# Theoretical review of spin physics (and problems for NICA)

International Workshop on Spin Physics at NICA (SPIN-Praha-2018), July 9, 2018



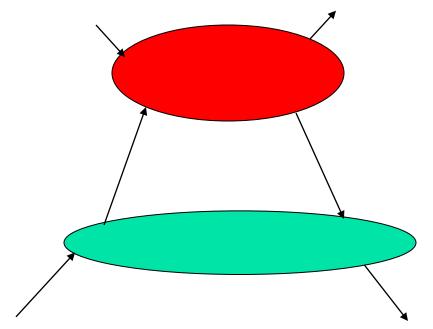
Oleg Teryaev JINR, Dubna

#### Outline

- QCD factorization and hadron spin structure: types of spin-dependent NPQCD functions
- TMDs and GPDs
- GPDs, pressure in proton and exclusive DY
- Single Spin Asymmetries in QCD Sources of (I)FSI
- Hadronic vs Heavy-Ion physics
- Problems for NICA

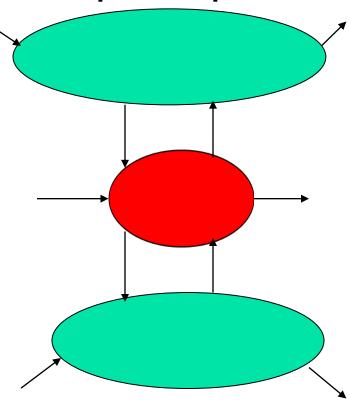
#### Factorization (lh-> DIS, DVCS)

 Short and hard distances separated (JINR – Efremov, Radyushkin; Higher twist – Efremov, OT; DVCS-Anikin, OT)





2 hadrons participate





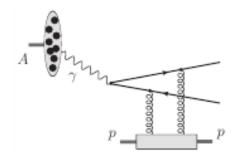
 Most general – Wigner function: nonsymmetric partonic and hadronic momenta with transverse components

The spin of both hadrons and partons

fixed

### Measurement of Wigner (GTMD) function

Small x – lp (Hatta, Xiao, Yuan'16) or Ap UP (Hagiwara, Hatta, Pasechnik, Tasevsky, OT'17) collisions



Larger x – UPC at SPD (R.Tsenov)!?

### Types of parton distributions -

- Too rich structure of Wigner function
- Simplifications Putting some (transverse) momenta to zero or average over some variables
- Hadronic moments equal inclusive
- Allow for proof of QCD factorization is some cases (perturbative corrections are taken into account by some kind of evolution)

#### Collinear vs k<sub>T</sub> factorization

- Collinear: NP longitudinal and pQCD transverse (GLAPD) evolution
- BFKL (also perturbative origin!) NP transverse and pQCD longitudinal evolution
- GI for off-shell partons?  $(xP + k_T)^2 < 0$
- Special BFKL vertices, effective action

#### TMD factorization

- BFKL (with non-linear unitarizaing modifications CGC, BK) – low x regions
- k<sub>T</sub> for larger x (relevant for SPD) TMD factorization
- Another approach to GI: transverse momentum only in parton distributions
- Transition? Application of effective action at larger x (talk of V. Saleev)
- Possible reason (Soffer,OT): convex x<sup>a</sup>(1-x)<sup>b</sup>
- Approximate validity of Regge ~ x<sup>a</sup> at rather large x~0.1

#### TMDs and GPDs

- Hadronic and partonic transverse momenta
- Variables k<sub>T</sub><sup>2</sup> vs t
- Models (AdS/QCD) using overlap of LCWF – relation (Maji, Mondal, Chakrabarti, OT'15)

$$\frac{\partial}{\partial |t|}[\ln(\text{GPD})] = \frac{(1-x)^2}{4} \frac{\partial}{\partial p_{\perp}^2}[\ln(\text{TMD})].$$

# Special interest to GPDs: pressure in proton

Universal concept at all scales

 Similarity to stable macroscopic objects in all known cases

 Transition to HIC – similarity to hadronic physics (c.f. "Ridge")

