Local polarimetry with π^0 in SPD at NICA

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Abstract. The Spin Physics Detector (SPD) will be installed in the second interaction point of the Nuclotron-based Ion Collider fAcility (NICA) at the Joint Institute for Nuclear Research in Dubna. The main goal is to study the spin structure of the proton and deuteron, and other spin-related phenomena with polarized proton and deuteron beams at a collision energy up to $\sqrt{s} = 27$ GeV and luminosity up to 10^{32} cm^{-2}s^{-1}. For local polarimetry and luminosity control in SPD, several detectors are proposed. This work presents an analysis of the possibilities of using the inclusive $p + p \to \pi^0 + X$ reaction, in the end-caps of the electromagnetic calorimeter (ECAL) for local polarimetry purposes. The accuracy of the azimuthal asymmetry of this reaction, as a measure of the beam polarization uncertainty, is investigated with Monte Carlo simulations in the frame of the SpdRoot code.

1. Introduction

The main objective of SPD is to investigate polarized phenomena in order to disentangle crucial issues of the nucleon spin physics. In this context, the polarimetry plays an important role. It is necessary to have a good monitoring of polarization and luminosity, trying to make the number of ions that are polarized in the needed direction as large as possible. At the same time, spin-dependent physical observables have to be extracted from the spin asymmetry measurements, which in turn, should be correctly scaled according to the degree of the beam polarization.

Measurement and monitory systems in NICA are planned to provide precise, relative and absolute determination of the polarization degree of the beams. However the major polarimetry information provided by the NICA facility needs to be cross-checked locally in each detector experiment. The local online monitoring of the beam polarization, with independence of the major polarimeters, should help to reduce the systematic errors coming from polarization variations.

In SPD, the transverse single-spin azimuthal asymmetry (TSSA) can be exploited to measure the degree to which the beam polarization is transverse (vertically or radially) or longitudinal. The main challenge of the local polarimetry at SPD is the lack of data from pp collisions in the energy range of a few MeV's up to $\sqrt{s} = 27$ GeV ($\sqrt{s}$ is the center-of-mass energy). Several detectors are suggested to participate in the local polarimetry. For example, the Beam-Beam

Counters (BBC) which are intended to measure the azimuthal asymmetry of inclusive charge particles, and the Zero Degree Calorimeter (ZDC) that will measure very forward neutrons. In addition, the use of the inclusive $p+p \to \pi^0 + X$ reaction, where π^0s are detected in the end-caps of the ECAL, is suggested in the conceptual design of SPD [1]. The later is investigated in this work.

The transverse asymmetry (A_{N}) is the ratio of the difference to the sum, of the spin dependent cross sections with opposite transverse polarizations. In this case, only one of two colliding protons is transversely polarized:

$$A_{\mathrm{N}}(p^{\uparrow} + p \to \pi^0 + X) = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}} \tag{1}$$

The A_{N} dependence of hadron cross-sections in $p^{\uparrow}p$ ($\overline{p}^{\uparrow}p$) reactions in the energy regime where perturbative QCD (pQCD) is applicable, is expected to be negligibly small in the lowest-order QCD approximation. According the leading-twist pQCD expectations the A_{N} should be suppressed at the partonic level as $\frac{\alpha_{\mathrm{s}} m_{\mathrm{q}}}{\sqrt{s}}$, where α_{s} is the coupling constant of QCD and m_{q} is the quark mass. Hence, spin asymmetries cannot be successfully described within a simple collinear and leading-twist parton model [2]. However, several experiments have shown non-negligible spin-dependent asymmetries of pions produced via transversely polarized proton beams. Such is the case of the sizeable asymmetry values observed in inclusive charge [3, 4] and neutral pions [5–8] produced in $p^{\uparrow}p$ ($\overline{p}^{\uparrow}p$) reactions, mainly at large x-Feynman, $x_{\mathrm{F}} = 2p_{\mathrm{L}}/\sqrt{s}$, where p_{L} is the momentum of the pion along the beam direction. This apparent contradiction with pQCD calculations has motivated new theoretical works along with additional experimental studies in order to interpret the non-zero asymmetries in hadron reactions where partonic QCD descriptions are more relevant, thus clarifying the transverse spin structure of the proton. Different, but mutually supportive approaches have been suggested in order to account for the experimental values of spin asymmetry, i.e. the Sivers effect [9], the Collins effect [10], the collinear twist-3 formalism [11], and sort of combination of them.

Results of the transverse single spin asymmetries π^0, π^+ and π^-, obtained by the collaborations E704 and E581, are shown on the left panel of figure 1. Data exhibit large A_{N} values for $x_{\mathrm{F}} \geq 0.3$. Their signs follow the polarization of the valence quarks fragmenting into the pions. The A_{N} of π^0 is almost half of that for charge pions, however, the advantage for π^0 is that it can be selected in a relative easy way through the invariant mass of the two-photon decay, not requiring the track reconstruction. This becomes a convenient option for polarimetry purposes.

The right panel of figure 1 shows the result of A_{N} vs. x_{F} in transversely polarized proton-proton collisions at forward rapidities in a wide energy range. The asymmetries are nearly independent of the collision energy. The p_{T} cut applied in those measurements was 0.5, 1.0, 1.2 and 2 GeV/c for $\sqrt{s} = 19.4$, 62.4, 200 and 500 GeV, respectively.

Unpolarized Monte Carlo simulation was used to estimate the expected statistical accuracy of the beam polarization. On this basis, the asymmetry can not be estimated. However, we can evaluate the statistical uncertainties from the π^0 yields of our simulation and combine them with the A_{N} values measured by the E704 Collaboration at Fermilab [5] as it is the closest to the SPD energy range. This enables us to estimate the accuracy of the beam polarization measurements for any expected amount of data.

