Kinetics of polarization and filtering

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Outline

- Motivation: possibilities to polarize antiproton beams.
- General form of a kinetic equation for spin observables.
- Spin evolution in scattering processes — equations and solutions.
- Spin evolution for the case when some particles are scattered out of the beam — equations and solutions.
- Kinetics of polarization in $p\bar{p}$ scattering — predictions within different models.
- A possibility to improve theoretical predictions for $p\bar{p}$ scattering cross sections.

PAX Collaboration proposed experiments with polarized antiprotons and protons.

Lenisa P., Rathmann F., et. al., arXiv:hep-ex/0505054.

- The transversity distribution is directly accessible via the double transverse spin asymmetry A_{TT} in the Drell-Yan production of lepton pairs.
- The relative phase of the magnetic and electric form factors of the proton in the time-like region can only be measured in the annihilation $p\bar{p} \to e^+e^-$ on a transversely polarized target.
- And more.

How to polarize antiproton beam?

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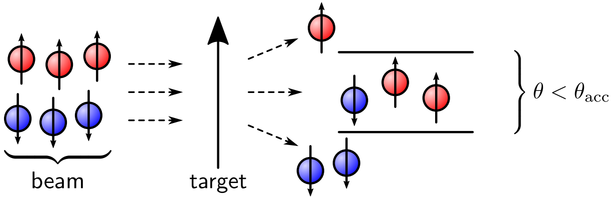
- The transversity distribution is directly accessible via the double transverse spin asymmetry A_{TT} in the Drell-Yan production of lepton pairs.
- The relative phase of the magnetic and electric form factors of the proton in the time-like region can only be measured in the annihilation $p\bar{p} \to e^+e^-$ on a transversely polarized target.
- And more.

There are two processes that result in polarization build-up: spin flip and spin filtering

However, spin flip cross sections are negligibly small to polarize antiprotons.

Milstein A. I., Strakhovenko V. M., Phys. Rev. E 72, 66503 (2005).
Milstein A. I., Salnikov S. G., Strakhovenko V. M., NIMB 266, 3453 (2008).
Oellers D., et al., Phys. Lett. B 674, 269 (2009). (COSY experiment with protons)



- Proposed in 1968 to polarize proton beams.

  Csonka P. L., Nucl. Instruments Methods 63, 247 (1968).

- Was proved to be applicable to pp scattering at $T_{\rm lab} = 49.3\,{\rm MeV}$.

  Augustyniak W., et al., Phys. Lett. B 718, 64 (2012). *(COSY experiment)*

- No experiments with antiprotons have been carried out so far.

Kinetic equation

For arbitrary operator $\mathcal{O}$ that doesn't depend on $\boldsymbol{p}$ and $\boldsymbol{r}$: $\dfrac{d}{dt}\mathcal{O}_H = i\,[H, \mathcal{O}_H]$

Average over $\psi_{\boldsymbol{k}}(\boldsymbol{r}) \sim \sqrt{N}\left(e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \dfrac{e^{ikr}}{r} F(\boldsymbol{n}, \boldsymbol{n}_0)\right)\chi_1\chi_2\,,$ $\boldsymbol{n}_0 = \boldsymbol{k}/k$, $\boldsymbol{n} = \boldsymbol{r}/r$

$$\frac{d}{dt}\langle\mathcal{O}\rangle = vN\,\mathrm{Sp}\left\{\rho(t)\left[\int d\Omega_{\boldsymbol{n}} F^+\mathcal{O}F - \frac{2\pi i}{k}\left(F^+(0)\mathcal{O} - \mathcal{O}F(0)\right)\right]\right\}$$

$$\mathrm{Sp}\left\{\rho(t)\left[\int d\Omega_{\boldsymbol{n}} F^+F - \frac{2\pi i}{k}\left(F^+(0) - F(0)\right)\right]\right\} = 0$$ – unitarity relation

- $\rho(t) = \rho_1(t)\cdot\rho_2(t)$ is the density matrix of the system
- 1st particle belongs to the beam
- 2nd particle belongs to the target
- $F(\boldsymbol{n}, \boldsymbol{n}_0)$ is the scattering amplitude (matrix in the spin space)
- N is the density of the target
- v is the beam velocity

These equations are valid if the number of particles is conserved ($N_B = \text{const}$).

Kinetic equation

For arbitrary operator $\mathcal{O}$ that doesn't depend on $\boldsymbol{p}$ and $\boldsymbol{r}$: $\frac{d}{dt}\mathcal{O}_H = i\,[H, \mathcal{O}_H]$

Average over $\psi_{\boldsymbol{k}}(\boldsymbol{r}) \sim \sqrt{N}\left(e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \frac{e^{ikr}}{r} F(\boldsymbol{n}, \boldsymbol{n}_0)\right)\chi_1\chi_2\,,$ $\quad \boldsymbol{n}_0 = \boldsymbol{k}/k$, $\boldsymbol{n} = \boldsymbol{r}/r$

$$\frac{d}{dt}\langle\mathcal{O}\rangle = vN\,\mathrm{Sp}\left\{\rho(t)\left[\int d\Omega_{\boldsymbol{n}} F^+\mathcal{O}F - \frac{2\pi i}{k}\left(F^+(0)\mathcal{O} - \mathcal{O}F(0)\right)\right]\right\}$$

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The kinetic equations can be generalized to the case of filtering ($N_B \neq$ const):

$$\frac{d}{dt}\langle\mathcal{O}\rangle = vN\,\mathrm{Sp}\left\{\rho(t)\left[\int_{\theta<\theta_{\mathrm{acc}}} d\Omega_{\boldsymbol{n}}\, F^+\left(\mathcal{O} - \langle\mathcal{O}\rangle\right)F - \frac{2\pi i}{k}\left(F^+(0)\left(\mathcal{O} - \langle\mathcal{O}\rangle\right) - \left(\mathcal{O} - \langle\mathcal{O}\rangle\right)F(0)\right)\right]\right\}$$

$$\frac{dn}{dt} = vN\,\mathrm{Sp}\left\{\rho(t)\left[\int_{\theta<\theta_{\mathrm{acc}}} d\Omega_{\boldsymbol{n}}\, F^+F - \frac{2\pi i}{k}\left(F^+(0) - F(0)\right)\right]\right\},\quad n(t) = \frac{N_B(t)}{N_B(0)}$$

