Gravity and QCD

AQFT-25, BLTP JINR, Dubna

August 12, 2025

Oleg Teryaev

BLTP JINR

Weakest and Strongest interactions?!

1. Holography (Extra-dimensional gravity -> field theories on the boundary)
2. Extremely large acceleration ($\sim 10^{30}$ g) and vorticity ($\sim 10^{28}\ \Omega_{Earth}$) in Heavy-Ion Collisions (~Superstrong gravity due to Equivalence Principle): NICA physics program extension
3. Studies in lattice QCD and models
4. "Emergent" conical singularity in flat space TD and quantum effects of gravity, instability at Unruh temperature
5. Gauge and gravity anomalies in HD
6. EMT appearance in gravity and hydrodynamics
7. Matrix elements of EMT operators (gravitational formfactors): hadron structure, relation to EP and its extensions (stability, viscosity)

Main Topics

- Axial Anomaly in HydroDynamics: Velocity as Gauge Field and and Vorticity as ia respective Strength
- Emergent conical geometry from thermodynamics in flat space.
- Unruh effect in medium
- Couplings of quarks and gluons to gravity: gravitational formfactors
- Pressure and viscosity in hadrons
- Some experimental applications: polarization, pressure in proton
- Conclusions

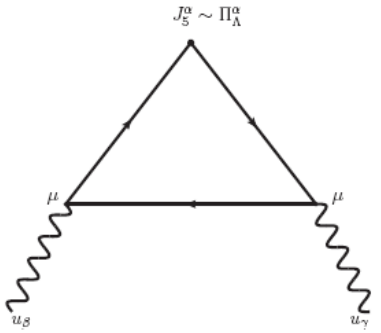
Heavy-Ion-Collisions: Strong interactions and superstrong "EP-gravity"

- $E_{EM}/E_G \sim e^2/(m/M_{Pl})^2$ $M_{Pl} \sim 10^{18}$ GeV
- For 2 particles with M_{Pl} mass at Compton wavelength distance $(1/M_{Pl})$: $E_G \sim (G = 1/M_{Pl}^2)\ M_{Pl}^2 / (1/M_{Pl}) = M_{Pl}$
  $g \sim (G = 1/M_{Pl}^2)\ M_{Pl} / (1/M_{Pl})^2 = M_{Pl}$
- Gravitational interaction is strongly suppressed wrt strong one $\sim (\Lambda/M_{Pl})^2$
- Equivalence Principle
- I: Acceleration <-> Gravity
- HIC: $a \sim \Lambda$, $a/g \sim \frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A} \sim 10^{30}$
- M_{Pl} -> Λ ("GeV Gravity" to be compared with 'TeV gravity" from extra dimensions)

Axial Anomaly in Hydrodynamics

- Chemical potential (from QCD phase diagram) + Lorentz invariance: new term in Lagrangian (A.G. Sadofyev, V.I. Shevchenko, V.I. Zakharov'10) analogous to gauge interactions
- 4-Velocity is a GAUGE FIELD
- Vorticity (= curl v) -----//------- STRENGTH
- Special role of anomalous graph: protected against PT and NP corrections
- Induced axial current
- Derivation from TD arguments: Son, Surowka'09

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$



Axial current and energy density from Zubarev density matrix (G.Y. Prokhorov, OT, V.I. Zakharov'19)

$$\hat{\rho} = \frac{1}{Z}\exp\left\{-\int_{\Sigma} d\Sigma_{\mu}[\hat{T}^{\mu\nu}(x)\beta_{\nu}(x) - \zeta(x)\hat{j}^{\mu}(x)]\right\}$$

$$\langle j^5_{\mu}\rangle = \left(\frac{1}{6}\left[T^2 - \frac{\omega^2}{4\pi^2}\right] + \frac{\mu^2}{2\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_{\mu} \qquad \beta_{\mu} = \frac{u_{\mu}}{T}$$

- Special role of Unruh temperature

$$\rho_{\mathrm{Den}} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17a^4}{960\pi^2} = \frac{1}{240}\left(T^2 - \left(\frac{a}{2\pi}\right)^2\right)(17a^2 + 28\pi^2 T^2)$$

- $T < T_U$: Negative energy – sign of phase transition?

