Общелабораторный семинар ЛЯП

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New quantum-mechanical effect and its application for a production of twisted particle beams

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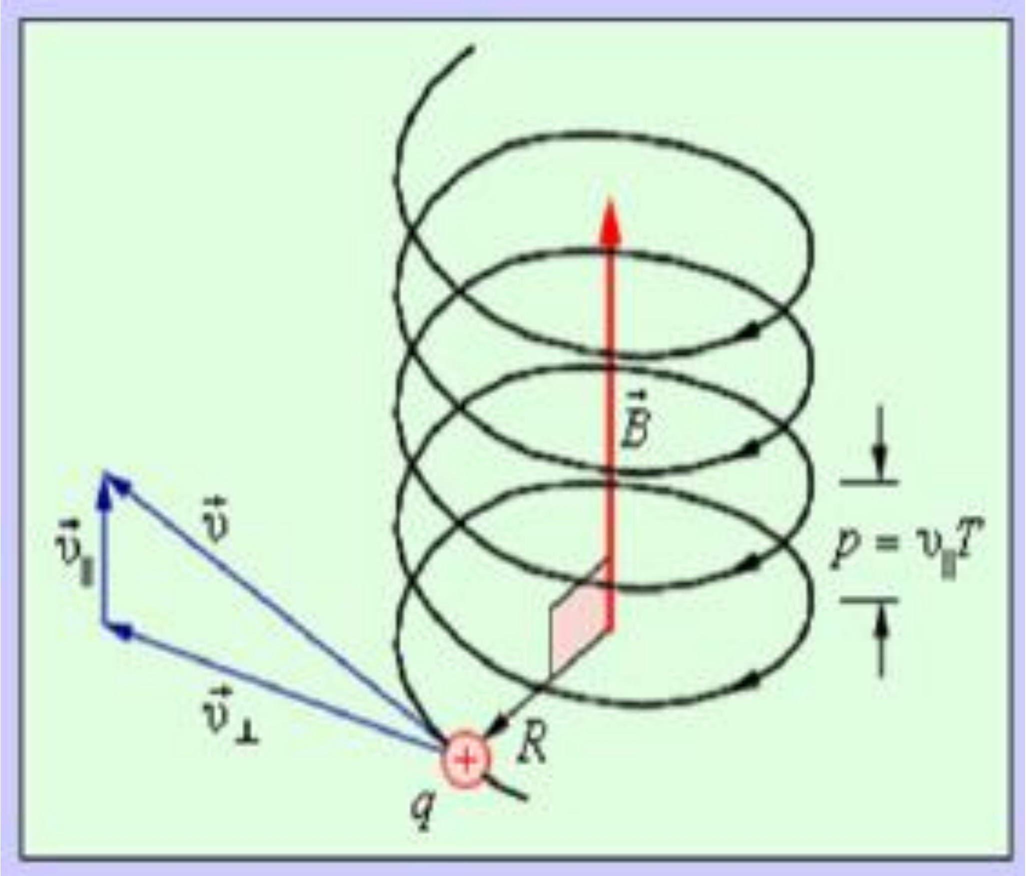
Introduction

The figure shows the classical motion of a particle in a (quasi)uniform magnetic field: the particle moves on a helix. In quantum mechanics, the particle motion takes also place in the *radial* direction and the *azimuthal* component of the particle momentum is not definite. Quantum-mechanical states found by *Landau* are Laguerre-Gauss beams:



$$\Psi = \mathcal{A} \exp(i\ell\phi) \exp(ip_z z), \qquad \int \Psi^\dagger \Psi \, r dr d\phi = 1,$$

$$\mathcal{A} = \frac{C_{n\ell}}{w_m} \left(\frac{\sqrt{2} r}{w_m} \right)^{|\ell|} L_n^{|\ell|} \left(\frac{2r^2}{w_m^2} \right) \exp\left(-\frac{r^2}{w_m^2} \right) \eta,$$

$$C_{n\ell} = \sqrt{\frac{2n!}{\pi (n+|\ell|)!}}, \qquad w_m = \frac{2}{\sqrt{|e|B}}.$$

