Conformal Anomaly in HQCD in Strong Magnetic Field

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Holographic QCD - phenomenological approach

- Perturbation methods are not applicable to describe QCD phase diagram
- Lattice methods do not work, because of problems with the chemical potential.
- AdS/CFT [What is wrong with exact AdS/CFT applications to QCD]
- Holographic QCD - phenomenological model(s)

- One of goals of Holographic QCD – describe QCD phase diagram

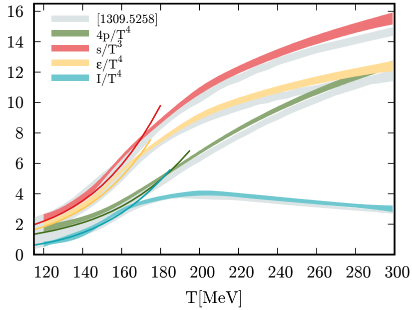
- **Requirements:**
  - reproduce the QCD results from perturbation theory at short distances
  - reproduce Lattice QCD results at large distances (~ 1 fm) and **small** μ_B

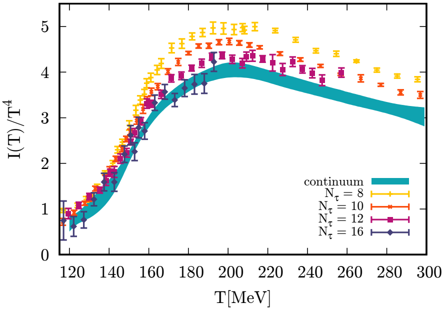
Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy





From:S. Borsanyi et al, arXiv:2502.10267

Left. Pressure (green), entropy (red), energy density (yellow) and **trace anomaly** (cyan) as functions of the temperature. EoS: $\frac{p(T_0=185)}{T_0^4} = 1.371$. The gray bands from

S. Borsanyi et al, arXiv: 1309.5258

Right Trace anomaly

Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

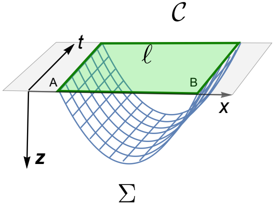
- QCD is not conformal, conformal invariance is restored only in high energy

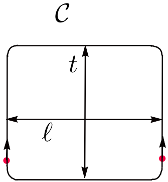
Holographic QCD vs exact AdS/CFT

Maldacena,1998

What is wrong with exact AdS/CFT applications to QCD:

- QCD is not conformal, conformal invariance is restored only in high energy
- No confinement in $BHAdS_5$





$$W_R[C] \underset{t\to\infty}{\sim} e^{-V(\ell)t}$$

$$V(\ell) \underset{\ell\to\infty}{\sim} \frac{1}{\ell}$$

corresponds to conformal invariance

Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S = \int d^5x \sqrt{-g}\left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2}\partial_M \phi \partial^M \phi - V(\phi)\right]$$

$$ds^2 = \frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-g(z)\,dt^2 + dx^2 + dy_1^2 + e^{c_B z^2} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

$A_{(1),m} = A_t(z)\delta_m^0,\ A_t(0) = \mu, \qquad F_{(B)} = dx \wedge dy^1$

Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$\mathfrak{b}(z) = e^{2\mathcal{A}(z)}$ $\Leftrightarrow$ quarks mass — **"Bottom-up approach"**

Heavy quarks (b, t):

$\mathcal{A}(z) = -cz^2/4$ — *Andreev, Zakharov'06*

$\mathcal{A}(z) = -cz^2/4 + pz^4$ — *IA, Hajilou, Rannu, Slepov, EPJ C (2023)83*

Light quarks (d, u)

