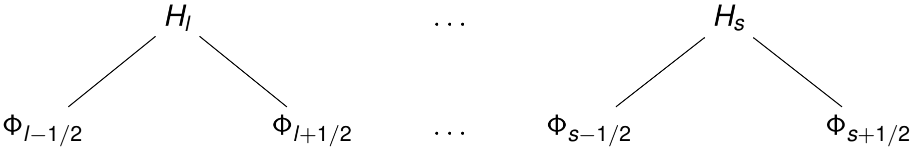
In AdS_4 bosonic and fermionic masses must satisfy

$$M_b{}^2 = M_f(M_f \pm \lambda), \qquad \Lambda = -\lambda^2$$

$$M_B{}^2 = m^2 + s(s-1)\lambda^2, \qquad M_f{}^2 = m^2 + (s - \frac{1}{2})^2\lambda^2$$

Helicities adjustment

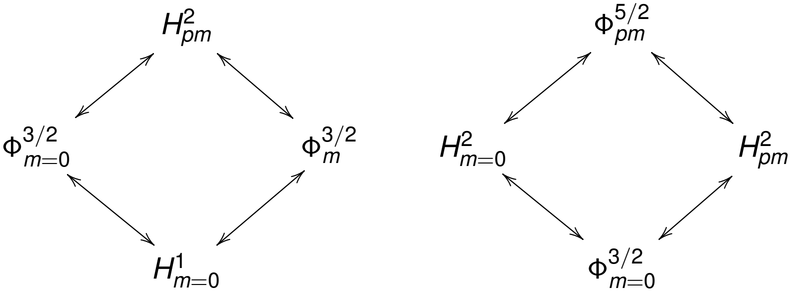
However, we need not only correct masses for all members of supermultiplet, but also a proper adjustment of helicities



Buchbinder, Khabarov, Snegirev, Z (2019a)

Massless plus partially massless

- There exists an interesting subclass of supermultiplets



- PM_2 supermultiplet was recently investigated in two different approaches
  Z 2412.04982 and Boulanger, Lhost, Thomee 2412.17713
- Possible connection with $N = 1$ conformal supergravity

Self-interaction: simple example

- We have considered a possible interaction of partially massless spin 2 ans massive spin 3/2.
- Note that both fields are unitary in de Sitter space.
- There exists a cubic vertex that has no more than one derivative and resembles very much similar vertex in spontaneouslly broken supergravity.
- The possibility to go beyond the cubic level is an open question.

Final remarks

- For me, the main reason for interest in this subject is the fact that in many aspects the partially massless fields occupy an intermediate position between massless and massive fields. So their investigation could gives us a useful experience on our way to massive higher spin fields.
- The very existence of partially massless supermultiplets is also one more argument in favour.
- Possible connection with conformal (super)gravity and in general with conformal higher spin fields is also interesting.