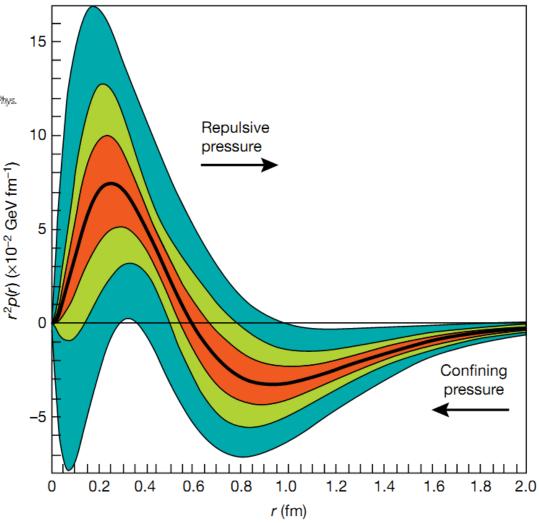
https://doi.org/10.1038/s41586-018-0060-z

#### The pressure distribution inside the proton

V. D. Burkert  $^{1*},$  L. Elouad<br/>rhiri  $^{1}$  & F. X. Girod  $^{1}$ 

 Teryaev, O. V. Gravitational form factors and nucleon spin structure. Front. Phys. 11, 111207 (2016).

 Anikin, I. V. & Teryaev, O. V. Dispersion relations and QCD factorization in hard reactions. Fizika B 17, 151–158 (2008).





### Pressure –related to D-term (Poyakov'03) and to holographic SR (OT'05)

Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

 Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta \mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|=1}^{1-|x|} dy \frac{G(x,y)}{1-u}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x-\xi+i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Also for exclusive DY! – OT'05 and work in progress

#### SR in energy plane (Anikin, OT'07)

Finite subtraction implied

$$\operatorname{Re}\mathcal{A}(\nu,Q^{2}) = \frac{\nu^{2}}{\pi} \mathcal{P} \int_{\nu_{0}}^{\infty} \frac{d\nu'^{2}}{\nu'^{2}} \frac{\operatorname{Im}\mathcal{A}(\nu',Q^{2})}{(\nu'^{2}-\nu^{2})} + \Delta \qquad \Delta = 2 \int_{-1}^{1} d\beta \frac{D(\beta)}{\beta-1}$$

$$\Delta_{\operatorname{CQM}}^{p}(2) = \Delta_{\operatorname{CQM}}^{n}(2) \approx 4.4, \qquad \Delta_{\operatorname{latt}}^{p} \approx \Delta_{\operatorname{latt}}^{n} \approx 1.1$$

 Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

#### From D-term to pressure

- Inverse -> 1<sup>st</sup> moment (model)
- Kinematical factor moment of pressure C~4</sup>> (2</sup>> =0) M.Polyakov'03

$$T^{Q}_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E} \int \frac{d^3\Delta}{(2\pi)^3} e^{i\vec{r}\cdot\vec{\Delta}} \langle p', S'|\hat{T}^{Q}_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$

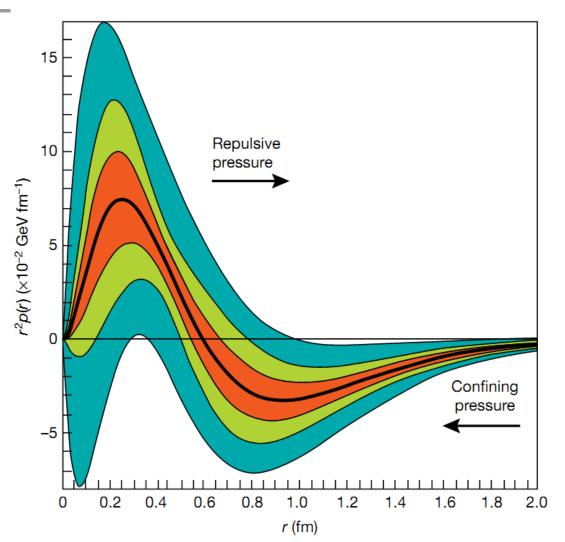
Stable equilibrium C>0:

https://doi.org/10.1038/s41586-018-0060-z

#### The pressure distribution inside the proton

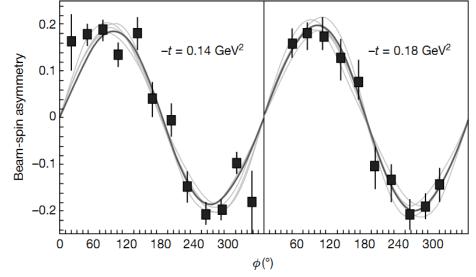
V. D. Burkert<sup>1</sup>\*, L. Elouadrhiri<sup>1</sup> & F. X. Girod<sup>1</sup>

