Small-p_T charmonium production in the Soft Gluon Resummation approach

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AQFT-25
August 12, 2025

Outline

- Introduction
- TMD factorization, TMD parton distributions
- Soft Gluon Resummation approach
- InEW, matching of factorization theorems
- Hadronization model: NRQCD, calculation results
- Hadronization model: ICEM, calculation results
- Summary

Introduction

Motivation

- **SPD NICA experiment: unpolarized and polarized** pp, pd, dd **collisions at the energy up to** $\sqrt{s} = 27$ **GeV**
- J/ψ production as a tool to study TMD gluon PDF in proton at small p_T and collinear gluon PDF at the large x.
- At the moment, heavy quarkonium hadronization models are normalized on description of unpolarized experimental data. As we have shown recently using the GPM [A. Karpishkov, M. Nefedov and V. Saleev, «Estimates for the single-spin asymmetries in the p↑p→J/ψX process at PHENIX RHIC and SPD NICA»,Phys. Rev. D **104** (2021) no.1, 016008], predictions for SSA strongly depend on hadronization model.
- Theoretical predictions for production cross sections and different spin-asymmetries depend on used factorization approach and region of it's applicability.

Introduction

This study

- J/ψ production as a tool to study gluon PDF in proton
  - In the small-p_T region, TMD PM and TMD PDFs
  - In the high-p_T region, CPM and Collinear PDFs
- InEW scheme for matching factorization theorems in the intermediate-p_T region
- Hadronization approaches: NRQCD and ICEM

TMD factorization and initial parton transverse momenta

- **Transverse Momentum Dependent (TMD) factorization**: $q_T, k_T \ll \mu_F \sim M$
- TMD parton distribution functions $F(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations:

$$\begin{cases} \dfrac{\partial \ln \hat{F}(x, \boldsymbol{b_T}, \mu_F, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu_F) & \text{with CS kernel } \tilde{K}(b_T, \mu_F) \\ \dfrac{\partial \tilde{K}(b_T, \mu)}{\partial \ln \mu} = -\gamma_K[\alpha_s(\mu_F)] & \text{with anomalous dimension } \gamma_K[\alpha_s(\mu_F)] \end{cases}$$

- Initial parton 4-momenta may be presented as:

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu$$

  - preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2} \right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2} \right)$$

TMD factorization and TMD PDFs

- General formula of TMD factorization [TMD Handbook, arXiv:2304.03302]:

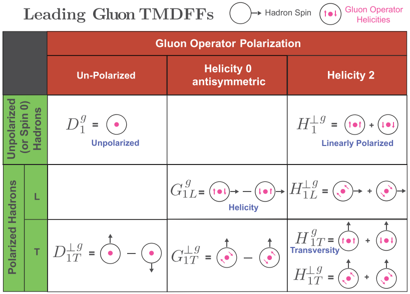
$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int d\boldsymbol{q_{1T}}\, d\boldsymbol{q_{2T}}\, F(x_1, \boldsymbol{q_{1T}}, \mu_F, \zeta_1)\, F(x_2, \boldsymbol{q_{1T}}, \mu_F, \zeta_2)\, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{p_T})$$

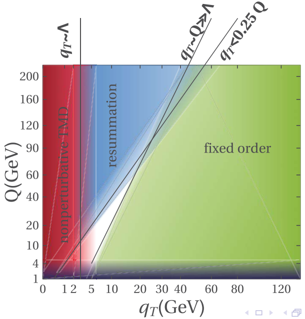
- To implement **CS** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int \frac{d\boldsymbol{b_T}}{(2\pi)^2}\, e^{i\boldsymbol{p_T}\boldsymbol{b_T}}\, \hat{F}(x_1, \boldsymbol{b_T}, \mu_F, \zeta_1)\, \hat{F}(x_2, \boldsymbol{b_T}, \mu_F, \zeta_2)$$

  - σ_0 is calculated as series with respect to small α_S

TMD factorization and TMD PDFs (*Pictures are from A. Vladimirov*)





Basic processes for theoretical study in the TMD factorization

$$pp \to \gamma^* X, \quad pp \to ZX, \quad pp \to HX,$$

$$pp \to \eta_{c,b} X, \quad pp \to \chi_{c0,2} X, \quad pp \to \chi_{b,0,2}$$

TMD factorization and TMD PDFs

TMD shape functions

- M. G. Echevarria, «Proper TMD factorization for quarkonia production: $pp \to \eta_{c,b}$ as a study case», JHEP 10 (2019) 144.
- S. Fleming, Y. Makris, and T. Mehen, «An effective field theory approach to quarkonium at small transverse momentum,» JHEP 04 (2020) 122, arXiv:1910.03586 [hep-ph].
- L. Maxia, D. Boer and J. Bor, «The impact of the TMD shape function on matching the transverse momentum spectrum in J/ψ production at the EIC», [arXiv:2504.19617 [hep-ph]].