Unruh efffect in vacuum and medium

- Vacuum: production of thermal particles in accelerated detector
- EP: SIMILAR to Hawking radiation
- Detector –> distant obeserver, inertial observer -> interior of BH
- Medium: no creation, $T > T_U$
- Our suggestion for $T < T_U$: new complementary mechanism of hadronization phase transition)

Same results: from geometry

Emergent conical geometry

[G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

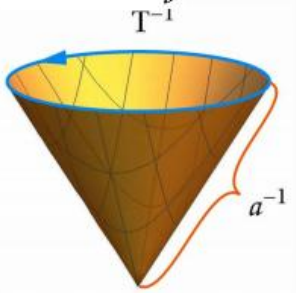
- The effects of acceleration can also be investigated from the point of view of an **accelerated observer**. In this case, the euclidean **Rindler coordinates** are to be used:

$$ds^2 = \rho^2 d\theta^2 + d\rho^2 + d\mathbf{x}_\perp^2 \Longrightarrow \mathcal{M} = \mathbb{R}^2 \otimes \mathcal{C}_\nu^2$$

Dictionary for translation *thermodynamic* characteristics in *geometrical*:

Inverse **acceleration** ⟺ **distance** from the **vertex**

Inverse proper **temperature** ⟺ **circumference**



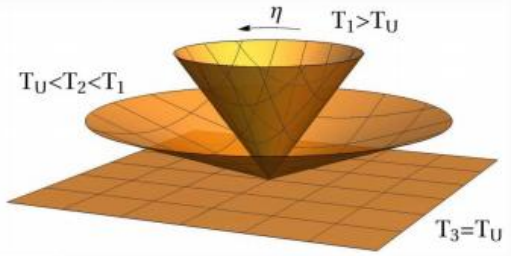
- **Two approaches** to calculate acceleration effects:
  1) **Geometrical** (Rindler, conical):
  2) **Statistical** (interaction with **boost**): $\alpha^\rho \hat{K}_\rho$

$$\rho_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}$$

Same results - **duality** of two approaches!

- **Novel phase transition** at the **Unruh** temperature in both approaches!

[G. Y. Prokhorov, O. V. Teryaev, and V. I. Zakharov. arXiv:2304.13151. (2023) and work in preparation]



Is curvature manifested?

Flat space limit: Kinematical Vortical Effect (KVE)

- Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains a contribution** to the axial current induced by the gravitational chiral anomaly:

Cheshire cat grin

Flat: $j^A_\mu = \lambda_1(\omega_\nu\omega^\nu)\omega_\mu + \lambda_2(a_\nu a^\nu)w_\mu$

Curved: $\nabla_\mu j^\mu_A = \mathscr{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathscr{N}$$

Direct check:

1) Spin **1/2** : $\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)\Big/32 = \frac{1}{384\pi^2}$

2) Spin **3/2** (*Rarita-Schwinger-Adler model*): $\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)\Big/32 = -\frac{19}{384\pi^2}$

- A **new** type of anomalous transport – the **Kinematical Vortical Effect (KVE)**.
- New global polarization (talk of N. Tsegelnik) source?

[G. Yu. Prokhorov, O. V. Teryaev, and V. I. Zakharov, Phys. Rev. Lett. 129, 151601, (2022)]

Real gravity action Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J^{\mu}_{q}|P\rangle A_{\mu}(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J^{\mu}_{q}|P\rangle = 2e_{q}P^{\mu} \qquad \sum_{q,G}\langle P|T^{\mu\nu}_{i}|P\rangle = 2P^{\mu}P^{\nu}$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J^{\mu}_{q}|P\rangle A_{\mu} = 2e_{q}M\,\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T^{\mu\nu}_{i}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism: action of classical gravity on quantum spin

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – "Falling elevator" – well known and checked with high accuracy (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun' ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); derived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Generalized for arbitrary gravitational fields (Yu.N. Obukhov, A.Ya. Silenko, OT'11)

