Lectures 15-16: OSC & CEC

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Bunched Beam Stochastic Cooling

Schottky Noise of Bunched Beam

$$P(\omega)=\frac{e^2}{T_0^{\,2}}\left[\sum_{k,l}\sum_{n=-\infty}^{\infty}\left\langle z_{0_k} z_{0_l}^{\;*}\right\rangle\delta\left(\omega-n\omega_0\right)+\sum_{k}\sum_{n=-\infty}^{\infty}\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}\left\langle\left|z_{m_k}\right|^2\right\rangle\delta\left(\omega-\omega_{nm}\right)\right] \quad , \quad \omega_{nm}=n\omega_0+m\omega_{s_k}.$$

$$z_{nm}=\frac{1}{T_s}\int_{-T_s/2}^{T_s/2}\exp\left(-i\frac{\omega_{nm}z(t)}{\beta c}\right)\mathrm{e}^{-im\omega_s t}\,dt.$$

where k, l enumerate particles

- For linear RF $z_{nm}=e^{-i\pi m/2}J_m\left(\pi\kappa_b n\right)$, $\kappa_b = L_b/2\pi R_0$

- Number of synchrotron lines around n-th harmonic grows $\propto n$
  - ⇨ Even very small tune spread will result in synchrotron band overlap for large m
- For large *n* the shape of the spectrum corresponds to the actual particle distribution on the momentum (see the proof in "Accelerator Physics At Tevatron Collider")
- For Gaussian distribution the coherent term exponentially decays with increase of *n* and disappears for sufficiently large frequencies.

Optimal Cooling Rate of Transient-Time Cooling of Bunched Beam

- For complete band overlap the diffusion does not depend on rel. momentum, x, but is dependent on the longitudinal position

  $\Rightarrow$ for Gaussian distribution

$$D(s)=\frac{2N_b}{T_0}\frac{Ce^{-s^2/2\sigma_s^2}}{\sqrt{2\pi}\sigma_s}\sum_{n=-\infty}^{\infty}\left|G(n\omega_0)\right|^2$$

$$\Rightarrow \qquad \overline{D}=\int_{-\infty}^{\infty}\frac{e^{-s^2/2\sigma_s^2}}{\sqrt{2\pi}\sigma_s}D(s)ds$$

- For rectangular band

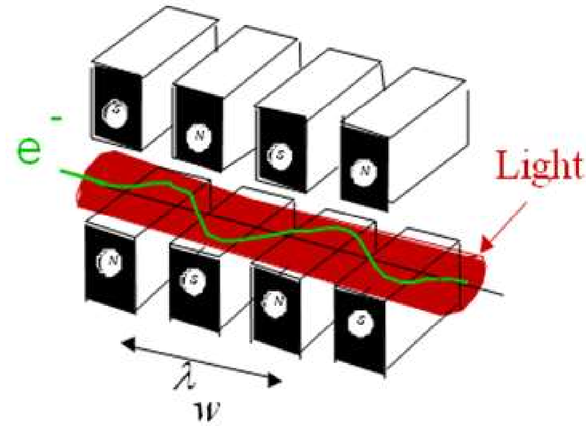
$$\lambda_{opt}\approx\frac{2\pi^2 W}{N n_\sigma^{\ 2}}\frac{\sqrt{\pi}\sigma_s}{C} \qquad W=\frac{n_{\max}-n_{\min}}{T_0}\,,\quad n_\sigma=\frac{x_{\max}}{\sigma_p}\,.$$

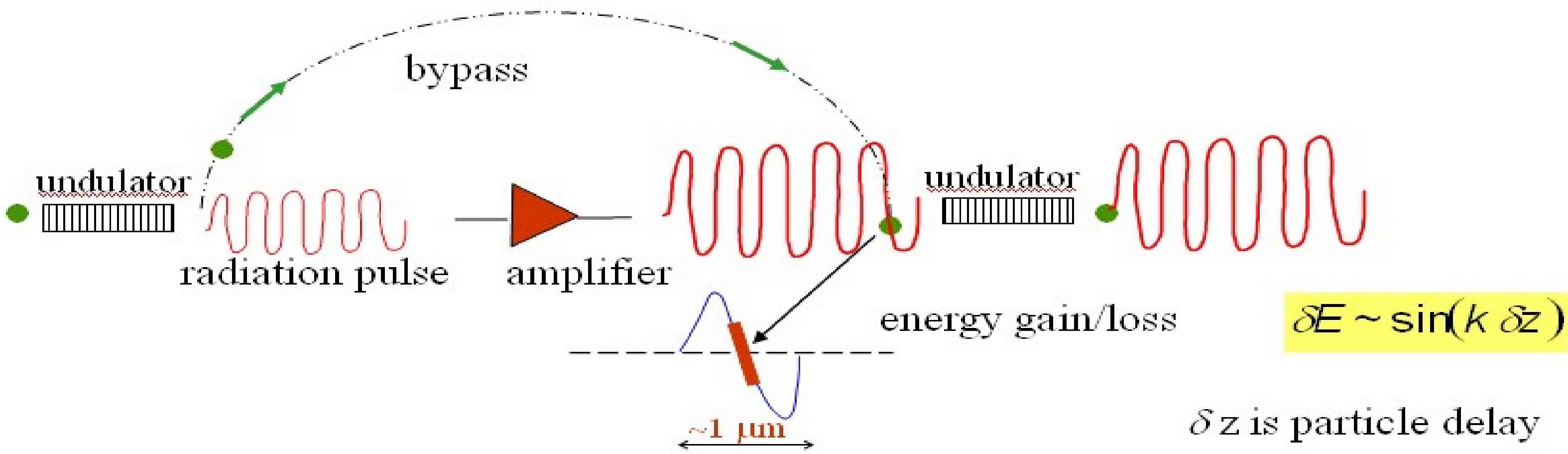
- Bandwidth for Gaussian band: $W=2\sqrt{\pi}\sigma_f$ $\quad \left(G(\omega)=G_0\exp\left(-\omega^2/2\left(2\pi\sigma_f\right)^2\right)\right)$

Optical Stochastic Cooling

Optical Stochastic Cooling

- Suggested by Zolotorev, Zholents and Mikhailichenko (1994)
- OSC obeys the same principles as the microwave stochastic cooling, but exploits the superior bandwidth of optical amplifiers ~ 10^{14} Hz
- Pickup and kicker must work in the optical range and support the same bandwidth as the amplifier
  - Microwave pickups cannot be scaled to μm
    - Distance to the beam is 10^3-10^4 λ
  - Undulators were suggested: both for pickup & kicker