$\mathcal{A}(z) = -a\ln(bz^2 + 1)$ — *Li, Yang, Yuan'17*

φ - dilaton, $\alpha(z) = e^{\varphi(z)}$ - **running** coupling in HQCD

Holographic Equation of Motions

$$
\begin{aligned}
\varphi'' + \varphi'\left(\frac{g'}{g} + \frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{3}{z} + c_B z\right) + \left(\frac{z}{L}\right)^2 \frac{\partial f_1}{\partial\varphi}\,\frac{(A_t')^2}{2\mathfrak{b}g} - \left(\frac{z}{L}\right)^2 \frac{\partial f_B}{\partial\varphi}\,\frac{q_B^2}{2\mathfrak{b}g} &= 0\,,\\
A_t'' + A_t'\left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f_1'}{f_1} - \frac{1}{z} + c_B z\right) &= 0\,,\\
g'' + g'\left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{3}{z} + c_B z\right) - \left(\frac{z}{L}\right)^2 \frac{f_1 (A_t')^2}{\mathfrak{b}} - \left(\frac{z}{L}\right)^2 \frac{q_B^2\, f_B}{\mathfrak{b}} &= 0\,,\\
\mathfrak{b}'' - \frac{3(\mathfrak{b}')^2}{2\mathfrak{b}} + \frac{2\mathfrak{b}'}{z} - \frac{4\mathfrak{b}}{3z^2}\left(-\frac{c_B z^2}{2} - \frac{c_B^2 z^4}{2}\right) + \frac{\mathfrak{b}\,(\varphi')^2}{3} &= 0\,,\\
c_B z^2\,(2g' + 3g)\left(\frac{\mathfrak{b}'}{\mathfrak{b}} - \frac{4}{3z} + \frac{2c_B z}{3}\right) - \left(\frac{z}{L}\right)^3 \frac{L\,q_B^2 f_B}{\mathfrak{b}} &= 0\,,\\
\frac{\mathfrak{b}''}{\mathfrak{b}} + \frac{(\mathfrak{b}')^2}{2\mathfrak{b}^2} + \frac{3\mathfrak{b}'}{\mathfrak{b}}\left(\frac{g'}{2g} - \frac{2}{z} + \frac{2c_B z}{3}\right) - \frac{g'}{3zg}(9 - 3c_B z^2) - \frac{2c_B}{3}(5 - c_B z^2) &\\
+ \frac{8}{z^2} + \frac{g''}{3g} + \frac{2}{3}\left(\frac{L}{z}\right)^2 \frac{\mathfrak{b}V}{g} &= 0\,,
\end{aligned}
$$

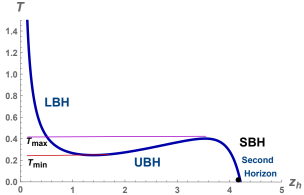
We choose (LQ): $f_1 = e^{-cz^2 - \mathcal{A}(z)} = (1 + bz^2)^a\, e^{-cz^2}$,

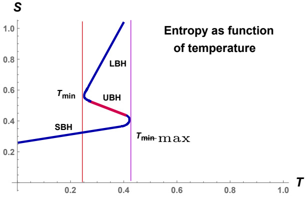
B.C.: $A_t(0) = \mu$, $A_t(z_h) = 0$; $g(0) = 1$, $g(z_h) = 0$; $\varphi(z_0) = 0$ We have to fix z_0.

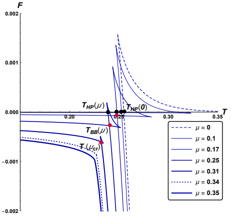
Origin of 1-st order phase transition in HQCD

- $g(z)$ blackenning function. The form of $g(z)$ depends on $\mathcal{A}(z)$.
- Due **non-monotonic** dependence of $T = T(z_h) = g'(z)/4\pi\Big|_{z=z_h}$ on z_h, the entropy $s = s(T)$ is **not monotonic**
- As a consequence the free energy $F = \int sdT$ undergoes the phase transition

1-st order phase transition describes transition from small black holes → large black holes