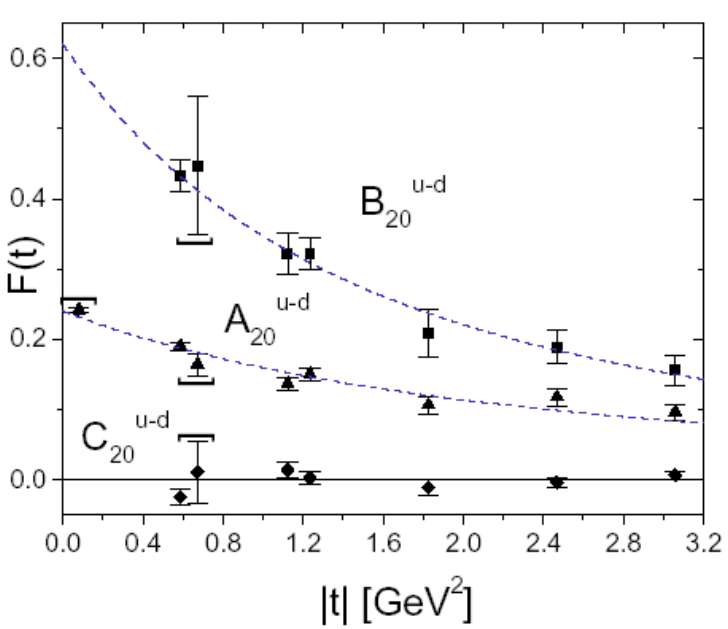
- For GEDM –checked with sometimes controversial results
- For AGM not checked on purpose but in fact checked in the same atomic spins experiments at % level (Silenko,OT'07)

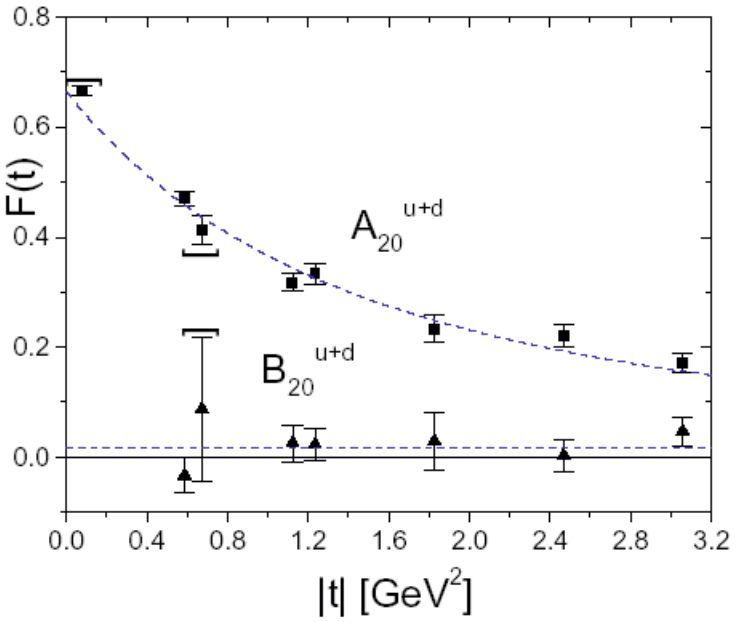
EP and quantum measurement

- *If spin is just a geometric vector, EP for Earth's rotation is "trivial": looking from stars, spin rotates with Earth's angular velocity like Foucault pendulum*
- *Non-trivial if quantum measurement (quite practical here) is performed in the rotating frame*
- *Cf with Unruh effect (measurement in accelerated frame is crusial)*

Action of gravity on quarks and gluons separately: Generalization of Equivalence principle

