Harmonic Analyticity as a Basis of $\mathcal{N} = 2$ Supersymmetric Higher Spins

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Supersymmetry and superfields

- Supersymmetry, despite lacking experimental confirmations, is in the heart of the modern of mathematical and quantum physics. It allowed to construct a lot of new theories with remarkable and surprising features: supergravities, superstrings, superbranes, $\mathcal{N} = 4$ super Yang-Mills theory (the first example of the ultraviolet-finite quantum field theory), etc. It also exhibited unexpected relations between these theories, e.g., the "gravity/gauge" duality.

- The natural approach to supersymmetric theories is the superfield methods.

- The natural generalization of Minkowski space x^m to supersymmetry is $\mathcal{N}$ **extended Minkowski superspace**

$$\mathcal{M}^{(4|4\mathcal{N})} = \left(x^m\,,\ \theta^\alpha_i\,,\ \bar\theta^{\dot\alpha\, i}\right),\ i = 1, \ldots, \mathcal{N}$$

  where $\theta^\alpha_i\,, \bar\theta^{\dot\alpha\, i}$ are anticommuting Grassmann coordinates, $\{\theta, \theta\}, \{\theta, \bar\theta\} = 0$.

- The supersymmetric theories are adequately formulated off shell in terms of superfields defined on various superspaces.

$\mathcal{N} = 1, 4D$ chirality

$\mathcal{N} = 1, 4D$ supersymmetry is an extension of the Poincaré symmetry by spinor generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 4\, P_{\alpha\dot{\beta}}\,,\ \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0\,,\ [P_m, Q_\alpha] = [P_m, \bar{Q}_{\dot{\alpha}}] = 0$$

$\mathcal{N} = 1, 4D$ superspace is an extension of Minkowski space by a doublet of Grassmann anticommuting spinorial coordinates $\theta^\gamma, \bar{\theta}^{\dot{\mu}}$

$$x^{\alpha\dot{\alpha}} \Rightarrow (x^{\alpha\dot{\alpha}}, \theta^\gamma, \bar{\theta}^{\dot{\mu}})\,, \theta^{\gamma\prime} = \theta^\gamma + \epsilon^\gamma\,, x^{\alpha\dot{\alpha}\prime} = x^{\alpha\dot{\alpha}} - 2i(\theta^\alpha \epsilon^{\dot{\alpha}} - \epsilon^\alpha \bar{\theta}^{\dot{\alpha}})$$

$\mathcal{N} = 1, 4D$ superfields are functions on $\mathcal{N} = 1$ superspace, $\Phi(x, \theta, \bar{\theta})$, with the transformation law,

$$\Phi'(x', \theta', \bar{\theta}') = \Phi(x, \theta, \bar{\theta})$$

There is one very essential distinction between Minkowski space and $\mathcal{N}=1$ superspace. While the former does not include any subspace where the whole $4D$ Poincaré symmetry could be linearly realized, the latter contains such smaller supermanifolds, $\mathcal{N}=1, 4D$ chiral superspaces $(x_L, \theta), (x_R, \bar\theta)$ with twice as less Grassmann coordinates:

$$x_L^{\alpha\dot\beta} = x^{\alpha\dot\beta} + 2i\theta^\alpha\bar\theta^{\dot\beta}\,, \quad \delta x_L^{\alpha\dot\beta} = -4i\theta^\alpha\bar\epsilon^{\dot\beta}\,, \quad x_R^{\alpha\dot\beta} = (x_L^{\alpha\dot\beta})^\dagger$$

The chiral superfields are carriers of the basic matter $\mathcal{N}=1$ multiplet:

$$\varphi(x_L,\theta) = \phi(x_L) + \theta^\alpha\psi_\alpha(x_L) + (\theta)^2F(x_L)\,, S_{free} \sim \int d^4x d^2\theta d^2\bar\theta\, \varphi(x_L,\theta)\bar\varphi(x_R,\bar\theta)$$

The chiral superfields can be looked upon as complex general $\mathcal{N}=1$ superfields subject to the covariant Grassmann analyticity condition (A. Galperin, E.I., V. Ogievetsky, 1981)

$$\varphi(x_L,\theta) = \Phi_L(x_L,\theta,\bar\theta)\,, \quad \frac{\partial}{\partial\bar\theta^{\dot\gamma}}\Phi_L = 0$$