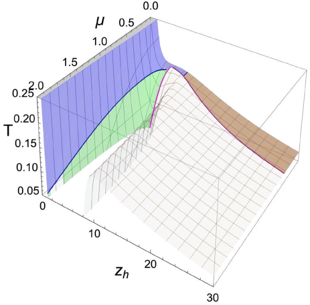




The swallow-tailed shape

- Physical quantities that probe backgrounds are smooth relative to z_h $\Rightarrow$ their dependence on T should be taken from stable region
- Non-monotonic dependence of $T = T(z_h)$ gives the 1-st PT for corresponding characteristic of QCD

1-st order phase transition in HQCD. 1/3

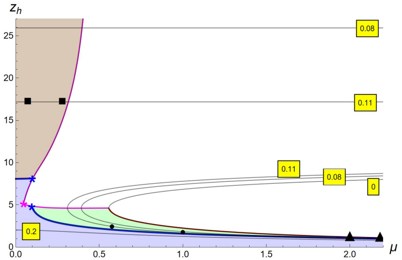


LQ

3D plot $T = T(\mu, z_h)$. The brown part of the surface corresponds to the hadronic phases, the blue one corresponds to the quark-gluon plasma and the green one to the quarkyonic phase. 3D plot $T = T(\mu, z_h)$

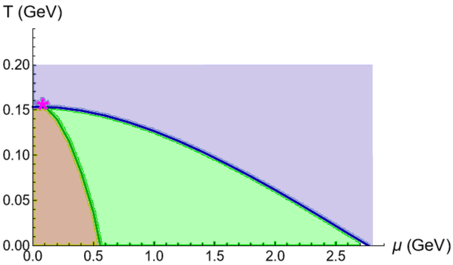
I.A., A.Hajilou, P.Slepov, M.Usova, PRD (2024) **110**, 126009; [arXiv:2402.14512]

1-st order phase transition in HQCD. 2/3

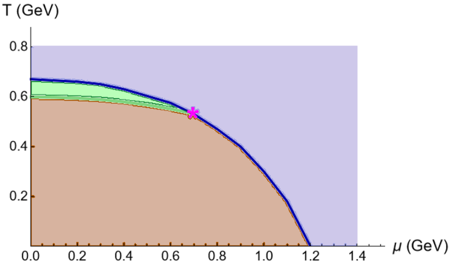


2D plots in (μ, z_h)-plane for light quarks. Hadronic, quarkyonic and QGP phases are denoted by brown, green and blue, respectively. Solid gray lines show the temperature indicated in rectangles. The intersection of the confinement/deconfinement and 1-st order phase transition lines is denoted by the blue stars. The magenta star indicates CEP

1-st order phase transition in HQCD. 3/3



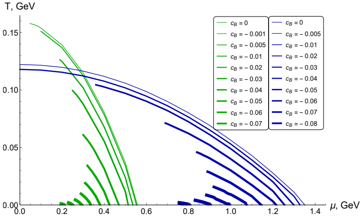
LQ



HQ

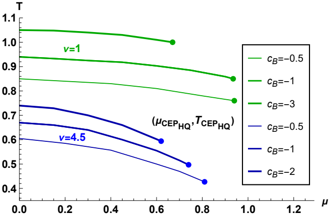
1-st order phase transition in HQCD, $B \neq 0$

Light quarks



I.A, Ermakov, Rannu, Slepov,EPJC'23

Heavy quarks



I.A, A. Hajilou, K.R., P.S.EPJC'23

HQCD: EoS

- **HQCD for light quarks**
- **How to get thermodynamical parameters from given numerical solution**

EoS for zero magnetic field

- Pressure $p = -F$, $F = F = -\int sdT\Big|_{\mu=const}$ - free energy
- Density

$$\rho = -\frac{\mu c}{\left(1 - e^{c\, z_h^2}\right)}$$

- Entropy [density of entropy]

$$s = \left(\frac{[L]}{z_h}\right)^3 \frac{\left(1 + bz_h^2\right)^{-3a}}{4[G_5]}$$