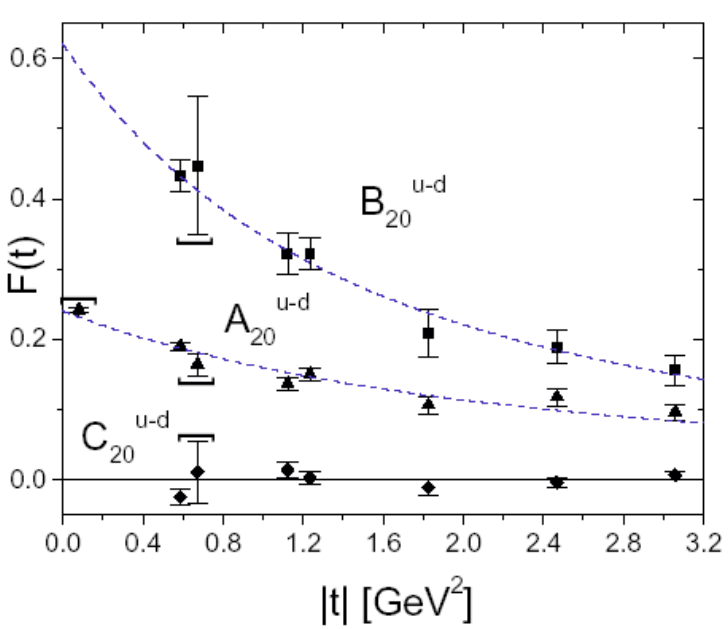
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)

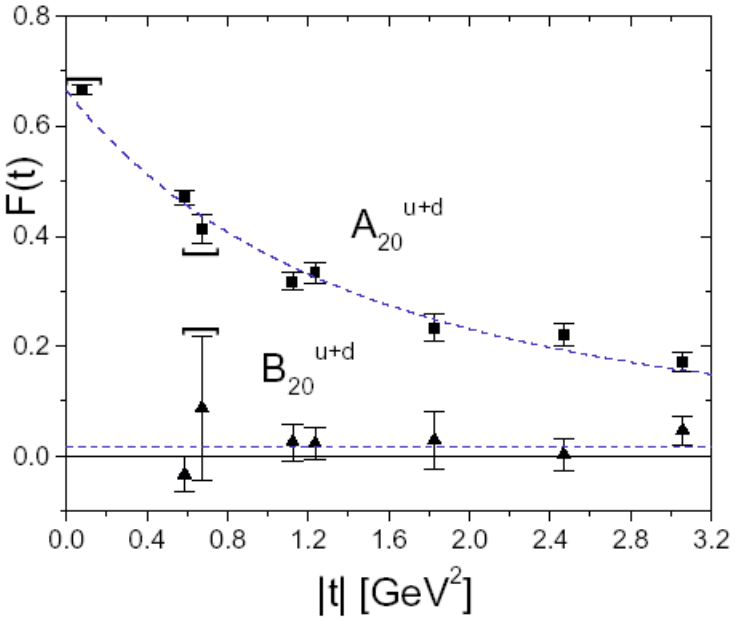




Generalization of Equivalence principle

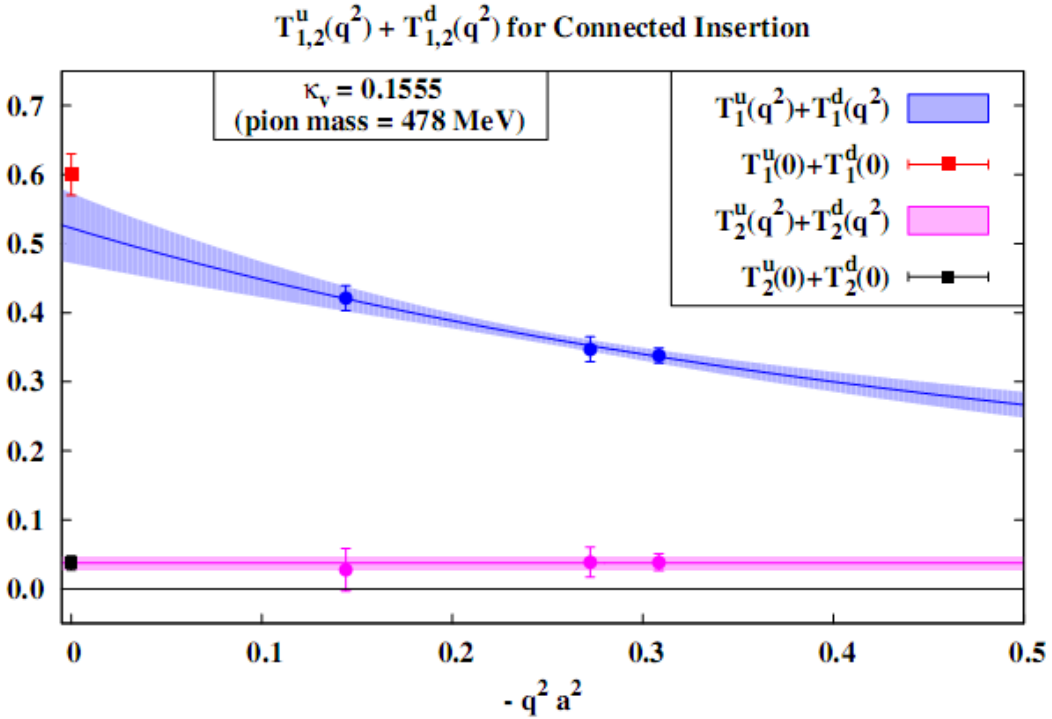
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





