Off-shell initial state effects in Drell-Yan process at NICA.

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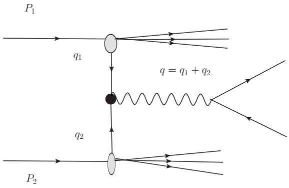
Outline.

1. Motivation
2. LO PRA Framework
3. Structure functions for DY process in PRA
4. Spectra and angular coefficients for DY process in PRA
5. Predictions for NICA

Motivation

- The study of leading twist Collinear Parton Distribution Functions (PDFs) and Transverse Momentum Dependent (TMD) PDFs of quarks and antiquarks in nucleon is main task of Spin Physics Experiments at NICA-SPD with polarized proton and deuteron beams.
- It is estimated, that measurements of asymmetries of the DY pair's production will supply complete information for tests of the quark-parton model of nucleons at the QCD twist-two level with minimal systematic errors.
- To do it, we should construct precise model which based on quantum field theory, namely QCD.
- It is well known, that fixed order QCD calculation is not enough for description DY-pair spectra at low transverse momenta of DY-pair. We should introduce soft-gluon resummation procedure or TMD PDFs to regularized spectra at the region of $q_T << Q$, where $Q = \sqrt{q^2}$ is the virtuality of DY-pair.

Motivation



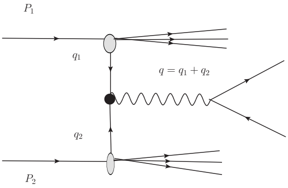
Sudakov (Light-cone) decomposition:
$n_-^\mu = 2P_1^\mu/\sqrt{S}$, $n_+^\mu = 2P_2^\mu/\sqrt{S}$, $n_\pm n_\pm = 0$, $n_\pm n_\mp = 2$:

$$k^\mu = \frac{1}{2}\left(k^+ n_-^\mu + k^- n_+^\mu\right) + k_T^\mu,$$

where $k^\pm = (n_\pm k) = k^0 \pm k^3$, $n_\pm k_T = 0$, and

$$kq = \frac{1}{2}(k^+ q^- + k^- q^+) - \mathbf{k}_T \mathbf{q}_T.$$

Motivation



$$q_1^\mu = \frac{1}{2}\left(q_1^+ n_-^\mu + q_1^- n_+^\mu\right) + q_{1T}^\mu,$$

and

$$q_2^\mu = \frac{1}{2}\left(q_2^+ n_-^\mu + q_2^- n_+^\mu\right) + q_{2T}^\mu,$$

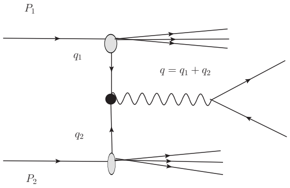
In CPM, TMD and
Multi-Regge-Kinematics (MRK):

$$q_1^+ \gg q_1^-$$

and

$$q_2^- \gg q_2^+$$

Motivation



and

$$q_1^\mu \simeq \frac{1}{2} q_1^+ n_-^\mu + q_{1T}^\mu,$$

$$q_1^2 \simeq q_{1T}^2 = -\mathbf{q_{1T}}^2$$

- We should deal with **off-shell initial-state** partons.
- The **gauge invariance** of QED/QCD amplitudes is under the question

Motivation

In CPM: $q_{1,2}^2 = q_{1,2T}^2 = 0$, gauge invariant amplitudes

In TMD: $|q_{1,2}^2|$ are neglected in amplitudes, $|q_{1,2T}^2| = {\mathbf{q}_{1,2T}}^2$ are included in TMD PDFs

In case of DY process we study region $q_T \ll Q$

Parton Reggeization Approach (PRA) is a hybrid scheme (CPM, TMD, **MRK**), which combines gauge-invariant matrix elements with off-shell (**Reggeized**) partons in the initial state with the unintegrated PDFs resumming doubly-logarithmic corrections $\sim \log^2(\mathbf{q}_T^2/Q^2)$.
PRA is based on *Lipatov's effective theory of Reggeized partons* and *modified-MRK-factorization of hard processes.*

$q_{1,2}^2 = q_{1,2T}^2 \neq 0$, + the **gauge invariance !**
In DY-pair production, **PRA** can be used at the **arbitrary** values of q_T/Q

LO PRA framework

LO PRA framework

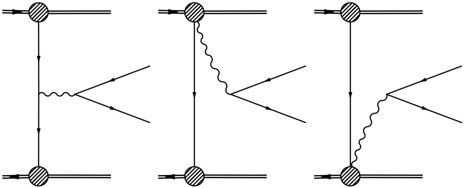
See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.