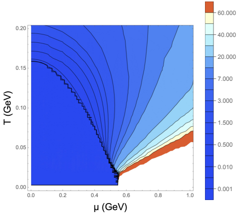
- Energy

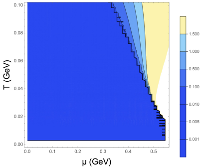
$$\epsilon = sT - p + \mu\rho$$

$[\epsilon] = GeV^4$, $[F] = GeV^4$, $[\mu\rho = GeV^4]$

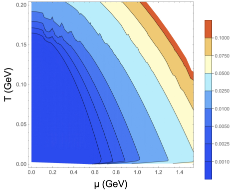
EoS for zero magnetic field.
Numerical calculations with our solution

- Entopy: s/T^3



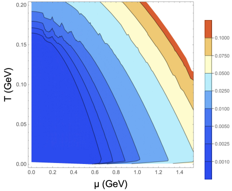


- Entopy: s

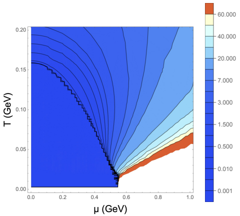


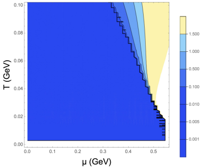
EoS for zero magnetic field.
Numerical calculations with our solution

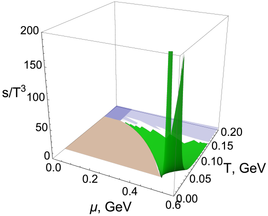
- Entopy: s

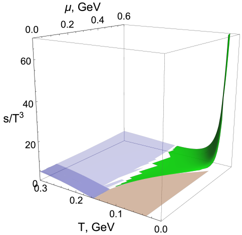


- Entopy: s/T^3



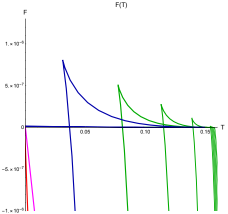


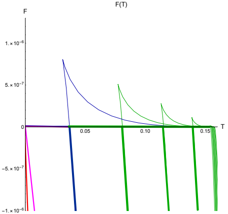




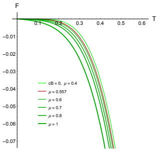
EoS for zero magnetic field.
Numerical calculations with our solution

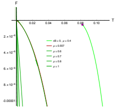
- Pressure:
  $-p = F = -\int s dT$

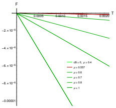




- Pressure on the right of 1-st order phase transition:

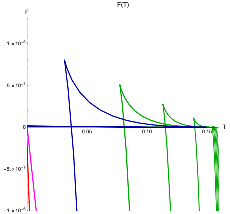


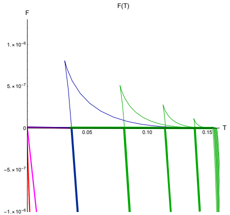




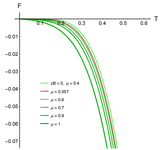
EoS for zero magnetic field.
Numerical calculations with our solution

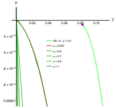
- Pressure:
  $-p = F = -\int s dT$

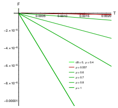


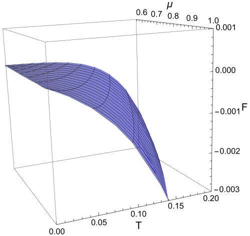


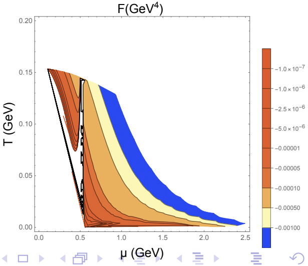
- Pressure on the right of 1-st order phase transition:





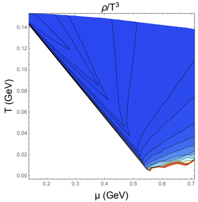




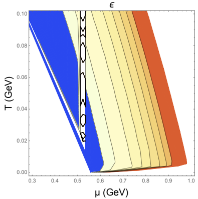


EoS for zero magnetic field.
Numerical calculations with our solution

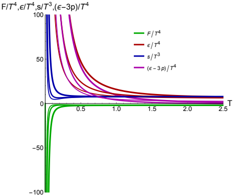
- Density



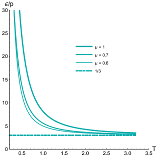
- Energy



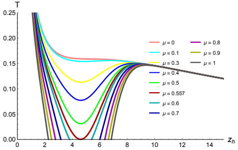
- All thermodynamical variables:



- Restoration of conformality:



- Origin: Temperature "van der Waals-type"dependence on z_h



EoS for non-zero magnetic field

- Pressure $p = -F$, $F = F = -\int s dT\Big|_{\mu=const}$ - free energy
- Density

$$\rho = -\frac{\mu\,(2c - c_B)}{2\left(1 - e^{(2c-c_B)z_h^2/2}\right)}$$

- Entropy [density of entropy]

$$s = \left(\frac{[L]}{z_h}\right)^3 \frac{\left(1 + b z_h^2\right)^{-3a}}{4[G_5]}$$

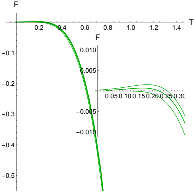
- Energy

$$\epsilon = sT - p + \mu\rho$$

$[\epsilon] = GeV^4$, $[F] = GeV^4$, $[\mu\rho = GeV^4]$

EoS for non0zero magnetic field.
Numerical calculations with our solution

- Free energy



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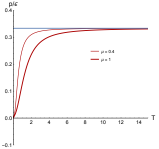

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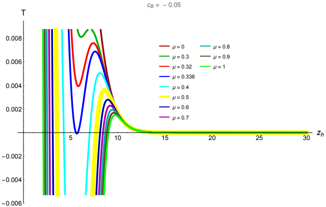


Holographic Running coupling

$\alpha(z) = e^{\varphi(z)}$ I.A. A.Hajilou, P.Slepov, M.Usova, 2402.14512

Light Quark Model

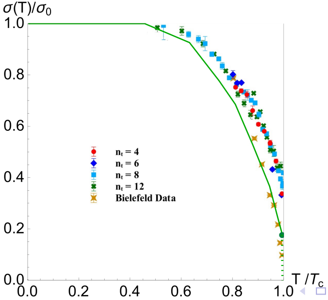
$\varphi(z)$ - dilaton field $\varphi(z)$ is defined up to a constant: $\varphi(z)\Big|_{z=z_0} = 0.$

There are 3 choices: a) $z_0 = 0$ b) $z_0 = f(z_h)$ c) $z_0 = z_h$

$$z_0 = 10\exp[-z_h/4] + 0.1$$

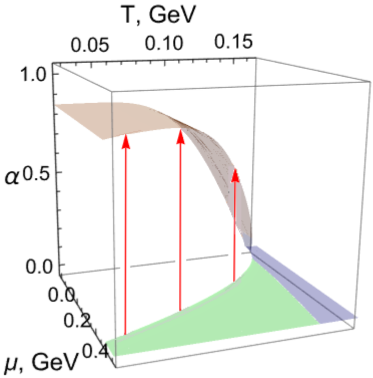
IA, K.Rannu, P.Slepov, JHEP'21

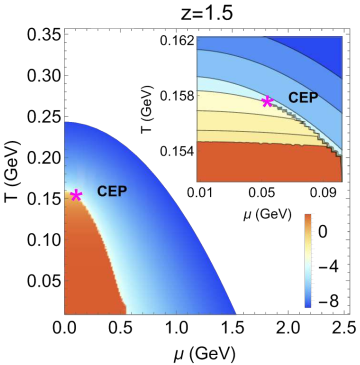
With this boundary condition the temperature dependence of σ_s fits the known lattice data Cordaso, Bicudo 1111.1317