Spin evolution for $S_1 = \frac{1}{2}$, $S_2 = 0$ ($N_B = \text{const}$)

$$\rho_1(t) = \frac{1}{2}\left(1 + \boldsymbol{\zeta}_1(t)\cdot\boldsymbol{\sigma}_1\right), \qquad \rho_2 = 1$$

$$F = f_0 + f_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\nu}, \qquad \boldsymbol{\nu} = [\boldsymbol{n} \times \boldsymbol{n}_0], \qquad f_0,\ f_1 \text{ are scalar functions}$$

- The unitarity relation reduces to the optical theorem:

$$\int d\Omega_{\boldsymbol{n}} \left[\,|f_0|^2 + |f_1|^2\,\nu^2\right] = \frac{4\pi}{k}\,\mathrm{Im}\ f_0(0)$$

- The spin relaxation is determined by the following equation:

$$\frac{d}{dt}\boldsymbol{\zeta}_1 = -\Omega\left[\boldsymbol{\zeta}_1 + (\boldsymbol{\zeta}_1\cdot\boldsymbol{n}_0)\boldsymbol{n}_0\right], \qquad \Omega = vN\int d\Omega_{\boldsymbol{n}}\nu^2|f_1|^2$$

The general solution of this equation is

$$\boldsymbol{\zeta}_1(t) = \underbrace{\left[\boldsymbol{\zeta}_1(0) - \boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right)\right]e^{-\Omega t}}_{\perp \text{ to } \boldsymbol{n}_0} + \underbrace{\boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right)e^{-2\Omega t}}_{\parallel \text{ to } \boldsymbol{n}_0}$$

During relaxation not only diminishing of $\zeta_1(t)$ takes place but also rotation of the direction of $\boldsymbol{\zeta}_1(t)$.

Spin evolution for $S_1 = \frac{1}{2}$, $S_2 = \frac{1}{2}$ ($N_B = \text{const}$)

$$\rho_1(t) = \frac{1}{2}\left(1 + \boldsymbol{\zeta}_1(t)\cdot\boldsymbol{\sigma}_1\right), \qquad \rho_2 = \frac{1}{2}\left(1 + \boldsymbol{\zeta}_2\cdot\boldsymbol{\sigma}_2\right)$$

$$F = f_0 + (f_1\boldsymbol{\sigma}_1 + f_2\boldsymbol{\sigma}_2)\cdot\boldsymbol{\nu} + T^{ij}\sigma_1^i\sigma_2^j\,, \qquad \boldsymbol{\nu} = [\boldsymbol{n}\times\boldsymbol{n}_0]$$

$$T^{ij} = f_3\delta^{ij} + f_4\left(n_0^i n_0^j + n^i n^j\right) + f_5\left(n_0^i n^j + n^i n_0^j\right)$$

- Unitarity relations:

$$\int d\Omega_{\boldsymbol{n}}\left[|f_0|^2 + \nu^2|f_1|^2 + \nu^2|f_2|^2 + T^{ab*}T^{ab}\right] = \frac{4\pi}{k}\,\mathrm{Im}\,f_0(0)$$

$$\int d\Omega_{\boldsymbol{n}}\left[2\,\mathrm{Re}\left(f_0^*T^{ab}\right) + 2\mathrm{Re}(f_1^*f_2)\nu^a\nu^b - T^{ij*}T^{\alpha\beta}\epsilon^{i\alpha a}\epsilon^{j\beta b}\right] = \frac{4\pi}{k}\,\mathrm{Im}\,T^{ab}(0)$$

- Spin evolution is determined by the following equation:

$$\frac{d}{dt}\boldsymbol{\zeta}_1 = A_1\boldsymbol{\zeta}_1 + B_1\left(\boldsymbol{\zeta}_1\cdot\boldsymbol{n}_0\right)\boldsymbol{n}_0 + A_2\left[\boldsymbol{\zeta}_1\times\boldsymbol{\zeta}_2\right] + B_2\left(\boldsymbol{\zeta}_2\cdot\boldsymbol{n}_0\right)\left[\boldsymbol{\zeta}_1\times\boldsymbol{n}_0\right] + A_3\boldsymbol{\zeta}_2 + B_3\left(\boldsymbol{\zeta}_2\cdot\boldsymbol{n}_0\right)\boldsymbol{n}_0$$

The coefficients A_i, B_i can be expressed via f_i. $A_1 < 0$, $A_1 + B_1 < 0$

Spin evolution for $S_1 = \frac{1}{2}$, $S_2 = \frac{1}{2}$ (special solutions)

$\zeta_2 = 0$

$$\boldsymbol{\zeta}_1(t) = \Big[\boldsymbol{\zeta}_1(0) - \boldsymbol{n}_0\,(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0)\Big]\, e^{A_1 t} + \boldsymbol{n}_0\,(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0)\, e^{(A_1+B_1)t} \qquad \text{(as for } S_2 = 0\text{)}$$

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$\zeta_2 \parallel \boldsymbol{n}_0$

$$\boldsymbol{\zeta}_1(t) = \left(e^{(A_1+B_1)t} - 1\right) \boxed{\frac{A_3+B_3}{A_1+B_1}\boldsymbol{\zeta}_2} + e^{A_1 t}\left[\cos(\omega t)\boldsymbol{\zeta}_1(0) + \sin(\omega t)\left[\boldsymbol{\zeta}_1(0)\times\boldsymbol{n}_0\right]\right]$$
$$+ \left[e^{(A_1+B_1)t} - e^{A_1 t}\cos(\omega t)\right]\boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right), \qquad \omega = (A_2+B_2)\,\zeta_2$$

