International Conference
Advances in Quantum Field Theory 2025

BLTP JINR
DUBNA, RUSSIA
11-15 AUGUST

AQFT-25

Naturalness Criteria for the Standard Model

Victor T. Kim

Petersburg Nuclear Physics Institute
NRC «Kurchatov institute»

Outline:

- **Brout-Englert-Higgs boson mass evolution and naturalness crirteria**
- **Brout-Englert-Higgs boson mass renormalization: logarithmic vs. quadratic**
- **Naturalness domain boundaries for the Standard Model**
- **Summary**

Running masses in the standard model

Running couplings: α_{QCD}, α_{EW}

Heavy quark (B-quark) running masses

Different mass parameterizations
(different approaches to include higher orders):
- pole (on-shell) mass
- running mass

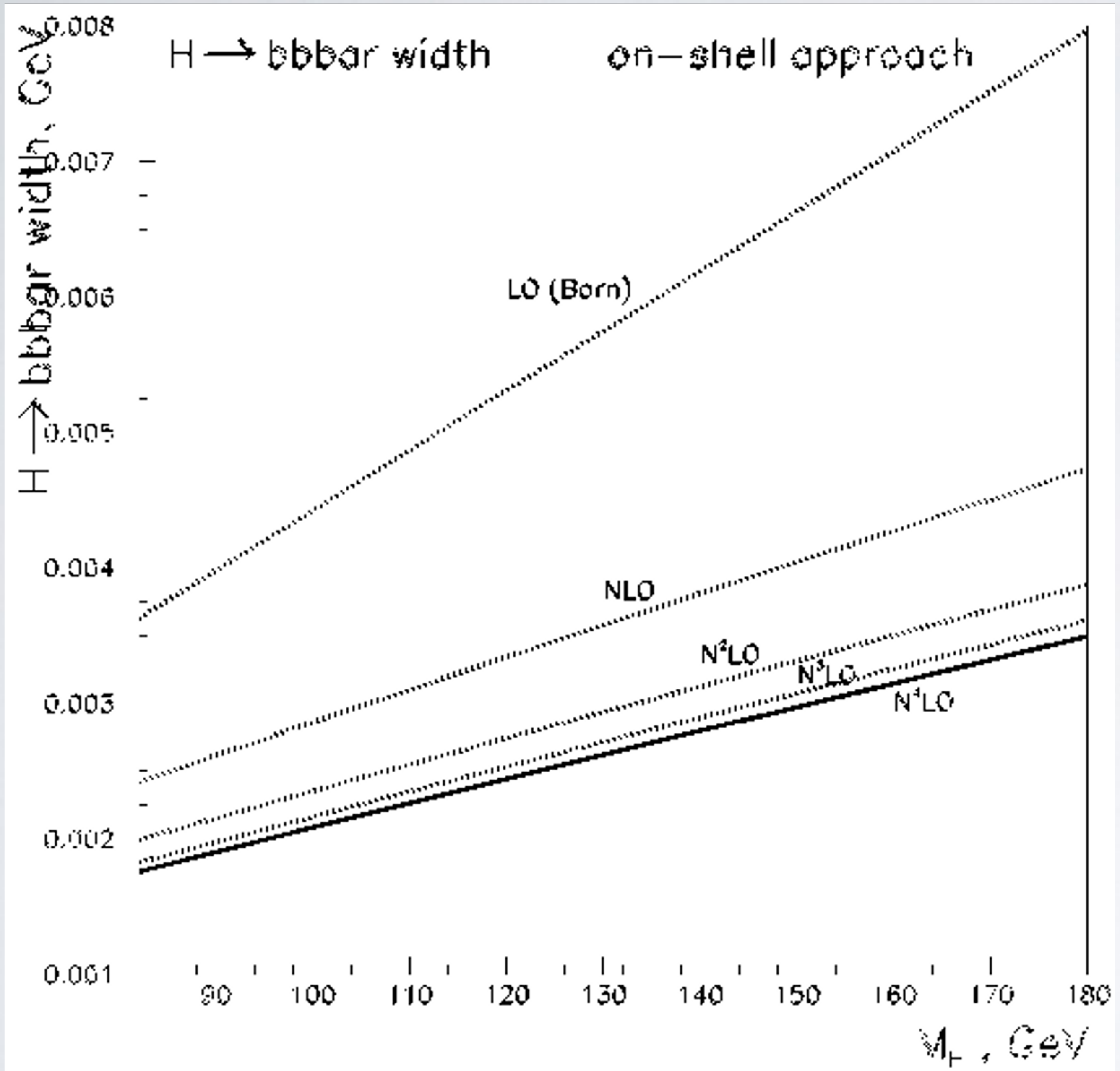
SM running masses
- fermions and vector bosons: logarithmic
- scalar Brout-Englert-Higgs boson: logarithmic
or/and quadratic ?
quadratic -> "non-naturalness"

