IR-effective quantum gravity in a theory with a cosmological constant: a renormalization group study of the gravitational potential at large distances.

E.Vasiljev, **M.Nalimov**, D.Shmygin.

The standard non-renormalizable action of quantum gravity without the contribution of matter

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, (R - 2\Lambda)\,.$$

is investigated.

Dimensional analysis, generally accepted in the study of continuous phase transitions in statistical physics, has obtained and the infrared-effective action for describing the theory at large distances was obtained. This action is renormalizable, the logarithmic dimension is d=5+1.

The quantum field renormalization group method is used to construct a $5+1-\epsilon$ expansion of the Green's functions of gravitons. In the one-loop approximation, a power-law behavior of the gravitational potential at large distances (the power differs from $1/r$) and an increase in effective masses are detected. All this is the result of taking into account the self-ineraction of massless gravitons in the IR region.

Notations: $g_{\mu\nu}$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu})$$

$$R_{\mu\nu} = \partial_\nu \Gamma^{\alpha}_{\mu\alpha} - \partial_\alpha \Gamma^{\alpha}_{\mu\nu} + \Gamma^{\alpha}_{\mu\lambda} \Gamma^{\lambda}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\nu} \Gamma^{\lambda}_{\alpha\lambda}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$g = \mathrm{det} g_{\mu\nu} \quad \kappa = 8\pi G$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \, (R - 2\Lambda) \, ,$$

Harmonic gadge $\partial_\nu(\sqrt{-g}g^{\mu\nu}) = 0$
introduce $\delta(\partial_\nu\sqrt{-g}g^{\mu\nu})$-functions and ghostes.

$$\sqrt{-g}g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

$\eta^{\mu\nu} = Const$

$$\partial_\nu h^{\mu\nu} = 0$$

To take into account the gauge degrees of freedom, fermion-like ghost fields $\bar{c}^\mu$, c^ν are introduced with the action:

$$S_{\text{ghost}} = \int d^4x\, \bar{c}^\alpha \mathcal{M} c_\alpha,$$

where $\mathcal{M} = \Box + h^{\mu\nu}\partial_\mu\partial_\nu$ — Faddeev-Popov operator.

Non-renormalizible.

Critical behavior – Ginzbudg-Landau hamiltonian.

$$S = \frac{1}{2}\partial\phi\partial\phi + \frac{\tau}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{\lambda_1}{6!}\phi^6 + \ldots$$

$$(\sqrt{-g})^d e^{\frac{1}{2}\mathbf{tr}\ln(1+\eta_{\mu\alpha}(h^{\nu\alpha}))} \quad 2\kappa^2 \to 1$$

$$S_1 := \frac{\delta S}{\delta g^{\mu\nu}} = \sqrt{-g}(\frac{1}{2}\eta(1+\eta h)^{-1}_{\mu\nu}(R-2\Lambda) + \frac{\delta R}{\delta g^{\mu\nu}}) + \bar{c}^{\alpha}\partial_\mu\partial_\nu c_\alpha$$

$$S_2 := \frac{\delta^2 S}{\delta g^{\mu\nu}\delta g^{\mu'\nu'}} = \sqrt{-g}(\frac{1}{2}\eta(1+\eta h)^{-1}_{\mu'\nu'}((\frac{1}{2}\eta(1+h)^{-1}_{\mu\nu}(R-2\Lambda)+\frac{\delta R}{\delta g^{\mu\nu}}))$$

$$+\sqrt{-g}(-\frac{1}{2}\eta(1+\eta h)^{-1}_{\mu\nu}\eta(1+\eta h)^{-1}_{\mu'\nu'}(R-2\Lambda)$$

$$+\frac{1}{2}\eta(1+\eta h)^{-1}_{\mu\nu}\frac{\delta R}{\delta g^{\mu'\nu'}} + \frac{\delta^2 R}{\delta g^{\mu\nu}\delta g^{\mu'\nu'}})$$

(new Einstein equation in tree approximation, there will be corrections to it from diagrammatic perturbation theory)
$S_1(h) = 0$ $g^{\mu\nu} \to \eta^{\mu\nu} + h^{\mu\nu} + \alpha^{\mu\nu}$ и подставить α в S_2,S_3.
$\alpha \sim \Lambda$, тогда $\alpha = 0$ где это возможно.
Then the symbols are raised and lowered by η. The contribution of α to S_2, S_3 is not taken into account.

$$\sqrt{-g} \approx 1 + \frac{1}{2}h + (\frac{1}{8}h^2 - \frac{1}{4}h^\mu_\nu h^\nu_\mu) + \frac{1}{6}h^\mu_\nu h^\nu_\alpha h^\alpha_\mu + \frac{1}{24}h^3 - \frac{1}{8}h h^\mu_\nu h^\nu_\mu$$