More recent lattice study (M. Deka,…K.-F. Liu et al. Phys.Rev. D91 (2015) no.1, 014505)

- *Sum of u and d for Dirac (T1) and Pauli (T2) FFs*



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!
- Works for other structures excluded for tota; EMT (“cosmological constant”, shear, shear viscosity)

Some experimental manifestations: Global polarization in HIC

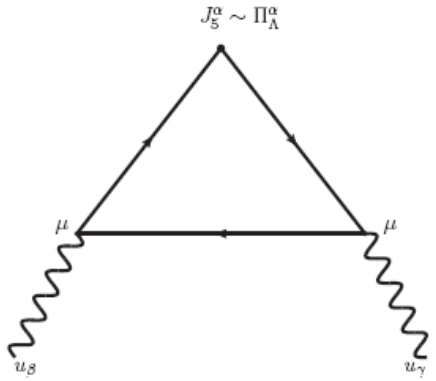
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- Differential rotation matters

Anomalous mechanism – polarization due to induced axial current similar to C(A)VE

- *4-Velocity is also a GAUGE FIELD (V.I. Zakharov et al):* $\mu \varrho = \mu J_0 V^0 \rightarrow \mu J_\gamma V^\gamma$

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- *Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)*
- *Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)*
- *4-velocity instead of gluon field!*



"Anomalous" mechanism

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

- Prediction of decrease with energy (due to chemical potential)
- Prediction of P~1%

BAZNAT, GUDIMA, SORIN, AND TERYAEV

PHYSICAL REVIEW C **88**, 061901(R) (2013)

$$\langle P_\Lambda \rangle \sim \frac{\langle \mu^2 \rangle \omega_c H}{2\pi^2 \langle N_\Lambda \rangle}.$$

For numerical estimate at NICA energies, we take (see Fig. 3) $H = 30\ \mathrm{fm}^2 (c = 1)$ and, as typical values, $\langle \mu^2 \rangle = 900\ \mathrm{MeV}^2$, $\langle N_\Lambda \rangle = 15$ to get $\langle P_\Lambda \rangle \sim 0.8\%$. This value is

One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

- Postdiction of larger polarization of antilambdas

ALEXANDER SORIN AND OLEG TERYAEV

PHYSICAL REVIEW C **95**, 011902(R) (2017)

The proportionality of the polarization to the square of the chemical potential related to C-even parity of axial current leads to the same sign of polarization of Λ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge, corresponding to each antihyperon and to a larger absolute value of polarization. Detailed numerical sim-