The same constraint can be rewritten in the basis $(x,\theta,\bar\theta)$ in terms of spinor covariant derivatives

$$\bar D_{\dot\gamma}\Phi_L(x,\theta,\bar\theta) = 0\,, \quad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}\,, \bar D_{\dot\gamma} = -\frac{\partial}{\partial\bar\theta^{\dot\gamma}} - 2i'\theta^\alpha\partial_{\alpha\dot\gamma}\,,$$

$$\{D_\gamma, D_\beta\} = \{\bar D_{\dot\gamma}, \bar D_{\dot\beta}\} = 0\,, \quad \{D_\gamma, \bar D_{\dot\beta}\} = -4i\partial_{\gamma\dot\beta}$$

The vanishing of anticommutators of the same chirality spinor derivatives is just the integrability conditions for $\mathcal{N}=1$ chirality. This chirality underlies all the gauge and supergravity $\mathcal{N}=1$ theories: the interacting case just corresponds to replacing all covariant derivatives by the gauge-covariant ones through adding proper superfield gauge connections

$$D_\gamma \ \Rightarrow \ \mathcal{D}_\gamma = D_\gamma + i\mathcal{A}_\alpha\,, \ \bar{D}_{\dot\gamma} \ \Rightarrow \ \bar{\mathcal{D}}_{\dot\gamma} = \bar{D}_{\dot\gamma} + i\bar{\mathcal{A}}_{\dot\gamma}\,, \ \partial_{\gamma\dot\beta} \ \Rightarrow \ \mathcal{D}_{\gamma\dot\beta} = \partial_{\gamma\dot\beta} + i\mathcal{A}_{\gamma\dot\beta}\,,$$

still preserving the flat integrability constraints

$$\{\mathcal{D}_\gamma, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\gamma}, \bar{\mathcal{D}}_{\dot\beta}\} = 0$$

The general $\mathcal{N}=1$ matter is also described by chiral superfields, implying a general Kähler target geometry for bosonic fields (Zumino, 1979).

For extended supersymmetries (with few sorts of Q generators) new kinds of Grassmann analyticities (different from chirality) can be defined. One of them, the harmonic $SU(2)$ analyticity, just forms the basis of the Harmonic Superspace approach.

Harmonic superspace

- In $4D$, the only self-consistent off-shell superfield formalism for $\mathcal{N}=2$ (and $\mathcal{N}=3$) theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z=(x^m\,,\ \theta^\alpha_i\,,\ \bar\theta^{\dot\alpha j}, u^{\pm i}),\quad u^{\pm i}\in SU(2)/U(1),\ u^{+i}u^-_i=1$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A=(x^m_A,\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm i}),\ \theta^{+\alpha,\dot\alpha}:=\theta^{\alpha,\dot\alpha i}u^+_i,\ x^m_A:=x^m-2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_iu^+_j$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}:\quad V^{++}(\zeta_A)\,,\ \ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A)\,,\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}:\quad H^{++m}(\zeta_A)\,,\ H^{++\hat\alpha+}(\zeta_A)\,,\ H^{++5}(\zeta_A)\,,\hat\alpha=(\alpha,\dot\alpha)$$

$\mathcal{N} = 2$ spin 1 multiplet

- An instructive example is Abelian $\mathcal{N} = 2$ gauge theory,

$$V^{++}(\zeta_A)\,, \quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\; D^{++} = \partial^{++} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$$

- Wess-Zumino gauge ($8 + 8$ off-shell degrees of freedom):

$$V^{++}(\zeta_A) = (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+(\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i u^-_k$$

- Invariant action:

$$S \sim \int d^{12}Z\;\left(V^{++}V^{--}\right),\; D^{++}V^{--} - D^{--}V^{++} = 0\,,\; \delta V^{--} = D^{--}\Lambda\,,$$
$$[D^{++}, D^{--}] = D^0, \quad D^0\,V^{\pm\pm} = \pm 2\,V^{\pm\pm}$$

Supersymmetry and higher spins

- Supersymmetric higher-spin theories provide a bridge between superstring theory and low-energy (super)gauge theories.
- Free massless bosonic and fermionic higher spin field theories: Fronsdal, 1978; Fang, Fronsdal, 1978.
- The natural tools to deal with supersymmetric theories are off-shell superfield methods. In the superfield approach the supersymmetry is closed on the off-shell supermultiplets and so is automatically manifest.
- The component approach to $4D$, $\mathcal{N} = 1$ supersymmetric free massless higher spin models: Courtright, 1979; Vasiliev, 1980.
- The complete off-shell $\mathcal{N} = 1$ superfield Lagrangian formulation of $\mathcal{N} = 1, 4D$ free higher spins: Kuzenko et al, 1993, 1994.