Largest
 ever
 (~Λ<sup>4</sup>QCD)
 ~10<sup>35</sup> pascals



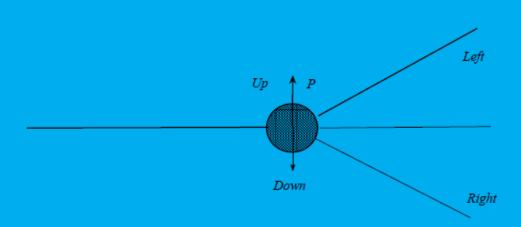
#### Experiment

- Jlab, TJNAF, CEBAF
- Very accurate data
- Imaginary part from Single Spin Asymmetry



### Single Spin Asymmetries: simplest example

Simplest example - (non-relativistic) elastic pion-nucleon scattering  $\pi \vec{N} \to \pi N$ 



 $M = a + ib(\vec{\sigma}\vec{n}) \vec{n}$  is the normal to the scattering plane.

Density matrix:  $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P}),$ 

Differential cross-section:  $d\sigma \sim 1 + A(\vec{P}\vec{n}), A = \frac{2Im(ab^*)}{|a|^2 + |b|^2}$ 

### Single Spin Asymmetries

#### Main properties:

- Parity: transverse polarization
- Imaginary phase can be seen from Tinvariance or technically - from the imaginary i in the (quark) density matrix

Various mechanisms – various sources of phases

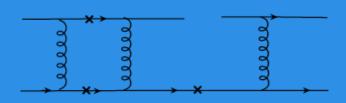
#### Phases in QCD

- QCD factorization soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem –
  phase due to on-shell intermediate states –
  positive kinematic variable (= their invariant
  mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal (1960):

Kane, Pumplin, Repko (78) Efremov (78)

#### Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts? Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like q - e scattering in DIS):

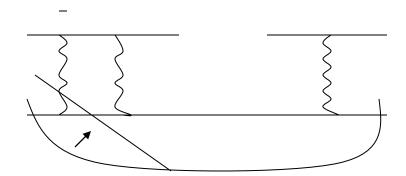


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

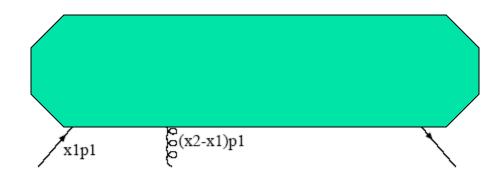
#### Short+ large overlaptwist 3

- Quarks only from hadrons
- Various options for factorization shift of SH separation (prototype of duality)



New option for SSA: Instead of 1-loop twist 2
 Born twist 3: Efremov, OT (85, Ferminonc poles); Qiu, Sterman (91, GLUONIC poles)

#### Quark-gluon correlators



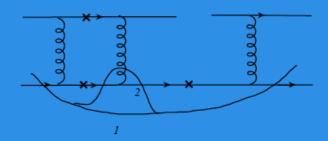
- Non-perturbative NUCLEON structure physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta quark momentum fractions are close to each other- gluonic pole; probed if:

Q >> P<sub>T</sub>>> M  

$$\chi_2 - \chi_1 = \delta = \frac{p_T^2 \chi_B}{O^2 z}$$

#### Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop → Born diagram

At Large distances - quark distribution → quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of  $\alpha_S$  to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \to \frac{Mb(x_1, x_2)p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

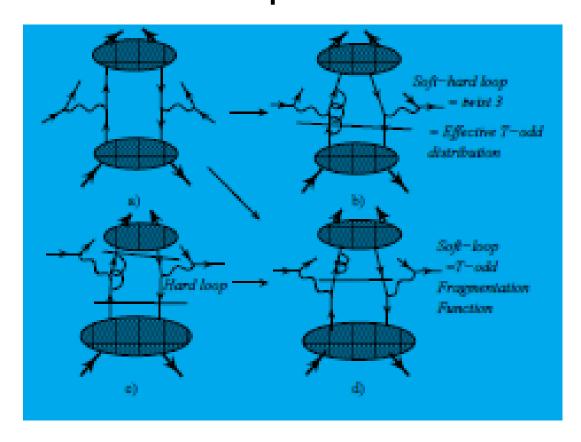
### Phases in QCD-Large distances in distributions

- Distributions: Sivers, Boer and Mulders no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process
- Brodsky -Hwang-Schmidt model: the same SH interactions as twist 3 but non-suppressed by Q: Sivers function leading (twist 2).
- Related in various complementary ways