- **The aim** of **PRA** is to improve the description of **multi-scale** correlational observables ($q_T \sim Q$) in comparison with the *fixed-order CPM* calculations.
- **The wider task** is to understand the role of transverse momentum in Initial-State Radiation (ISR) and put it under theoretical control at the level of quantum field theory.
- To provide predictions with controllable accuracy and understand our formalism better we can go to NLO in **PRA**.

1. M. Nefedov and V. Saleev, "DIS structure functions in the NLO approximation of the Parton Reggeization Approach," EPJ Web Conf. **158** (2017) 03011.
2. M. Nefedov and V. Saleev, "On the one-loop calculations with Reggeized quarks," Mod. Phys. Lett. A **32** (2017) no.40, 1750207.

LO PRA framework

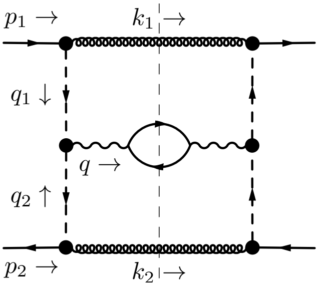
In order for **gauge invariance to be satisfied**, in addition to the annihilation diagram, one must also take into account the *direct interaction of the photon with the proton or its fragments*:



- Such a consideration in the general case does not lead to a simple factorization formula.
- However, in the **multi-Regge limit**, factorization is possible, since particles in the central region in rapidity perceive the particles flying forward and backward as Wilson lines along the light cone, i.e. the specific structure of the systems X_1 and X_2 is not important.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Auxiliary hard CPM subprocess:

$$q(p_1) + \bar{q}(p_2) \to g(k_1) + l\bar{l}(P_{\mathcal{A}}) + g(k_2),$$

Modified MRK approximation: $\underline{z_{1,2}}$ and $\underline{\mathbf{q}_{T1,2}^2}$ – arbitrary ($q_1^+ = z_1 p_1^+, \quad q_2^- = z_2 p_2^-$):

$$\overline{|\mathcal{M}|^2}_{\mathrm{mMRK}} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{qq}(z_1) P_{qq}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $q_{1,2}^2 = -\mathbf{q}_{T1,2}^2/(1 - z_{1,2})$, has correct **collinear** and **Multi-Regge** limits!

Conjecture: mMRK approximation is reasonable zero-approxiamtion for exact $\overline{|\mathcal{M}|^2}$ *away from collinear limit.*
At least, it is better than collinear limit itself.

Factorization formula

Substituting the $\overline{|\mathcal{M}|^2}_{\text{mMRK}}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_q(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_{\bar{q}}(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – "tree-level" **unintegrated PDFs**, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{\text{PRA}} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Note the usual **flux-factor** Sx_1x_2 for **off-shell** initial-state partons.

LO unintegrated PDF

The "tree-level" unPDF:

$$\tilde{\Phi}_q(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int\limits_x^1 dz\ P_{qq}(z)\cdot\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right).$$

contains collinear divergence at $t \to 0$ and IR divergence at $z \to 1$.
In the "dressed" unPDF collinear divergence is regulated by **Sudakov formfactor** $T(t,\mu^2)$:

$$\Phi_i(x,t,\mu^2) \quad = \quad \frac{T_i(t,\mu^2)}{t}\times\frac{\alpha_s(t)}{2\pi}\int\limits_x^1 dz\ \theta_z^{\text{cut}} P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},t\right)$$

where: $\theta_z^{\text{cut}} = \theta\left((1-\Delta_{KMR}(t,\mu^2)) - z\right)$, and the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess.

LO unintegrated PDF

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2)}{t} \times \frac{\alpha_s(t)}{2\pi} \int_x^1 dz\, \theta_z^{\text{cut}} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, t\right)$$

$$\simeq \boxed{\frac{\partial}{\partial t}\left[T_i(t,\mu^2)\cdot x f_i(x,t)\right]} \leftarrow \text{derivative form of unPDF}$$

$\Rightarrow$ **LO normalization condition:** (since $T(0,\mu^2,x) = 0$ and $T(\mu^2,\mu^2,x) = 1$)

$$\boxed{\int_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2)} \leftarrow$$

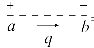
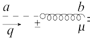
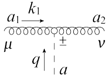
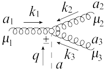
Holds approximately for standard KMR formula!