- An off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown for long even for free theories.

- This gap was filled in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 12 (2021) 016. An off-shell manifestly $\mathcal{N} = 2$ supersymmetric unconstrained formulation of $4D, \mathcal{N} = 2$ super Fronsdal theory for integer spins was constructed in the harmonic superspace approach.

- Manifestly $\mathcal{N} = 2$ supersymmetric off-shell cubic couplings of $4D, \mathcal{N} = 2$ to the matter hypermultiplets were further constructed in I. Buchbinder, E. Ivanov, N. Zaigraev, 2022, 2023.

- Quite recently, we generalized HSS non-conformal construction to the case of $\mathcal{N} = 2$ superconformal multiplets and their hypermultiplet coupling (arXiv:2404.19016 [hep-th], JHEP 08 (2024) 120).

- Our papers opened a new domain of applications of the harmonic superspace formalism, that time in $\mathcal{N} = 2$ higher-spin theories.

$\mathcal{N} = 2$ spin 2: linearized $\mathcal{N} = 2$ supergravity

- Analogs of $V^{++}(\zeta_A)$ are the following set of analytic gauge potentials:

$$\left(h^{++m}(\zeta_A)\,,\; h^{++5}(\zeta_A)\,,\; h^{++\hat{\mu}+}(\zeta_A)\right), \quad \hat{\mu} = (\mu\,,\dot{\mu})\,,$$
$$\delta_\lambda h^{++m} = D^{++}\lambda^m + 2i\left(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\right),$$
$$\delta_\lambda h^{++5} = D^{++}\lambda^5 - 2i\left(\lambda^{+\alpha}\theta^+_\alpha - \bar{\theta}^+_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\right), \delta_\lambda h^{++\hat{\mu}+} = D^{++}\lambda^{+\hat{\mu}}$$

- Wess-Zumino gauge:

$$h^{++m} = -2i\theta^+\sigma^a\bar{\theta}^+\Phi^m_a + \left[(\bar{\theta}^+)^2\theta^+\psi^{m\,i}u^-_i + c.c.\right] + \ldots$$
$$h^{++5} = -2i\theta^+\sigma^a\bar{\theta}^+C_a + \ldots\,, \quad h^{++\mu+} = \ldots$$

- The residual gauge freedom:

$$\lambda^m \;\Rightarrow\; a^m(x)\,,\; \lambda^5 \;\Rightarrow\; b(x)\,,\; \lambda^{\mu+} \;\Rightarrow\; \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu}l_{(\nu}{}^{\mu)}(x)$$

- The physical fields are $\Phi^m_a, \psi^{m\,i}_\mu, C_a$ ($(\mathbf{2}, \mathbf{3/2}, \mathbf{3/2}, \mathbf{1})$ on shell). In the "physical" gauge:

$$\Phi^m_a \sim \Phi_{\beta\dot{\beta}\alpha\dot{\alpha}} \Rightarrow \Phi_{(\beta\alpha)(\dot{\beta}\dot{\alpha})} + \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi$$

$\mathcal{N}=2$ spin 3 and higher spins

- The spin 3 triad of analytic gauge superfields is introduced as :

$$\{h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)\,,\; h^{++\alpha\dot\alpha}(\zeta),\; h^{++(\alpha\beta)\dot\alpha+}(\zeta),\; h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)\}\,,$$

$$\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],$$

$$\delta h^{++\alpha\dot\alpha} = D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],$$

$$\delta h^{++(\alpha\beta)\dot\alpha+} = D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\; \delta h^{++(\dot\alpha\dot\beta)\alpha+} = D^{++}\lambda^{+(\dot\alpha\dot\beta)\alpha}$$

- The bosonic physical fields in WZ gauge are collected in

$$h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + \dots \quad h^{++\alpha\dot\alpha} = -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + \dots$$

- The physical gauge fields are $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}$ (spin 3 gauge field), $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ (spin 2 gauge field) and $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_{\gamma}$ (spin 5/2 gauge field). The rest of fields are auxiliary. On shell, $\mathbf{(3, 5/2, 5/2, 2)}$.

- The general case with the maximal integer spin **s** is spanned by the analytic gauge potentials

$$h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta),\, h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta),\, h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta),\, h^{++\dot\alpha(s-1)\alpha(s-2)+}(\zeta),$$

  where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot\alpha(s) := (\dot\alpha_1 \ldots \dot\alpha_s)$

- The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1})$.