#### SSAs in SIDIS

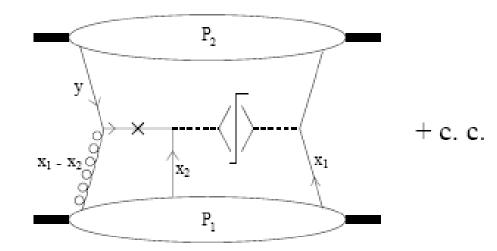
Various opportunities for phases

generation



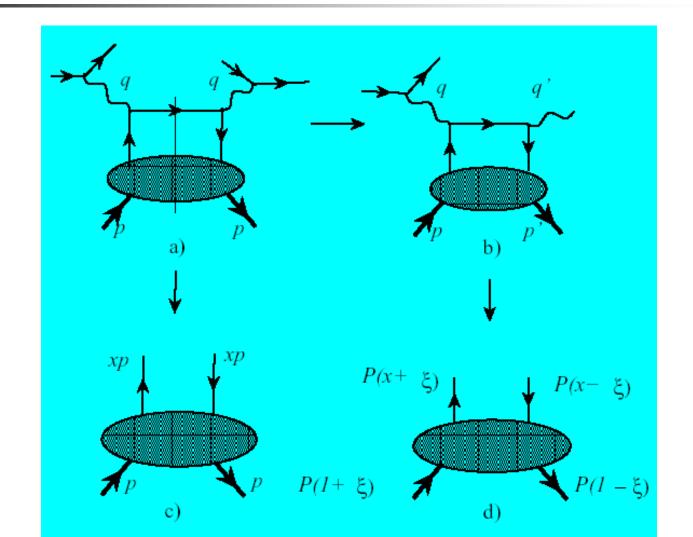
#### SSA in DY

- TM integrated DY with one transverse polarized beam— unique SSA — glu onic pole (Anikin,OT —factor 2)
- Important for lower M (SPD)



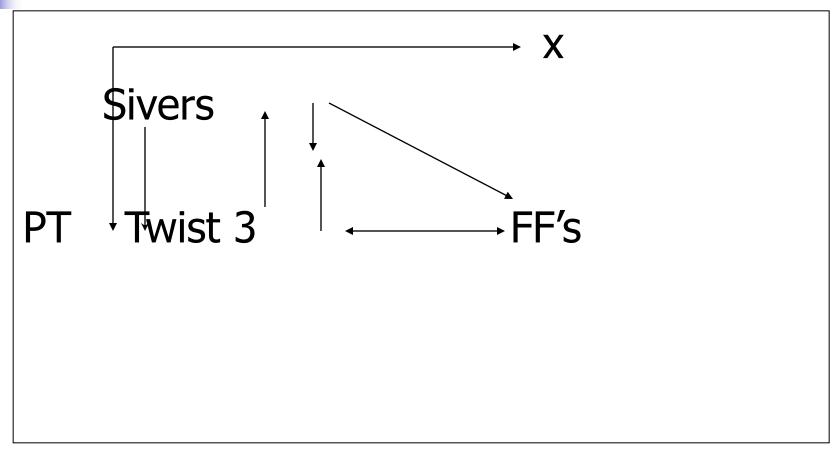
$$A = g \frac{\sin 2\theta \cos \phi \left[ T(x, x) - x \frac{dT(x, x)}{dx} \right]}{M \left[ 1 + \cos^2 \theta \right] q(x)}$$

### GPDs – another source of T-odd effects





#### Kinematical domains for SSA's



#### **Λ-polarisation**

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: '87): Randomization – smearing – no direction normal to the scattering plane

#### Global polarization

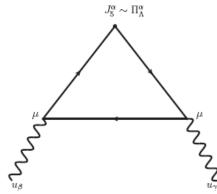
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07): polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

# Anomalous mechanism – polarization similar to CM(V)E

 4-Velocity is also a GAUGE FIELD (V.I. Zakharov)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)
- 4-velocity instead of gluon field!