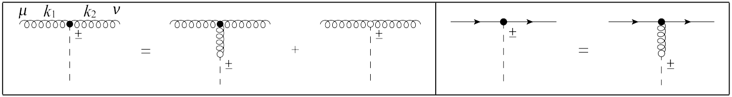
The normalization can be made exact by introducing the x-dependence to Sudakov factor.

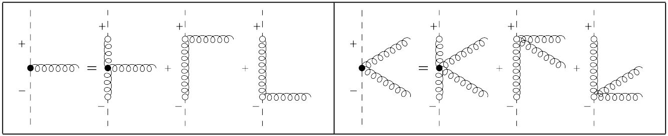
Gauge-invariant off-shell amplitudes

$\overline{|\mathcal{A}_{\rm PRA}|^2}$ is obtained from Lipatov's **gauge-invariant effective theory for MRK processes in QCD** [Lipatov 1995; Antonov, Lipatov, Kuraev, Cherednikov 2005].

Some Feynman rules for **Reggeized gluons:**

 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu}\right) \frac{q^2}{k_1^{\mp}}$
	$ig_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3}\right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}}\right]$





Gauge-invariant off-shell amplitudes

Some Feynman rules for Reggeized quarks [Lipatov, Vyazovsky, 2001]; [Fadin, Sherman, 1976]:

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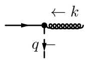
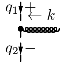
<td></td>
<td>$ig_s T^a \left(\gamma_\mu + \hat{q}\frac{n^+_\mu}{k^+}\right)$</td>
<td></td>
<td>$ig_s T^a \left(\gamma_\mu + \hat{q}_2\frac{n^+_\mu}{k^+} + \hat{q}_1\frac{n^-_\mu}{k^-}\right)$</td>
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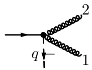
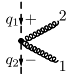
<td></td>
<td>$ig_s^2(n^+_{\mu_1}n^+_{\mu_2})\hat{q}\left[\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+} + \frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right]$</td>
<td></td>
<td>$$ig_s^2\left[\hat{q}_2(n^+_{\mu_1}n^+_{\mu_2})\left(\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+} + \frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right) - \hat{q}_1(n^-_{\mu_1}n^-_{\mu_2})\left(\frac{T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-} + \frac{T^{a_1}T^{a_2}}{k_2^-(k_1+k_2)^-}\right)\right]$$</td>
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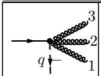
<td></td>
<td colspan="3">$$ig_s^3\hat{q}(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left[\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+} + (1 \leftrightarrow 2 \leftrightarrow 3)\right]$$</td>
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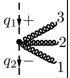
<td></td>
<td colspan="3">$$ig_s^3\left[\hat{q}_2(n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left(\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+} + (1 \leftrightarrow 2 \leftrightarrow 3)\right) + \hat{q}_1(n^-_{\mu_1}n^-_{\mu_2}n^-_{\mu_3})\left(\frac{T^{a_3}T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-(k_1+k_2+k_3)^-} + (1 \leftrightarrow 2 \leftrightarrow 3)\right)\right]$$</td>
</tr>
</table>

Reggeized amplitudes

The first use of Lipatov's effective vertices for Reggeized gluons:
P. Hagler, R. Kirschner, A. Schafer, L. Szymanowski and O. Teryaev, "Heavy quark production as sensitive test for an improved description of high-energy hadron collisions," Phys. Rev. D **62** (2000) 071502.

The first use for Reggeized quarks:
V. A. Saleev, "Prompt photon photoproduction at HERA within the framework of the quark Reggeization hypothesis," Phys. Rev. D **78** (2008) 114031
V. A. Saleev, "Deep inelastic scattering and prompt photon production within the framework of quark Reggeization hypothesis," Phys. Rev. D **78** (2008) 034033

The first use for description of DY process:
M. A. Nefedov, N. N. Nikolaev and V. A. Saleev, "Drell-Yan lepton pair production at high energies in the Parton Reggeization Approach," Phys. Rev. D **87** (2013) no.1, 014022

Structure functions for DY process in PRA

Structure functions for DY process in PRA

$$\frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} = \frac{\alpha^2}{64\pi^3 S Q^4} L_{\mu\nu} W^{\mu\nu},$$

where y is the rapidity of virtual photon (or l^+l^- lepton pair), $d\Omega = d\phi d\cos\theta$ is the spatial angle of producing positive lepton in the rest frame of virtual photon.