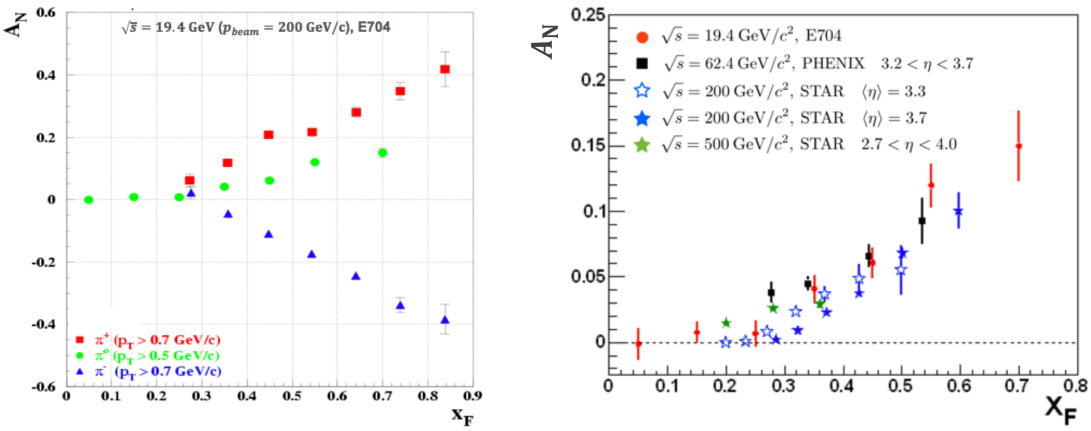

Figure 1. Transverse-single spin asymmetry $A_{\rm N}$ versus $x_{\rm F}$ for inclusive pion production, using 200 GeV ($\sqrt{s}$ = 19.4 GeV) polarized proton beams, $p^{\uparrow\downarrow}p \to \pi^{\pm,0}$ [5] (left), and for $A_{\rm N}(\pi^0)$ at different collision energies [5, 7, 12, 13] (right).

2. Monte Carlo simulations

The Monte Carlo simulations of inelastic pp interactions, were performed using the offline software of the SPD experiment, SpdRoot version 4.1.6, which stems from the FairRoot software. The simulation step makes use of Geant4 tools [14] to transport the particles through the detector geometry. The multipurpose generator Pythia 8 [15], was used to produce $\sim 10^8$ pp collisions at $\sqrt{s}$ = 27 GeV. This amount of generated events corresponds to the number of collisions produced in $\sim 47s$ ($\sigma^{pp}_{27\rm GeV}$ = 40 mb, $L = 10^{32}$ cm^{-2}s^{-1}). The generation was configured for minimum bias by activating all the soft QCD processes, except for the elastic topologies which in principle are excluded by the inelastic process.

The local polarimetry is an online procedure that requires fast reconstruction of the particle under consideration, which means that the information on the vertex position along the beam axis is unknown. Collision events were generated assuming Gaussian vertex distributions with σ_z = 30 cm and $\sigma_{x,y}$ = 0.1 cm [16]. However, due to the lack of precise vertex position, nominal collision center $(0, 0, 0)$ is assumed to obtain the momentum direction.

Photon candidates are reconstructed from energy deposits (clusters) in the ECAL, in an effort to address a more realistic scenario. Clusters belonging to the ECAL end-caps are selected with E^{γ}_{min} = 400 MeV in order to filter out some background. This is a rather conservative criterion based on the measurement of the passage of vertical muons through the SPD-ECAL allowing to estimate the mean of the MIP (minimum ionizing particle) signal to be 240 MeV, with an energy resolution of 9.6% [1]. The two-photon decays are required to have $p_{\rm T} > 0.5$ GeV/c.

According to the current geometrical parameters of the ECAL, implemented in SpdRoot, the active part of the ECAL end-caps, covers the region of intermediate and near-forward pseudorapydity, $1.3 < |\eta| < 3.8$.

Two analysis methods are proposed in this work to extract the statistical uncertainty of the asymmetry. The first one is based on the azimuthal cosine modulation of the TSSA($A_{\rm N}$) which is deduced from the signal of the invariant mass distribution. The second method uses the raw π^0 yield in discrete azimuthal bins and makes the necessary corrections for the background. For both, π^0 candidates are reconstructed from the two-photon invariant mass spectra that are fitted with the combination of a normalized Gaussian signal and a 2^{nd} degree polynomial background. The first method is based on the π^0 yields obtained by integrating the background–subtracted peak within 3σ from the mean value of the π^0 position. The second method, based on the raw π^0 yield, includes all counts under the normal Gaussian plus polynomial distribution within a defined mass windows. The π^0 yield error is the error on the integral of the fitting function, based on the parameter uncertainties and the covariance matrix from the fit.

3. The estimate of the relative error $\frac{\sigma_P}{P}$ of the beam polarization

The $A_{\rm N}$ is defined as the ratio $\frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$ of the difference to the sum of cross sections with opposite transverse polarizations (up$^{\uparrow}$/down$^{\downarrow}$) of the colliding particles. This transverse asymmetry has to do with the spin of only one beam that is polarized along an axis perpendicular to the momentum. The azimuthally dependent cross section to first order, can be written as,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 (1 + P \cdot A_{\rm N} \cos(\phi)), \tag{2}$$

where $(\frac{d\sigma}{d\Omega})_0$ is the average cross section, which is essentially unpolarized cross section. P is the vertical beam polarization and $A_{\rm N}$ is the transverse asymmetry [12, 17, 18]. The azimuthally dependent cross section relies on the spin dependence of π^0 yield ($N_{\pi^0}(\phi)$) in presence of polarized beams. A simplified approach of the $A_{\rm N}$ dependence on the azimuth of yields can be represented through the cosine function:

$$N_{\pi^0}(\phi) = a \cdot (1 + b \cdot \cos(\phi + \phi_0)), \tag{3}$$

where the term $b = P \cdot A_{\rm N}$ is the amplitude of azimuthal angular modulation of the cross section of the outgoing scattered particles with respect to the transverse spin direction of the polarized proton. The free phase ($\phi_0 \neq 0$) in the equation 3 is a way to check for deviations of the beam polarization from the vertical direction, while in real situations it might be also added to account for the relative luminosity effects that could deviate from $R = L^{\uparrow}/L^{\downarrow} = 1$, being R the relative luminosity between the polarized crossings having spin up and spin down, respectively.