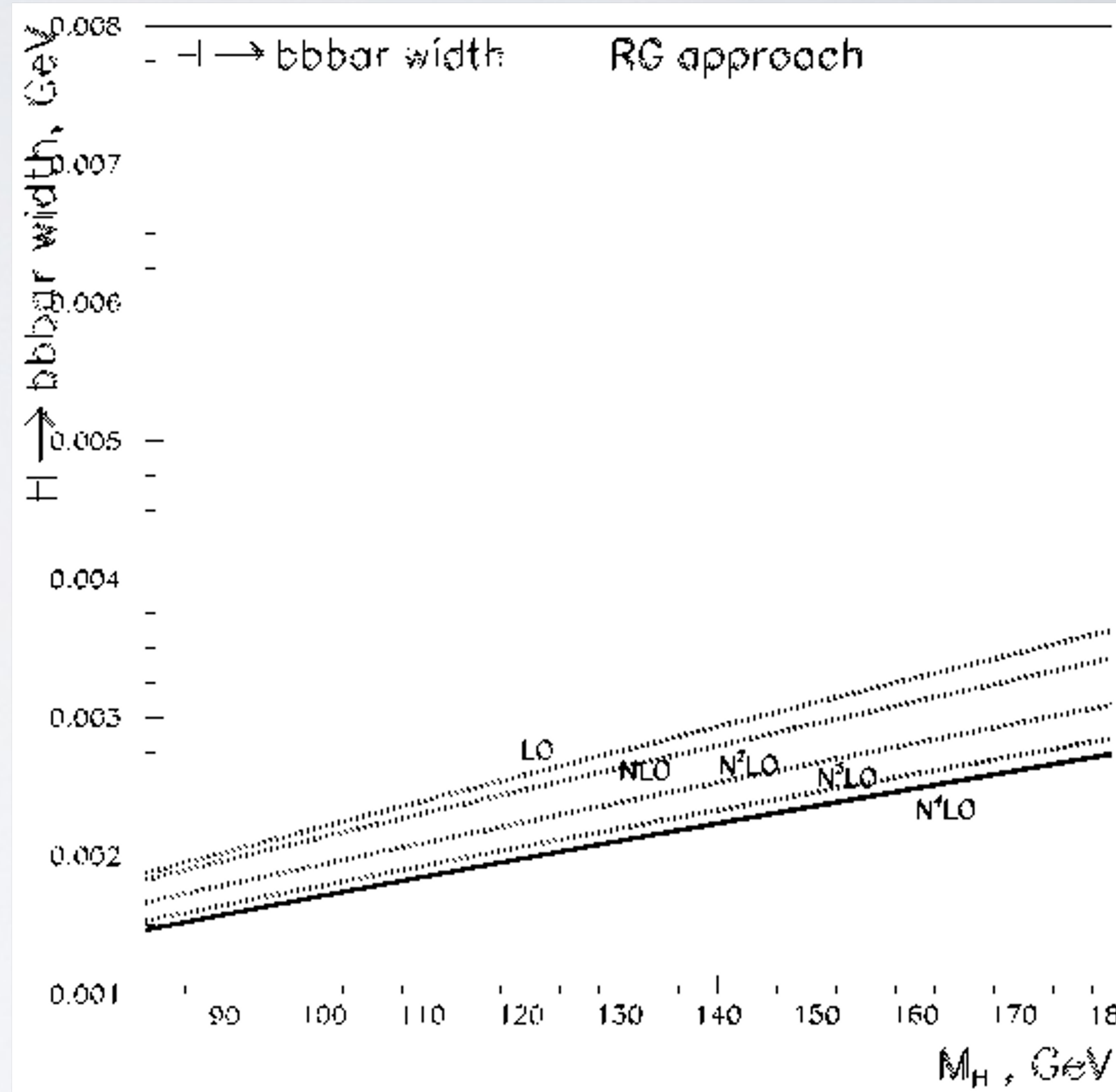
Brout-Englert-Higgs boson decay width

Width of BEH boson decay into b-quarks (up to N⁴LO)

P. Baikov, K. Chetyrkin, J. Kuhn (2006)

A.P.Bakulev, S.V.Mikhailov, N.G.Stefanis(2007,2008), A.L. Kataev, VK (2007,2008)





b-quark mass running: - 4.5 GeV Upsilon mass

- 2.8 GeV BEH boson mass

Brout-Englert-Higgs boson of the Standard Model

Brout-Englert-Higgs boson discovery of CMS and ATLAS in 2012 is most important physics result at LHC upto now

ATLAS, Phys. Lett. B 716 (2012) 1
CMS, Phys. Lett. B 716 (2012) 30

Brout-Englert-Higgs-Guralnik-Hagen-Kibble mechanism of spontaneous symmetry breaking

R. Brout, F. Englert, Phys. Rev. Lett. 13 (1964) 321
P.W. Higgs, Phys. Rev. Lett. 13 (1964) 508
G.S. Guralnik, C.R. Hagen, T.W.B. Kibble, Phys. Rev. Lett. 13 (1964) 585

P. Higgs & F. Englert: Nobel Prize (2013)

Higgs boson is only scalar elementary particle known up to now

The Standard Model with 125 GeV BEH-boson

Brout-Englert-Higgs boson:
if only logarithmic mass evolution

Brout-Englert-Higgs boson defines electroweak vacuum density
(meta)stable vacuum up to Planck scales

F. Bezrukov, M. Kalmykov, B. Kiehl & M. Shaposhnikov, JHEP 10 (2012) 140
within MSbar-scheme

One may conclude: M. Shaposhnikov et al.
(Almost) no need for a New Physics up to Planck scales
Only needs:
- (~ 1 GeV) BSM neutral leptons to explain Dark Matter
- strong CP-problem
- neutrino masses
- baryon-antibaryon asymmetry

...
- and still needs to explain:
  why there is naturalness (New Physics?!)