The convolution of hadronic and leptonic tensors reads as a sum of contributions of the so-called helicity structure functions $W_{T,L,\Delta\Delta}$:

$$\begin{aligned}\frac{d\sigma}{dQ^2 dq_T^2 dy d\Omega} &= \frac{\alpha^2}{64\pi^3 S Q^2}\Big[W_T \cdot (1+\cos^2\theta) + W_L \cdot (1-\cos^2\theta) + \\ &+ W_{\Delta\Delta} \cdot \sin^2\theta \cos 2\phi\Big].\end{aligned}$$

$$\begin{aligned}\frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} &= \frac{\alpha^2}{4Q^2}\Big[F_{UU}^{(1)} \cdot \big(1+\cos^2\theta\big) + F_{UU}^{(2)} \cdot \big(1-\cos^2\theta\big) + \\ &+ F_{UU}^{(\cos 2\phi)} \cdot \sin^2\theta \cos(2\phi)\Big],\end{aligned}$$

were $x_{A,B} = Qe^{\pm y}/\sqrt{S}$

Structure functions for DY process in PRA

In the analysis of experimental data, the angular distribution of leptons is represented in terms of two sets of the angular coefficients:

$$\frac{dN}{d\Omega} \quad = \quad (1+\cos^2\theta) + A_0\left(\frac{1}{2}-\frac{3}{2}\cos^2\theta\right) + \frac{A_2}{2}\sin^2\theta\cos 2\phi$$

and

$$\frac{dN}{d\Omega} \quad = \quad \frac{4}{\lambda+3}\left(1+\lambda\cos^2\theta+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right),$$

with the normalization

$$\int\left(\frac{dN}{d\Omega}\right)d\Omega = \frac{16\pi}{3}.$$

One set consists of the coefficients

$$A_0 = \frac{W_L}{W_{TL}}, \quad A_2 = \frac{2W_{\Delta\Delta}}{W_{TL}}, \quad W_{TL} = W_T + W_L/2,$$

and the other one is defined by

$$\lambda = \frac{2-3A_0}{2+A_0}, \quad \nu = \frac{2A_2}{2+A_0}.$$

Structure functions for DY process in PRA

The squared amplitude of the subprocess ($Q_i\bar{Q}_i \to l^+l^-$):

$$\overline{|M(Q_i\bar{Q}_i \to l^+l^-)|^2} = \frac{16\pi^2}{3Q^4}\alpha^2 e_i^2 L^{\mu\nu} w_{\mu\nu}^{PRA},$$

where the partonic tensor of Reggeized quarks reads:

$$\begin{aligned} w_{\mu\nu}^{PRA} &= x_1x_2\Big[-Sg^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu)\frac{(2x_1x_2S - Q^2 - t_1 - t_2)}{x_1x_2S} + \\ &+ \frac{2}{x_2}(q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1}(q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \\ &+ \frac{4(t_1 - x_1x_2S)}{Sx_2^2}P_1^\mu P_1^\nu + \frac{4(t_2 - x_1x_2S)}{Sx_1^2}P_2^\mu P_2^\nu\Big]. \end{aligned}$$

We define the quark helicity structure functions $w_{T,L,\Delta\Delta}$ respectively to the hadron helicity structure functions $W_{T,L,\Delta\Delta}$. Upon direct calculations we obtain

$$w_T^{PRA} = Q^2 + \frac{(\mathbf{q}_{1T} + \mathbf{q}_{2T})^2}{2}, \quad w_L^{PRA} = (\mathbf{q}_{1T} - \mathbf{q}_{2T})^2, \quad w_{\Delta\Delta}^{PRA} = \frac{(\mathbf{q}_{1T} + \mathbf{q}_{2T})^2}{2}$$

Structure functions for DY process in PRA

The helicity structure functions $W^{PRA}_{T,\dots}$ at the fixed values of variables S, Q, q_T, y can be presented via corresponding Reggeized quark helicity functions $w^{PRA}_{T,\dots}$:

$$W^{PRA}_{T,\dots}(S,Q,q_T,y) = \frac{8\pi^2 S}{3Q_T^4}\int dt_1 \int d\phi_1 \sum_q \Phi_q^p(x_1,t_1,\mu^2)\Phi_{\bar{q}}^p(x_2,t_2,\mu^2)w^{PRA}_{T,\dots},$$