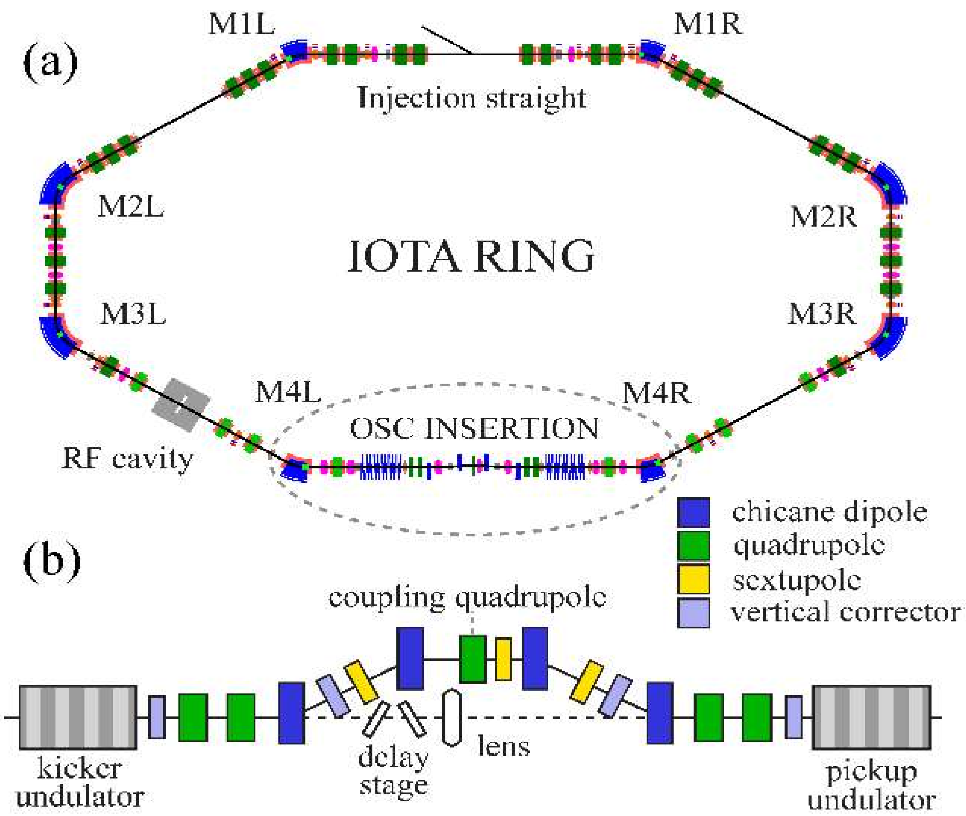




- ⊥ cooling is due to coupling between different degrees of freedom

Optical Stochastic Cooling

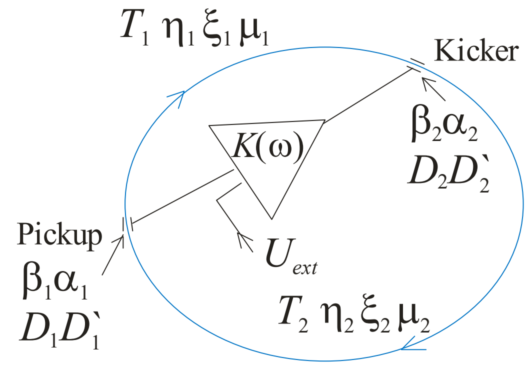
- OSC was experimentally tested at FNAL at IOTA (summer 2021)
  - 100 MeV electrons in 40 m ring
  - Multipurpose ring (Integrable optics, high space charge, OSC, …)
    - reasonably small price for OSC
- Cooling of hadrons requires a beyond "state of art" optical amplifier
  - Power?
  - Small signal delay?
  - Large duty-factor: 0.01 – 0.1



Schematic of the IOTA OSC system. a, Schematic of the IOTA ring and the location of the OSC insertion. **b,** Diagram of the OSC insertion including the undulators, chicane and light optics.

Amplifier	λ [nm]	Δf/f	D.F.
Ti-Sapphaire	800	0.2	CW
Dye	300-900	0.2	CW
Parametric	350-1500	0.2	$\sim 10^{-6}$ @10 W

Basics of OSC - Damping Rates



- Pickup-to-Kicker Transfer Matrix
  - Vertical plane is uncoupled and we omit it

$$\mathbf{M}^{pk} = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ \theta_x \\ s \\ \Delta p / p \end{bmatrix}$$

$\mathbf{M}^{pk}$ - pickup-to-kicker matrix
$\mathbf{M}^{kp}$ - kicker-to-pickup matrix
$\mathbf{M} = \mathbf{M}^{pk}\mathbf{M}^{kp}$ – ring matrix

- Partial slip factor (pickup-to-kicker) describes a longitudinal particle displacement in the course of synchrotron motion

$$\tilde{M}_{56} = M_{51}D_1 + M_{52}D_1' + M_{56}$$

- Linearized longitudinal kick in pickup wiggler

$$\frac{\delta p}{p} = k\xi_0 \Delta s = k\xi_0\left(M_{51}x_1 + M_{52}\theta_{x_1} + M_{56}\frac{\Delta p}{p}\right) \to k\xi_0\left(Dx_1 + D'\theta_{x_1} + M_{56}\right)\frac{\Delta p}{p}$$

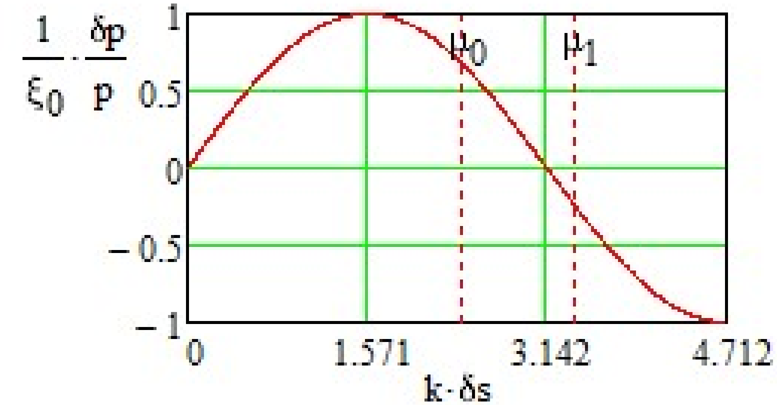
- Cooling rates (per turn)

$$\lambda_x = \frac{k\xi_0}{2}\left(M_{56} - \tilde{M}_{56}\right)$$
$$\lambda_s = \frac{k\xi_0}{2}\tilde{M}_{56}$$