3.1. Method 1: Estimate of $\frac{\sigma_P}{P}$ from the cosine modulation amplitude

Each ECAL end-cap covers 2π in azimuth, with a hole for the beam pipe in the center. In this analysis, each end-cap was divided in 8 azimuthal sectors (ϕ bins). For each ϕ bin, counts N_{π^0} were determined in 6 $x_{\rm F}$ intervals assuming Poisson distribution of N_{π^0}, so that the statistical uncertainty of yields obeys $\sigma_{\rm N} = \sqrt{N}$. The invariant mass distributions of $\gamma\gamma$ pairs for six $x_{\rm F}$ intervals in one of the eight ϕ bins ($90^\circ - 135^\circ$) is shown in figure 2. The fit of the normalized gaussian signal after the background subtraction is represented with a blue line, while the 2^{nd} degree polynomial fitted background is represented with green line. In the interval $x_{\rm F} = [0.5, 0.6]$ there is not enough π^0 signal allowing us to make a proper fit, as illustrated in the sixth panel of the figure 2.

Based on Eq. 3, the N_{π^0} yields for each $x_{\rm F}$ interval were plotted as function of ϕ and fitted with a cosine function with parameters a_0, a_1 and a_2:

$$f(x) = a_0 \cdot (1 + a_1 \cdot cos(a_2 + x)) \tag{4}$$

The cosine fits of the simulated yields for five $x_{\rm F}$ intervals, at $\sqrt{s} = 27$ GeV, are shown in figure 3. It must be noted that the lower π^0 yields correspond to the higher $x_{\rm F}$ values.

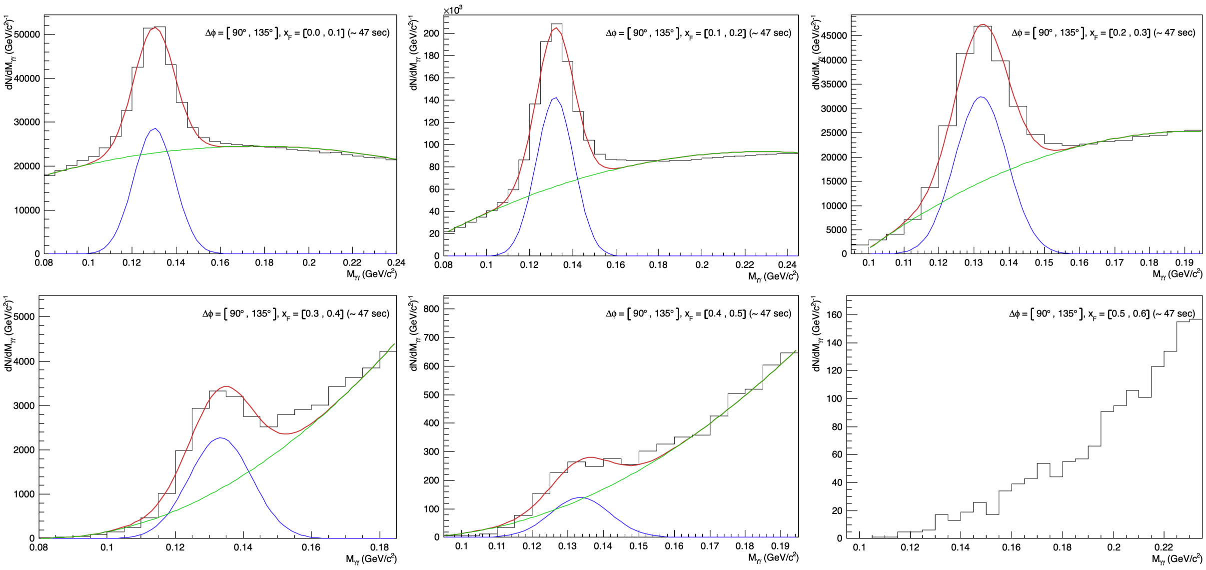

Figure 2. Invariant mass of photon pairs, for 6 x_{F} intervals in the azimuthal bin $\Delta\phi = [90^{\circ}, 135^{\circ}]$ in the ECAL end-cap at $z > 0$. The photon pair transverse momentum is $p_{\mathrm{T}} > 0.5$.