- **In the vacuum, particles (electron, photon, and others) can be in twisted states with nonzero orbital angular momenta). Wave functions of such particles also define Laguerre-Gauss beams:**

$$\Psi = \mathcal{A}\exp(i\Phi),\ \mathcal{A} = \frac{C_{n\ell}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right),$$

$$\Phi = \ell\phi + \frac{kr^2}{2R(z)} - \Phi_G(z), \quad C_{n\ell} = \sqrt{\frac{2n!}{\pi(n+|\ell|)!}},\ w(z) = w_0\sqrt{1+\frac{4z^2}{k^2w_0^4}},$$

$$R(z) = z + \frac{k^2w_0^4}{4z},\ \Phi_G(z) = (2n+|\ell|+1)\arctan\left(\frac{2z}{kw_0^2}\right),\ \int\Psi^\dagger\Psi\, r\,\mathrm{d}r\,\mathrm{d}\phi = 1$$

Floettmann K and Karlovets D (2020 **Quantum mechanical formulation of the Busch theorem *Phys. Rev. A 102*** *043517***)** **have shown that a twisted beam penetrating from a solenoid to the vacuum remains twisted after the penetration. This conclusion has been confirmed by a different method in L. Zou, P. Zhang, A. J. Silenko, Production of twisted particles in magnetic fields, J. Phys. B: At. Mol. Opt. Phys. 57** 045401 (2024).