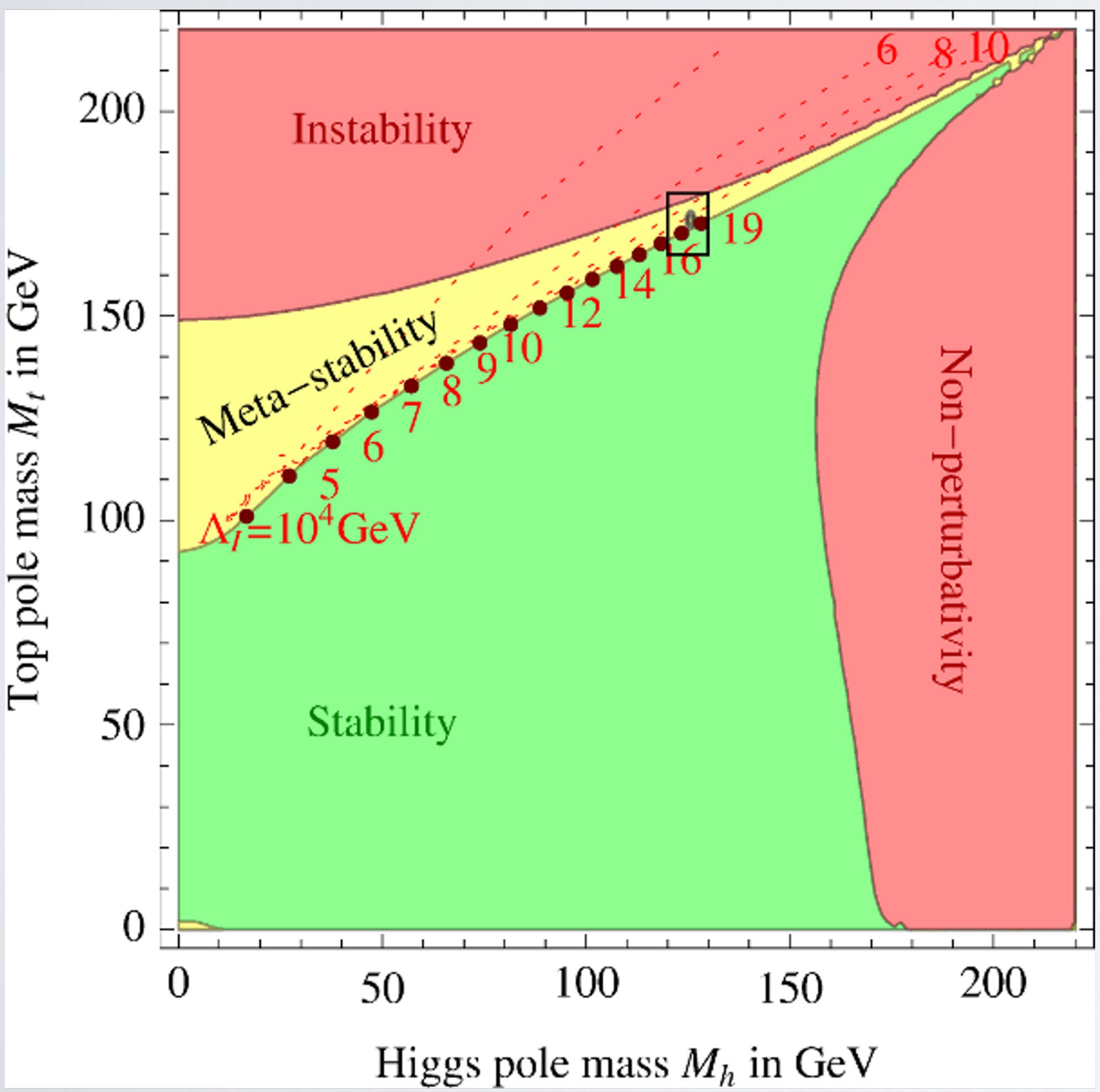
The Standard Model with 125 GeV Higgs boson