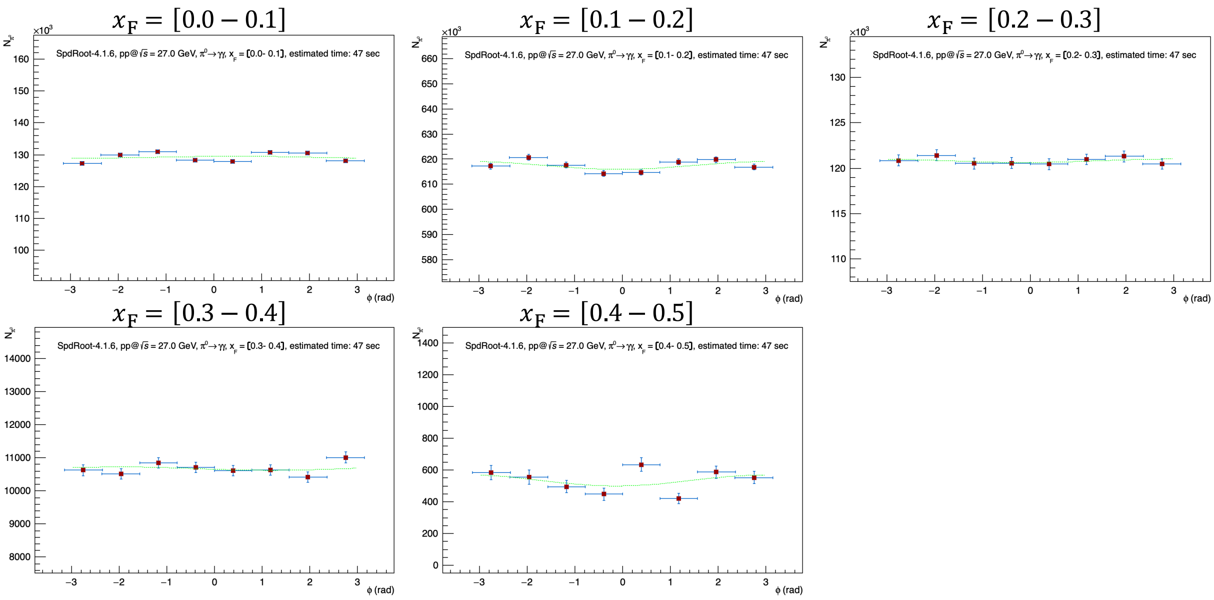

Figure 3. Cosine modulation fittings of $N_{\pi^0}(\phi)$ in x_{F} intervals at $\sqrt{s} = 27$ GeV.

This method relies on estimating the accuracy of the polarization measurement, based on the statistical uncertainty of the fitting parameter corresponding to the azimuthal modulation amplitude a_1 (Eq. 4). This parameter contains the product of polarization P and asymmetry A_{N}:

$$a_1 = P \cdot A_{\mathrm{N}} \tag{5}$$

As can be seen in figure 3 the π^0 yields for each x_{F} range, are almost uniformly distributed in ϕ bins. In such circumstances, the proposed analysis method relies on the cosine fit of a basically flat distribution. In order to assess the reliability of this analysis strategy, a simple toy model was created where two distributions were generated within the same limits $(-\pi, \pi)$, one is uniform ($y = 1$) and the other one outlines a more realistic scenario with a cosine distribution, $1 + [0] \cdot cos(x)$. Both distributions were fitted with the same cosine modulation function, $f(x) = p_0 \cdot (1 + p_1 \cdot cos(x + p_2))$. The uncertainty of the modulation coefficient, p_1, was extracted from both fits and finally the two case studies were compared.

In the left panel of figure 4 the uncertainties of the modulation coefficients, obtained from the described case studies were plotted for ten modulation sizes ranging from 0.5 to 0.8. The uncertainties extracted from the fit of the flat distribution (red circles) remain constant, while the uncertainties from the cosine modulation case (black circles) gradually decline with the increasing of the modulation size. The ratio of both uncertainties is depicted in the right panel of figure 4. It can be observed that the uncertainties extracted from fitting the flat distribution slightly exceeds the cosine modulated one. This excess represents 2.5% at a modulation size equal to 0.2. This suggests that σ_{A_N} can be estimated with this method in a reasonably precise way, if we bear in mind that the expected A_N values for π^0 does not exceed 0.2 over the x_F range that concerns us, as can be seen in figure 1.

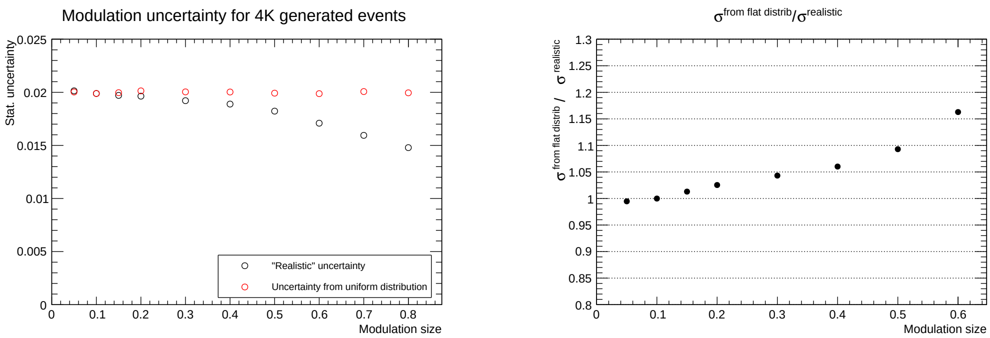

Figure 4. Simple toy model to study the reliability of extracting the statistical uncertainty of the modulation coefficient from the cosine fit of a uniform distribution. Left: uncertainty of the modulation coefficient in both cases (details on the text) vs. modulation size. Right: ratio of both uncertainties vs. modulation size.

In order to predict the uncertainty of the polarization P for each x_F bin, the terms a_1 and A_N in equation 5 are assumed to be uncorrelated variables, so that its statistical uncertainties can be propagated as follows,

$$\left(\frac{\sigma_P}{P}\right)_\mathrm{i} = \sqrt{\left(\frac{\sigma_{a_1}}{a_1}\right)^2_\mathrm{i} + \left(\frac{\sigma_{A_\mathrm{N}}}{A_\mathrm{N}}\right)^2_\mathrm{i}} \tag{6}$$

where i is the individual x_F bin. Since we can not measure A_N in non-polarized MC simulations, we propose to evaluate $\frac{\sigma_{A_\mathrm{N}}}{A_\mathrm{N}}$ with the asymmetry (A_N) and its the respective uncertainty (σ_{A_N}) value reported in the experiment E704 [5] for the same x_F values of this study. The term σ_{a_1} is the statistical uncertainty of the fit parameter a_1 obtained in this analysis. In addition, a_1 stems from the product of the polarization that, in our particular case was assumed $P = 0.7$ and the asymmetry that, considering the worst scenario can be taken as $A_\mathrm{N} = 0.15$ (see Fig. 1). In consequence, $a_1 = P \cdot A_\mathrm{N} = 0.105$.