Holographic Running coupling for $T \neq 0, \mu \neq 0$

I.A. Hajilou, Slepov, Usova, PRD, 2024

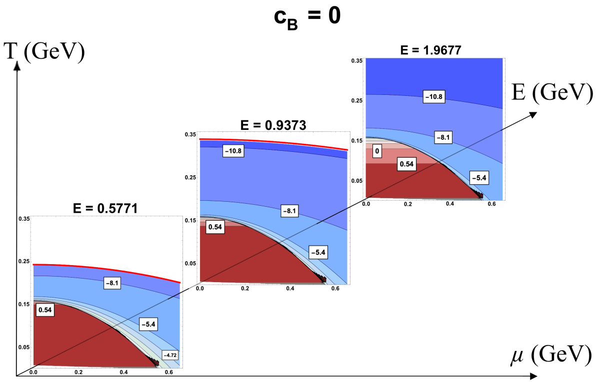




3D plot of $\alpha(z; \mu, T)$ **Density plot** of $\log \alpha(z; \mu, T)$

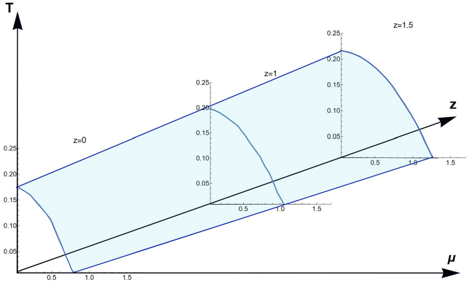
Light quark model, $z = 0$

Running Coupling for Light Quark Model



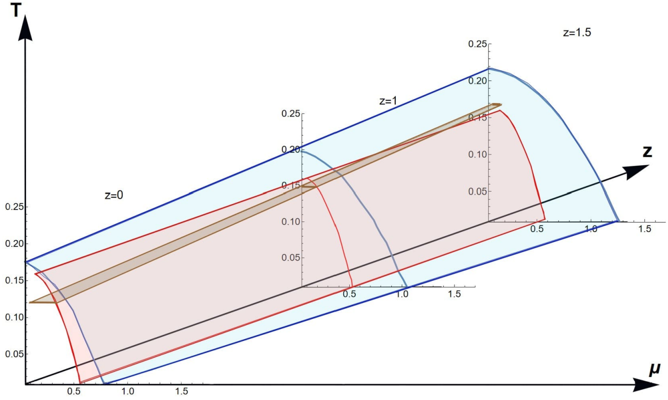
Density plots of $\log \alpha(E; \mu, T)$ at different energy scales.
All values of E on the top of each panel show fixed value of energy E-coordinate.

Automodel behaviour of Running coupling



$$\alpha(T, \mu) = f_{up}(T^2 + c\mu^2)$$

Automodel Behaviour of Running Coupling



$$\alpha(T,\mu) = f_{below}(T^2 + c\mu^2)$$

Jet Quenching as function of T and μ for LQ

- $\langle W^A(\mathcal{C})\rangle \sim e^{-\frac{1}{4\sqrt{2}}\hat{q}L^- L_\perp^2}$
- $\hat{q} = \frac{L^2}{\pi\alpha' a}$,
- $a = \int_0^{z_h} dz \frac{z^2 e^{-2\mathcal{A}_s(z)}}{\sqrt{g(z)(1-g(z))}}$.