Soft Gluon Resummation approach

Relevant publications based on SGR approach

- J.C. Collins , D.E. Soper, G. Sterman «Transverse momentum distribution in Drell-Yan pair and W and Z boson production», Nuclear Physics B250 (1985) 199-224.
- S. Catani, D. de Florian, M. Grazzini and P. Nason, «Soft gluon resummation for Higgs boson production at hadron colliders», JHEP **07** (2003), 028.
- C. Balazs, E. L. Berger, S. Mrenna and C. P. Yuan, «Photon pair production with soft gluon resummation in hadronic interactions», Phys. Rev. D **57** (1998), 6934-6947
- P. Sun, C. P. Yuan and F. Yuan, «Heavy Quarkonium Production at Low Pt in NRQCD with Soft Gluon Resummation,» Phys. Rev. D **88** (2013), 054008
- D. Boer and W. J. den Dunnen, «TMD evolution and the Higgs transverse momentum distribution», Nucl. Phys. B **886** (2014), 421-435
- D. Boer and C. Pisano, «Impact of gluon polarization on Higgs boson plus jet production at the LHC», Phys. Rev. D **91** (2015) no.7, 074024
- S. Catani and M. Grazzini, «QCD transverse-momentum resummation in gluon fusion processes», Nucl. Phys. B **845** (2011), 297-323
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Soft Gluon Resummation approach

- Soft and collinear gluon resummation approach by [J. Collins, D. Soper, 1981]:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1, \mu'_{b^*}, b_T^*)\, \hat{F}(x_2, \mu'_{b^*}, b_T^*)$$

- Sudakov factor in LL–LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_P(b_T, \mu_F, Q) = \frac{C_A}{\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \alpha_s(\mu') \left[\ln \frac{Q^2}{\mu'^2} - \left(\frac{11 - 2N_f/C_A}{6} \right) \right] + \mathcal{O}(\alpha_s)$$

- Sudakov factor expression is valid only on region $b_0/Q \leqslant b_T \leqslant b_{T,\,\mathrm{max}}$ which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu'_b = \frac{Q b_0}{Q b_T + b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\,\mathrm{max}})^2}}$$

Soft Gluon Resummation approach

- Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T\, b_T\, J_0(p_T b_T)\; e^{-S_P(b_T,\mu_F,Q)}\; e^{-S_{NP}(b_T)}\; \hat{F}(x_1,\mu'_{b^*},b_T^*)\; \hat{F}(x_2,\mu'_{b^*},b_T^*)$$

- **Nonperturbative** quark factor obtained in SIDIS data fitting: [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T,Q) = \left[g_1 \ln\frac{Q}{2Q_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10xx_0}{x_0+x}\right)\right] b_T^2$$

  - it should be Casimir-scaled by C_A/C_F for gluons

- In the leading order of α_S, the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x,\mu'_{b^*},b_T^*) = f(x,\mu'_{b^*}) + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}})$$

Matching of small-p_T and high-p_T regions within Inverse-Error Weighting Scheme

- Matched cross-section as a weighed sum of CPM and TMD terms [M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

$$d\sigma = \mathcal{W}\, d\sigma^{\mathsf{TMD}} + \mathcal{Z}\, d\sigma^{\mathsf{CPM}}$$

- Normalized weights for each of the two terms:

$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \qquad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}$$

$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2 + \left(\frac{m}{Q}\right)^2, \qquad \Delta\mathcal{Z} = \left(\frac{M}{p_T}\right)^2 \left(1 + \ln^2 \frac{Q_T}{p_T}\right)$$

- Uncertainty due to the matching procedure:

$$\Delta d\sigma = \frac{d\sigma}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta\mathcal{W} \cdot \Delta\mathcal{Z}}{\sqrt{\Delta\mathcal{W}^2 + \Delta\mathcal{Z}^2}} d\sigma$$

Hadronization model: NRQCD, approximations

- J/ψ wave function as a series with respect to relative constituent quarks velocity v:

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \ldots$$

- Hard cross section factorisation:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle \mathcal{O}^{\mathcal{C}}[n]\rangle$$

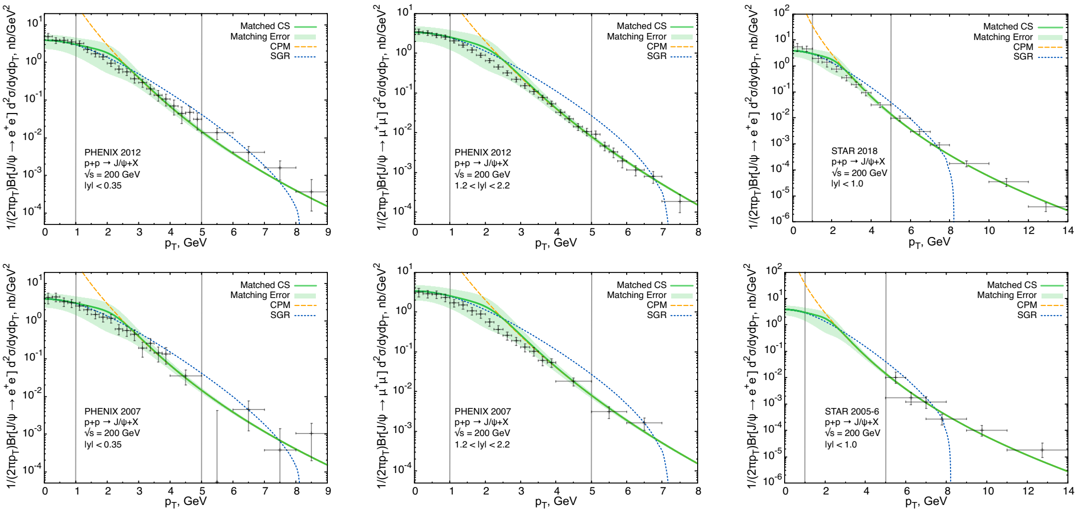
- Nonperturbative long-distance matrix elements (LDME) $\langle \mathcal{O}^{\mathcal{C}}[n]\rangle$:

color singlet LDMEs — potential models, data for leptonic decay

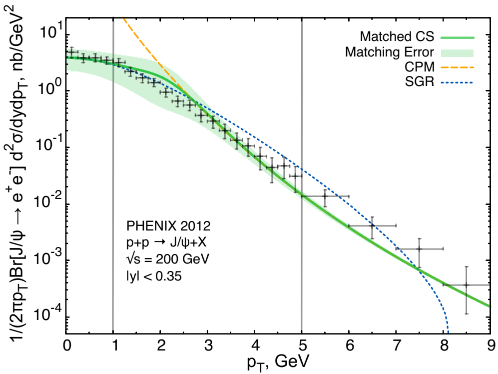
color octet LDMEs — lattice QCD calculation or **experimental data fitting**

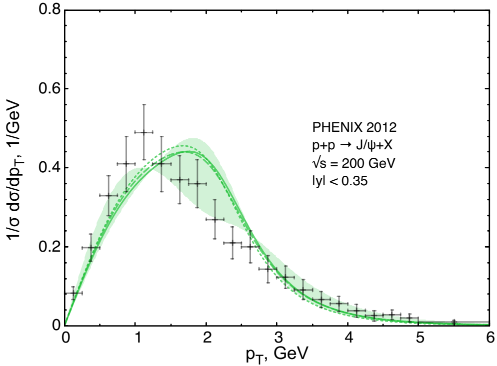
- **Prompt** = Direct + Feed-down contributions from $\psi(2S) \to J/\psi + X$ and $\chi_{cJ} \to J/\psi + \gamma$
  - P-wave feed-down is estimated at 30% at $\sqrt{s} = 200$ GeV
- **Gluon-gluon fusion** & **quark-antiquark annihilation** subprocesses
  - $q\bar{q}$ subprocesses are estimated at 10% at $\sqrt{s} = 200$ GeV within the ICEM [Saleev, Chernyshev, 2022]
- Calculations in the LO approximation of the pQCD with respect to α_s

Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)



Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)