Positive values of $A_{\rm N}$ are expected for $x_{\rm F} > 0.3$ (Fig. 1). This fact, makes the last two values of $(\frac{\sigma_P}{P})_{\rm i}$, in $x_{\rm F}$ bins $[0.3-0.4]$ and $[0.4-0.5]$, of particular relevance to this analysis, since it is known that errors in asymmetry must be dominated by uncertainties of the polarization measurements, in particular for high statistics. At the same time, one of the largest uncertainties in a polarimeter comes from the asymmetry calibration.

With the two points of $(\frac{\sigma_P}{P})_{\rm i}$, at $x_{\rm F} = [0.3-0.4]$ and $[0.4-0.5]$ respectively, and considering that the beam polarization does not depend on $x_{\rm F}$, the relative error of the polarization can be finally estimated as follows,

$$\frac{\sigma_P}{P} = \frac{1}{\sqrt{\sum_{i=1}^{2} \frac{1}{(\frac{\sigma_P}{P})_{\rm i}}}} \tag{7}$$

where i means the i^{th} $x_{\rm F}$ bin. The result of evaluating the equation 13 was scaled to different expected times of data taking, i. e. 2 min., 5 min., 10 min., 20 min., 30 min. and 1 hour. It was assumed that the beam intensity and the beam polarization remain stable over the proposed periods of times.

3.2. Method 2: Calculation of $\sigma_{A_{\rm N}}$ after background correction

In experiments where typical asymmetry measurements take place, the measured quantity is the raw asymmetry $A_{\rm N}^{raw}$. Its statistical uncertainty must be corrected for dilution due to the background under the π^0 peak in the invariant mass spectrum, so that the $A_{\rm N}^{sig}$ can be determined. The background asymmetry for this correction is usually based on the asymmetry measured in a combined mass region placed $\pm 3\sigma$ away from the signal peak position, which is then statistically subtracted from the $A_{\rm N}^{raw}$ taking into account the ratio $r = N^{bkg}/N^{raw}$. This, results in the signal asymmetry $A_{\rm N}^{sig}$ as follows,

$$A_{\rm N}^{sig}(\phi) = \frac{A_{\rm N}^{raw} - r \cdot A_{\rm N}^{bkg}(\phi)}{1-r}. \tag{8}$$

N^{bkg} represents, in each ϕ bins, the counts under the invariant mass regions selected to make the background correction, while N^{raw} is the total of counts under the peak at the nominal π^0 mass.

The following equation outlines the way in which the asymmetry is obtained for both, $A_{\rm N}^{raw}$ and $A_{\rm N}^{bkg}$, assuming $N^{\uparrow} \sim N^{\downarrow} = N$, $R \sim 1$:

$$\frac{1}{P}\frac{N^{\uparrow}(\phi) - RN^{\downarrow}(\phi)}{N^{\uparrow}(\phi) + RN^{\downarrow}(\phi)} = A_{\rm N}\cos(\phi), \tag{9}$$

If we consider Poisson distribution of counts, that is $\sigma_N = \sqrt{N}$, the statistical uncertainty of raw and background asymmetry in each ϕ bin can be written in a simplified way, as follows,

$$\sigma_{A_{\rm N}}(\phi) = \frac{1}{P\langle|cos(\phi)|\rangle}\frac{1}{\sqrt{2N}} \tag{10}$$

The term $\langle|cos(\phi)|\rangle = \frac{\int_{\phi_1}^{\phi_2} cos(\phi) d\phi}{\phi_2 - \phi_1}$ in equation 10 is the average of the cosine of azimuth in the ϕ bin, while $\frac{1}{\langle|cos(\phi)|\rangle}$ fulfils the roll of azimuthal acceptance correction factor.

Finally the statistical uncertainty on the π^0 asymmetry after subtraction of background asymmetry is given by equation 11.

$$\sigma_{A_{\mathrm{N}}^{sig}}(\phi) = \frac{\sqrt{\sigma^2_{A_{\mathrm{N}}^{raw}}(\phi) + r^2 \cdot \sigma^2_{A_{\mathrm{N}}^{bkg}}(\phi)}}{1 - r} \tag{11}$$

Under this approach, the yield of π^0 candidates in our simulation was determined for each x_F bin in each of the eight azimuthal sectors, following the same reasoning explained for the $Method$ 1 in 3.1, but in this case rather than extracting the $N_{\pi^0}^{sig}$ by subtracting the background, $N_{\pi^0}^{raw}$ was obtained by counting all photon pair candidates under the peak. The invariant mass distributions, in our case, does not allow to select two mass background regions at both sides of the peak in order to perform background corrections. For this reason, it was selected the background region defined by the 2^{nd} degree polynomial fit under the same mass limits that were used for N^{raw}. So that, $N^{raw} = N^{sig} + N^{bkg}$, and $r = N^{bkg}/N^{raw}$. Figure 5 illustrates how the regions for obtaining N^{raw} and N^{bkg} were separately defined for five x_{F} intervals in the azimuthal bin $[90° - 135°]$.