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$\zeta_2 \perp \boldsymbol{n}_0$

$$\boldsymbol{\zeta}_1(t) = \left(e^{A_1 t} - 1\right)\boxed{\frac{A_3}{A_1}\boldsymbol{\zeta}_2} + e^{(A_1+B_1/2)t}\cos(\Omega t)\boldsymbol{\zeta}_1(0) + e^{(A_1+B_1/2)t}\sin(\Omega t)\times$$
$$\times\left\{\frac{A_2}{\Omega}\,[\boldsymbol{\zeta}_1(0)\times\boldsymbol{\zeta}_2] + \frac{B_1}{2\Omega}\left[(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0)\,\boldsymbol{n}_0 - \frac{(\boldsymbol{\zeta}_1(0)\cdot[\boldsymbol{\zeta}_2\times\boldsymbol{n}_0])}{\zeta_2^2}\,[\boldsymbol{\zeta}_2\times\boldsymbol{n}_0]\right]\right\}$$
$$+ \left(e^{A_1 t} - e^{(A_1+B_1/2)t}\cos(\Omega t)\right)\frac{(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{\zeta}_2)}{\zeta_2^2}\boldsymbol{\zeta}_2\,, \qquad \Omega = \sqrt{A_2^2\zeta_2^2 - B_1^2/4}$$

Spin evolution for $S_1 = \frac{1}{2}$, $S_2 = \frac{1}{2}$ (special solutions)

$\zeta_2 = 0$

$$\boldsymbol{\zeta}_1(t) = \left[\boldsymbol{\zeta}_1(0) - \boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right)\right] e^{A_1 t} + \boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right) e^{(A_1+B_1)t} \qquad \text{(as for } S_2 = 0\text{)}$$

$\boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0$

$$\boldsymbol{\zeta}_1(t) = \left(e^{(A_1+B_1)t} - 1\right) \boxed{\frac{A_3 + B_3}{A_1 + B_1}\boldsymbol{\zeta}_2} + e^{A_1 t}\left[\cos(\omega t)\boldsymbol{\zeta}_1(0) + \sin(\omega t)\left[\boldsymbol{\zeta}_1(0)\times\boldsymbol{n}_0\right]\right]$$
$$+ \left[e^{(A_1+B_1)t} - e^{A_1 t}\cos(\omega t)\right]\boldsymbol{n}_0\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right), \qquad \omega = (A_2 + B_2)\,\zeta_2$$

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$$\boldsymbol{\zeta}_1(t) = \left(e^{A_1 t} - 1\right)\boxed{\frac{A_3}{A_1}\boldsymbol{\zeta}_2} + e^{(A_1+B_1/2)t}\cos(\Omega t)\boldsymbol{\zeta}_1(0) + e^{(A_1+B_1/2)t}\sin(\Omega t)\times$$
$$\times\left\{\frac{A_2}{\Omega}\left[\boldsymbol{\zeta}_1(0)\times\boldsymbol{\zeta}_2\right] + \frac{B_1}{2\Omega}\left[\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{n}_0\right)\boldsymbol{n}_0 - \frac{\left(\boldsymbol{\zeta}_1(0)\cdot\left[\boldsymbol{\zeta}_2\times\boldsymbol{n}_0\right]\right)}{\zeta_2^2}\left[\boldsymbol{\zeta}_2\times\boldsymbol{n}_0\right]\right]\right\}$$
$$+\left(e^{A_1 t} - e^{(A_1+B_1/2)t}\cos(\Omega t)\right)\frac{\left(\boldsymbol{\zeta}_1(0)\cdot\boldsymbol{\zeta}_2\right)}{\zeta_2^2}\boldsymbol{\zeta}_2, \qquad \Omega = \sqrt{A_2^2\zeta_2^2 - B_1^2/4}$$

Kinetics of polarization in $e^+\bar{p}$ scattering

- All of the coefficients were calculated analytically. For the most interesting case $\xi = -\dfrac{\alpha}{v} \to -\infty$:

$$A_1 = -2\pi N v \left(\frac{\alpha\mu_0}{m_p}\right)^2 \left(\pi^2 \xi^2 \left(\frac{25}{3} - 6\ln 2\right) + \ln\frac{l_{\max}}{|\xi|}\right), \quad A_3 = 4\pi N v \left(\frac{\alpha\mu_0}{m_p}\right)^2 \pi^2 \xi^2 \left(\frac{1}{3} + 2\ln 2\right)$$

$$B_1 = -2\pi N v \left(\frac{\alpha\mu_0}{m_p}\right)^2 \left(\pi^2 \xi^2 (2\ln 2 - 1) + \ln\frac{l_{\max}}{|\xi|}\right), \quad B_3 = 4\pi N v \left(\frac{\alpha\mu_0}{m_p}\right)^2 \pi^2 \xi^2 (7 - 10\ln 2)$$

$$\boldsymbol{\zeta}_1(t) = P_0 \boldsymbol{\zeta}_2 \left(1 - e^{-\Omega t}\right), \qquad \Omega = v N l f \frac{\gamma_B^2}{V_B} \cdot \sigma$$

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$$\boldsymbol{\zeta}_1(t) = P_0 \boldsymbol{\zeta}_2 \left(1 - e^{-\Omega t}\right), \qquad \Omega = vNlf\frac{\gamma_B^2}{V_B}\cdot\sigma$$

$$P_0 = 0.75\,, \qquad \sigma = 0.8\,\mathrm{mb}\,, \qquad \text{if } \boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0$$

$$P_0 = 0.8\,, \qquad \sigma = 0.72\,\mathrm{mb}\,, \qquad \text{if } \boldsymbol{\zeta}_2 \perp \boldsymbol{n}_0$$

Time of polarization is about 10^9 years

$$v/c = 0.0019\,, \quad V_b/c = 0.95\,, \quad N = 1.4\cdot 10^4\,\mathrm{cm}^{-3}\,, \quad l = 2\,\mathrm{m}\,, \quad \rho = 2\,\mathrm{mm}\,, \quad f = 0.71\mathrm{MHz}$$

Milstein A. I., Salnikov S. G., Strakhovenko V. M., NIMB 266, 3453 (2008).