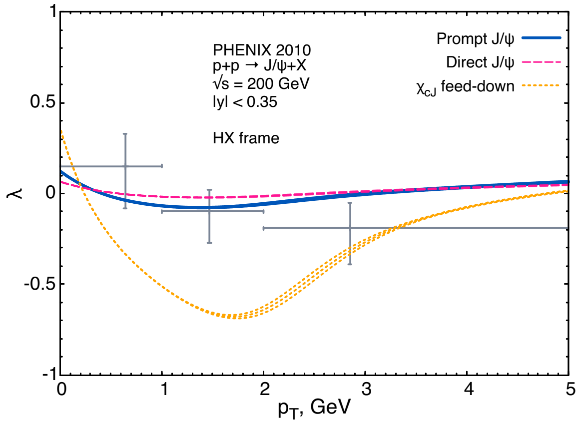
At $\sqrt{s} = 200$ GeV:

- quark-antiquark annihilation contribution $< 3\%$
- χ_{cJ} decays contribution $\approx 7\%$

Extraction of CO LDME at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM)

CO LDME	Our fit	LO CPM [Cho, Leibovich (1996)]	NLO CPM, global fit [Butenschön, Kniehl (2011)]
$\langle \mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle$, GeV^3	$(9.7 \pm 0.5) \cdot 10^{-2}$	–	$(3.0 \pm 0.4) \cdot 10^{-2}$
$\langle \mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle$, GeV^5	$(1.3 \pm 0.2) \cdot 10^{-2}$	–	$(-9.1 \pm 1.6) \cdot 10^{-3}$
$M_3^{J/\psi}$, GeV^3	$(1.1 \pm 0.1) \cdot 10^{-1}$	$(6.6 \pm 1.5) \cdot 10^{-2}$	$(1.8 \pm 0.6) \cdot 10^{-2}$
$\langle \mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle$, GeV^3	$(2.0 \pm 1.6) \cdot 10^{-3}$	$(6.6 \pm 2.1) \cdot 10^{-3}$	$(1.7 \pm 0.5) \cdot 10^{-3}$
$\langle \mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle$, GeV^3	$(8.6 \pm 2.9) \cdot 10^{-3}$	$(3.3 \pm 0.5) \cdot 10^{-3}$	–
χ^2/n.d.f.	0.76	0.9	3.74

Polarization test for CO LDME at PHENIX



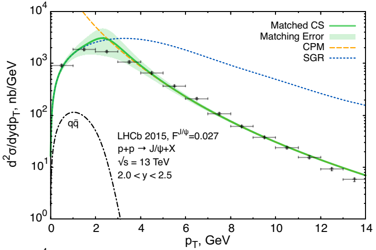
Hadronization model: ICEM, approximations

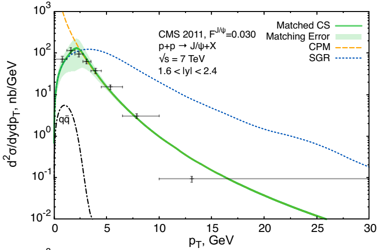
- Improved Color Evaporation Model, scale hierarchy $m_c \gg \lambda \gg \Lambda_{\mathrm{QCD}}$ [Y.-Q. Ma, R. Vogt (2016)]:
  - $c\bar{c}$-pair production on $\mathcal{O}(\frac{\Lambda}{m_c})$ scale
  - soft gluon exchange and emission on $\mathcal{O}(\lambda)$ scale
  - $\lambda \sim q_T$, q_T is intrinsic transverse momentum of parton
- General cross-section expression:

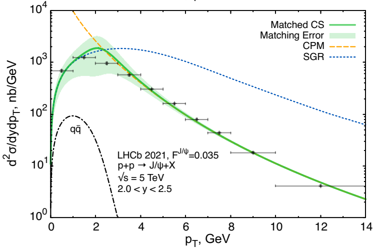
$$d\sigma(p^{J/\psi}) = F^{J/\psi} \times \int_{M_{J/\psi}}^{2m_D} dM_{c\bar{c}} \int d^4 p^{c\bar{c}} \frac{d\sigma\left(p^{c\bar{c}}\right)}{dM_{c\bar{c}}} \delta^{(4)}\left(p^{J/\psi} - \frac{M_{J/\psi}}{M_{c\bar{c}}} p^{c\bar{c}}\right) + \mathcal{O}\left(\lambda^2/m_c^2\right)$$