#### Energy dependence

Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \, \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Large chemical potential: appropriate for NICA/FAIR energies

### One might compare the prediction below with the right panel figures

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in
heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

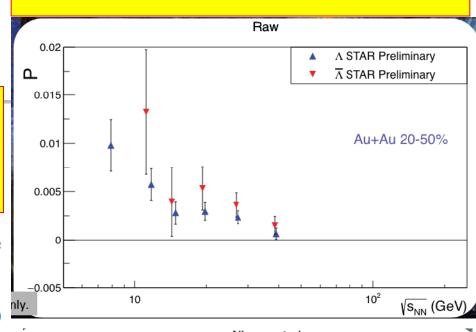
One would expect that polarization is proportional to the anomalously induced axial current [7]

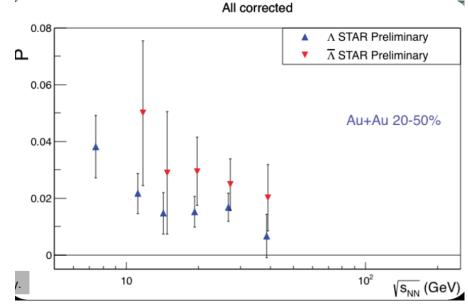
$$j_A^{\mu} \sim \mu^2 \left( 1 - \frac{2\mu n}{3(\epsilon + P)} \right) \epsilon^{\mu\nu\lambda\rho} V_{\nu} \partial_{\lambda} V_{\rho},$$
 (6)

where n and  $\epsilon$  are the corresponding charge and energy densities and P is the pressure. Therefore, the  $\mu$  dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

M. Lisa, for the STAR collaboration, QCD Chirality Workshop, UCLA, February 2016; SQM2016, Berkeley, June 2016





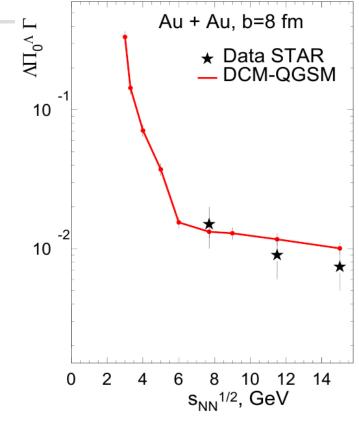
#### Another NATURE article

- Global / hyperon polarization in nuclear collisions
- The STAR Collaboration
- Journal name:Nature Volume: 548, Pages:62–65 Date published: (03 August 2017

### Energy dependence (Baznat, Gudima, Sorin, OT)

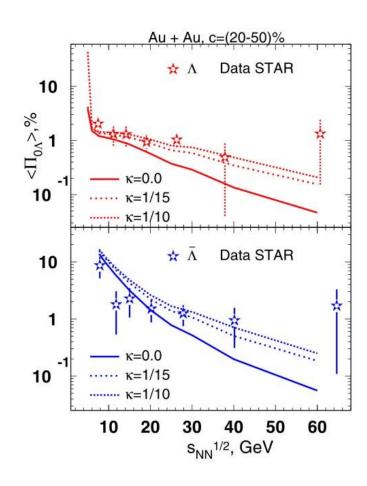
Growth at low energy

Close to STAR data!



Baryon-antibaryon successfully described - but a lot of work ahead

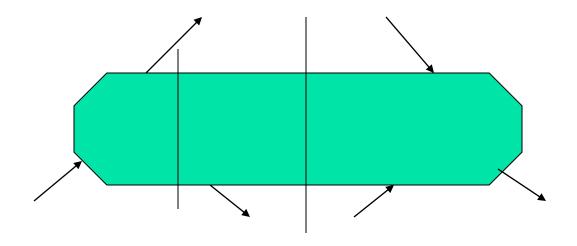
#### Λ vs Anti Λ



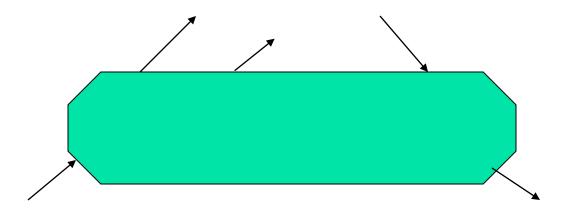
#### Fracture functions

- Common NP ingredient for FRAgmentation and struCTURE
- Structure functions parton distributions
- Fracture functions fractural (conditional,correlational,entangling?) parton distributions
- May be T-odd (Collins'95 –polarized beam jets; OT'01-T-odd Diffractive Distributions)
- Related by crossing to dihadron fragmentation functions