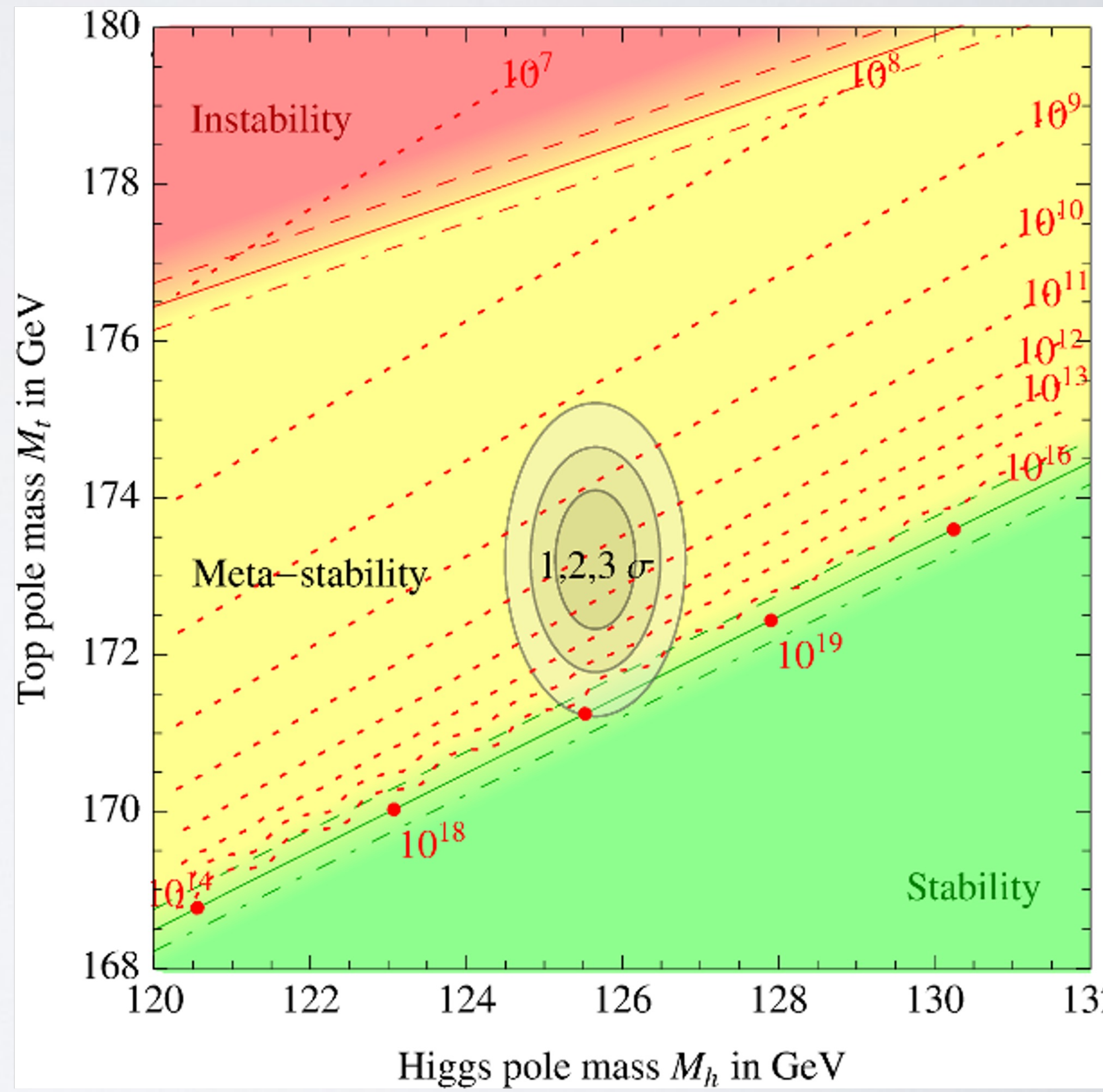
Brout-Englert-Higgs boson mass defines electroweak vacuum density: meta-stable vacuum

G. Degrassi et al., JHEP 08 (2012) 098

D. Butazzo et al., JHEP 12 (2013) 089

A.V.Bednyakov, B.A.Kniehl, A.F.Pikelner, O.L.Veretin, PRL 115 (2015) 201802





(Non-)Naturalness of the standard model

Logarithmic evolution of theory parameters: weak dependence between low and very large scales -> concept of "Naturalness"

- **Scalar field is simple, but "non-natural": scalar mass evolution is quadratic, not only logarithmic**

  K. Wilson, Phys. Rev. D3 (1971) 1818
  L. Susskind, Phys. Rev. D20 (1979) 2619

- **Scalar field is not protected by a symmetry, while fermions are protected by chiral symmetry**

  G. 't Hooft, Proc. Cargese Summer Inst. (1980) 135-157

for reviews see, e.g., G.F. Giudice (2008), L. Alvarez-Gaume, M.A.Vazquez-Mozo (2012)

Quantifying Naturalness: Criteria

G. ‘t Hooft, Proc. Gargese Inst. (1980) pp.135-157

P.A.M. Dirac 139 (1937) 323, Proc. Roy. Soc. A165 (1948) 199

Naturalness Criterion by G. ‘t Hooft:
physics parameter can be small at any scale only if setting this parameter to zero would increase the symmetry of the system

E.g., naturalness for fermions: m=0 leads to chiral symmetry

However, nothing like that for scalar particles …

dilaton symmetry W. Bardeen (1995)

