J/ψ production in the Soft Gluon Resummation approach using the ICEM

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Outline

- Introduction
- Improved Color Evaporation model (ICEM)
- TMD factorisation and Soft Gluon Resummation approach
- Collinear parton model (CPM)
- InEW scheme for matching CPM and TMD predictions
- J/ψ production at small and large p_T within current data
- Extracting of the ICEM $F^{J/\psi}$ parameter
- ICEM predictions for SPD NICA energy

Introduction

- J/ψ production as a tool to study gluon PDF in proton
  - In the small-p_T region, TMD PM and TMD PDFs
  - In the large-p_T region, CPM and Collinear PDFs
- Hadronization approaches: NRQCD and ICEM
- V. Saleev and K. Shilyaev, «Production of S-wave charmonia in the soft gluon resummation approach using the NRQCD», [arXiv:2502.16461 [hep-ph]]
- J/ψ production in the Soft Gluon Resummation approach using the ICEM

Improved Color Evaporation model

- Improved Color Evaporation Model, scale hierarchy $m_c \gg \lambda \gg \Lambda_{\mathrm{QCD}}$ [Y.-Q. Ma, R. Vogt (2016)]:
  - $c\bar{c}$-pair production on $\mathcal{O}(m_c)$ scale
  - soft gluon exchange and emission on $\mathcal{O}(\lambda)$ scale
  - $\lambda \sim q_T$, q_T is intrinsic transverse momentum of parton

- Momentum shift for $c\bar{c} \to J/\psi$ transition (with invariant mass $M_{c\bar{c}}$ of the produced $c\bar{c}$-quark pair):

$$\langle p^{J/\psi} \rangle = \frac{M_{J/\psi}}{M_{c\bar{c}}} p^{c\bar{c}} + \mathcal{O}\left(\lambda^2/m_c\right)$$

- General cross-section expression:

$$d\sigma(p^{J/\psi}) = F^{J/\psi} \times \int_{M_{J/\psi}}^{2m_D} dM_{c\bar{c}} \int d^4p^{c\bar{c}} \frac{d\sigma\left(p^{c\bar{c}}\right)}{dM_{c\bar{c}}} \delta^{(4)}\left(p^{J/\psi} - \frac{M_{J/\psi}}{M_{c\bar{c}}} p^{c\bar{c}}\right) + \mathcal{O}\left(\lambda^2/m_c^2\right)$$

with $F^{J/\psi}$ as a hadronisation factor assumed to be independent on the factorisation approach.

General remarks on our calculations of prompt J/ψ production

- Only **prompt** production of J/ψ within the ICEM
  - no direct opportunity to estimate the feed-down ratio without the higher states production data
- Study of both **gluon-gluon fusion** $g+g \to c+\bar{c}+X$ and **quark-antiquark annihilation** $q+\bar{q} \to c+\bar{c}+X$
  - rise of quark-antiquark subprocess contribution with the decrease of $\sqrt{s}$, center-of-mass energy
- Assumption of $c\bar{c}$-quark pair production in color-octet state
  - followed by soft gluon interactions and hadronisation into some observable state
- Dependence of $F^{J/\psi}$ on center-of-mass energy $\sqrt{s}$
- Calculations in the LO approximation of the pQCD with respect to α_s
  - for $g+g\,(q+\bar{q}) \to c+\bar{c}+g$ processes at $p_T \gg M_{J/\psi}$

TMD factorisation and initial parton transverse momenta

- **Transverse Momentum Dependent (TMD) factorisation**: $q_T, k_T \ll \mu_F \sim M$
- TMD parton distribution functions $F(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations:

$$\begin{cases} \dfrac{\partial \ln \hat{F}(x, \boldsymbol{b_T}, \mu_F, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu_F) & \text{with CS kernel } \tilde{K}(b_T, \mu_F) \\ \dfrac{\partial \tilde{K}(b_T, \mu)}{\partial \ln \mu} = -\gamma_K[\alpha_s(\mu_F)] & \text{with anomalous dimension } \gamma_K[\alpha_s(\mu_F)] \end{cases}$$

- Partons' momenta decomposition:

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu$$

  - preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2} \right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2} \right)$$

- Relevant $2 \to 2$ subprocesses:
  - gluon-gluon fusion $g + g \to c + \bar{c}$ and quark-antiquark annihilation $q + \bar{q} \to c + \bar{c}$