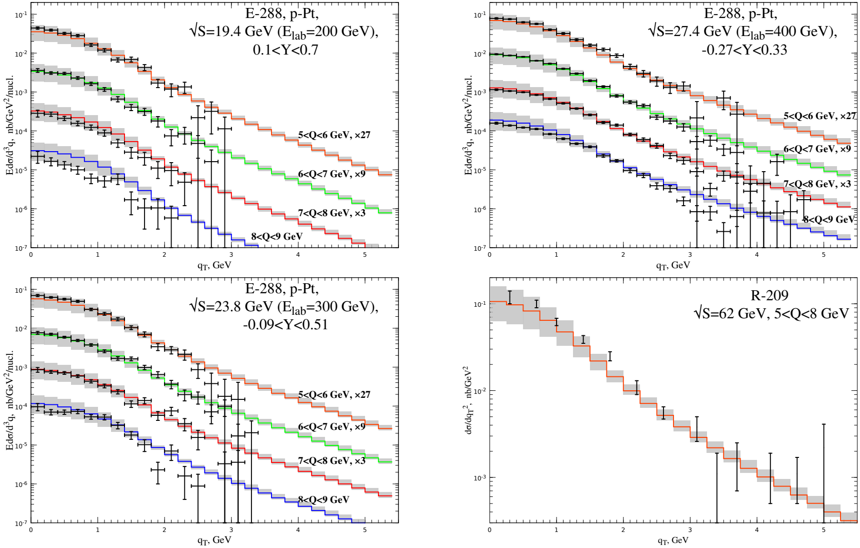
where $Q_T^2 = Q^2 + q_T^2$.
For the calculation of cross-sections, the π^2-resummation K-factor is included:

$$K = \exp\left(C_F \frac{\alpha_s(\mu_K^2)}{2\pi}\pi^2\right),$$

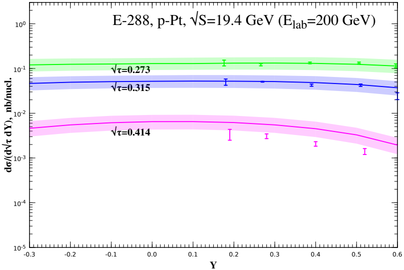
with $\mu_K^2 = Q^{2/3}Q_T^{4/3}$, and $\mu_F = Q_T$. The theoretical uncertainty was obtained by independent variation of this scales. Typical values of K-factor are 1.3 – 1.8.

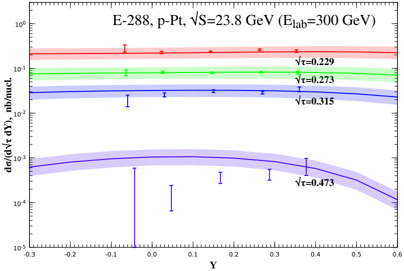
Spectra and angular coefficients for DY process in PRA

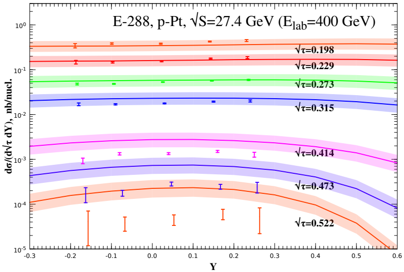
Spectra and angular coefficients for DY process in PRA



Spectra and angular coefficients for DY process in PRA

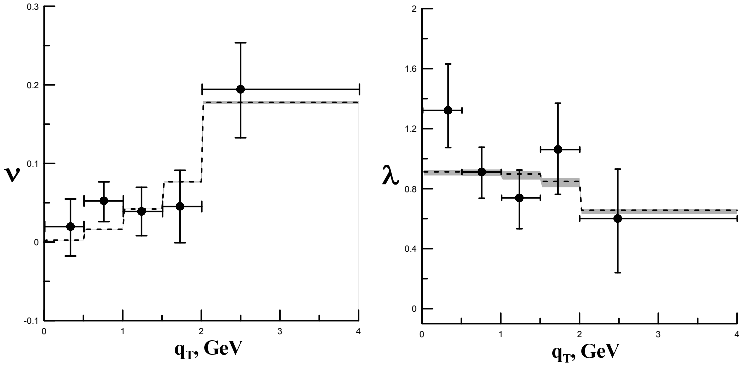






$$\tau = \frac{Q^2}{S} = x_A \times x_B$$

Spectra and angular coefficients for DY process in PRA



Angular coefficients ν and λ as function of q_T. The histogram corresponds to LO calculation in PRA. The data are from NuSea Collaboration ($E_{lab} = 800$ GeV, $\sqrt{S} = 39$ GeV, $4.5 < Q < 15$ GeV, $0 < x_F < 0.8$).