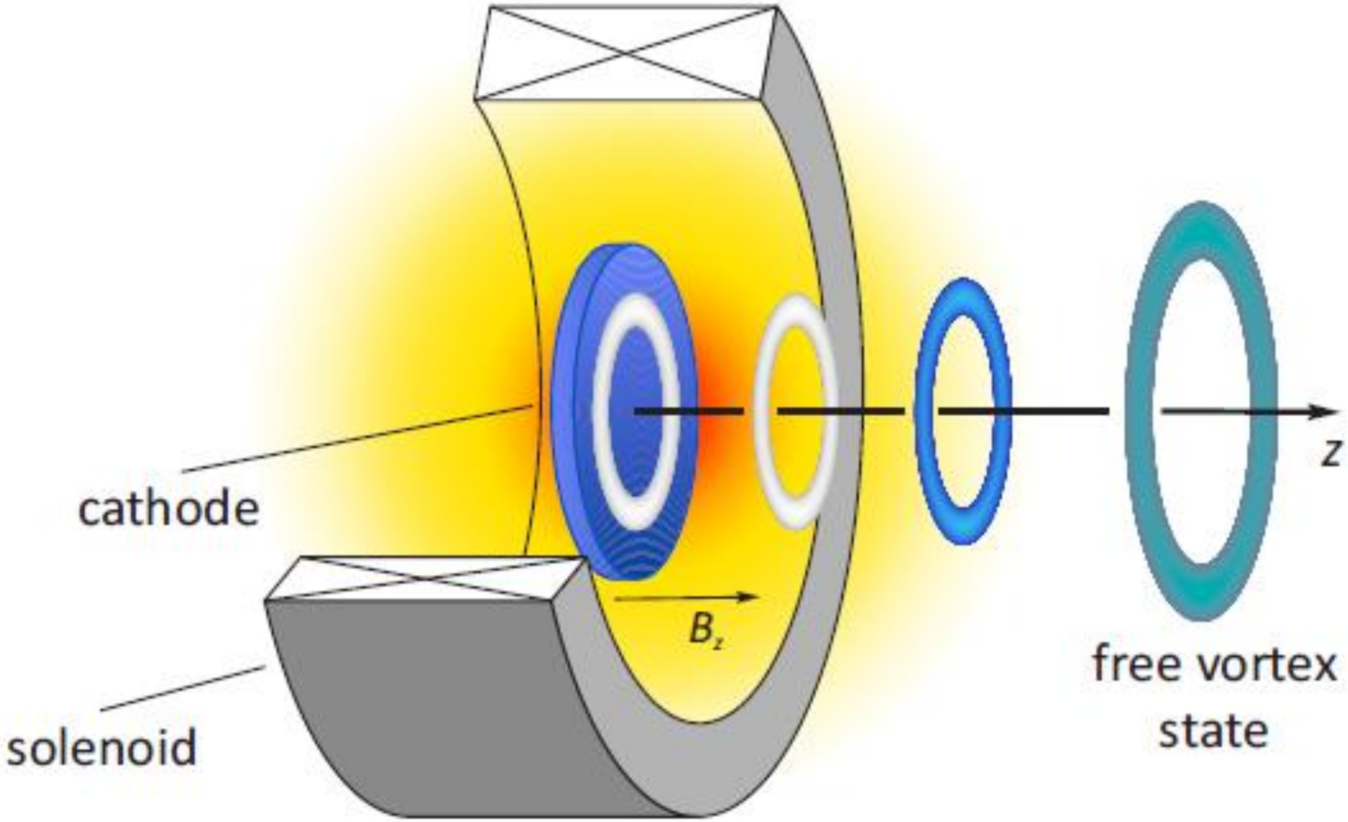


FIG. 1. Illustration of the immersed cathode technique. Electrons with a ring-shaped probability density are released from a cathode which is immersed in a solenoid field. When the electrons leave the solenoid field an orbital angular momentum is imparted and a free vortex state is generated.

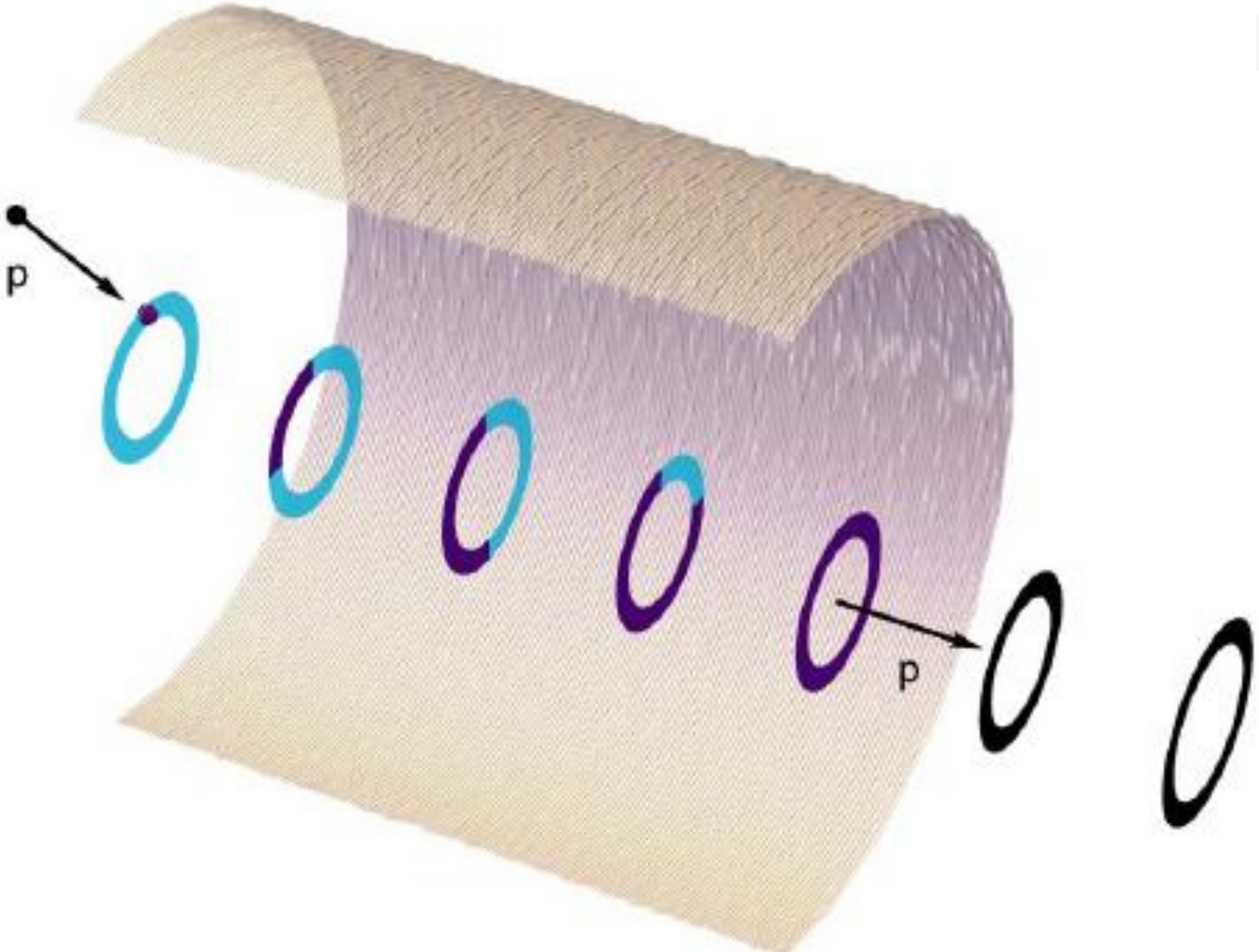
(The figure from the paper by Floettmann and Karlovets)