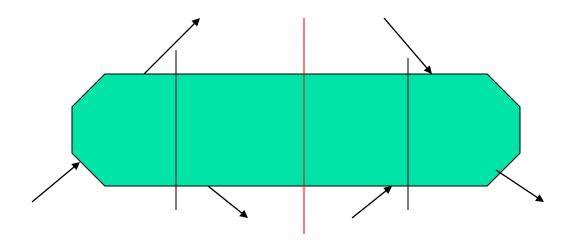




# T-odd fracture function for hyperons polarization

- May be formally obtained from spindependent T-odd DIS (cf OT'99 for pions SSAwork in progress)
- Transverse spin in DIS either transverse spin or transverse momentum of hyperon in SIDIS
- Both longitudinal and transverse polarizations appear
- SPD extra hadrons (pions) with low TM





#### **Problems for NICA**

- SPD LoI: TMDs@DY
- TMDs J/Ψ, γ
- GPDs: Exlusive DY-type (smaller x-section but lower background)
- GPDs from TMDs (pressure?!)
- Fracture SSAs with extra hadrons
- Relation of HIC/hadronic spin (MPD/SPD) polarization for hadrons, light and heavy ions

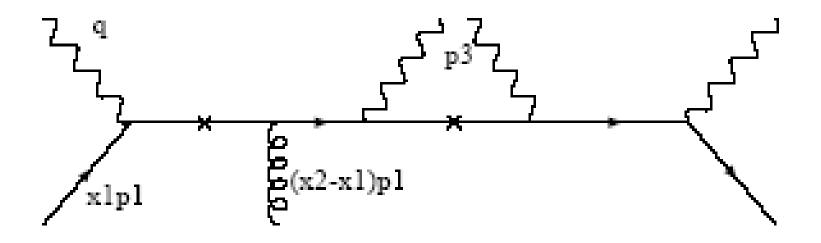


#### BACKUP



Frac´tur`al
 a.1.Pertaining to, or consequent on, a fracture.

# Twist 3 partonic subprocesses for SIDVCS



# Real and virtual photons - most clean tests of QCD

- Both initial and final real :Efremov, O.T. (85)
- Initial quark/gluon, final real : Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial real, final-virtual (or quark/gluon) –
   Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05; smooth transition from fermionic via hard to GLUONIC poles).

# Sivers function and formfactors

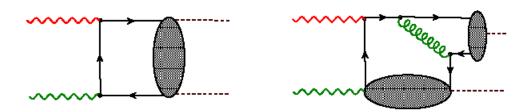
- Relation between Sivers and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?
- Solution: SSA in DY

#### SSA in exclusive limit

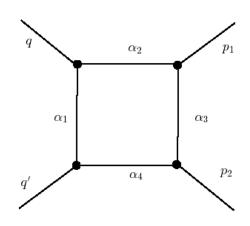
- Proton-antiproton valence annihilation cross section is described by Dirac FF squared
- The same SSA due to interference of Dirac and Pauli FF's with a phase shift
- Exclusive large energy limit; x -> 1: (d/dx)T(x,x)/q(x) -> Im F2/F1
- No suppression of large x − large E704 SSA
- Positivity: Twist 4 correction to q(x) may be important

# mechanisms for exclusive amplitudes (Anikin, Cherednikov, Stefanis, OT, 08)

2 pion production : GDA (small s) vs
 TDA+DA (small t)

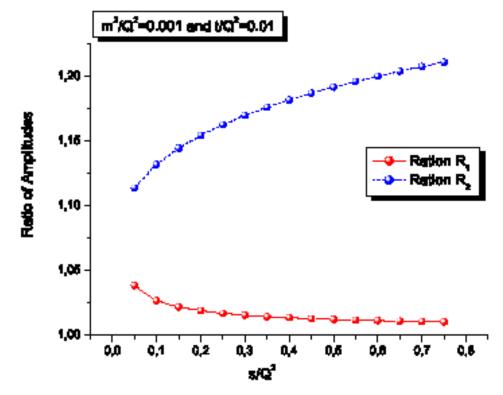


 Scalar model asymptotics(Efremov, Ginzburg, Radyushkin...)