- The on-shell spin contents of $\mathcal{N}=2$ higher-spin multiplets;

$$\underline{\textit{spin}\ 1}:\ 1,\ (1/2)^2,\ (0)^2$$
$$\underline{\textit{spin}\ 2}:\ 2,\ (3/2)^2,\ 1$$
$$\underline{\textit{spin}\ 3}:\ 3,\ (5/2)^2,\ 2$$
$$\ldots\ldots.$$
$$\underline{\textit{spin}\ s}:\ s,\ (s-1/2)^2,\ s-1$$

- Each spin enters the direct sum of these multiplets twice, in accord with the general Vasiliev theory of $4D$ higher spins. The off-shell contents of the spin **s** multiplet: $8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_B + 8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_F$.

Hypermultiplet couplings

- The construction of interactions in the theory of higher spins is a very important (albeit difficult) task.
- There is an extensive literature on the construction of cubic higher spin interactions (e.g., Bengtsson et al, 1983; Fradkin, Metsaev, 1991; Metsaev, 1993; Manvelyan, Mkrtchyan, Ruehl, 2010, 2011, and many others)
- Supersymmetric $\mathcal{N} = 1$ generalizations of the bosonic cubic vertices with matter were explored in terms of $\mathcal{N} = 1$ superfields by Gates, Koutrolikos, Kuzenko, I. Buchbinder, E. Buchbinder and many others.
- In JHEP 05 (2022) 104 we have constructed the off-shell manifestly $\mathcal{N} = 2$ supersymmetric cubic couplings $(\frac{1}{2}, \frac{1}{2}, \mathbf{s})$ of an arbitrary higher integer superspin $\mathbf{s}$ gauge $\mathcal{N} = 2$ multiplet to the hypermultiplet matter in $4D, \mathcal{N} = 2$ harmonic superspace.
- In our approach $\mathcal{N} = 2$ supersymmetry of cubic vertices is always manifest and off-shell, in contrast, e.g., to the non-manifest light-cone formulations (Metsaev, 1905.11357, 1909.05241).

- The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S=\int d\zeta^{(-4)}\,\mathcal{L}^{+4}_{free}=-\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\mathcal{D}^{++}q^{+}_{a}\,,\ a=1,2$$

- Analytic gauge potentials for any spin **s** with the correct transformation rules are recovered by proper gauge-covariantization of the harmonic derivative $\mathcal{D}^{++}$. The simplest option is gauging of $U(1)$,

$$\delta q^{+a}=-\lambda_0 Jq^{+a},\quad Jq^{+a}=i(\tau_3)^a_{\ b}q^{+b}\,,$$
$$\mathcal{D}^{++}\ \Rightarrow\ \mathcal{D}^{++}+\hat{\mathcal{H}}^{++}_{(1)}\,,\quad \hat{\mathcal{H}}^{++}_{(1)}=h^{++}J\,,$$
$$\delta_\lambda\hat{\mathcal{H}}^{++}_{(1)}=[\mathcal{D}^{++},\hat{\Lambda}]\,,\quad \hat{\Lambda}=\lambda J\ \Rightarrow\ \delta_\lambda h^{++}=\mathcal{D}^{++}\lambda$$

- In $\mathcal{N}=2$ supergravity, that is for $\mathbf{s}=2$,

$$S_{(2)}=-\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\big(\mathcal{D}^{++}+\mathcal{H}_{(2)}\big)q^{+}_{a}\,,\quad \delta\mathcal{H}_{(2)}=[\mathcal{D}^{++},\hat{\Lambda}_{(2)}],$$
$$\mathcal{H}_{(2)}=h^{++M}(\zeta)\partial_M,\ \hat{\Lambda}_{(2)}=\lambda^M(\zeta)\partial_M,\ M:=(\alpha\dot{\beta},5,\hat{\mu}+)$$

- For higher **s** everything goes analogously. For $\mathbf{s}=3$

$$S_{(3)}=-\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\big(\mathcal{D}^{++}+\mathcal{H}_{(3)}J\big)q^{+}_{a}\,,$$
$$\delta\mathcal{H}_{(3)}=[\mathcal{D}^{++},\hat{\Lambda}_{(3)}],\quad \mathcal{H}_{(3)}=h^{++\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}},\quad \hat{\Lambda}_{(3)}=\lambda^{\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}$$