Y. Bai, H. Lv, X. Fu, et al. Vortex beam: generation and detection of orbital angular momentum, Chin. Opt. Lett. 20, 012601 (2022).

Conversion from a state with zero orbital angular momentum to a twisted state in a solenoid

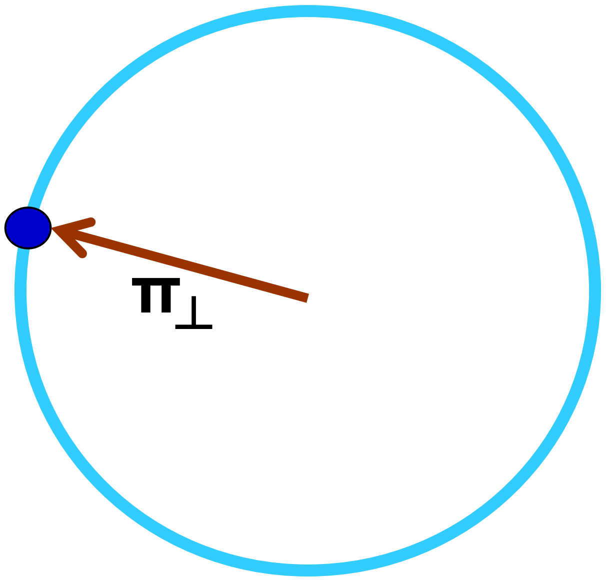
Conversion from a state with the OAM *L*=0 to a twisted state with *L*≠0 in a solenoid is a new quantum-mechanical effect.

L. Zou, P. Zhang, A. J. Silenko, Production of twisted particles in magnetic fields, J. Phys. B: At. Mol. Opt. Phys. 57, 045401 (2024), section 6 and appendix.



An evolution of a quantum state of the charged particle at its passage through the solenoid. The purple color denotes angular sectors where the particle can be found. Sky-blue color marks angular sectors where the probability of finding the particle is negligible. All these sectors rotate with an angular velocity determined with some uncertainty. The last purple ring characterize a Landau state which is converted into a LG one (black rings) after the penetration to the vacuum.