$\Leftrightarrow$

$$\lambda_x + \lambda_s = \frac{k\xi_0}{2}M_{56}^{pk}$$

Basics of OSC - Cooling Range



- Cooling force depends on Δs nonlinearly

$$\frac{\delta p}{p} = k\xi_0 \Delta s \quad \Rightarrow \quad \frac{\delta p}{p} = \xi_0 \sin(k\delta s)$$

where $k\delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

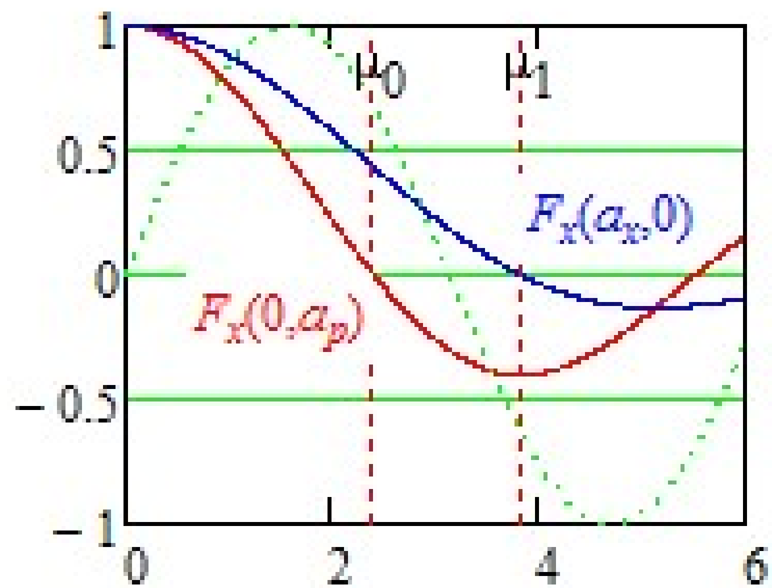
and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to ⊥ and L motions measured in units of laser phase

- Averaging yields the form-factors for damping rates

$$\lambda_{s,x}(a_x,a_p) = F_{s,x}(a_x,a_p)\lambda_{s,x}$$

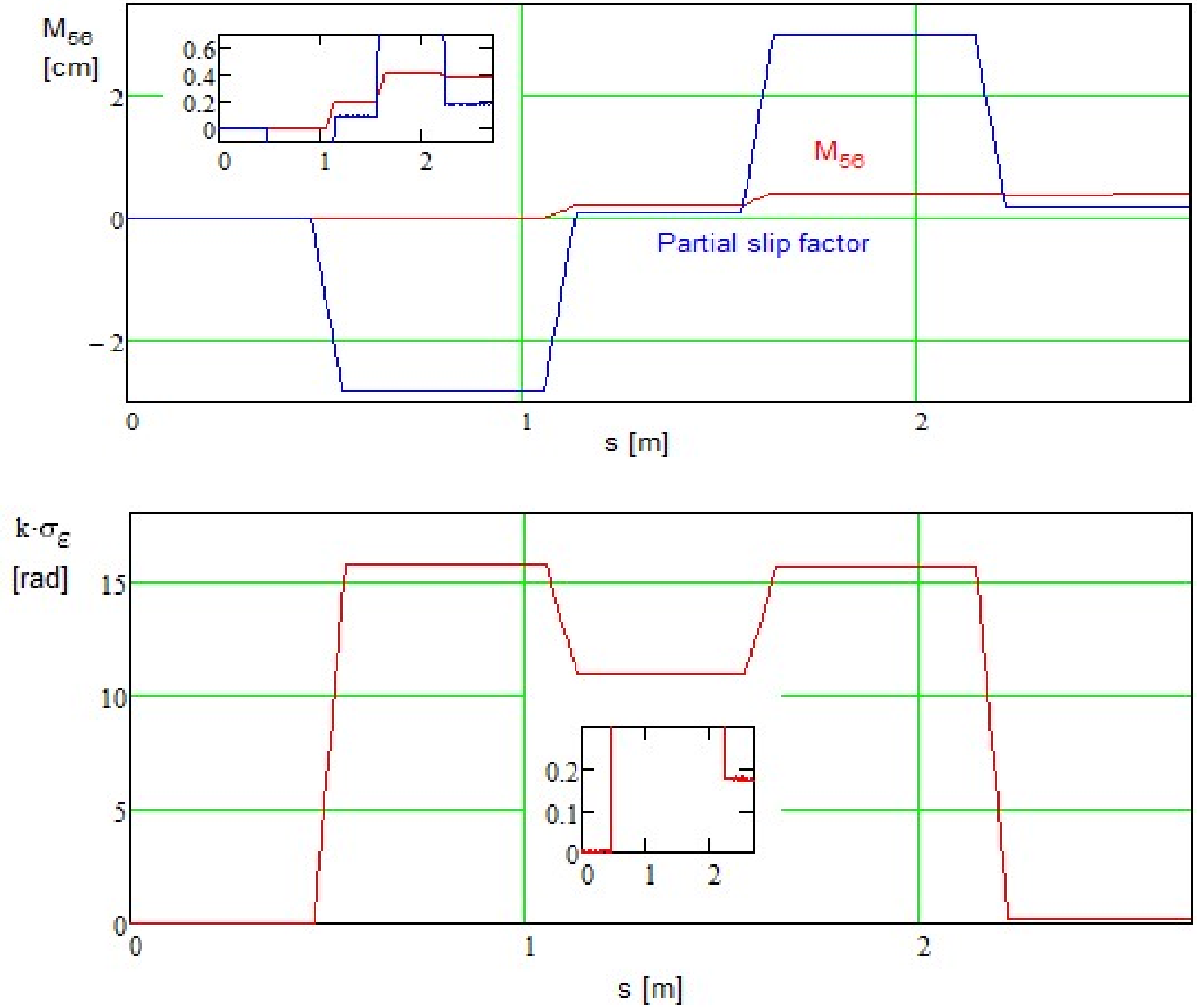
$$F_x(a_x,a_p) = \frac{2}{a_x} J_0(a_p) J_1(a_x)$$