One might compare the prediction below with the right panel figures

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

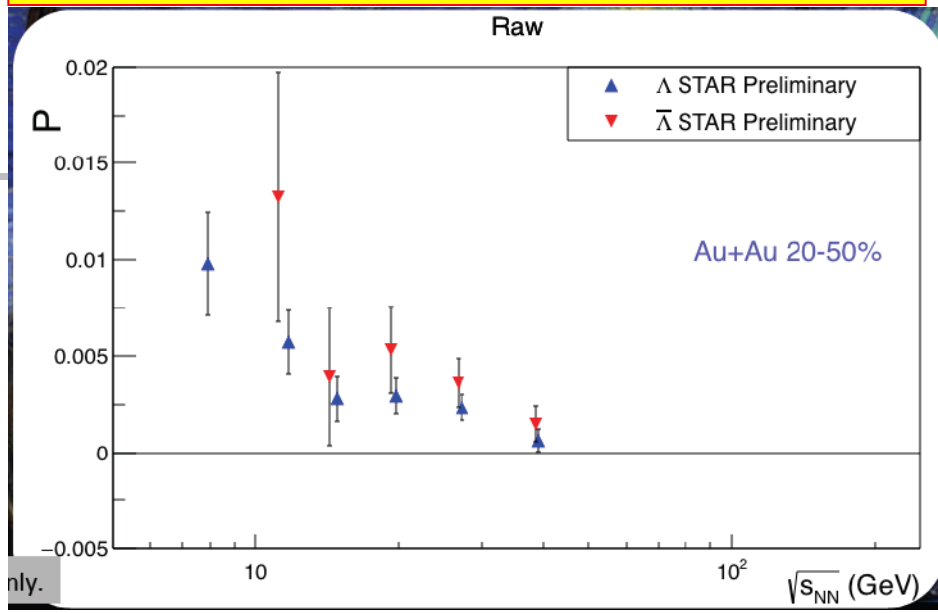
One would expect that polarization is proportional to the anomalously induced axial current [7]

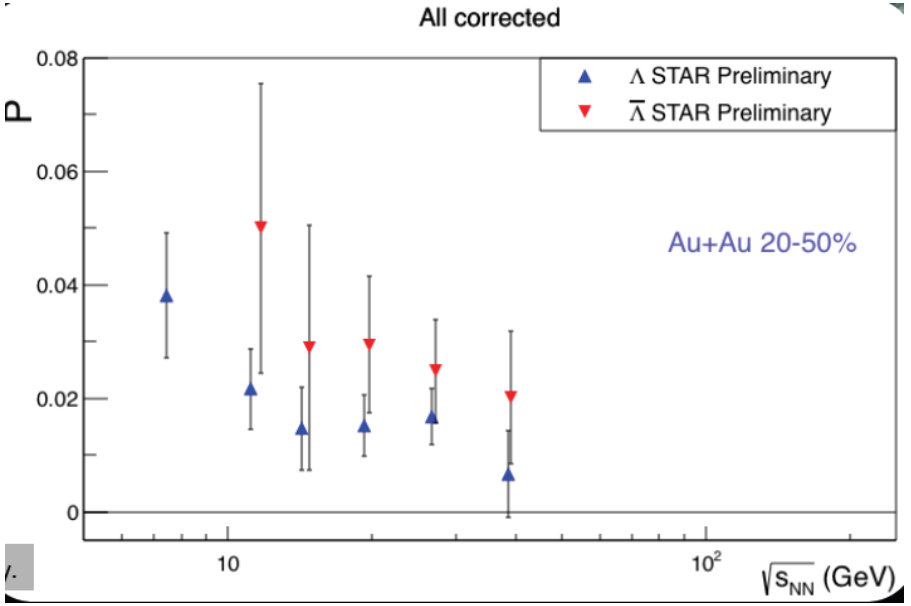
$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