Spin evolution in pp ($p\bar{p}$) scattering ($N_B \neq \text{const}$)

Particles are scattered out of the beam if the scattering angle $\theta > \theta_{\text{acc}}$ ($\theta_{\text{acc}} \ll 1$).

$$\rho_1(t) = \frac{1}{2}\left(1 + \boldsymbol{\zeta}_1(t)\cdot\boldsymbol{\sigma}_1\right), \qquad \rho_2 = \frac{1}{2}\left(1 + \boldsymbol{\zeta}_2\cdot\boldsymbol{\sigma}_2\right)$$

$$F = \underbrace{f_C}_{\text{singular}} + f_0 + f_1\,\underbrace{(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)}_{\boldsymbol{S}}\cdot\boldsymbol{\nu} + T^{ij}\sigma_1^i\sigma_2^j\,, \qquad \boldsymbol{\nu} = [\boldsymbol{n}\times\boldsymbol{n}_0]$$

$$\frac{dn}{dt} = -vN\sigma\cdot n$$

$$\sigma = \sigma_0 + \sigma_1\,(\boldsymbol{\zeta}_1\cdot\boldsymbol{\zeta}_2) + (\sigma_2 - \sigma_1)\,(\boldsymbol{\zeta}_1\cdot\boldsymbol{n}_0)\,(\boldsymbol{\zeta}_2\cdot\boldsymbol{n}_0)\,, \qquad \sigma_i = \sigma_i^C + \sigma_i^{\text{h}} + \sigma_i^{\text{int}}$$

Kinetic equation

$$\frac{d}{dt}\boldsymbol{\zeta}_1 = A_1\left[(\boldsymbol{\zeta}_1\cdot\boldsymbol{\zeta}_2)\,\boldsymbol{\zeta}_1 - \boldsymbol{\zeta}_2\right] + B_1\,(\boldsymbol{n}_0\cdot\boldsymbol{\zeta}_2)\left[(\boldsymbol{\zeta}_1\cdot\boldsymbol{n}_0)\,\boldsymbol{\zeta}_1 - \boldsymbol{n}_0\right] + A_2\left[\boldsymbol{\zeta}_1\times\boldsymbol{\zeta}_2\right] + B_2\,(\boldsymbol{n}_0\cdot\boldsymbol{\zeta}_2)\left[\boldsymbol{\zeta}_1\times\boldsymbol{n}_0\right]$$

$$A_1 = vN\cdot\sigma_1\,, \qquad A_2 = vN\cdot\mathcal{R}_1$$

$$B_1 = vN\cdot(\sigma_2 - \sigma_1)\,, \qquad B_2 = vN\cdot(\mathcal{R}_2 - \mathcal{R}_1)$$

- $\boldsymbol{\zeta}_2 = 0 \quad \Longrightarrow \quad \boldsymbol{\zeta}_1 =$ const
- $\boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0$ or $\boldsymbol{\zeta}_2 \perp \boldsymbol{n}_0$

$$\frac{d}{dt}\boldsymbol{\zeta}_1 = \lambda\left((\boldsymbol{\zeta}_1\cdot\hat{\boldsymbol{\zeta}}_2)\boldsymbol{\zeta}_1 - \hat{\boldsymbol{\zeta}}_2\right) + \omega\left[\boldsymbol{\zeta}_1\times\hat{\boldsymbol{\zeta}}_2\right], \qquad \hat{\boldsymbol{\zeta}}_2 = \boldsymbol{\zeta}_2/\zeta_2$$

$$\lambda = vN\zeta_2\cdot\sigma_1\,, \qquad \omega = vN\zeta_2\cdot\mathcal{R}_1\,, \qquad \text{if } \boldsymbol{\zeta}_2\perp\boldsymbol{n}_0$$

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Spin evolution in pp ($p\bar{p}$) scattering (special solutions)

- $\zeta_2 = 0 \quad \Longrightarrow \quad \zeta_1 =$ const
- $\boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0$ or $\boldsymbol{\zeta}_2 \perp \boldsymbol{n}_0$

$$\frac{d}{dt}\boldsymbol{\zeta}_1 = \lambda\Big((\boldsymbol{\zeta}_1\cdot\hat{\boldsymbol{\zeta}}_2)\boldsymbol{\zeta}_1 - \hat{\boldsymbol{\zeta}}_2\Big) + \omega\left[\boldsymbol{\zeta}_1 \times \hat{\boldsymbol{\zeta}}_2\right], \qquad \hat{\boldsymbol{\zeta}}_2 = \boldsymbol{\zeta}_2/\zeta_2$$

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$$\lambda = vN\zeta_2\cdot\sigma_2\,, \qquad \omega = vN\zeta_2\cdot\mathcal{R}_2\,, \qquad \text{if } \boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0$$

$\zeta_1(0) = 0$ or $\zeta_1(0) \parallel \zeta_2$

$$\frac{d}{dt}\zeta_1 = \lambda\left(\zeta_1^2 - 1\right) \qquad \Longrightarrow \qquad \zeta_1(t) = -\tanh\big(\lambda t - \operatorname{artanh}\zeta_1(0)\big)$$