$$F_p(a_x,a_p) = \frac{2}{a_p} J_0(a_x) J_1(a_p)$$



- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_0 \approx 2.405$

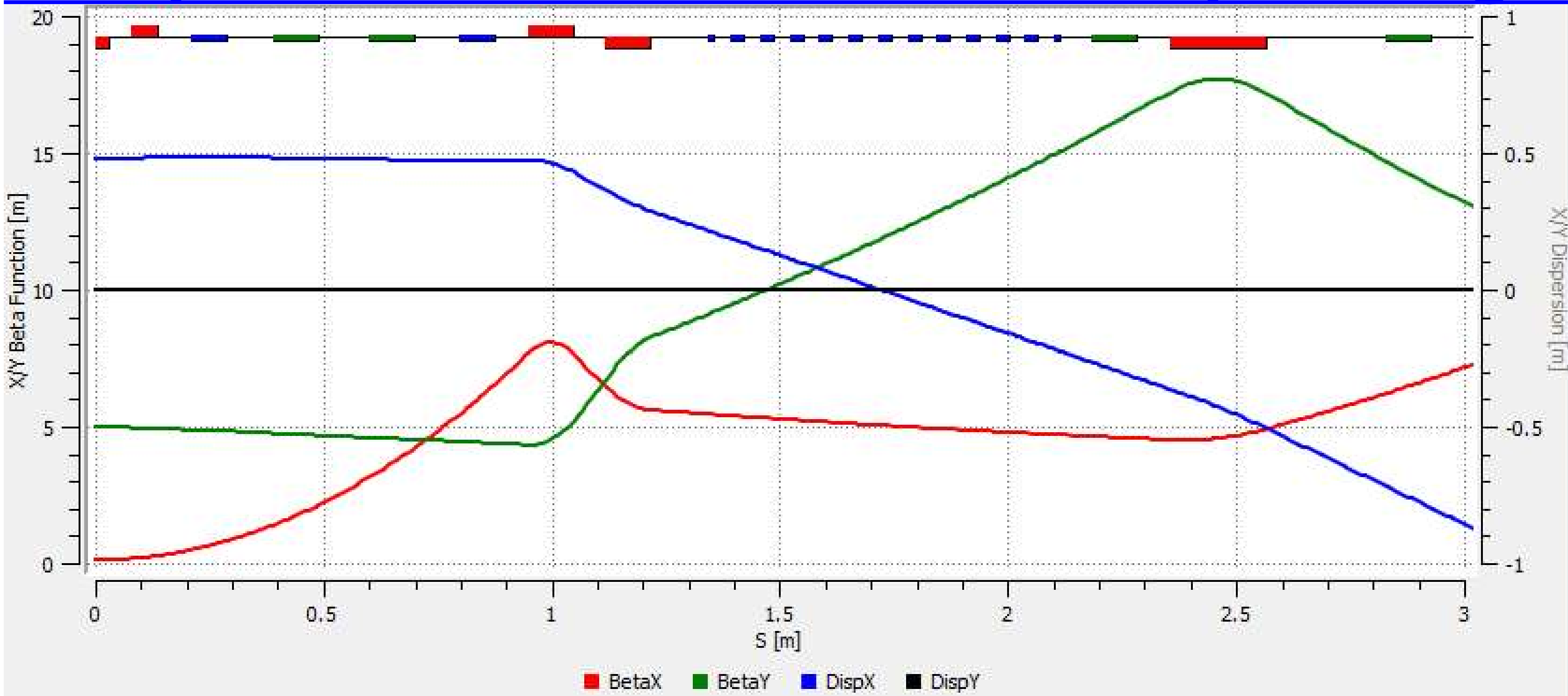
Linear Sample Lengthening on the Travel through Chicane



Sample lengthening due to momentum spread (top) and due to betatron motion (bottom, H. emittance for x-y coupled case)

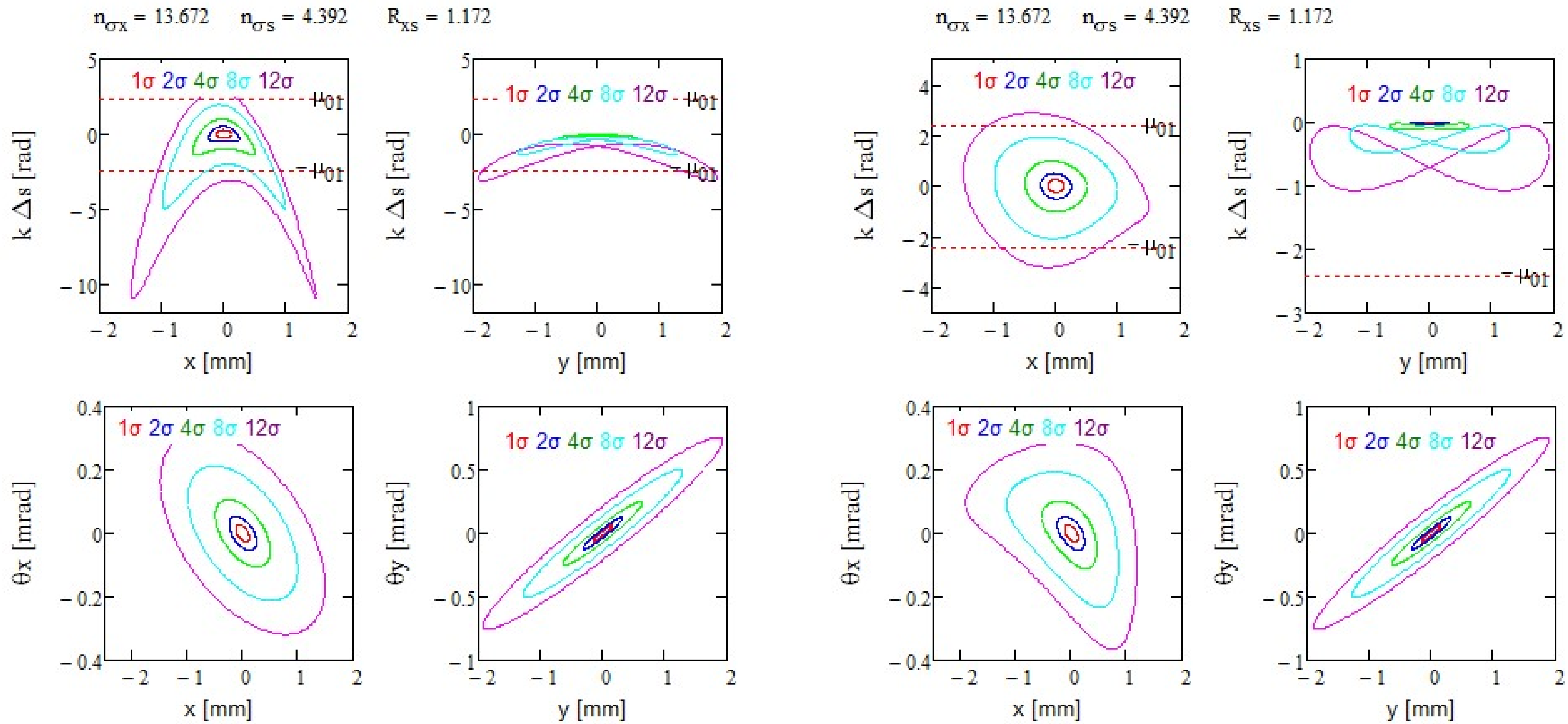
- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening, $\Delta(BL)/(BL)_{dipole}<10^{-3}$