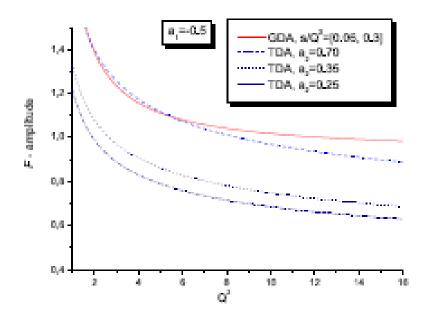
### Duality in scalar model

"Right" (TDA, red) and "wrong" (GDA, blue) asymptotics / exact result (>1- negative "Higher Twist"



### Duality in QCD

 Qualitatively- surprisingly good, quantitatively - model-dependent



### Duality and helicity amplitudes

- Holds if different mechanisms contribute to SAME helicity amplitudes
- Scalar- only one; QCD L and T photons
- Other option : Different mechanisms different helicity amplitudes ("unmatching")
- Example -> transition from perturbative phase to twist 3 (m -> M)

# Twist 3 factorization (Efremov, OT '84, Ratcliffe, Qiu, Sterman)

 Convolution of soft (S) and hard (T) parts

$$d\sigma_s = \int dx_1 dx_2 \frac{1}{4} Sp[S_{\mu}(x_1, x_2) T_{\mu}(x_1, x_2)]$$

 Vector and axial correlators: define hard process for both double (g<sub>2</sub>) and single asymmetries

$$T_{\mu}(x_1, x_2) = \frac{M}{2\pi} (\hat{p}_1 \gamma^5 s_{\mu} b_A(x_1, x_2) - i \gamma_{\rho} \epsilon^{\rho \mu s p_1} b_V(x_1, x_2))$$

#### Twist 3 factorization -II

Non-local operators for quark-gluon correlators

$$b_{A}(x_{1}, x_{2}) = \frac{1}{M} \int \frac{d\lambda_{1} d\lambda_{2}}{2\pi} e^{i\lambda_{1}(x_{1} - x_{2}) + i\lambda_{2}x_{2}} \langle p_{1}, s | \bar{\psi}(0) \hat{n} \gamma^{5}(D(\lambda_{1})s) \psi(\lambda_{2}) | p_{1}, s \rangle,$$

$$b_V(x_1, x_2) = \frac{i}{M} \int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1 - x_2) + i\lambda_2 x_2} \epsilon^{\mu s p_1 n} \langle p_1, s | \bar{\psi}(0) \hat{n} D_{\mu}(\lambda_1) \psi(\lambda_2) | p_1, s \rangle$$

Symmetry properties (from Tinvariance)

$$b_A(x_1, x_2) = b_A(x_2, x_1), \ b_V(x_1, x_2) = -b_V(x_2, x_1)$$

#### Twist-3 factorization -III

Singularities

$$b_A(x_1, x_2) = \varphi_A(x_1)\delta(x_1 - x_2) + b_A^r(x_2, x_1),$$

$$b_V(x_1, x_2) = \frac{\varphi_V(x_1)}{x_1 - x_2} + b_V^r(x_1, x_2)$$

- Very different: for axial Wandzura-Wilczek term due to intrinsic transverse momentum
- For vector-GLUONIC POLE (Qiu, Sterman '91)
  - large distance background

#### Sum rules

EOM + n-independence (GI+rotational invariance) –relation to (genuine twist
 3) DIS structure functions

$$\int_0^1 x^n \bar{g}_2(x) dx = \int_0^1 x^n (\frac{n}{n+1} g_1(x) + g_2(x)) dx =$$

$$-\frac{1}{\pi(n+1)} \int_{|x_1, x_2, x_1 - x_2| \le 1} dx_1 dx_2 \sum_f e_f^2 [\frac{n}{2} b_V(x_1, x_2) (x_1^{n-1} - x_2^{n-1}) +$$

$$b_A^r(x_1, x_2) \phi_n(x_1, x_2)], \quad \phi_n(x, y) = \frac{x^n - y^n}{x - y} - \frac{n}{2} (x^{n-1} - y^{n-1}), \quad n = 0, 2...$$

#### Sum rules -II

To simplify – low moments

$$\int_0^1 x^2 \hat{g}_2(x) dx = \frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \le 1} dx_1 dx_2 \sum_f e_f^2 b_V(x_1, x_2) (x_1 - x_2)$$

Especially simple – if only gluonic pole kept:

$$\int_0^1 x^2 \bar{g}_2(x) dx = -\frac{1}{3\pi} \int_{|x_1, x_2, x_1 - x_2| \le 1} dx_1 dx_2 \sum_f e_f^2 \varphi_V(x_1)$$
$$= -\frac{1}{3\pi} \int_{-1}^1 dx_1 \sum_f e_f^2 \varphi_V(x_1) (2 - |x_1|)$$