$\zeta_1(0) \nparallel \zeta_2$

$$\boldsymbol{\zeta}_1(t) = -\tanh\Big(\lambda t - \operatorname{artanh}\big(\boldsymbol{\zeta}_1(0)\cdot\hat{\boldsymbol{\zeta}}_2\big)\Big)\cdot\hat{\boldsymbol{\zeta}}_2 - \frac{\sin(\omega t)\cdot\left[\hat{\boldsymbol{\zeta}}_2\times\boldsymbol{\zeta}_1(0)\right] + \cos(\omega t)\cdot\left[\hat{\boldsymbol{\zeta}}_2\times\left[\hat{\boldsymbol{\zeta}}_2\times\boldsymbol{\zeta}_1(0)\right]\right]}{\cosh\Big(\lambda t - \operatorname{artanh}\big(\boldsymbol{\zeta}_1(0)\cdot\hat{\boldsymbol{\zeta}}_2\big)\Big)\sqrt{1-\big(\boldsymbol{\zeta}_1(0)\cdot\hat{\boldsymbol{\zeta}}_2\big)^2}}$$

$$\boldsymbol{\zeta}_1(t) = -\tanh(\lambda t) \cdot \hat{\boldsymbol{\zeta}}_2$$

$$\lambda = \begin{cases} N_a f \zeta_2 \cdot \sigma_1 , & \text{if } \boldsymbol{\zeta}_2 \perp \boldsymbol{n}_0 \\ N_a f \zeta_2 \cdot \sigma_2 , & \text{if } \boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0 \end{cases}$$

$$n(t) = n(0) \cdot e^{-t/\tau_B}$$

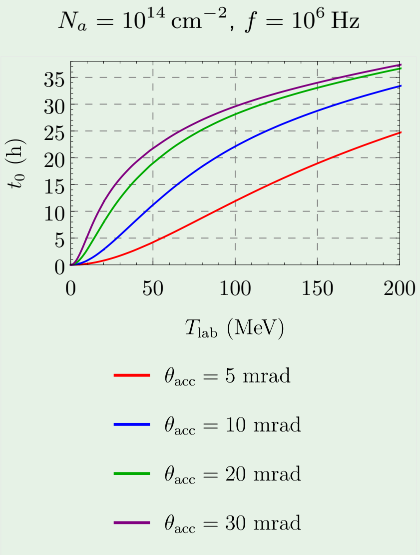
$$\tau_B \approx 1/(N_a f \sigma_0)$$

N_a is the areal density of the target,
f is the beam revolving frequency.

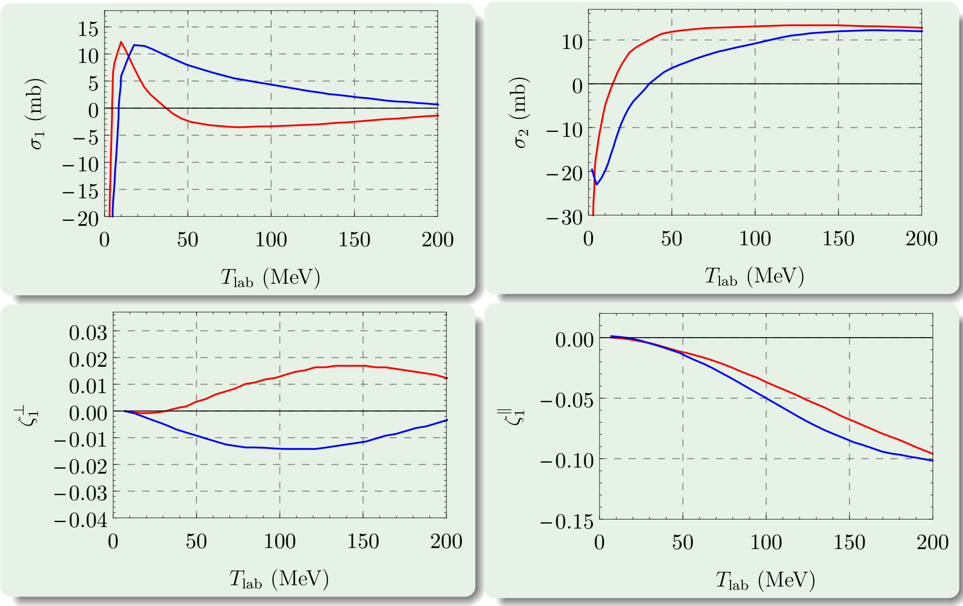
The optimum ratio between the beam polarization and the number of particles at $t = t_0 \approx 2\tau_B$.

$$\zeta_1(t_0) \approx \begin{cases} -2\zeta_2 \dfrac{\sigma_1}{\sigma_0} , & \text{if } \boldsymbol{\zeta}_2 \perp \boldsymbol{n}_0 \\ -2\zeta_2 \dfrac{\sigma_2}{\sigma_0} , & \text{if } \boldsymbol{\zeta}_2 \parallel \boldsymbol{n}_0 \end{cases}$$