Quantifying Naturalness: Criteria

K. Wilson, Phys. Rev. D3 (1971) 1818
L. Susskind, Phys. Rev. D20 (1979) 2619
G. ‘t Hooft, Proc. Gargese Inst. (1980) pp.135-157

G.F. Guiduce (2008) [arXiv:0801.2562 [hep-ph]]
L. Alvarez-Gaume, M.A. Vazquez-Mozo, Invitation to Quant. Field Theory (2012)

E.g., naturalness for fermions: m=0 leads to chiral symmetry

However, nothing like that for scalar particles ...

dilaton symmetry W. Bardeen (1995)

The Standard Model: Brout-Englert-Higgs boson roles

The two main roles of BEH boson in the SM:

- **providing mass** to the SM particles by Brout-Englert-Higgs mechanism

- **restoring unitarity** for EW vector boson scattering: Brout-Englert-Higgs boson cancels quadratic growth of longitudinal components for EW vector bosons with collision energy

---

- **if Higgs boson could be very light** -> no noticeable growth with collision energy

- **if Higgs boson could be very heavy** -> strong growth of EW vector boson interaction -> new SM dynamics: nonpertubative strong EW interaction can lead to heavy EW resonances

Regularization and Renormalization

The two main regularization methods:

- **Dimensional regularization**
  "Geometrical": regulator depends on geometry of space-time

- **Cut-off regularization**
  Covariant Pauli-Villars
  "Physical"

Собственная энергия (пропагатор):

$$-i\Sigma(p^2) = -\frac{\lambda}{2}\int \frac{d^4k}{2\pi^4}\frac{i}{k^2-\mu^2+i\epsilon} = \frac{-i\lambda\pi^{\frac{n}{2}}\Gamma\left(1-\frac{n}{2}\right)}{32\pi^4(\mu^2)^{1-\frac{n}{2}}}$$

Регуляризованная собственная энергия:

$$\begin{aligned}
-i\Sigma(p^2) &= \frac{\lambda}{2}\int \frac{d^4k}{2\pi^4}\frac{\Lambda^4}{(k^2-\mu^2+i\epsilon)(k^2-\Lambda^2+i\epsilon)^2} \\
&= \frac{-i\lambda\Lambda^4}{32\pi^2}\int_0^1 \frac{\alpha d\alpha}{\alpha\Lambda^2+(1-\alpha)\mu^2} \\
&= \frac{-i\lambda}{32\pi^2}\left[\Lambda^2-\mu^2 log\left(\frac{\Lambda^2}{\mu^2}\right)\right]
\end{aligned}$$

Скалярные бозоны: Размерная регуляризация

Размерность пространства $D = n - \varepsilon < 4\,, \quad \varepsilon > 0$

$$-i\Sigma(p^2) = \frac{\lambda}{2}\int \frac{d^n k}{2\pi^4}\frac{1}{k^2-\mu^2+i\epsilon} = \frac{-i\lambda\pi^{\frac{n}{2}}\Gamma\left(1-\frac{n}{2}\right)}{32\pi^4(\mu^2)^{1-\frac{n}{2}}}$$

$$\Gamma\left(1-\frac{n}{2}\right) = \frac{\Gamma\left(3-\frac{n}{2}\right)}{\left(1-\frac{n}{2}\right)\left(2-\frac{n}{2}\right)}$$

Полюс: $n = 4$ (и при $n = 2$)

При $n \to 4$ только логарифмическая расходимость (!):

$$-i\Sigma(0) = \frac{i\lambda\mu^2}{16\pi^2}\frac{1}{(4-n)}$$

N.B. Вклад $n \to 2$ дает квадратичную расходимость (!)