### Gluonic poles and Sivers function

- Gluonic poles effective Sivers functions-Hard and Soft parts talk, but **SOFTLY**
- Implies the sum rule for effective Sivers function  $x f_T(x) = \frac{1}{2M} T(x, x) = \frac{1}{4} \phi_v(x)$ (soft=gluonic pole dominance assumed in the whole allowed x's region of quark-gluon correlator)

$$x f_{T}(x) = \frac{1}{2M} T(x, x) = \frac{1}{4} \phi_{V}(x)$$

$$\int_{0}^{1} dx x^{2} g_{2}(x) = \frac{4}{3\pi} \int_{0}^{1} dx x f_{T}(x)(2-x)$$

### Compatibility of SSA and DIS

- Extractions of and modeling of Sivers function: "mirror" u and d
- Second moment at % level
- Twist -3  $g_2$  similar for neutron and proton and of the same sign no mirror picture seen –but supported by colour ordering!
- Scale of Sivers function reasonable, but flavor dependence differs qualitatively.
- Inclusion of pp data, global analysis including gluonic (=Sivers) and fermionic poles
- HERMES, RHIC, E704 –like phonons and rotons in liquid helium; small moment and large E704 SSA imply oscillations
- JLAB –measure SF and g2 in the same run

#### **CONCLUSIONS**

- 3<sup>rd</sup> way from SF to GP proof of Torino recipe supplemented by colour correlations
- Effective SF small in pp factorization in terms of twist 3 only
- Large x E704 region relation between SF, GP and time-like FF's

# Outlook (high energies)

- TMD vs UGPD
- T-odd UGPD?
- T-odd (P/O) diffractive distribiutions (analogs - also at small energies)
- Quark-hadron duality: description of gluon coupling to "exotic" objects in diffractive production via their decay widths

# Relation of Sivers function to GPDs

- Qualitatively similar to Anomalous Magnetic Moment (Brodsky et al)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (hep-ph/0612205 ):  $x f_{\tau}(x) : xE(x)$
- Burkardt SR for Sivers functions is now related to Ji SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$

# How gravity is coupled to nucleons?

- Current or constituent quark masses ? neither!
- Energy momentum tensor like electromagnertic current describes the coupling to photons

### Equivalence principle

- Newtonian "Falling elevator" well known and checked
- Post-Newtonian gravity action on SPIN known since 1962 (Kobzarev and Okun') – not yet checked
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

#### **Gravitational formfactors**

$$\langle p'|T_{q,g}^{\mu\nu}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}/2M]u(p)$$

• Conservation laws - zero Anomalous Gravitomagnetic Moment :  $\mu_G = J$  (g=2)

$$\begin{split} P_{q,g} &= A_{q,g}(0) & A_{q}(0) + A_{q}(0) = 1 \\ J_{q,g} &= \frac{1}{2} \left[ A_{q,g}(0) + B_{q,g}(0) \right] & A_{q}(0) + B_{q}(0) + A_{g}(0) + B_{g}(0) = 1 \end{split}$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity – similar t-dependence to EM FF

### Electromagnetism vs Gravity

#### Interaction – field vs metric deviation

$$M = \langle P'|J^{\mu}_{q}|P\rangle A_{\mu}(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

Static limit

$$\langle P|J_q^{\mu}|P\rangle = 2e_q P^{\mu}$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^{\mu}P^{\nu}$$
$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^{\mu}|P\rangle A_{\mu} = 2e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M\phi(q)$$

Mass as charge – equivalence principle

### Gravitomagnetism

• Gravitomagnetic field – action on spin – ½ from  $M = \frac{1}{2} \sum_{q,G} \langle P' | T^{\mu\nu}_{q,G} | P \rangle h_{\mu\nu}(q)$ 

$$ec{H}_J = rac{1}{2} rot ec{g}; \; ec{g}_i \equiv g_{0i}$$
 spin dragging twice smaller than EM

- Lorentz force similar to EM case: factor 1/2 cancelled with 2 from  $h_{00} = 2\phi(x)$  Larmor frequency same as EM  $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging the same Equivalence principle  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

## Sivers function and Extended Equivalence principle

- Second moment of E zero SEPARATELY for quarks and gluons –only in QCD beyond PT (OT, 2001) supported by lattice simulations etc.. ->
- Gluon Sivers function is small! (COMPASS, STAR, Brodsky&Gardner)
- BUT: gluon orbital momentum is NOT small: total about 1/2, if small spin – large (longitudinal) orbital momentum
- Gluon Sivers function should result from twist 3 correlator of 3 gluons: remains to be proved!

# Generalization of Equivalence principle

 Various arguments: AGM 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM, confirmed recently)