Superconformal couplings

- Free conformal higher-spin actions in $4D$ Minkowski space were pioneered by Fradkin & Tseytlin, 1985; Fradkin & Linetsky, 1989, 1991. Since then, a lot of works on (super)conformal higher spins followed (e.g., Segal, 2003, Kuzenko, Manvelyan, *et al*, 2017, 2023 ......).
- (Super)conformal higher-spin theories are considered as a basis for all other types of higher-spin models. Non-conformal ones follow from the superconformal ones through couplings to the superfield compensators.
- In (Buchbinder, Ivanov, Zaigraev, arXiv:2404.19016 [hep-th]), we extended the off-shell $\mathcal{N}=2, 4D$ higher spins and their hypermultiplet couplings to the superconformal case. Rigid $\mathcal{N}=2, 4D$ superconformal symmetry plays a crucial role in fixing the structure of the theory.
- $\mathcal{N}=2, 4D$ SCA preserves harmonic analyticity and is a closure of the rigid $\mathcal{N}=2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha}i} u^+_i\,,\ \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha}i}\right) u^-_i\,, \hat{\alpha} = (\alpha, \dot{\alpha})\,,$$

$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}} k_{\beta\dot{\beta}} \theta^{\hat{\beta}}\,,\ \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha}\,,\ \delta_k u^{+i} = (4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}}) u^{-i}$$

- What about the conformal properties of various analytic higher-spin potentials? No problems with the spin **1** potential V^{++}:

$$\delta_{sc} V^{++} = -\hat{\Lambda}_{sc} V^{++}\,, \quad \hat{\Lambda}_{sc} := \lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+} + \lambda^{++}_{sc}\partial^{--}$$

- The cubic vertex $\sim q^{+a} V^{++} J q^{+}_{a}$ is invariant up to total derivative if

$$\delta_{sc} q^{+a} = -\hat{\Lambda}_{sc} q^{+a} - \frac{1}{2}\Omega q^{+a}\,, \quad \Omega := (-1)^{P(M)}\partial_M \lambda^M$$

- Situation gets more complicated for $\mathbf{s} \geq 2$. Requiring $\mathcal{N}=2$ gauge potentials for $\mathbf{s}=2$ to be closed under $\mathcal{N}=2$ SCA necessarily leads to

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,,$$
$$\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^{-}_{\alpha} + h^{++\dot{\alpha}+}\partial^{-}_{\dot{\alpha}} + h^{(+4)}\partial^{--}$$
$$\delta_{k_{\alpha\dot{\alpha}}} h^{(+4)} = -\hat{\Lambda} h^{(+4)} + 4i h^{++\alpha+}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}} + 4i\theta^{+\alpha} h^{++\dot{\alpha}+} k_{\alpha\dot{\alpha}}$$

It is impossible to avoid introducing the extra potential $h^{(+4)}$ for ensuring conformal covariance. The extended set of potentials embodies $\mathcal{N}=2$ **Weyl multiplet** ($\mathcal{N}=2$ conformal SG gauge multiplet).

- For $\mathbf{s} \geq 3$ the gauge-covariantization of the free q^{+a} action requires adding the gauge superfield differential operators of rank $\mathbf{s-1}$ in ∂_M,

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)} (J)^{P(s)}, \quad P(s) = \frac{1 + (-1)^{s-1}}{2}$$

- For $\mathbf{s} = 3$:

$$\hat{\mathcal{H}}_{(s=3)} = h^{++MN} \partial_N \partial_M + h^{++}, \quad h^{++MN} = (-1)^{P(M)P(N)} h^{++NM}$$

- $\mathcal{N} = 2$ SCA mixes different entries of h^{++MN}, so we need to take into account all these entries, as distinct from non-conformal case where it was enough to consider, e.g., $h^{++\alpha\dot{\alpha}M}$.

- The spin **3** gauge transformations of q^{+a} and h^{++MN} leaving invariant the action $\sim q^{+a}(D^{++} + \kappa_3 \hat{\mathcal{H}}_{(s=3)}) q^+_a$ are

$$\delta^{(s=3)}_\lambda q^{+a} = -\kappa_3 \hat{\mathcal{U}}_{(s=3)} J q^{+a},$$
$$\delta^{(s=3)}_\lambda \hat{\mathcal{H}}^{++}_{(s=3)} = \left[\mathcal{D}^{++}, \hat{\mathcal{U}}_{(s=3)}\right],$$

Here $\hat{\mathcal{U}}_{(s=3)}$ is some differential operator of rank 2.