Compensation of Non Linear Sample Lengthening



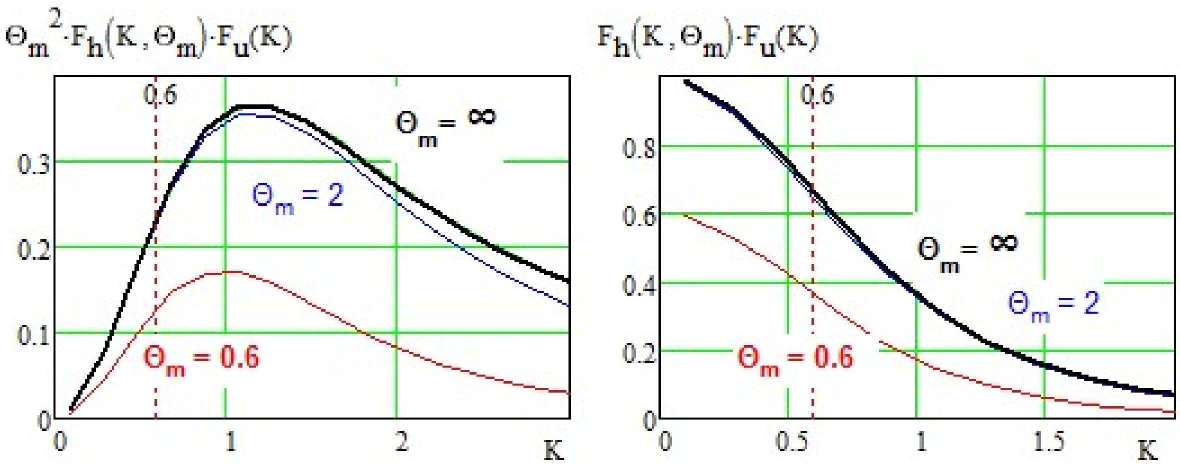
- Nonlinear lengthening
  - It mainly comes from the betatron angles, $\Delta L_{x,y} = \int \left(\theta_{x,y}{}^{2} / 2 \right) ds$, and is larger for horizontal plane
  - It is large and has to be compensated
- Compensation is achieved by two pairs of sextupoles located between dipoles of each dipole pair of the chicane (marked by green boxes)
  - Strengths of sextupoles are large: SdL_y=-7.5 kG/cm, SdL_x=1.37 kG/cm. It results in considerable limitation of the ring dynamic aperture.

Compensation of Non Linear Sample Lengthening



Phase space distortion for the cases of uncompensated (left) and compensated (right) sample lengthening (reference emittance is equal to the horizontal emittance of x-y uncoupled case)

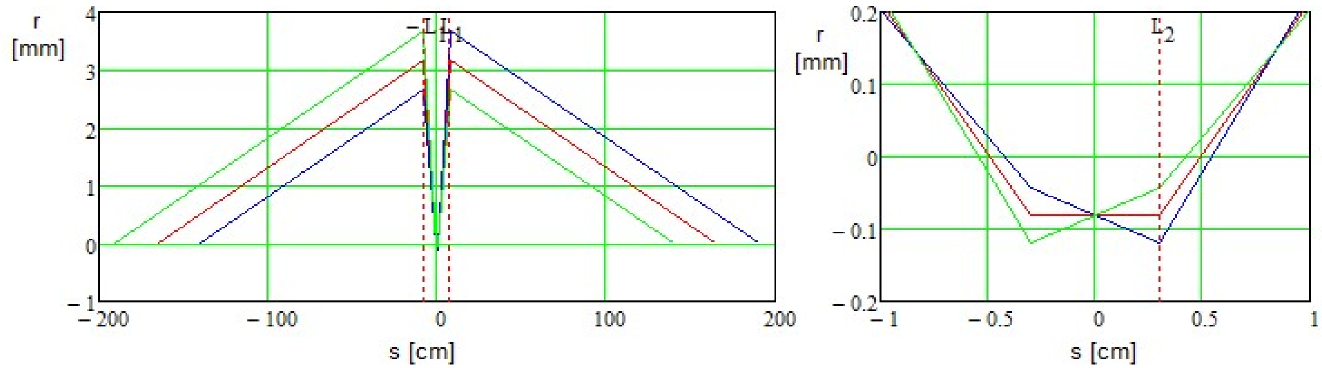
Dependence of Cooling Efficiency on Undulator Parameter



- Particle motion in undulator becomes comparable to the size of the focused radiation
  - It reduces cooling efficiency
- An increase of the undulator parameter also increases undulator magnetic field and, consequently, the equilibrium emittance and undulator focusing
- Chosen undulator parameter K=1.038 corresponds to the 7 period undulator with B_0=1 kG. It results in moderate increase of equilibrium emittance of ~5%.

The Depth of Field for Focusing of Radiation

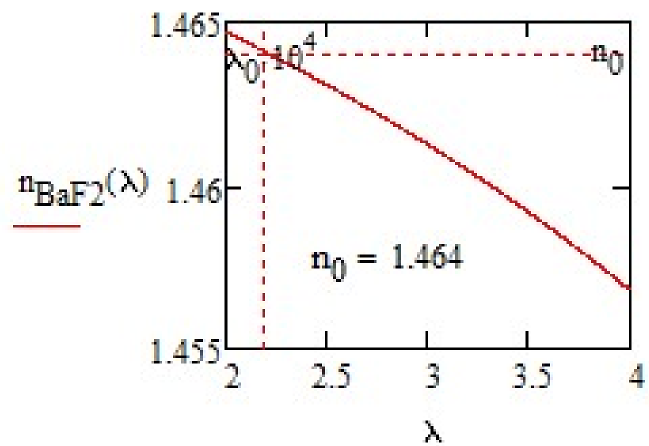
- Two possibilities
  - Four lens system with complete suppression of depth of field



$$\begin{pmatrix}1 & L_1\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\-\frac{1}{F_1} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\-\frac{1}{F_2} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\-\frac{1}{F_1} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1\\0 & 1\end{pmatrix} = p\cdot\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

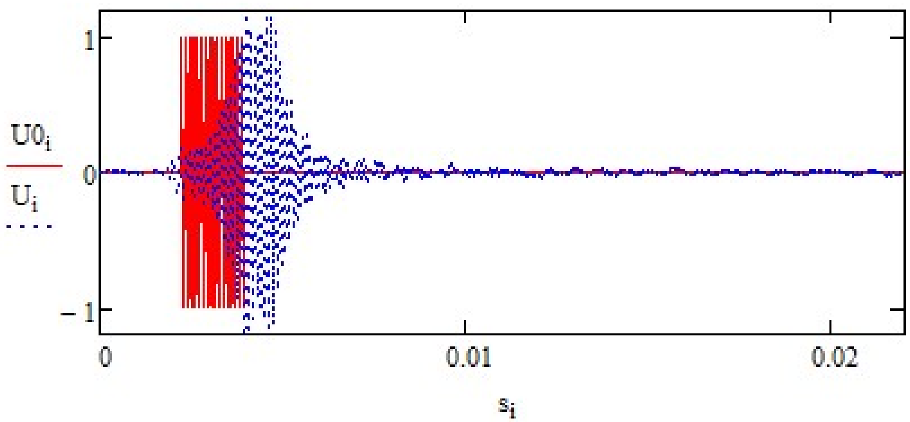
$L_2 := 33$ cm $L_1 := L_{tot} - L_2 = 140$ cm $F_1 := L_2 = 33$ cm $F_2 := \frac{L_2^{\,2}}{2\cdot(L_2 - L_1)} = -5.089$ cm