M. Lisa, for the STAR collaboration , QCD Chirality Workshop, UCLA, February 2016;
SQM2016, Berkeley, June 2016





LETTER

NATURE | www.nature.com/nature

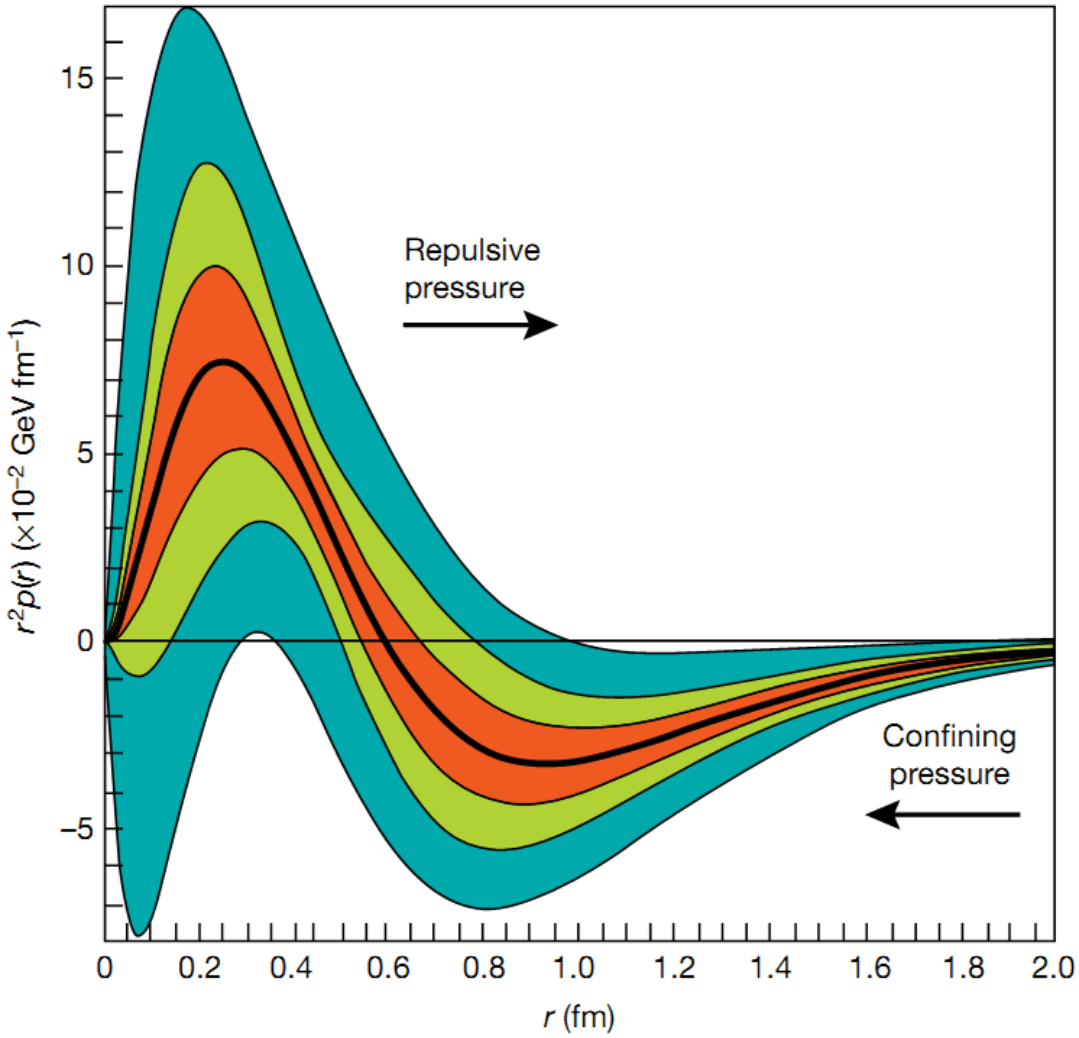
https://doi.org/10.1038/s41586-018-0060-z

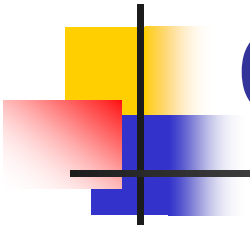
The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

5. Teryaev, O. V. Gravitational form factors and nucleon spin structure. Front. Phys. 11, 111207 (2016)

15. Anikin, I. V. & Teryaev, O. V. Dispersion relations and QCD factorization in hard reactions. Fizika B 17, 151–158 (2008)





Conclusions

- "Equivalent" gravity due to (super)strong inertial effects is manifested via gauge and gravity anomalies
- Statistical description in flat space implies "emergent" conical geometry.
- Unruh effect in QGP may be related with complementary mechanism of hadronization
- Misterious (approximate) validity of equivalence principle for quarks and gluions separately in NP QCD
- Gravity effects are measurable in experiments with heavy ions and hadrons