- All the potentials except $h^{++\alpha\dot{\alpha}M}$ can be put equal to zero using the original extensive gauge freedom:

$$S^{(s=3)}_{int|fixed} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\, q^{+a}h^{++\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}}Jq^+_a \qquad (1)$$

- Using the linearized gauge transformations of $h^{++\alpha\dot{\alpha}M}$

$$\begin{aligned}
\delta_\lambda h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= \mathcal{D}^{++}\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 4i\lambda^{+(\alpha\beta)(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})} + 4i\theta^{+(\alpha}\bar{\lambda}^{+\beta)(\dot{\alpha}\dot{\beta})},\\
\delta_\lambda h^{++(\alpha\beta)\dot{\alpha}+} &= \mathcal{D}^{++}\lambda^{+(\alpha\beta)\dot{\alpha}} - \lambda^{++(\alpha\dot{\alpha}}\theta^{+\beta)},\\
\delta_\lambda h^{++(\dot{\alpha}\dot{\beta})\alpha+} &= \mathcal{D}^{++}\lambda^{+(\dot{\alpha}\dot{\beta})\alpha} - \lambda^{++\alpha(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})},\\
\delta_\lambda h^{(+4)\alpha\dot{\alpha}} &= \mathcal{D}^{++}\lambda^{++\alpha\dot{\alpha}} - 4i\bar{\theta}^{+\dot{\alpha}}\lambda^{+\alpha++} + 4i\theta^{+\alpha}\lambda^{+\dot{\alpha}++},
\end{aligned}$$

we can find WZ gauge for the spin 3 gauge supermultiplet

$$\begin{aligned}
h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= -4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}_{\rho\dot{\rho}} + (\bar{\theta}^+)^2\theta^+\psi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i\\
&\quad +(\theta^+)^2\bar{\theta}^+\bar{\psi}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})ij}u^-_i u^-_j\,,\\
h^{++(\alpha\beta)\dot{\alpha}+} &= (\theta^+)^2\bar{\theta}^+_{\dot{\nu}}P^{(\alpha\beta)(\dot{\alpha}\dot{\nu})} + (\bar{\theta}^+)^2\theta^+_\nu T^{(\alpha\beta\nu)\dot{\alpha}} + (\theta^+)^4\chi^{(\alpha\beta)\dot{\alpha}i}u^-_i\,,\\
h^{(+4)\alpha\dot{\alpha}} &= (\theta^+)^2(\bar{\theta}^+)^2D^{\alpha\dot{\alpha}}
\end{aligned}$$

- In the **bosonic sector**: the spin $\mathbf{s} = 3$ gauge field, $SU(2)$ triplet of conformal gravitons, singlet conformal graviton, spin 1 gauge field and non-standard field which gauges self-dual two-form symmetry:

$$\Phi^{(\alpha\beta\rho)(\dot\alpha\dot\beta\dot\rho)},\ V^{(\alpha\beta)(\dot\alpha\dot\beta)(ij)},\ P^{(\alpha\beta)(\dot\alpha\dot\nu)},\ D^{\alpha\dot\alpha},\ T^{(\alpha\beta\gamma)\dot\alpha}$$

In the **fermionic sector**: conformal spin $5/2$ and spin $3/2$ gauge fields:

$$\psi^{(\alpha\beta\rho)(\dot\alpha\dot\beta)i}, \quad \chi^{(\alpha\beta)\dot\alpha i}$$

- They carry total of $40 + 40$ off-shell degrees. Starting from $\mathbf{s} = 3$, all the component fields are gauge fields, no auxiliary fields are present.

- The sum of conformal spin 2 and spin 3 actions

$$S = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\right) q^+_a$$

  is invariant with respect to the (properly modified) spin $\mathbf{3}$ transformations to the leading order in κ_3 and to any order in κ_2. Thus the cubic vertex $(\mathbf{3}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ is invariant under the gauge transformations of conformal $\mathcal{N} = 2$ SG and we obtain the superconformal vertex of the spin $\mathbf{3}$ supermultiplet on *generic* $\mathcal{N} = 2$ Weyl SG background.

- The whole consideration can be generalized to the general integer higher-spin s case: $8(2s-1)_B + 8(2s-1)_F$ d.o.f. off shell.
- The superconformal cubic vertices $(\mathbf{s}, \frac{1}{2}, \frac{1}{2})$ can in fact be made invariant with respect to gauge transformations of the whole tower of the higher-spin $\mathcal{N}=2$ gauge superfields.
- The action of an infinite tower of integer $\mathcal{N}=2$ superconformal higher spins interacting with the hypermultiplet in an arbitrary $\mathcal{N}=2$ conformal supergravity background reads:

$$S_{full} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}\right) q^{+}_{a}$$

where

- $$\hat{\mathcal{H}}^{++} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{H}}^{++}_{(s)}(J)^{P(s)}$$