Но зависит от числа петель L: $D = 4 - 2/L$

При $L \to \infty \quad D = n \to 4$

M. Veltman, Acta Phys. Pol. B12 (1981) 437

Naturalness of the Standard Model at 1-loop

M. Veltman, Acta Phys. Pol. B12 (1981) 437

renormalization scheme dependence for scalar particles:

$$m_H^2 = m_0{}_H^2 + C_L(\lambda_i, m_i) \cdot \log(\frac{\Lambda_{UV}^2}{m^2}) + C_X(\lambda_i, m_i) \cdot \Lambda_{UV}^2$$

"physical" schemes $\rightarrow C_X \neq 0$

schemes with dim. regularization ($\overline{\text{MS}}$, ...) $\rightarrow C_X = 0$

MSbar reproduces quadratic divergence at D = 2, L=1

$$m_H^2 = m_{H0}^2 + \delta m_H^2$$

$$v = 246\,\text{GeV}$$

$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}\left(24y_t^2 - 6(2y_W^2 + y_Z^2 + y_H^2)\right) \sim 8.2\frac{\Lambda^2}{16\pi^2}$$

$$y_i \equiv \frac{m_i}{v}$$

Non-naturalness of BEH boson at Λ > 550 GeV (Veltman criterion):

$$\delta m_H^2 \approx m_H^2 \qquad (\Lambda = 550\,\text{GeV},\ m_H = 125\,\text{GeV})$$

The Standard Model: Higgs boson mass evolution

M. Veltman, Acta Phys. Pol. B12 (1981) 437

quadratic mass divergences within MSbar renormalization:

Dim = 4 – 2/L

$m_R^2 = m_B^2 + \xi\, \Lambda^2,$

where $\xi = \xi(mH, mt, mW, mZ)$

Veltman's condition for absence of quadratic mass divergences:

$$\xi(mH, mt, mW, mZ) = 0$$

Veltman's condition holds up to 3-loops:

but in higher orders it cannot be hold in self-consistent way

M.S. Al-sarhi, I. Jack, D.R.T. Jones, Zeit fur Physik Pol. C55 (1992) 283

Veltman's condition and Higgs effective potential

M.B. Einhorn, D.R.T. Jones, Phys. Rev. D42 (1992) 5206

Quardratic divergence cancellation: Veltman's condition

Масса бозона Браута-Энглера-Хиггса:

$$m^2 = m_0^2 + \Delta m^2 = m_0^2 + \xi(\lambda_0, g)\,\Lambda^2$$

Условие сокращения квадратичной расходимости Вельтмана:

$$\xi(\lambda_0, g) = 3(3g_2^2 + g_1^2 + 2\lambda_0 - 4y_t^2)/(32\pi^2) = 0$$

g_1, g_2, y_t - константы СМ: групп $U(1)$, $SU(2)$, юкавская константа t-кварка

M. Veltman, Acta Phys. Pol. B12 (1981) 437

Нарушается с 4-х петель $L \geq 4$

M.S. Al-sarhi, I. Jack, D.R.T. Jones, Z.Phys. C55 (1992) 283

Naturalness criteria of the Standard Model

M. Veltman, Acta Phys. Pol. B12 (1981) 437

$$m_H^2 = m_{H0}^2 + \delta m_H^2$$

$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}\left(24y_t^2 - 6(2y_W^2 + y_Z^2 + y_H^2)\right) \sim 8.2\frac{\Lambda^2}{16\pi^2}$$

$$y_i \equiv \frac{m_i}{v} \qquad v = 246\,\mathrm{GeV}$$

Veltman's criterion: $\left|\frac{m^2 - m_0^2}{m_0^2}\right| = \left|\frac{\Delta m^2}{m_0^2}\right| \le q = 1$

Non-naturalness of BEH boson at Λ > 550 GeV:

$$\delta m_H^2 \approx m_H^2 \qquad (\Lambda = 550\,\mathrm{GeV},\ m_H = 125\,\mathrm{GeV})$$

Log-Derivative criterion: $\left|\frac{\lambda_0}{m^2}\frac{\partial m^2}{\partial \lambda_0}\right| \le q \simeq 10$

R. Barbieri, G.F. Giudice, Nucl. Phys. B306 (1988) 63
J.R.Ellis,K.Enquist,D.V.Nanopoulos,F.Zwirner, Mod.Phys.Lett. A1(1986)57

Naturalness Criterion

$$m_0^2 = m^2 - \xi(\lambda, g)\Lambda^2$$
$$\lambda_0 = \lambda + \frac{\beta(\lambda, g)}{2}\log\left(\frac{\Lambda^2}{m^2}\right)$$