An evolution of a phase of rotation in the transversal plane



The exact relativistic FW Hamiltonian is given by

$$i\frac{\partial\Psi_{FW}}{\partial t}=\mathcal{H}_{FW}\Psi_{FW},\qquad \mathcal{H}_{FW}=\beta\sqrt{m^2+\pi^2-e\Sigma\cdot\boldsymbol{B}}$$

The operator *r* does not commute with the FW Hamiltonian and $[\mathcal{H}_{FW},\omega]\neq 0.$

$$\omega \equiv \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \phi] = \frac{1}{2}\left\{\frac{1}{\epsilon'}, \frac{\pi_\phi}{r}\right\},$$

$$\frac{\pi_\phi}{r} = \frac{p_\phi}{r} - \frac{eB}{2} = -\frac{i\hbar}{r^2}\frac{\partial}{\partial\phi} - \frac{eB}{2}.$$

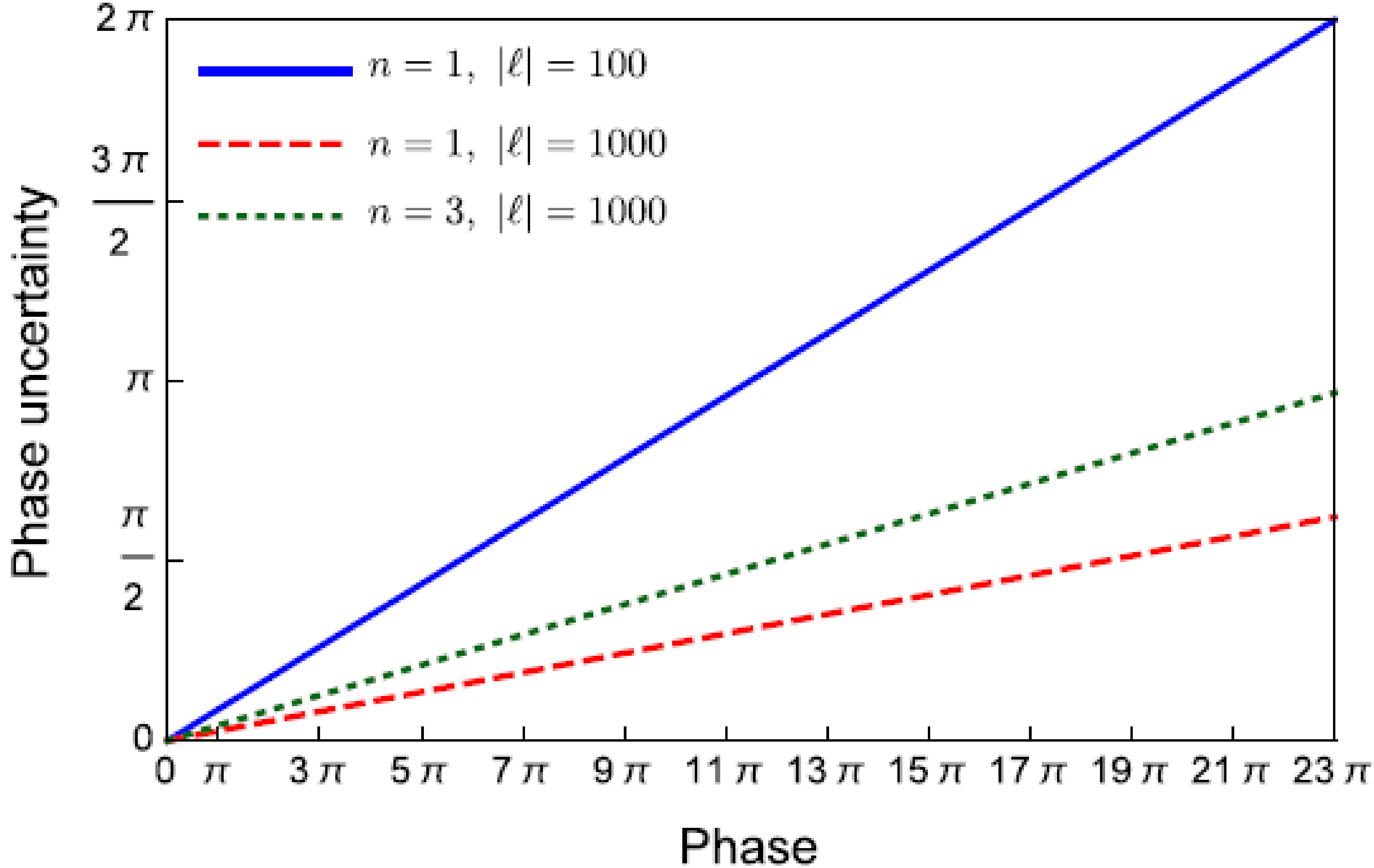
Specifically, $\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{i}{\hbar}[\mathcal{H}_{FW}, \omega] = -\frac{1}{4}\left\{\frac{1}{\epsilon'}, \left\{\frac{L_z}{\epsilon'}, \left\{p_r, \frac{1}{r^3}\right\}\right\}\right\}, \quad L_z = -i\frac{\partial}{\partial\phi}.$