- Ascribing the proper gauge transformation to $\hat{\mathcal{H}}^{++}$, one can achieve gauge invariance to any order in the couplings constants

$$\delta_\lambda q^{+a} = -\hat{\mathcal{U}}_{hyp} q^{+a} \tag{2}$$

$$\delta_\lambda \hat{\mathcal{H}}^{++} = \left[\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}, \hat{\mathcal{U}}_{gauge}\right], \qquad \hat{\mathcal{U}}_{gauge} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s$$

- It mixes different spins, so it is a non-Abelian deformation of the spin $\mathbf{s}$ transformation laws. In the lowest order, it is reduced to the sum of linearized transformations of all integer spins $\mathbf{s} \geq 1$.

- The invariance under $\mathcal{N} = 2$ conformal supergravity transformations is automatic. So we have constructed the fully consistent gauge-invariant and conformally invariant interaction of hypermultiplet with an infinite tower of $\mathcal{N} = 2$ higher spins in an arbitrary $\mathcal{N} = 2$ conformal supergravity background.

Towards AdS background

- It is most interesting to explicitly construct $\mathcal{N} = 2$ higher spins in the AdS background, with the superconformal symmetry $SU(2,2|2)$ being broken to the AdS supersymmetry $OSp(2|4;R)$.

- One way is to start from the covariant formalism on the $AdS_4, \mathcal{N} = 2$ superspace defined as the coset $OSp(2|4;R)/[SO(2) \times SL(2,C)]$, thus generalizing the $AdS_4, \mathcal{N} = 1$ duperfield approach by Ivanov & Sorin, 1979, 1980. This way was chosed by Kuzenko, Tartaglino-Mazzucchelli, 2008, based on the so called projective superspace techniques. No clear connection with the harmonic analiticity was found on this way.

- Our approach proceeds from the realization of the superconformal symmetry in $\mathcal{N} = 2$ HSS (as described on the previous slides) and identifies AdS supersymmetry $OSp(2|4;R)$ as its subalgebra, $OSp(2|4;R) \subset SU(2,2|2)$. So the super AdS supersymmetry is already implied by the superconformal symmetry. Once again, the harmonic Grassmann analyticity plays the defining role.

- The embedding of the $\mathcal{N}=2$ AdS superalgebra into $SU(2,2|2)$ is realized through the identification (Bandos, Ivanov, Lukierski, Sorokin, 2002)

$$\Psi^i_\alpha = Q^i_\alpha + c^{ik} S_{k\alpha}, \quad \bar{\Psi}^i_{\dot\alpha} = \overline{\Psi^i_\alpha} = \bar{Q}_{\dot\alpha i} + c_{ik} \bar{S}^k_{\dot\alpha},$$
$$c^{ik} = c^{ki} \qquad \overline{c^{ik}} = c_{ik} = \varepsilon_{il}\varepsilon_{kj} c^{lj}$$

- The $SU(2,2|2)$ commutation relations imply for super AdS generators

$$\{\Psi^i_\alpha, \Psi^k_\beta\} = c^{ik} L_{(\alpha\beta)} + 4i\varepsilon_{\alpha\beta}\varepsilon^{ik} T, \quad T := c_{lm} T^{lm}, \quad [T, \Psi^i_\alpha] \sim c^{ik}\Psi_{k\alpha},$$
$$\{\Psi^i_\alpha, \bar{\Psi}_{\dot\beta k}\} = 2\delta^i_k R_{\alpha\dot\beta}, \quad R_{\alpha\dot\beta} = P_{\alpha\dot\beta} + \frac{1}{2} c^2 K_{\alpha\dot\beta}, \quad c^2 := c^{ik} c_{ik} \sim \frac{1}{R^2_{AdS}},$$
$$[R_{\alpha\dot\alpha}, R_{\gamma\dot\gamma}] \sim c^2 \left(\varepsilon_{\alpha\gamma} L_{\dot\alpha\dot\gamma} + \varepsilon_{\dot\alpha\dot\gamma} L_{\alpha\gamma}\right), \quad [R_{\alpha\dot\beta}, \Psi^i_\beta] \sim \varepsilon_{\alpha\beta} \bar{\Psi}^i_{\dot\beta} \ (\text{and c.c.})$$