TMD factorisation and TMD PDFs

- General formula of TMD factorisation [TMD Handbook, arXiv:2304.03302]:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, F(x_1, \boldsymbol{q_{1T}}, \mu_F, \zeta_1)\, F(x_2, \boldsymbol{q_{1T}}, \mu_F, \zeta_2)\, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{p_T})$$

- To implement **CS** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int \frac{d\boldsymbol{b_T}}{(2\pi)^2}\, e^{i\boldsymbol{p_T}\boldsymbol{b_T}}\, \hat{F}(x_1, \boldsymbol{b_T}, \mu_F, \zeta_1)\, \hat{F}(x_2, \boldsymbol{b_T}, \mu_F, \zeta_2)$$

  - σ_0 is calculated as series with respect to the small α_s

Soft Gluon Resummation approach, perturbative evolution

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper (1981)]:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T,Q)}\; \hat{F}(x_1,\mu'_{b^*},b^*_T)\, \hat{F}(x_2,\mu'_{b^*},b^*_T)$$

- Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T,\mu_F,Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{Q^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} + \frac{1}{2}\right)\right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\text{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Q b_0}{Q b_T + b_0} \quad \text{and} \quad b^*_T(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\,\text{max}})^2}}$$

Soft Gluon Resummation approach, nonperturbative content

- Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T,Q)}\; \hat{F}(x_1,\mu'_{b^*},b_T^*)\, \hat{F}(x_2,\mu'_{b^*},b_T^*)$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T,Q) = \left[g_1 \ln\frac{Q}{2Q_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10xx_0}{x_0+x}\right)\right] b_T^2$$

  - it should be Casimir-scaled by C_A/C_F for initial gluons

- In the leading order of α_s, the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x,\mu'_{b^*},b_T^*) = f(x,\mu'_{b^*}) + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}})$$

Collinear Parton Model factorisation

- **CPM factorisation**: $q_T \ll Q \sim M \ll k_T$
- Collinear parton distribution functions $f(x, \mu_F) \Rightarrow$ **DGLAP evolution equation**
- Partons' momenta:

$$q_1^\mu = x_1 p_1^\mu, \qquad q_2^\mu = x_2 p_2^\mu$$

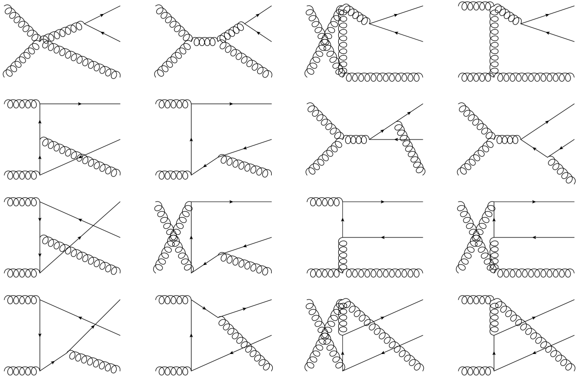
  - components of partons' momenta:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, 0, 0, \frac{x_1\sqrt{s}}{2}\right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, 0, 0, -\frac{x_2\sqrt{s}}{2}\right)$$

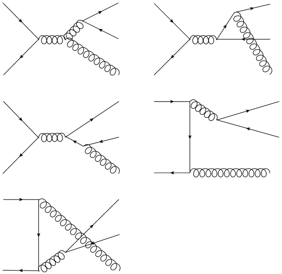
- Relevant $2 \to 3$ subprocesses:
  - gluon-gluon fusion $g + g \to c + \bar{c} + g$
  - quark-antiquark annihilation $q + \bar{q} \to c + \bar{c} + g$

Collinear Parton Model factorisation, diagrams

$$g + g \to c + \bar{c} + g$$



$$q + \bar{q} \to c + \bar{c} + g$$



Matching of small-p_T and high-p_T regions within Inverse-Error Weighting Scheme

- Matched cross-section as a weighted sum of CPM and TMD terms [M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

$$d\sigma = \mathcal{W}\, d\sigma^{\mathsf{TMD}} + \mathcal{Z}\, d\sigma^{\mathsf{CPM}}$$

- Normalised weights for each of the two terms:

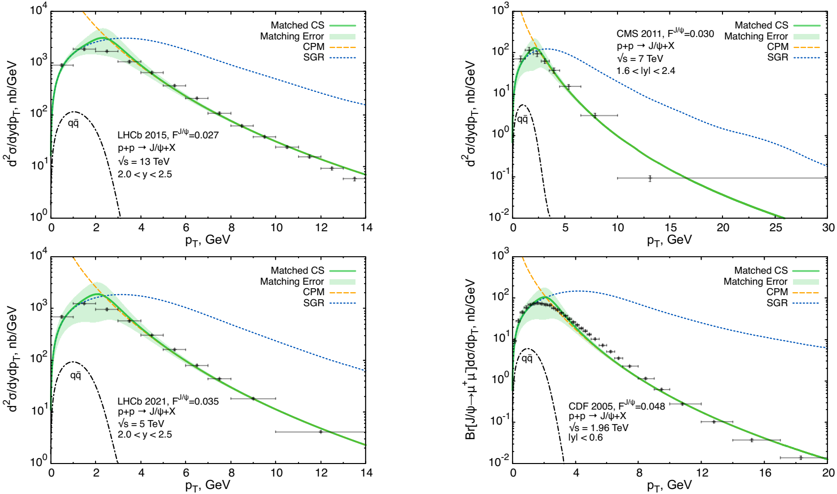
$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \qquad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}$$

$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2 + \left(\frac{m}{Q}\right)^2, \qquad \Delta\mathcal{Z} = \left(\frac{m}{p_T}\right)^2 \left(1 + \ln^2 \frac{\sqrt{Q^2 + p_T^2}}{p_T}\right)$$

- Uncertainty due to the matching procedure:

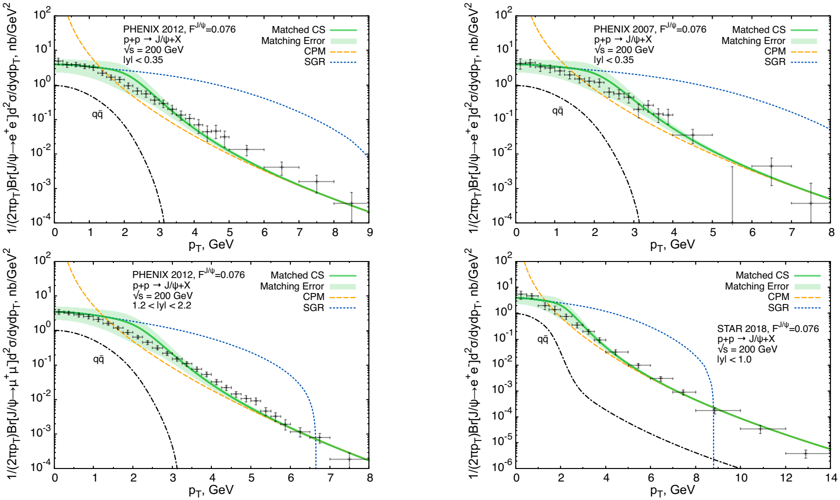
$$\Delta d\sigma = \frac{d\sigma}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta\mathcal{W} \cdot \Delta\mathcal{Z}}{\sqrt{\Delta\mathcal{W}^2 + \Delta\mathcal{Z}^2}} d\sigma$$

Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM) $\sqrt{s} = 1.96 - 13$ TeV

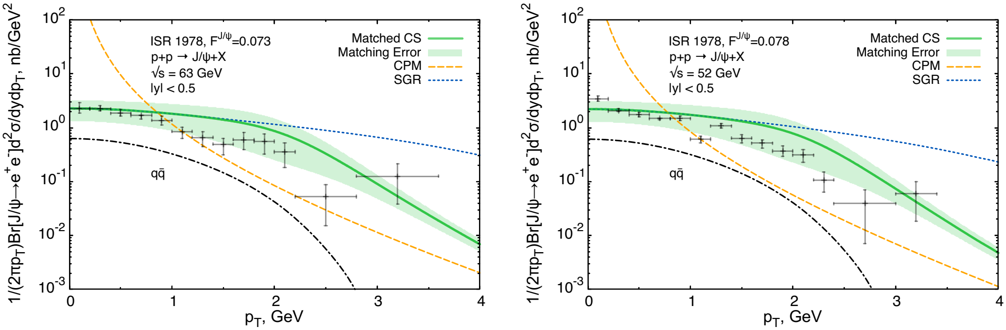


Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)