Predictions for NICA

Predictions for NICA

Predictions of TMD-factorization:

$$F_{UU}^{(1)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot f_1^q(x_1, q_{T1}) f_1^{\bar{q}}(x_2, q_{T2}) + O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(\cos 2\phi)} \sim \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \cdot h_1^{\perp q}(x_1, q_{T1}) h_1^{\perp \bar{q}}(x_2, q_{T2}) \times$$

$$\times \quad \frac{2(\mathbf{q}_T\mathbf{q}_{T1})(\mathbf{q}_T\mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1}\mathbf{q}_{T2})}{\mathbf{q}_T^2 \Lambda^2} + O\left(\frac{q_T^2}{Q^2}\right),$$

where $f_1^q(x, q_T)$ and $h_1^{\perp q}(x, q_T)$ are unpolarized and Boer-Mulders leading twist TMD PDFS.

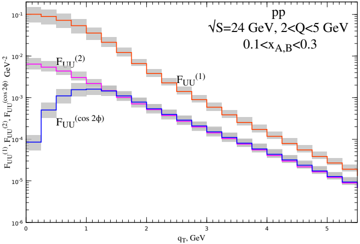
PRA predicts:

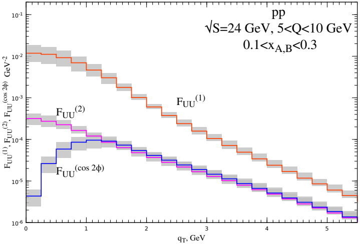
$$F_{UU}^{(1)} \sim O(1), \ F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right),$$

in agreement with TMD-factorization, but also:

$$F_{UU}^{(\cos 2\phi)} \sim F_{UU}^{(2)} \sim O\left(\frac{q_T^2}{Q^2}\right)!$$

Predictions for NICA





$$\frac{F_{UU}^{(2)}}{F_{UU}^{(1)}} \sim \frac{\Lambda^2}{Q^2},$$

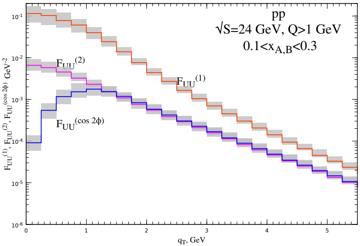
where Λ – hadronic scale.

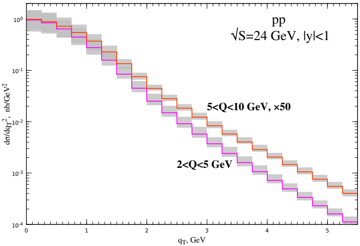
$$\frac{F_{UU}^{(\cos 2\phi)}}{F_{UU}^{(1)}} \sim \frac{q_T^2}{Q^2}, \qquad F_{UU}^{(\cos 2\phi)}(q_T = 0) = 0,$$

$$F_{UU}^{(2)} \simeq F_{UU}^{(2)} \simeq F_{UU}^{(\cos 2\phi)}, \qquad q_T \gg Q$$

Observation of sizable $F_{UU}^{(\cos 2\phi)}$ at small q_T would be very difficult to describe in the formalism, which combines **factorization** and **QED gauge-inveriance**!

Predictions for NICA





Conclusions

- PRA can be applied for study of DY process at energy range of NICA!
- PRA takes into account effects of transverse momenta and off-shell effects in the initial state **in a gauge-invariant way.**
- PRA predicts some $O\left(q_T/Q\right)$ quantities at $q_T << Q$, such as $F_{UU}^{(2)}$ and $F_{UU}^{(\cos 2\phi)}$.
- PRA can be applied at all values of q_T/Q
- In PRA, we can do NLO calculations at the all q_T/Q, and specially, at the $q_T \leq Q$.
- Proton polarization effects can be included.

Thank you for your attention!