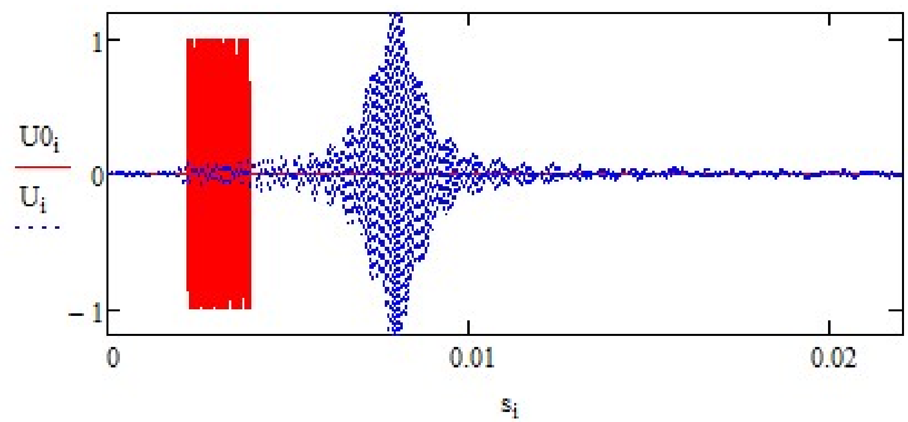
- Lenses are manufactured from barium fluoride
  - Antireflection coating protects from humidity damage
- Excellent material with very small second order dispersion



First Order Dispersion Effects in Optical Lenses

- The first order dispersion, $dn/d\lambda$, results in 1.5% difference between phase and group velocities in the lens material
  - It has to be accounted in the total lens thickness
  - Significant separation of radiation of the first and higher harmonics

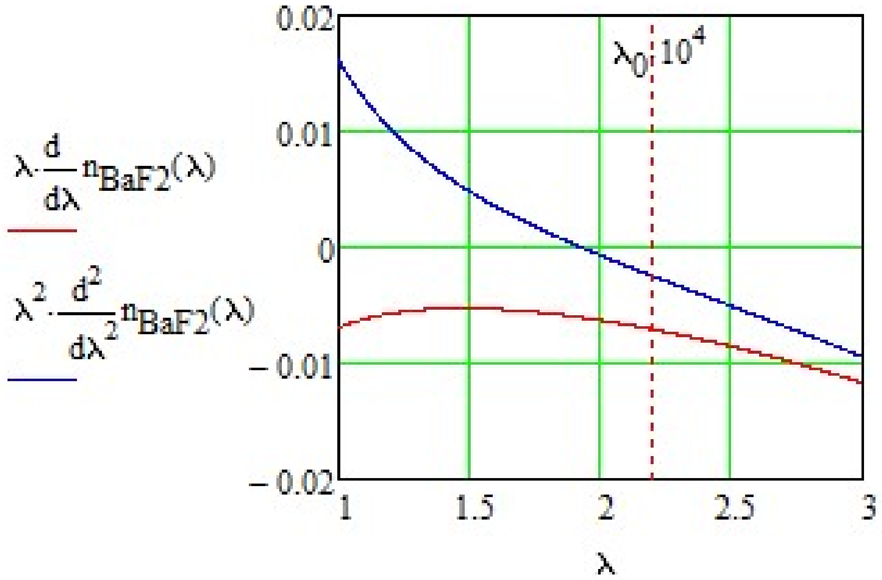


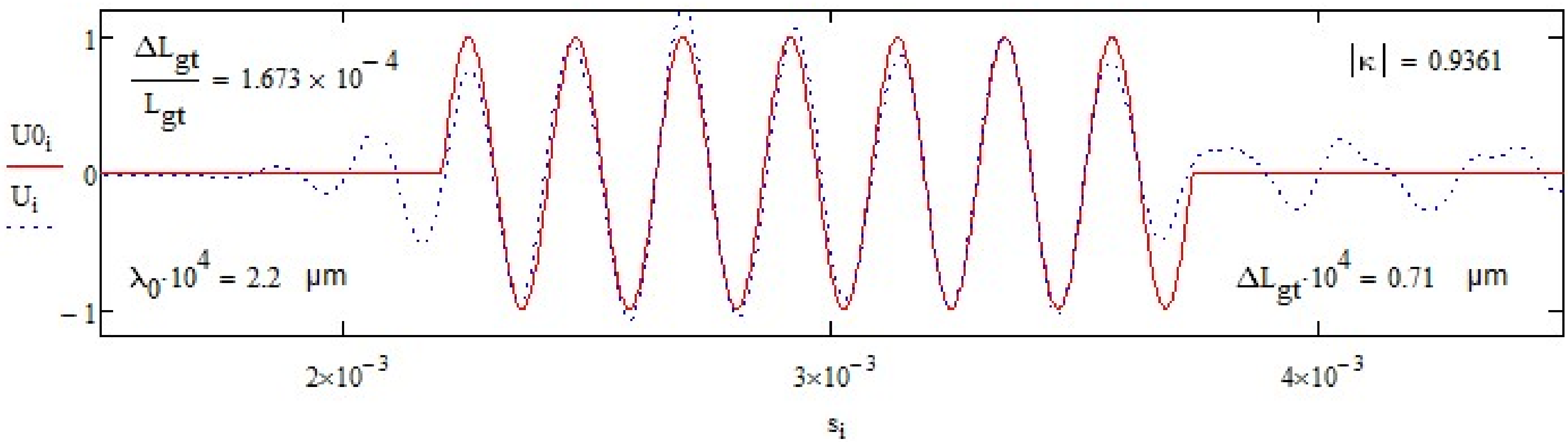


Overlap of radiation for the second and third harmonics of undulator radiation

- Dependence of focusing strength on wave length makes reasonably small reduction of the damping rates
  - Accurate numbers will be obtained soon