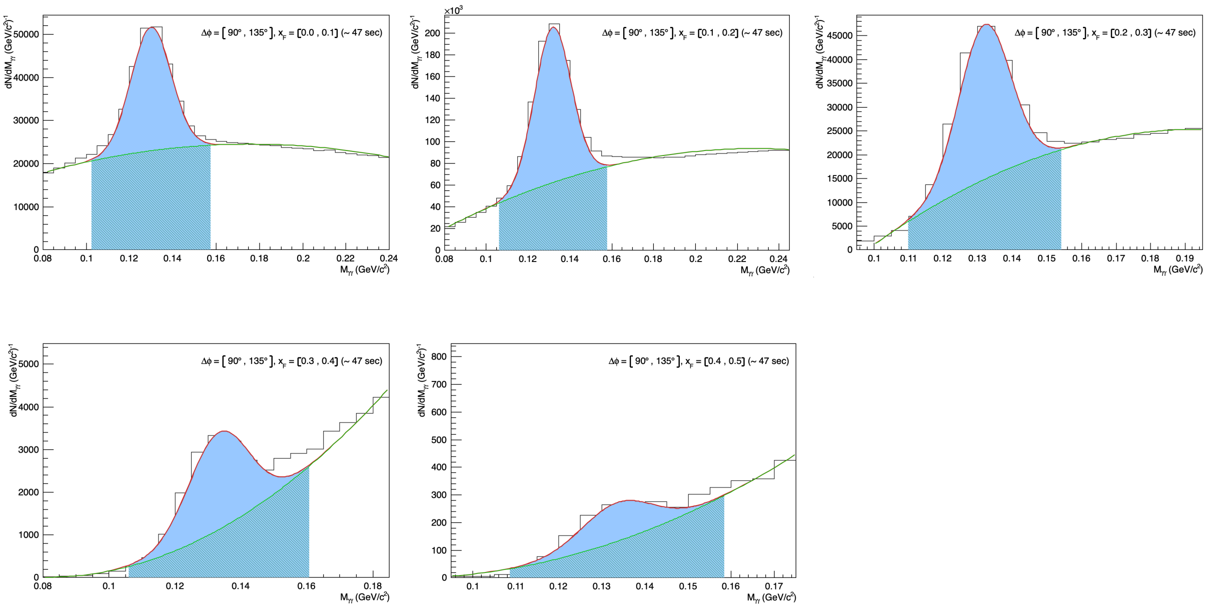

Figure 5. Invariant mass of photon pairs, for 5 x_{F} intervals in the azimuthal bin $\Delta\phi = [90°, 135°]$ in the ECAL end-cap at $z > 0$. Illustration of the mass regions were N^{raw} (light blue shadow) and N^{bkg} (green hatched lines) were obtained.

Since counts N^{raw}, N^{bkg} $vs.$ x_{F}, are distributed uniformly in 8 ϕ bins, raw and background statistical uncertainties, $\sigma_{A_{\mathrm{N}}^{raw}}$ and $\sigma_{A_{\mathrm{N}}^{bkg}}$, were estimated for each ϕ bin. Then, the statistical uncertainty of the π^0 signal asymmetry was calculated according the equation 11 for each ϕ bin. The eight resulting $\sigma_{A_{\mathrm{N}}^{sig}}(\phi)$, were statistically combined to estimate the uncertainty of the π^0

signal asymmetry as function of x_F:

$$\sigma_{A_\mathrm{N}^{sig}}(x_\mathrm{F}) = \frac{1}{\sqrt{\sum_{i=1}^{8} \frac{1}{\sigma^2_{A_\mathrm{N}^{sig}}(\phi_\mathrm{i})}}} \tag{12}$$

The x_F dependence of $\sigma_{A_\mathrm{N}^{sig}}$ based on this method is shown in figure 6. The result of 100 million events, shown in red, is equivalent to a data collection period of 47 seconds, while the statistical uncertainty of the asymmetry expected to be measured in 5 minutes is shown in light blue. These values of $\sigma^{sig}_{A_\mathrm{N}}$ make it possible to compare this estimation with the results that came out from the experimental data of asymmetry for $\sqrt{s} = 19.4$ GeV, reported by the collaboration E704 in Fermilab [5].

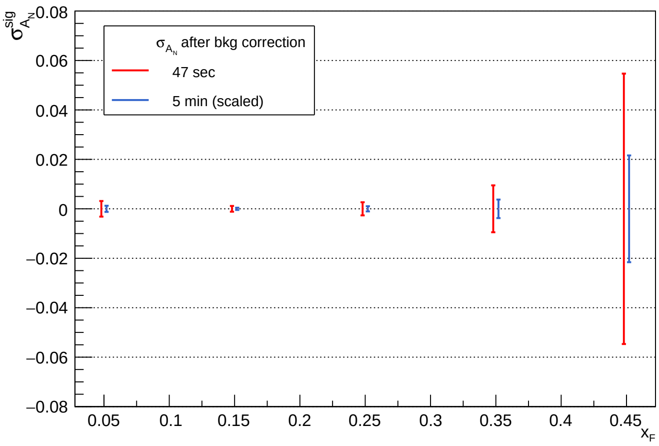

Figure 6. Statistical uncertainty of A^{sig}_N for inclusive π^0 in simulated *pp* collisions at $\sqrt{s} = 27$ GeV, after background correction of the raw asymmetry.

The relative errors $\frac{\sigma_{A_\mathrm{N}}}{A_\mathrm{N}}$ of the asymmetry have been calculated as the ratio of the statistical uncertainties $\sigma^{sig}_{A_\mathrm{N}}$ obtained in this simulation to the experimental values of A^{exp}_N reported in [5]. This renders feasible to have an estimate of the accuracy that can be expected in local polarimetry using the inclusive π^0 detected in the end-caps of the ECAL in SPD. In the figure 7 it is shown $\sigma^{sig}_{A_{\mathrm{N_i}}}/A^{exp}_{\mathrm{N_i}}$ as obtained in this simulation (equivalent to 47 seconds), compared with its projection for 5 minutes of data taking and with the experimental relative errors $\sigma^{exp}_{A_{\mathrm{N_i}}}/A^{exp}_{\mathrm{N_i}}$ extracted from [5].

Since the experiment E704 obtained asymmetries $A_\mathrm{N} > 0$ for $x_\mathrm{F} > 0.3$ (Fig. 1), we must rely on only two $\sigma^{sig}_{A_{\mathrm{N_i}}}/A^{exp}_{\mathrm{N_i}}$ values to proceed with the analysis, the smallest of which is at $0.3 < x_\mathrm{F} < 0.4$ as shown in figure 7. Roughly speaking, at $0.3 < x_\mathrm{F} < 0.4$ it might be expected a better accuracy of the asymmetry for inclusive π^0 detected in the end-caps of the ECAL in SPD.