Logarithmic sensitivity
Transformation matrix
RG mixing -> matrix non-degeneracy -> matrix

V.K., G. Pivovarov, Phys. Rev. D78 (2008) 016001
V.K., Phys. Part. Nucl. 55 (2024) 156

$$\begin{pmatrix} m_0^2 \\ \lambda_0 \end{pmatrix} = A \times \begin{pmatrix} m^2 \\ \lambda \end{pmatrix} \qquad A = \begin{pmatrix} \frac{\partial\lambda}{\partial\lambda_0} & \frac{\partial\lambda}{\partial m_0^2} \\ \frac{\partial m^2}{\partial\lambda_0} & \frac{\partial m^2}{\partial m_0^2} \end{pmatrix}$$

$$A = \frac{1}{\det(A^{-1})}\begin{pmatrix} 1 & \frac{\beta(\lambda,g)}{2m^2} \\ \xi'(\lambda,g)\Lambda^2 & 1 + \frac{\beta'(\lambda,g)}{2}\log\left(\frac{\Lambda^2}{m^2}\right) \end{pmatrix}$$

$$\det(A^{-1}) = 1 - \xi'(\lambda, g)\frac{\beta(\lambda, g)}{2}\frac{\Lambda^2}{m^2} + \frac{\beta'(\lambda, g)}{2}\log\left(\frac{\Lambda^2}{m^2}\right)$$

BEH-boson mass evolution and naturalness criterion

Logarithmic sensitivity
Transformation matrix
RG mixing -> matrix non-degeneracy -> matrix
V.K., G. Pivovarov, Phys. Rev. D78 (2008) 016001
V.K., Phys. Part. Nucl. 55 (2024) 156

$$m_H^2 = m_{H0}^2 + \delta m_H^2$$

$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}\left(24y_t^2 - 6(2y_W^2 + y_Z^2 + y_H^2)\right) \sim 8.2\frac{\Lambda^2}{16\pi^2}$$ **+ C log(Λ^2/m^2)**

Non-naturalness from BEH Higgs boson mass at Λ ~ O (10 TeV)

Previously without logs: Λ ~ O (1 TeV)

Naturalness of the Standard Model

Quantifying Naturalness

Naturalness log-derivative criterion:

weak sensitivity physical parameters for small variation of bare ones

J.R.Ellis, K.Enquist, D.V.Nanopoulos, F.Zwirner, Mod.Phys.Lett. A1(1986)57

R. Barbieri, G.F. Giudice, Nucl. Phys. B306 (1988) 63

Using BG condition with both quadratic and logarithmic contributions leads to extension of Naturalness domain of SM: up ~ O(10 TeV) instead of ~ O (1 TeV)

V.K., G. Pivovarov, Phys. Rev. D78 (2008) 016001

V.K., Phys. Part. Nucl. 55 (2024) 156

Regular way for scalar boson mass evolution with quadratic mass divergences

G. Pivovarov, Phys. Rev. D81 (2010) 076077

K. Fujikawa, Int. Mod. Phys. A (2016)

The SM with the "non-natural" Brout-Englert-Higgs boson

Proper physical consideration with quadratic evolution for Brout-Englert-Higgs boson mass:

Higgs boson observables (mass, self-coupling, EW vacuum density) gets critical values at later scales than in popular "standard" naturalness treatments

-> at those scales ~ O(10 TeV) one should expect new physics manifestations:
- **new strong EW dynamics**
- **or/and New Physics beyond Standard Model**

Summary

- The Standard Model without quadratic evolution for Brout-Englert-Higgs boson mass requires (!) New Physics to have Naturalness

- Naturalness domain of the Standard Model with quadratic evolution for Brout-Englert-Higgs boson mass may be larger than generally accepted: instead of ~ O (1 TeV) it is up ~ O(10 TeV)

- Present LHC physics: new physics is unavoidable either as a new dynamics of SM or/and a New Physics. Besides search direct search of New Physics it requires ‘non-naturalness’ studies