Second Order Dispersion Effects in Optical Lenses





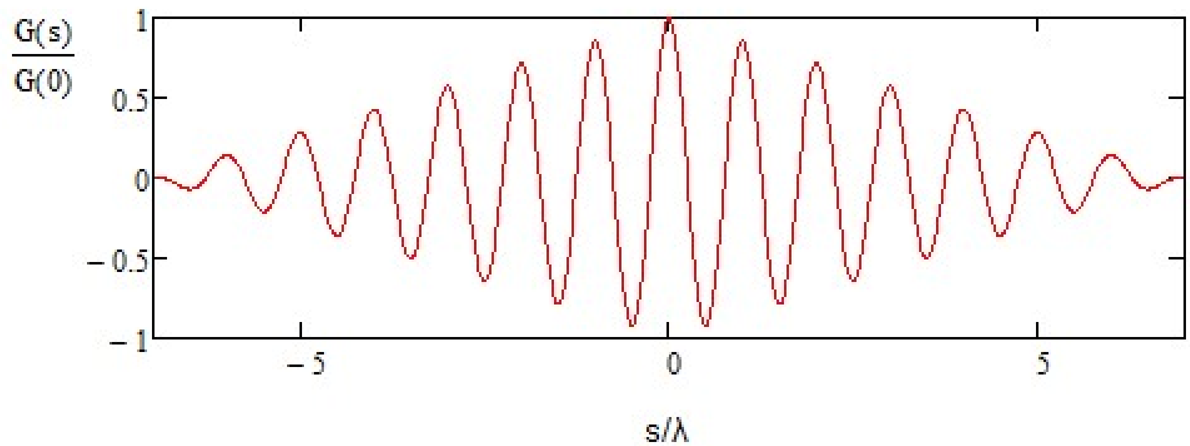
The second order dispersion, $d^2n/d\lambda^2$, results in lengthening of the light packet and, consequently, 6% loss of cooling rates

Effect of Beams Overlap on Cooling Rates

- Transfer matrix for the light is equal to the negative identity matrix
- Transfer matrices for particles are close to the negative identity matrix. It compensates separation of light and particles due to betatron motion

Bandwidth of Optical Stochastic Cooling

- In the absence of dispersion in the lenses and OA the bandwidth is determined by
  - Number of wiggles in the undulators



- Radiation focused back to the kicker wiggler has "correct" freq. for resonant particle interaction

  ⇨ frequency dependence of SR θ does not directly affect the bandwidth

- It is desirable to have the bandwidth of optical amplifier larger than the bandwidth of radiation (both forward and at the aperture of optical system)
- Total rms bandwidth for Fermilab OSC test is ~3% (~10% effective);

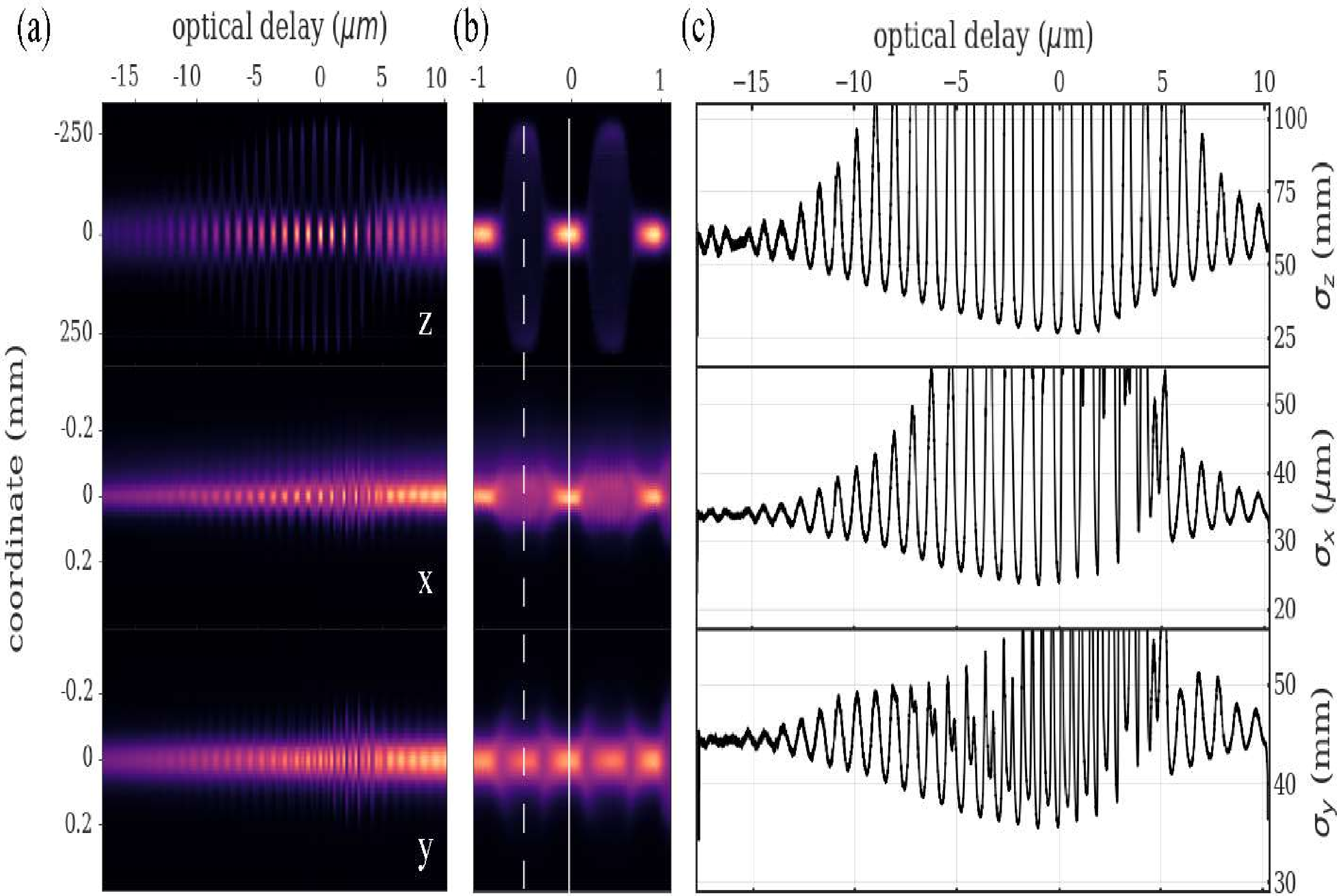
Can Parametric Optical Amplifier help?

- POA looks as a good choice
  - Large amplification & small delay
- However, there are problems
  - typical duty factor <~10^{-5}
  - Amplification duration (pump length) ~30 fs (10 μm)
    - 100 μJ*50 MHz=5 kW
  - Looks to be impossible to obtain reasonable duty factor (1-10%) with reasonable pumping power

$$\lambda_{opt} = \frac{2\pi^3 \sigma_f}{n_\sigma^{\ 2}} \frac{\sigma_s}{N_p C} \frac{\sigma_g}{\sigma_s}$$