$$n(t_0) \approx 0.14 \cdot n(0)$$

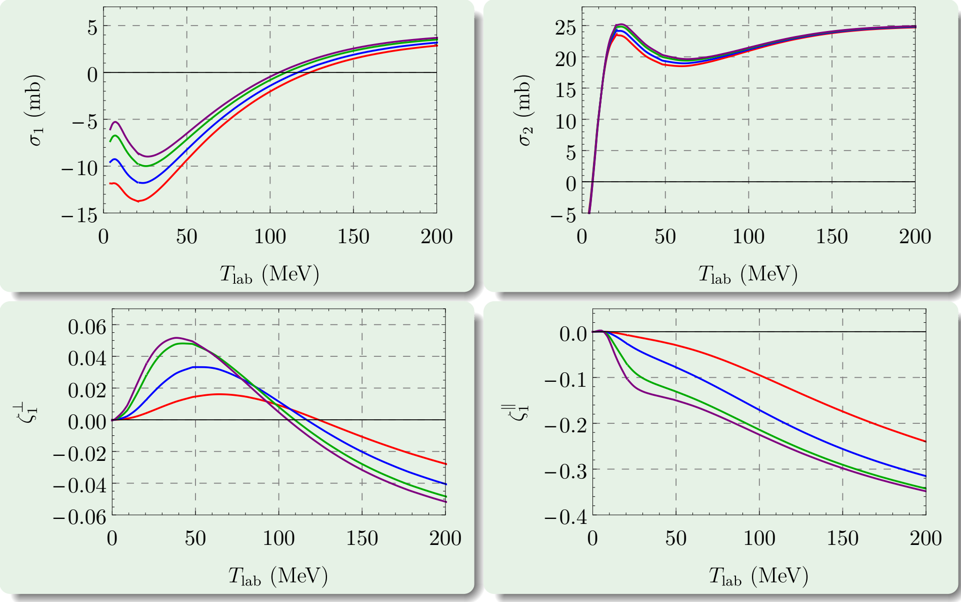


Jülich A, D models ($\theta_{\mathrm{acc}} = 10\,\mathrm{mrad}$)



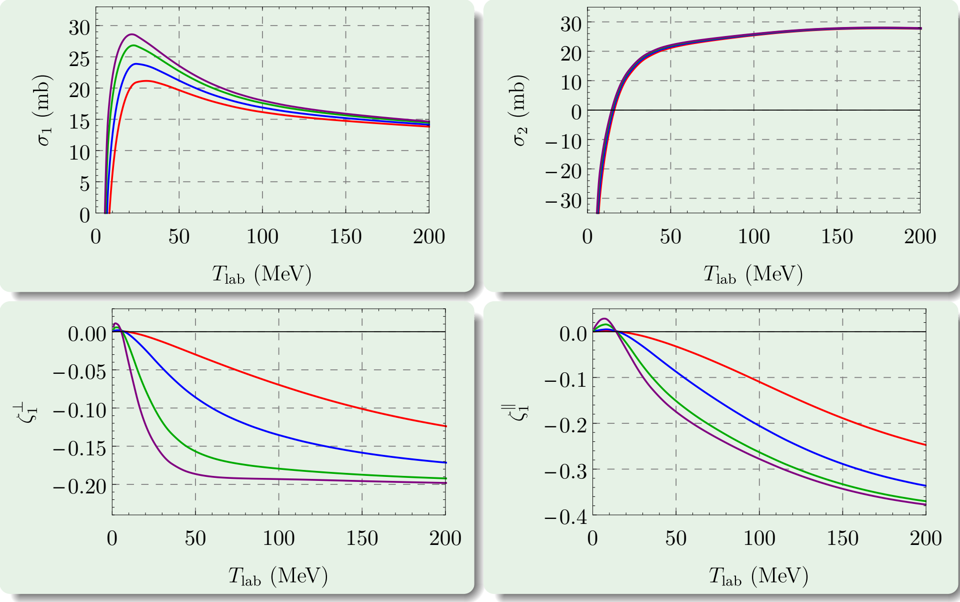
Uzikov Y., Haidenbauer J., Phys. Rev. C 79, 24617 (2009).

Paris model (2009)



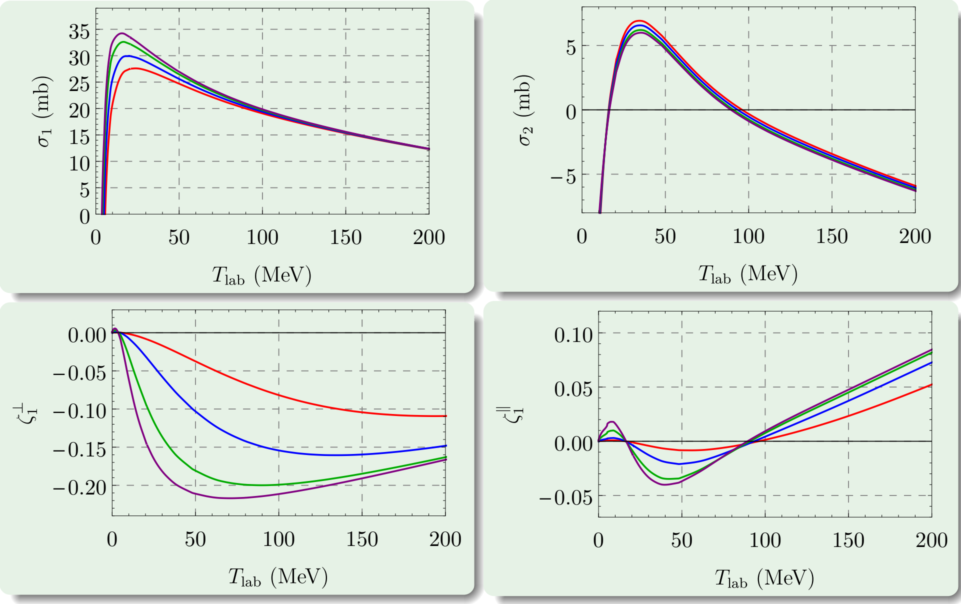
Dmitriev V. F., Milstein A. I., Strakhovenko V. M., NIMB 266, 1122 (2008).

Nijmegen model (1994)



Dmitriev V. F., Milstein A. I., Salnikov S. G., Phys. Lett. B 690, 427 (2010).

Nijmegen model (2012)



Zhou D., Timmermans R. G. E., Phys. Rev. C 87, 54005 (2013).

How to improve the theoretical predictions?

- There are several optical potential models describing $N\bar{N}$ interaction at low energies.
- The parameters of these models were obtained by fitting the experimental data mainly for scattering of unpolarized particles and some single-spin observables.
- The predictions for some spin-dependent observables are essentially different.
- No experiments with polarized antiprotons have been carried out so far.