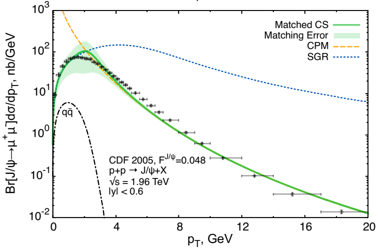
  with dependence of $F^{J/\psi}$ on center-of-mass energy $\sqrt{s}$
- Only **prompt** production of J/ψ within the ICEM
- Study of both **gluon-gluon fusion** $g + g \to c + \bar{c}$ and **quark-antiquark annihilation** $q + \bar{q} \to c + \bar{c}$
  - rise of quark-antiquark subprocess contribution with the decrease of center-of-mass energy $\sqrt{s}$
- Assumption of $c\bar{c}$-quark pair production in color-octet state
- Calculations in the LO approximation of the pQCD with respect to α_s, $g + g \to c + \bar{c} + g$, ...

Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM) $\sqrt{s} = 1.96 - 13$ TeV

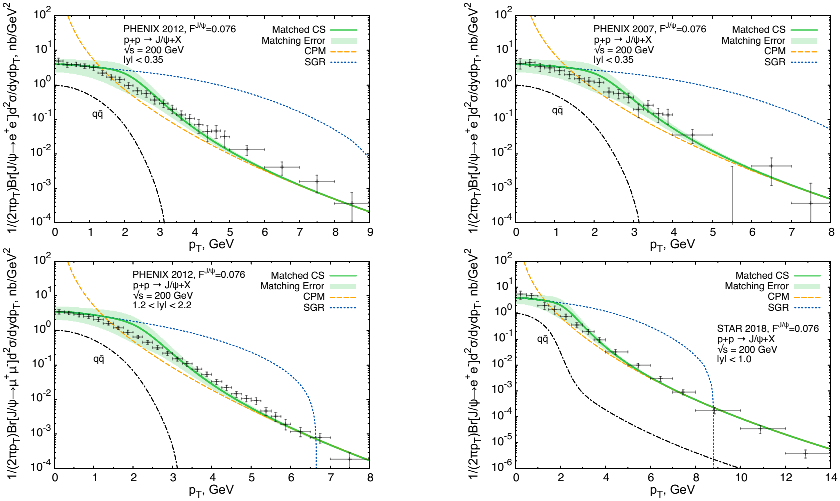




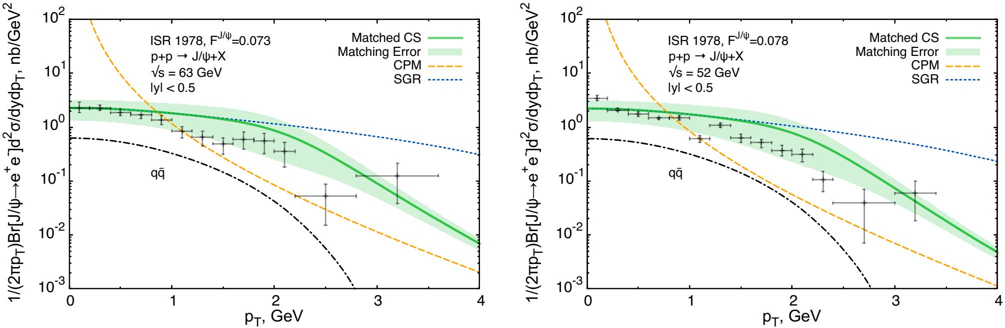




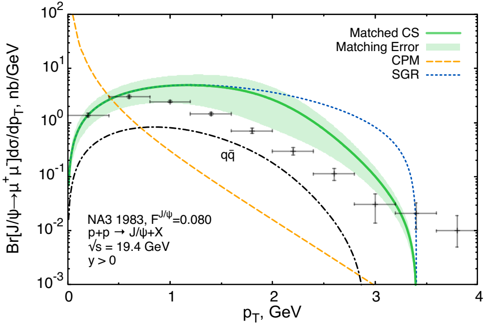
Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) and at $p_T > 5$ GeV (CPM) $\sqrt{s} = 200$ GeV