- The first step toward constructing an off-shell $\mathcal{N}=2$ AdS higher spin theory is to define the super AdS invariant Lagrangian of hypermultiplet, such that it respects no full superconformal invariance, but only the super AdS.
- One needs to define the AdS covariant version of the analyticity-preserving harmonic derivative $\mathcal{D}^{++}$. The appropriate $\mathcal{D}^{++}_{AdS}$ acting on $q^{+a}=(q^+,\tilde{q}^+)$ has the structure

$$\mathcal{D}^{++}_{AdS}=\partial^{++}-4i\hat{\theta}^{+\alpha}\hat{\bar{\theta}}^{+\dot{\alpha}}\nabla_{\alpha\dot{\alpha}}+h^{++}\hat{T}+\mathcal{O}(c)$$

$$\nabla_{\alpha\dot{\alpha}}=(1+\frac{1}{2}c^2x^2)\partial_{\alpha\dot{\alpha}},\quad h^{++}=i|c|\left[(\hat{\theta}^+)^2-(\hat{\bar{\theta}}^+)^2\right]+\mathcal{O}(c),$$

$$\hat{T}\,(q^+,\,\tilde{q}^+)=(q^+,\,-\tilde{q}^+)\,,$$

  where $\hat{\theta}^+_\alpha,\hat{\bar{\theta}}^+_{\dot{\alpha}}$ are some redefinitions of the original Grassmann coordinates and $\mathcal{O}(c)$ stand for terms vanishing in the limit $c^{ik}\to 0$.
- An extra term $\sim\hat{T}$ in $\mathcal{D}^{++}_{AdS}$ is necessary for breaking superconformal invariance and it produces a mass of q^+ proportional to $1/R^2_{AdS}$. In the properly defined flat limit this term becomes the central charge extension of flat D^{++} and $\hat{T}$ goes just into the derivative ∂_5.

- More details on the AdS invariant q^+ Lagrangians will be given in our work with Nikita Zaigraev (to appear soon).
- An interesting new result is the analyticity-preserving Weyl transformation of the hypermultiplet Lagrangian.
- We start from the free q^+ action, $S_{free} = -\frac{1}{2}\int d\zeta^{(-4)} q^{+a}\mathcal{D}^{++}q^+_a$. It is superconformally invariant and hence invariant under super $\mathcal{N}=2$ super AdS$_4$ group. Then we make Weyl-type rescaling of q^+,

$$q^{+a} = G^{\frac{1}{2}}\,\hat{q}^{+a}\,, \quad G = \frac{\left(1+\frac{(c^{+-})^2}{m^2}\right)}{\left(1+\frac{m^2x^2}{2}\right)^2}\left(1+\theta\ terms\right), c^{+-} = c^{ik}u^+_i u^-_k\,,$$

so that $\hat{q}^{+a}$ is a scalar under the $\mathcal{N}=2$ super AdS$_4$ group. The $\hat{q}^+$ action takes the form manifestly invariant under this group

$$S_{free} = -\frac{1}{2}\int d\zeta^{(-4)}\,G\,\hat{q}^{+a}\mathcal{D}^{++}\hat{q}^+_a\,, \quad \delta_{osp}\hat{q}^{+a} = 0\,,$$

- The new integration measure $d\zeta^{(-4)}G$ is invariant under $OSp(2|4;R)$. So one can add to the Lagrangian any proper function of $\hat{q}^{+a}$ without breaking of $OSp(2|4;R)$.
- In particular, one can add an arbitrary $\mathcal{L}^{+4}(\hat{q}^{+a},u^-)$ and so gain a wide class of the hyper-Kähler sigma model actions on the AdS$_4$ background.

Summary and outlook

The theory of $\mathcal{N} = 2$ supersymmetric higher spins $s \geq 3$ opens a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N} = 2$ theories with maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories **is the harmonic Grassmann analyticity** (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell before fixing WZ-type gauges).

Under way:

- The linearized actions of conformal higher-spin $\mathcal{N} = 2$ multiplets ($\mathcal{N} = 2$ analogs of the square of Weyl tensor)?

- Quantization, induced actions,...

- $\mathcal{N} = 2$ supersymmetric half-integer spins?

- An extension to AdS background? Superconformal compensators? The $\mathcal{N} = 2$ AdS_4 supergroup $OSp(2|4; R) \subset SU(2,2|2)$, so the conformal invariance already implies AdS_4 invariance (in progress).

- From the linearized theory to its full nonlinear version? At present, the latter is known only for $s \leq 2$ ($\mathcal{N} = 2$ super Yang - Mills and $\mathcal{N} = 2$ supergravities). This problem seemingly requires accounting for **ALL** higher $\mathcal{N} = 2$ superspins simultaneously. New supergeometries?

THANK YOU FOR ATTENTION!