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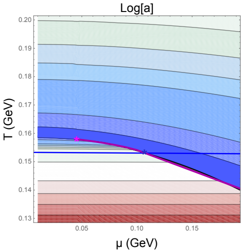
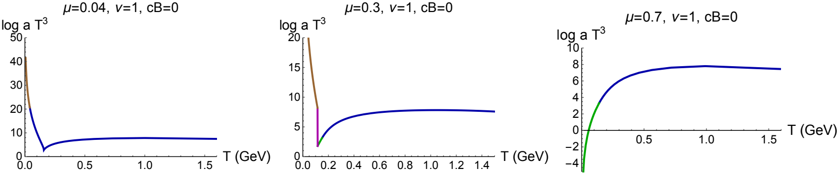


We see that junction in JQ is exactly on the line of the first order transition (as should be)

Jet Quenching in conformal regime for LQ

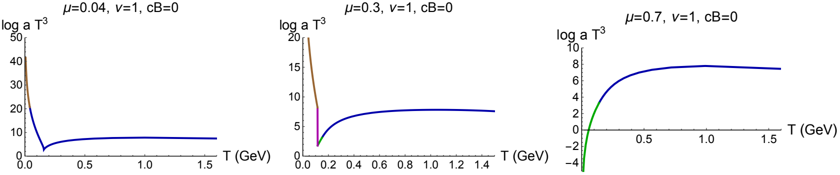


A B C

LQ

The dependence of $\log aT^3$ on T for the isotropic LQ model, with $\nu = 1$ and $c_B = 0$, at fixed chemical potentials: (A) $\mu = 0.04$ GeV, (B) $\mu = 0.3$ GeV, (C) $\mu = 0.7$ GeV.

Jet Quenching in conformal regime for LQ



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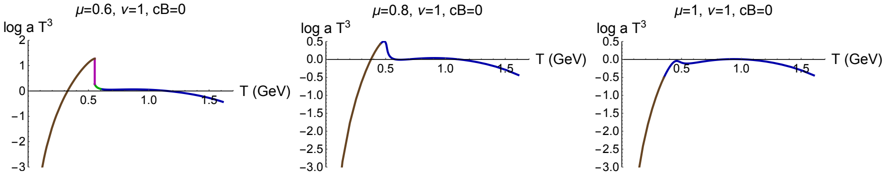


HQ

Conclusion

- Conformal anomaly goes to zero for LQ on the right of PD.
- Phase structure in (T, μ, B, ν)-space;
  B - characterizes the magnetic field,
  ν -anisotropy
- Dependence of the phase structure in (T, μ, B, ν) - space on quark mass
- Jumps in physical quantities (jet quenching, energy loss, etc.) at the first-order phase transition and their dependence on B and anisotropy parameter ν.
- Smooth variation of physical quantities (running couplings, jet quenching parameter, etc.) across the confinement-deconfinement transition.