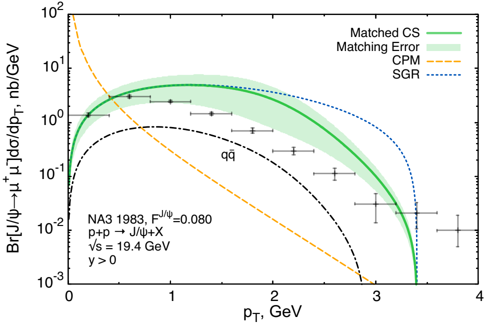
$\sqrt{s} = 200$ GeV

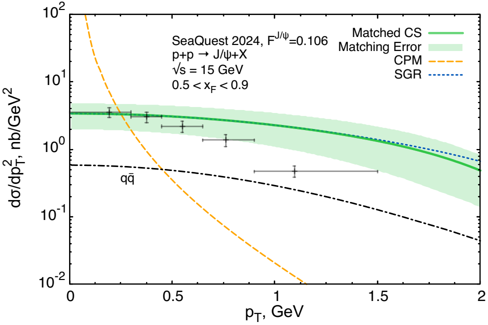


Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR)
$\sqrt{s} = 52 - 63$ GeV

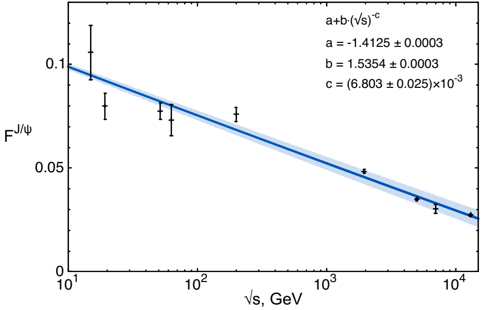


Extraction of $F^{J/\psi}$ at $p_T < 0.5$ GeV (SGR)
$\sqrt{s} = 15 - 19.4$ GeV

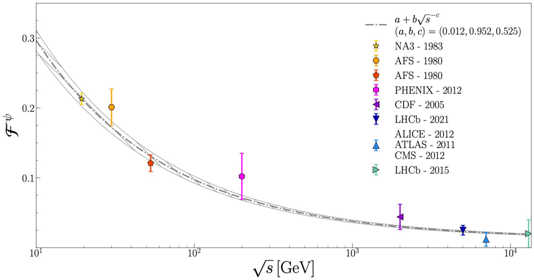




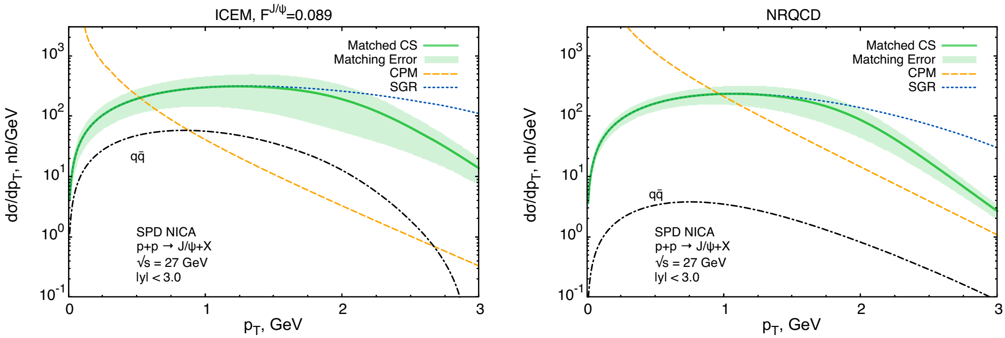
Dependence of $F^{J/\psi}$ on center-of-mass energy $\sqrt{s}$



[A. Chernyshev, V. Saleev (2022)]



Predictions for SPD NICA, comparison with the previous NRQCD calculations



Summary

- We have used the Soft Gluon Resummation approach to calculate small-p_T J/ψ production in the TMD factorisation
- ICEM's hadronisation factor $F^{J/\psi}$ crucially depends on the center-of-mass energy $\sqrt{s}$, showing decrease when $\sqrt{s}$ increases
- Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describes experimental data for unpolarised J/ψ production at $\sqrt{s}$ from 15 GeV up to 13 TeV in the TMD domain of $p_T < 1$ GeV
- We have tested the factorisation matching scheme InEW for description of intermediate p_T domain in the ICEM, calculation agrees with experimental data within the uncertainty band
- We estimate the perspective region for extraction of gluon TMD PDF in J/ψ production as $p_T \leqslant 1$ GeV

THANK YOU FOR ATTENTION!