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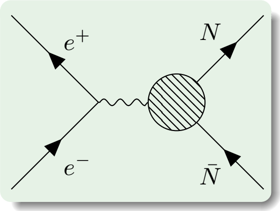
$$\Downarrow$$

Idea

Try to include other available experimental data in fitting procedures.

$N\bar{N}$ production at small distance (QCD) $\Longrightarrow$ $N\bar{N}$ interaction at large distance (optical potential)

Possible states of $N\bar{N}$ pair: ${}^3S_1 - {}^3D_1$, coupled by the tensor forces $\Longrightarrow$ $u_1^I(r)$, $u_2^I(r)$



$$G_M^I = G_s^I \left[u_1^I(0) + \frac{1}{\sqrt{2}} u_2^I(0) \right]$$

$$\frac{2M}{Q} G_E^I = G_s^I \left[u_1^I(0) - \sqrt{2}\, u_2^I(0) \right]$$

$$\sigma_{\text{el}}^I = \frac{2\pi\beta\alpha^2}{Q^2} \left| G_s^I \right|^2 \left[\left| u_1^I(0) \right|^2 + \left| u_2^I(0) \right|^2 \right] \qquad -\text{"elastic"}$$

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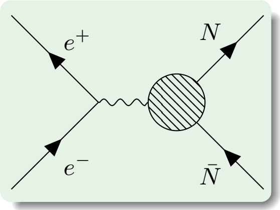

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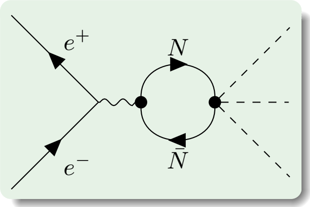
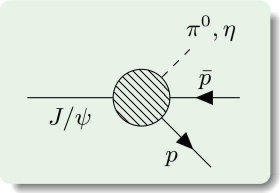


$N\bar{N}$ contribution to the total hadronic cross section

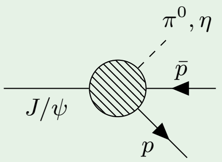
$$\sigma_{\mathrm{tot}}^I = -\frac{2\pi\alpha^2}{M^2 Q^2} \left| G_s^I \right|^2 \mathrm{Sp} \left[\mathrm{Im}\, \mathcal{D}(0, 0|E) \right] \qquad \text{– "total"}$$

- Valid also below the $N\bar{N}$ threshold



$J/\psi \to p\bar{p}\pi^0(\eta)$: ${}^3S_1 - {}^3D_1$ $N\bar{N}$ states dominate near the threshold of $p\bar{p}$ production

$$\frac{d\Gamma}{dM_{p\bar{p}}} = \frac{\mathcal{G}_I^2 p k^3}{2^5\, 3\pi^3 m_{J/\psi}^4} \left(\left| u_1^I(0) \right|^2 + \left| u_2^I(0) \right|^2 \right)$$



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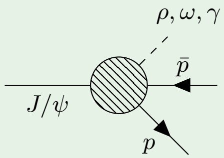


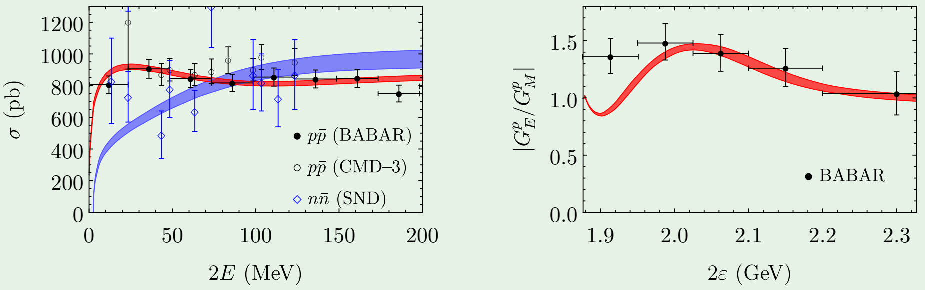
$J/\psi \to p\bar{p}\gamma(\rho,\omega)$: $^1S_0$ $N\bar{N}$ state dominates near the threshold of $p\bar{p}$ production

$$\frac{d\Gamma}{dM_{p\bar{p}}} = \frac{\mathcal{G}_I^2 p k^3}{2^4\,3\pi^3 m_{J/\psi}^4}\left|u^I(0)\right|^2$$

Simple optical potential model of $N\bar{N}$ interaction

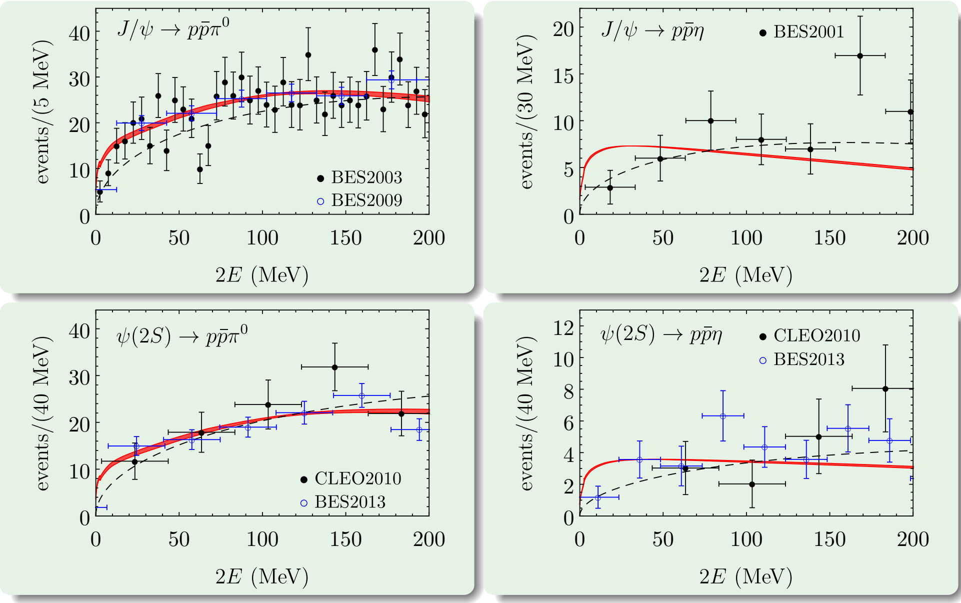
- Long-range pion-exchange potential, short-range potential well for each partial wave.
- Experimental data taken into account include:
  - $N\bar{N}$ scattering cross sections (elastic, charge-exchange $p\bar{p} \leftrightarrow n\bar{n}$ and annihilation)
  - $p\bar{p}$ and $n\bar{n}$ production in e^+e^- annihilation
  - ratio of electromagnetic form factors of the proton $|G_E^p/G_M^p|$
  - $p\bar{p}$ invariant mass spectra of the decays $J/\psi \to p\bar{p}\pi^0(\eta)$ and $J/\psi \to p\bar{p}\gamma(\omega)$