Positive values of A_N from the E704 experiment are linear rising (Fig. 8). Bearing in mind the proportionality $\frac{\Delta P}{P} \sim \frac{\Delta A_\mathrm{N}}{A_\mathrm{N}}$, and assuming that the beam polarization does not depend on x_F,

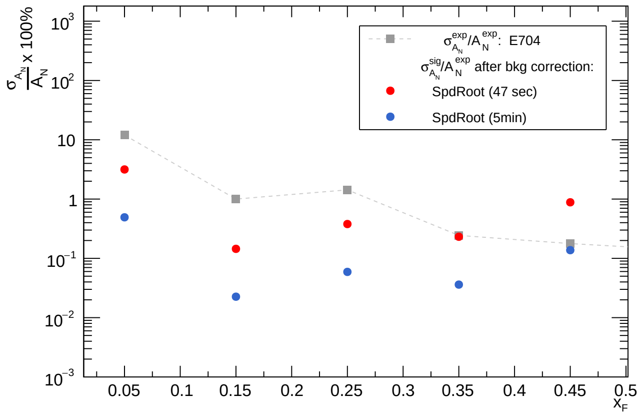

Figure 7. Relative error of $A_{\rm N}$ of the MC simulations with SpdRoot compared with data from E704. Dashed line is used only to guide the eyes.

the relative error of the polarization can be estimated, as follows,

$$\frac{\sigma_{\rm P}}{P} = \frac{1}{\sqrt{\sum_{i=1}^{n} \frac{1}{(\frac{\sigma_{A_{\rm N}}}{A_{\rm N}})_i^2}}}, \tag{13}$$

where i^{th} is the $x_{\rm F}$ bin and n is the number of values of $A_{\rm N} > 0$. The result of evaluating equation 13 with the three experimental values $\sigma^{exp}_{A_{\rm N_i}}/A^{exp}_{\rm N_i}$ in the range $x_{\rm F} = [0.3 - 0.6]$ shown in figure 8, is 9.9% accuracy of the beam polarization, which is very close to $\sim$10% reported in the E704 publication in 1991 [5]. However, as it has been mentioned above, in this simulation there are only two points at $x_{\rm F} > 0.3$ where the π^0 yield can be extracted from a properly fitted signal. Evaluating equation 13 with the two values of $\sigma^{exp}_{A_{\rm N_i}}/A^{exp}_{\rm N_i}$ shown in figure 8 in the range $0.3 < x_{\rm F} < 0.5$, then $\frac{\sigma_P}{P}$ is 14.3 %, which we find still reasonable. Thus, following the same reasoning, the equation 13 was evaluated with the relative errors $\sigma^{sig}_{A_{\rm N_i}}/A^{exp}_{\rm N_i}$ for $0.3 < x_{\rm F} < 0.5$, estimated in the present simulation based on the background correction method and scaled for different data taking times.

3.3. Effect of the spin angle smearing on the asymmetry

In NICA, the online polarization control is proposed to be performed when the collider operates in spin transparency mode, and the direction of the polarization can be defined by means of the solenoid magnetic field measurement. An estimation of how TSSA would be affected by variations in the spin direction under the magnetic field was done with a toy Monte Carlo simulation.

It was considered a proton travelling a distance $Z = 60$ cm under the magnetic field $B = 1$T, we may calculate the angle ϕ_{max} of the spin precessing around a vector perpendicular to the beam

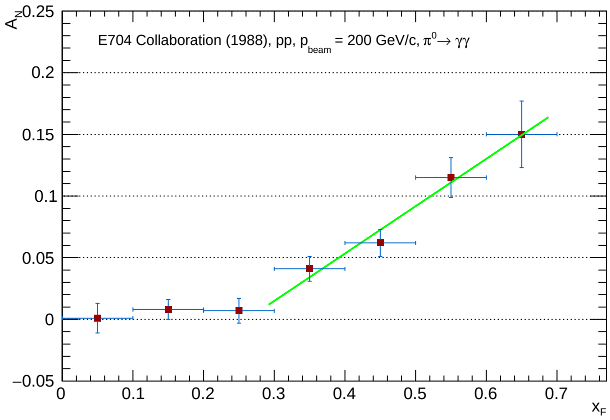

Figure 8. Experimental results of $A_{\rm N}$ as function of $x_{\rm F}$ for inclusive π^0 production by 200 GeV polarized proton beams, from the experiment E704 [5].

direction, as follows,

$$\phi_{\rm max} = \frac{g_{\rm p} \cdot \mu_{\rm N} \cdot B}{\beta\gamma\hbar c} Z, \qquad Z = 60cm, \quad \phi_0 = \phi_0(Z) \tag{14}$$

In Eq. 14, we have well known constants, as the g-factor of protons $g_{\rm p} = 5.586$, the nucleon magneton $\mu_{\rm N}$ and the β , γ Lorentz factors.

A cosine function, $1 + [0] \cdot \cos(x + [1])$, was created to randomly generate ϕ. The parameter [0], accounting for the amplitude of the modulating cosine function, is assigned discrete input values which are fixed in the interval [0.01 - 0.1]. Those fix modulation values ($A_{\rm inp}$) correspond to asymmetry values in the range [0.014 - 0.143], on the assumption that $P = 0.7$. The parameter [1] in turn, was set according to $Z/60 \cdot \phi_{\rm max}$, being $\phi_{\rm max} = 0.0372$ rad (2.13°) while Z was described by a Gaussian distribution, with $\sigma = 30$, in the limits $Z = \pm 60$ cm.