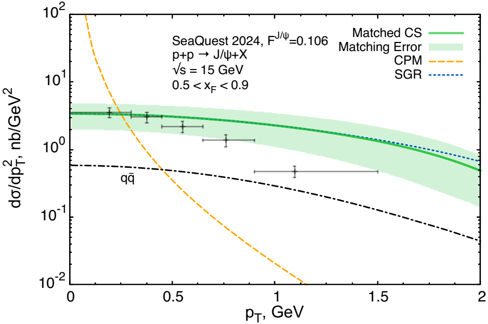


Extraction of $F^{J/\psi}$ at $p_T < 1$ GeV (SGR) $\sqrt{s} = 52 - 63$ GeV

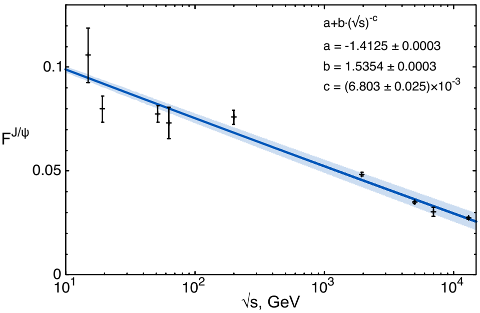


Extraction of $F^{J/\psi}$ at $p_T < 0.5$ GeV (SGR)
$\sqrt{s} = 15 - 19.4$ GeV

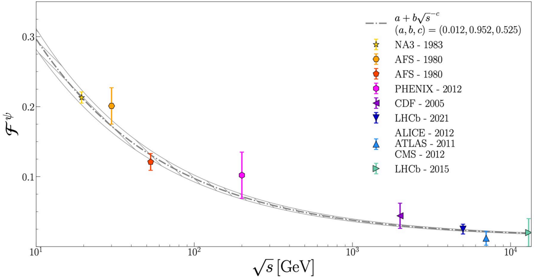




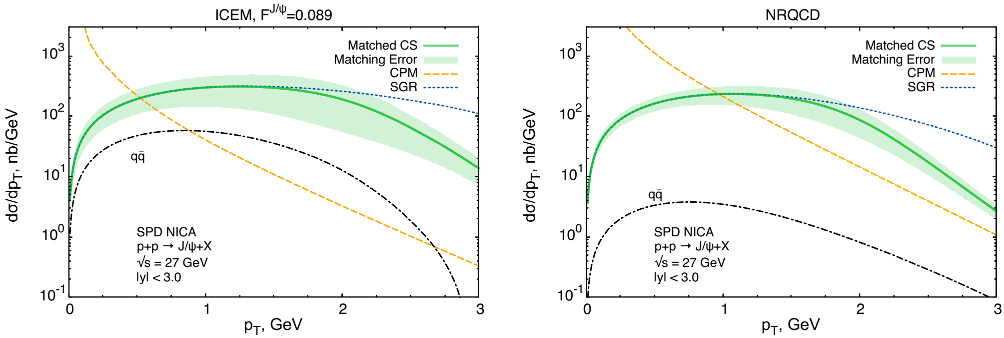
Dependence of $F^{J/\psi}$ on center-of-mass energy $\sqrt{s}$



[A. Chernyshev, V. Saleev (2022)]



Predictions for SPD NICA, comparison of ICEM and NRQCD calculations



Summary

Our published results

- V. Saleev and K. Shilyaev, «Production of S-wave charmonia in the soft gluon resummation approach using the NRQCD», [arXiv:2502.16461 [hep-ph]], to be published in Mod. Phys. Lett. A (2025).
- V. Saleev and K. Shilyaev, «Production of S-wave charmonia in the soft gluon resummation approach using the ICEM», in preparation.
- V. Saleev and K. Shilyaev, «Small-p_T Production of η_c Mesons within the Soft Gluon Resummation Approach», Phys. Atom. Nucl. **88** (2025) no.2, 338-341
- V. A. Saleev, «Challenges and Problems in Charmonium Production at the SPD NICA», Phys. Part. Nucl. **55** (2024) no.6, 1460-1466

Summary

- We have used the Soft Gluon Resummation approach to calculate small-p_T J/ψ production in the TMD factorisation
- NRQCD: CO LDMEs are necessary to describe J/ψ production using the TMD and CPM factorisations, where they are major contributions
- ICEM: hadronisation factor $F^{J/\psi}$ crucially depends on the center-of-mass energy $\sqrt{s}$, showing decrease when $\sqrt{s}$ increases
- Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describes experimental data for unpolarised J/ψ production at $\sqrt{s}$ from 15 GeV up to 13 TeV in the TMD domain of $p_T < 1$ GeV
- We have tested the factorisation matching scheme InEW for description of intermediate p_T domain in the ICEM, calculation agrees with experimental data within the uncertainty band
- We estimate the perspective region for extraction of gluon TMD PDF in J/ψ production as $p_T \leqslant 1$ GeV

THANK YOU FOR ATTENTION!