Cross sections of $e^+e^- \to p\bar{p}$ and $e^+e^- \to n\bar{n}$ and the ratio $|G_E^p/G_M^p|$



Dmitriev V. F., Milstein A. I., Salnikov S. G., Phys. Rev. D 93, 34033 (2016).

$p\bar{p}$ invariant mass spectra of ψ decays ($^3S_1 - {}^3D_1$)



Dmitriev V. F., Milstein A. I., Salnikov S. G., Phys. Lett. B 760, 139 (2016).

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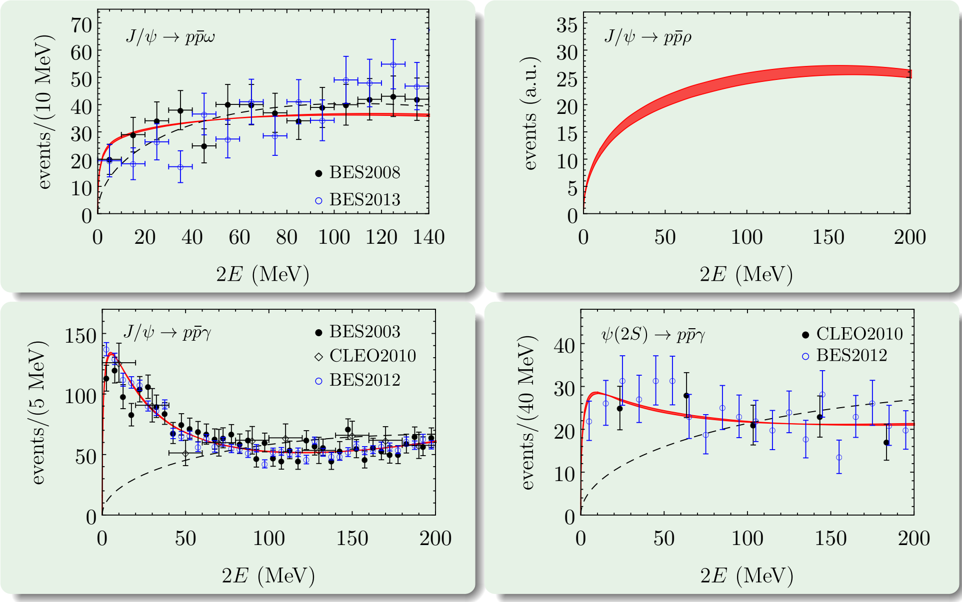

Milstein A. I., Salnikov S. G., Nucl. Phys. A 966, 54 (2017).

Conclusions

- An approach was demonstrated how to obtain kinetic equations for any spin observables.
- The equations for spin evolution in different scattering processes were obtained and the solutions of these equations were discussed.
- Kinetics of polarization in $p\bar{p}$ scattering was studied within several optical potential models. The predictions obtained are very different!
- An idea is proposed how to improve the models of $N\bar{N}$ interaction. Further investigations are required.

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THANK YOU FOR ATTENTION

Spin evolution for $S_1 = \frac{1}{2}$, $S_2 = \frac{1}{2}$ (general solution)

Let us decompose the polarization into orthogonal components:

$$\boldsymbol{\zeta}_1(t) = \alpha\,(\boldsymbol{\zeta}_2 \cdot \boldsymbol{n}_0)\,\boldsymbol{n}_0 \;+\; \beta\,(\boldsymbol{\zeta}_2 \cdot \boldsymbol{n}_0)\,[\boldsymbol{n}_0 \times \boldsymbol{\zeta}_2] \;+\; \gamma\,\big[\boldsymbol{n}_0 \times [\boldsymbol{n}_0 \times \boldsymbol{\zeta}_2]\big]$$

The kinetic equation can be written as

$$\frac{d}{dt}\psi(t) = U\psi + \xi$$

$$\psi = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \quad U = \begin{pmatrix} A_1 + B_1 & -A_2\,[\boldsymbol{\zeta}_2 \times \boldsymbol{n}_0]^2 & 0 \\ A_2 & A_1 & A_2 + B_2 \\ 0 & -\,(A_2 + B_2)\,(\boldsymbol{\zeta}_2 \cdot \boldsymbol{n}_0)^2 & A_1 \end{pmatrix}, \quad \xi = \begin{pmatrix} A_3 + B_3 \\ 0 \\ -A_3 \end{pmatrix}$$

$$\psi(t) = e^{Ut}\,\big[\psi(0) + U^{-1}\xi\big] - U^{-1}\xi$$

Eigenvalues of U have negative real part: $\operatorname{Re}\lambda_i < 0$