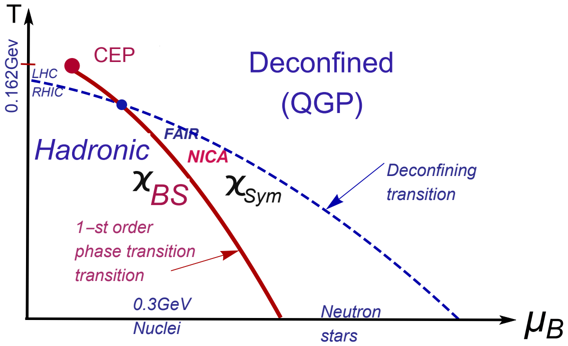
Main refs 1/2

- И.Я. Арефьева, “Голографическое описание кварк-глюонной плазмы, образующейся при столкновениях тяжелых ионов”, УФН, **184:6** (2014)
- И.А., "QGP time formation in holographic shock waves model of heavy ion collisions“ TMPh, 184 (2015), 398–417
- I.A., A. Golubtsova, “Shock waves in Lifshitz-like spacetimes,” JHEP’04, 011 (2015)
- I.A. and K. Rannu, “Holographic Anisotropic Background with Confinement-Deconfinement Phase Transition,” JHEP **05** (2018) 206
- I.A., A. Golubtsova, G. Policastro, “Exact holographic RG flows and the $A_1 \times A_1$ Toda chain,” JHEP **05** (2019) 117
- I.A., K. Rannu, P. Slepov, “Orientation Dependence of Confinement-Deconfinement Phase Transition in Anisotropic Media,” Phys. Lett. **B 792** (2019) 470
- I. A., A. Patrushev, P. Slepov “Holographic Entanglement Entropy in Anisotropic Background with Confinement-Deconfinement Phase Transition”, JHEP **07** (2020) 043
- I.A., K. Rannu, P. S. Slepov, “Anisotropic solutions for a holographic heavy-quark model with an external magnetic field”, TMPh **206** (2021) 400
- I.A., A. Golubtsova, E. Gourgoulhon, “Holographic drag force in 5d Kerr-AdS black hole,” JHEP **04** (2021) 169
- I. A., K. Rannu, P. Slepov, "Holographic Anisotropic Model for Light Quarks with Confinement-Deconfinement Phase Transition”, JHEP **06** (2021) 90
- I.A., K. Rannu, P. Slepov, “Holographic Anisotropic Model for Heavy Quarks in Anisotropic Hot Dense QGP with External Magnetic Field,” JHEP **07**(2021) 161
- I.A., A.Ermakov, P.Slepov, “Direct photons emission rate and electric conductivity in twice anisotropic QGP holographic model with first-order phase transition,” EPJC **82** (2022) 85
- I.A., A. Ermakov, K. Rannu and P. Slepov, “Holographic model for light quarks in anisotropic hot dense QGP with external magnetic field,” EPJC **83** (2023) 79

Main refs 2/2

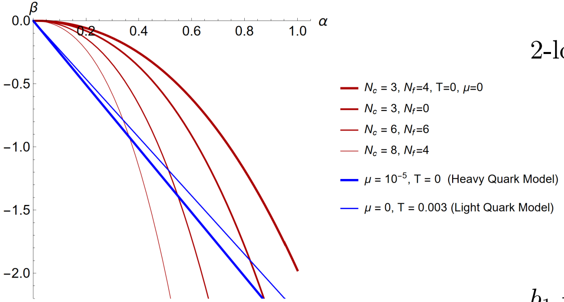
- I.A, A. Hajilou, A. Nikolaev and P. Slepov, ‘Holographic QCD running coupling for light quarks in strong magnetic field,” Phys. Rev. D **110** (2024) no.8, 8
- I.A, A. Hajilou, P. Slepov, M. Usova, ”Running coupling for holographic QCD with heavy and light quarks: Isotropic case”, Phys. Rev. D **110**, 126009 (2024)
- И.Я. Арефьева, А. Хаджилу, П.С. Слепов, М.К. Усова, ”Бета-функция в голографических моделях КХД”, ТМФ (2024), т. 221, № 3, стр. 615–628
- I. A, A. Hajilou, P. Slepov, M. Usova, ”Beta-functions and RG flows for holographic QCD with heavy and light quarks: Isotropic case”, Phys. Rev. D **111**, 046013 (2025)

Backup. The expected more detailed QCD phase diagram



- Parameter of the chiral symmetry breaking $<\bar{\psi}\psi>$
  - $<\bar{\psi}\psi> = 0 \iff \chi$-symmetry
  - $<\bar{\psi}\psi> \neq 0 \iff$ broken χ-symmetry

Backup. β function



Beta-function $\beta(\alpha)$ for QCD at 2-loop level for $T = 0$, $\mu = 0$ at different N_c and N_f in red lines, and holographic β-function for light quarks at $\mu = 0$, $T = 0.003$ (light blue) and heavy quarks $\mu = 10^{-5}$, $T = 0$ (dark blue); $[\mu] = [T] =$ GeV.

2-loops QCD β-function:

$$\beta(\alpha) = -b_0\,\alpha^2 - b_1\,\alpha^3,$$

$$b_0 = \frac{1}{2\pi}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

$$b_1 = \frac{1}{8\pi^2}\left(\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_f}\right)\right)$$

N_c # of colors, N_f #of flavors.

I.A, A. Hajilou, P. Slepov and M. Usova, Phys.Rev.D (2025) 111