Once the random ϕ has been extracted, histograms of $d{\rm N}/d\phi$ *vs* $A_{\rm inp}$ were created and fitted with a new cosine function, from where a new amplitude parameter, $A_{\rm rec}$, has been extracted and compared with $A_{\rm inp}$. As a result, the asymmetry was reconstructed with a statistical accuracy of ± 0.006 in all the $x_{\rm F}$ range. When referring to the particular $x_{\rm F}$ range where sizeable values of asymmetry are expected, the asymmetry modification was evaluated as $|A_{\rm rec} - A_{\rm inp}|/A_{\rm inp}$, resulting in 10 % at $x_F = 0.35$ and 4 % at $x_F = 0.7$.

4. Results and Discussion

In this work two approaches were addressed to estimate the degree of accuracy with which the local polarization P of proton beams could be measured in SPD, based on the TSSA of forward π^0 from simulated pp collisions at $\sqrt{s} = 27$ GeV. The first method (Sec 3.1) was intended to estimate $\frac{\sigma_P}{P}$ using the presence of P as a multiplying factor contained in the fit parameter a_1 representing the amplitude of the cosine function used to describe the yield N_{π^0} modulation for five $x_{\rm F}$ intervals (Eq.5). The error propagation of terms contained in equation 5 to derive $\frac{\sigma_P}{P}$, was possible by evaluating $\sigma^{exp}_{A_{\rm N}}/A^{exp}_{\rm N}$ with the experimental results of the collaboration E704, and using the statistical uncertainty σ_{a_1} of the modulation coefficients obtained in this work from the cosine fitting of distributions N_{π^0} *vs.* x_{F} in azimuthal bins. Since a non-polarize simulation, results in A_{N} consistent with zero, the reliability of extracting $\sigma_{A_{\mathrm{N}}}$ from the cosine fit of flat distributions was verified with a toy model (Fig.4). Hence, the statistical average of the $\frac{\sigma_P(x_{\mathrm{F}})}{P}$ was performed to obtain the final $\frac{\sigma_P}{P}$ prediction.

The second method, outlined in section 3.2, calculated $\sigma^{sig}_{A_{\mathrm{N}}}(x_{\mathrm{F}})$ after have corrected the raw asymmetry and its statistical uncertainty using the background asymmetry as well as the relative ratio of background-to-raw yields of π^0 candidates, selected under the peak of the invariant mass distribution. The relative errors were calculated by dividing the $\sigma^{sig}_{A_{\mathrm{N}}}(x_{\mathrm{F}})$ obtained with this method by the A^{exp}_{N} data of the collaboration E704 [5]. Finally, they were combined in order to estimate $\frac{\sigma_P}{P}$ according to the equation 13, and scaled for different estimated periods of data collection.

The final results of relative errors of polarization, obtained by both methods, are shown in table 1. With both methods, $\frac{\sigma_P}{P}$ gets smaller for longer estimated time of data taking. No major difference is observed between the results of both methods.

	$\frac{\sigma_P}{P}$ (%)	
Estimated time	Method 1	Method 2
2 min	15.5	14.0
5 min	9.8	8.8
10 min	6.9	6.3
20 min	4.9	4.4
30 min	4.0	3.6
1 h	2.8	2.6

Table 1. Comparison of estimated relative error of P, for different expected times of data taking, using two methods. Method 1: cosine modulation fitting of $N^{sig}_{\pi 0}(\phi)$ without correction for the background. Method 2: background correction of $\sigma^{raw}_{A_{\mathrm{N}}}$.

5. Conclusions

The energy and position of π^0 decayed photons in the end-caps of the SPD ECAL are quantities which are accessible online, with no necessity of particle identification or vertex reconstruction. On these bases, the accuracy of the beam polarization has been estimated for *pp* collisions at 27 GeV, through Monte Carlo simulations in the frame of the SpdRoot-4.1.6 software. It is expected that the π^0 decays registered in the ECAL end-caps of SPD provide an accuracy of the beam polarization of ∼9 % for 27 GeV after 5 minutes of data taking.

References

[1] Abazov V M *et al.* (SPD) 2021 *arXiv e-prints* (*Preprint* `2102.00442`)

[2] Kane G L, Pumplin J and Repko W 1978 *Phys. Rev. Lett.* **41** 1689–1692

[3] Klem R D *et al.* 1976 *Phys. Rev. Lett.* **36**(16) 929–931

[4] Adams D L *et al.* (FNAL E704) 1991 *Phys. Lett. B* **264** 462–466

[5] Adams D L *et al.* (FNAL E581/E704) 1991 *Phys. Lett. B* **261**(1-2) 201–206

[6] Antille J *et al.* 1980 *Phys. Lett. B* **94**(4) 523–526

[7] Abelev B I *et al.* (STAR) 2008 *Phys. Rev. Lett.* **101**(22) 222001 (*Preprint* `0801.2990v2`)
[8] Chiu M (PHENIX) 2011 *J. Phys.: Conf. Series* **295** 012060
[9] Sivers D W 1990 *Phys. Rev. D* **41** 83
[10] Collins J 1993 *Nucl. Phys. B* **396** 161–182
[11] Qiu J w and Sterman G F 1991 *Phys. Rev. Lett.* **67** 2264–2267
[12] Adare A *et al.* (PHENIX) 2014 *Phys. Rev. D.* **90**(1) 012006 (*Preprint* `1312.1995v1`)
[13] Adam J *et al.* (STAR) 2021 *Phys. Rev. D* **103**(9) 092009 (*Preprint* `2012.11428v4`)
[14] Agostinelli S *et al.* (GEANT4) 2003 *Nucl. Instrum. Meth. A* **506** 250–303
[15] Bierlich C *et al.* 2022 (*Preprint* `2203.11601`)
[16] Igamkulov Z *et al.* 2019 *Phys. Part. Nuclei* **16**(6) 744
[17] Dilks C J 2018 Ph.D. thesis Penn State U.
[18] Kleinjan D W 2014 Ph.D. thesis UC Riverside