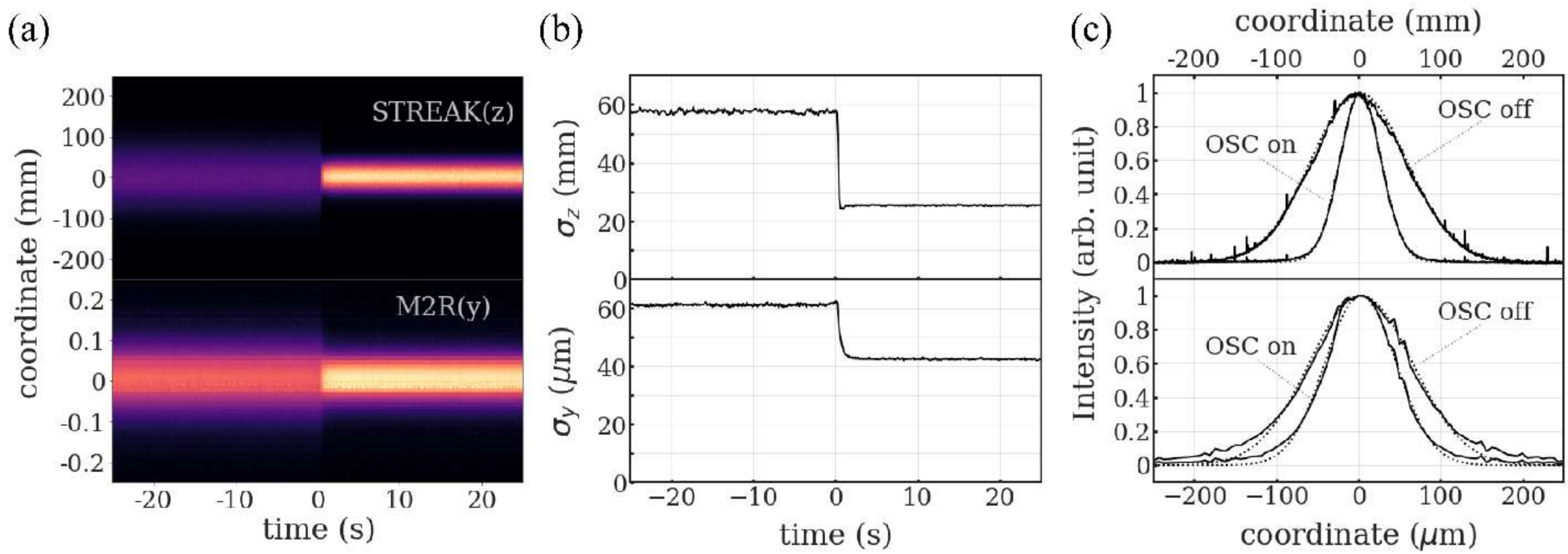
We will lose orders of magnitude in the damping rate due to small gain length, σ_g

OSC in IOTA



|Projected beam distributions for a delay scan in the 3D OSC configuration. a,

OSC in IOTA

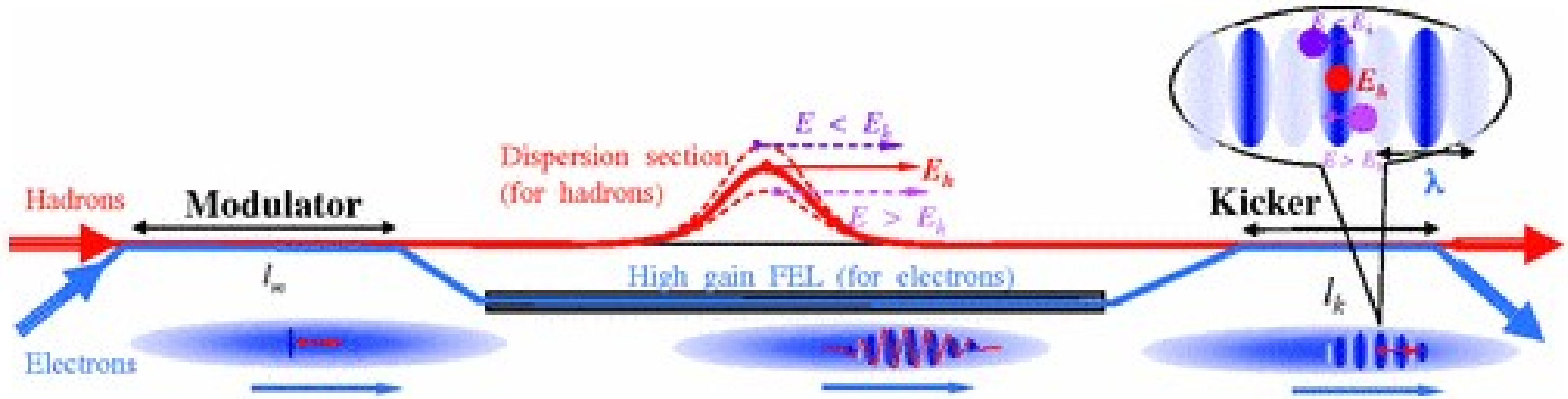


Fast toggle of the OSC system

- The non-Gaussian tails in the transverse distribution appear due to scattering on the residual gas
- Very small electron beam current (~1 μA) reduces the IBS. Multiple IBS reduces the difference in the beam size with cooling on and off. Touschek does not create considerable tails.

Coherent Electron Cooling

- Idea proposed by Derbenev in 1980
- First realistic proposal by Derbenev and Litvinenko in 2007



  - Based on the amplification in FEL
    - Gain in frequency by ~10^4 relative to microwave SC
    - Loss ~100 due to bandwidth: 50% -> 0.5%
    - Loss due to short length of FEL electron bunch ~100
- Two proposals address the bandwidth problem (0.5%->~50%).
  They use: (1) plasma cascade instability, and
  (2) micro-bunching in a chicane (first observed in SLAC SASE FEL
- Major problems with CEC
  - Saturation of amplifier Δn/n ~1/2
  - Noise in electron beam

Problems

1. Using electrostatic analogy find the maximum longitudinal force in electron cooling for pan cake distribution ($\sigma_{vx}=\sigma_{vy}$, $\sigma_{vz}=0$). Assume that the transverse velocity of a proton is equal to zero and non-magnetized electron cooling.
2. In the shortwave approximation ($k\sigma_{tr}>>1$) find the maximum cooling force in the coherent electron cooling. Assume Gaussian beam in the transverse direction with equal rms transverse sizes, and neglect plasma oscillations.

- Prove the rate-sum theorem. Let the revolution symplectic matrix $\mathbf{M}$ be perturbed: $\mathbf{M}=(\mathbf{I}+\mathbf{P})\mathbf{M}_0$, where the perturbation $\mathbf{P}$ is small, but not necessarily symplectic. Then in the first order of perturbation theory, the complex tune shifts are: $\Delta\mu_l/(2\pi)=-(4\pi)^{-1}\,\mathbf{v}_l^{\dagger}\mathbf{U}\,\mathbf{P}\,\mathbf{v}_l$ and the sum of all growth rates is independent on the eigenvectors so that $\mathrm{Im}\left(\sum_l \Delta\mu_1\right)=\mathrm{Tr}(\mathbf{P})/2$. Here l changes from 1 to the number of degrees of freedom (3 - for 3-dimensional motion).