The dispersion of ω is defined by $D(\omega)=\langle(\omega - \langle\omega\rangle)^2\rangle$ and is nonzero:

$$\langle\omega\rangle = -\frac{eB}{E}, \qquad D(\omega) = \frac{e^2B^2\left[(2n+1)|\ell|+1\right]}{4E^2(\ell^2-1)},$$

where E is the energy. The relative root-mean-square deviation reads

$$\delta = \frac{\sqrt{D(\omega)}}{|\langle\omega\rangle|} = \sqrt{\frac{(2n+1)|\ell|+1}{4(\ell^2-1)}}.$$



The growth of the phase uncertainty versus the phase of the particle revolution. The semiclassical case ($|\ell| \gg 1, n \sim 1$) is considered and three sets of quantum numbers n, $|\ell|$ are used.

In particular, $\delta \approx 0.087$ when $|\ell| = 100, n = 1$; $\delta \approx 0.027$ when $|\ell| = 1000, n = 1$; and $\delta \approx 0.042$ when $|\ell| = 1000, n = 3$. These values have been confirmed by numerical calculations. Thus, the root-mean-square uncertainty of the phase is equal to 2π after approximately 11.5, 37.0, and 23.8 revolutions, respectively. While our derivation is valid for any direction of L_z, we consider only the basic states $\text{sgn}(e)\ell \leqslant 0$.

Our results underline that there is a fundamental difference between the classical and quantum-mechanical pictures of the evolution of particle states. Classical physics is based on mechanical determinism. In this case, the inversion of time $t \to -t$ leads to a correct mechanical process where the initial and final states are swapped. Therefore, the acquisition of a nonzero intrinsic OAM by a plane wave or a beam carrying no OAM and flying through the solenoid is an effect forbidden in the classical theory. In contrast, the main specific features of QM (e.g. in the Copenhagen interpretation [58]) is its *indeterminism* [59]. In the quantum-mechanical picture, the inversion of time does not lead to the initial state. Therefore, the problem of the evolution of quantum states of charged particles during penetration through a solenoid should be reconsidered. The indeterminism and uncertainty of QM make the appearance of the intrinsic OAM of the charged particle passing through the solenoid possible. Both classical and quantum-mechanical pictures satisfy the strongly restricted law of conservation of only the canonical OAM relative to the solenoid axis.

Evaluation of the effect considered

The well-known relativistic classical formula for the particle orbit is given by

$$r^2 = \frac{2|L_z|}{|eB|} = \frac{|\mathcal{L}_z|}{|eB|},$$

where L_z and $\mathcal{L}_z$ are the canonical and kinetic angular momenta, respectively. We can calculate the lead of the helix:

$$d = 2\pi r \frac{v}{|v_\phi|} = 2\pi r^2 \frac{m\gamma v}{|\mathcal{L}_z|} = \frac{2\pi m\gamma v}{|eB|} = \frac{2\pi h\gamma v}{c\Lambda|eB|},$$

where Λ is the Compton wavelength. Let *B* be 1T. For the positron with $\gamma=10$, $v\approx c$, $d\approx 0.11$m. For the proton with $\gamma\approx 1$, $v=0.1c$, $d\approx 2.0$m. When $v=0.01c$, $d\approx 0.20$m. When $L_z=1000\hbar$, $r\approx 1.1\times 10^{-6}$m.


Thank you for your attention