где $h = h^\mu_\mu$.

$$g_{\mu\nu} = ((g^{\mu\nu}))^{-1} \approx \eta_{\mu\nu}(1 + \frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^\gamma_\delta h^\delta_\gamma) - (1 + \frac{1}{2}h)h_{\mu\nu} + h^\alpha_\mu h_{\alpha\nu}$$

$$g^{\mu\nu} \approx \eta^{\mu\nu}(1 - \frac{1}{2}h + \frac{1}{8}h^2 + \frac{1}{4}h^\gamma_\delta h^\delta_\gamma) + h^{\mu\nu}(1 - \frac{1}{2}h)$$

$$R \approx -\frac{1}{2}h^{\mu\lambda}\Box h_{\mu\lambda}$$

effective action

$$S \approx -\frac{1}{2}h^{\mu\nu}\Box h_{\mu\nu} - ((\frac{1}{4}h^2 - \frac{1}{2}h^\mu_\nu h^\nu_\mu) + \frac{1}{3}h^\mu_\nu h^\nu_\alpha h^\alpha_\mu + \frac{1}{12}h^3 - \frac{1}{4}hh^\mu_\nu h^\nu_\mu)\Lambda$$

$$+S_{ghost})$$

The first task is to detect the massless (IR singular) modes. We split all fields into matrices with zero diagonal, traceless diagonal matrices and matrices proportional to $\eta^{\mu\nu}$ (the linearity of the space is respected). Traceless fields are not massless ($\mathrm{tr} h^{\mu\nu}h_{\mu\nu} \neq 0$). We restrict ourselves to diagonal fields proportional to $\eta^{\mu\nu}$.

The fields transversality

$$P^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

split the space of transverse fields using orthogonal projectors into traceless

$$\mathcal{P}_{nt} = \frac{1}{2}[P^{\mu\mu'}P^{\nu\nu'} + P^{\mu\nu'}P^{\nu\mu'}] - \frac{1}{\text{tr}P}P^{\mu\nu}P^{\mu'\nu'}$$

and diagonal

$$\mathcal{P}_d = \frac{1}{\text{tr}P}P^{\mu\nu}P^{\mu'\nu'}$$

Linear integration space

$$h = \mathcal{P}_d \Box^{-1} A$$

.

$h^2 - 2h^{\mu}_{\nu}h^{\nu}_{\mu} = 0$: as generators of such fields we can propose $h^{\mu\nu} = \eta^{\mu\nu}h$, μ, ν take the values {0,1,2,3} with one (arbitrary) missing sign (the corresponding rows and columns of the matrix are equal to zero) "quantum virtual gravitational waves"(used $\mathrm{tr}P^{\mu\mu} = d - 1$ c $d = 3$). These fields form a linear space, but only if we discard the cross terms, which are considered IR unimportant. The cross modes remain massive. We have 4 independent massless modes in our theory.

Also, according to the power count, interaction with ghosts turns out to be unimportant (there are two extra derivatives), and they can be excluded from consideration.
Canonical dimension of massless modes: $d/2-1$,as well as ghosts, massive– $d/2$. According to the power-law account in the IR effective theory, first of all, all self-action of massive modes should be discarded as IR insignificant. Moreover, the interaction of massive modes with massless modes turns out to be less IR significant than the interaction of massless modes with each other. Therefore, massive modes can be excluded from consideration.
Effective action

$$S_{IR} = -\frac{1}{2} h^{\mu\nu} \Box h_{\mu\nu} - \frac{1}{6} \lambda h^3$$

Renormalized IR effective action

$$S_R = -\frac{1}{2} Z_h^2 h^{\mu\nu} \Box h_{\mu\nu} - Z_\lambda Z_h^3 \mu^{\epsilon/2} \lambda h^3$$

RG in a "log"dimension $6 - \epsilon$, physical $\epsilon = 2$
One-loop results

$$Z_h = 1 - \frac{1}{6\pi^3\epsilon}\lambda^2, \quad Z_\lambda = 1 + \frac{257}{2}\frac{\lambda^2}{\pi^3\epsilon}$$

(four massless modes)

RG

$$\beta_{\lambda^2} = \lambda^2(-\epsilon + \frac{257}{2}\frac{\lambda^2}{\pi^3}) \quad \gamma_h = \frac{1}{6\pi^3}\lambda^2$$

β – IR stable fixed point

$$\lambda_*^2 = \frac{2\pi^3}{257}\epsilon$$

anomalous field h dimension

$$\gamma_{h*} = -\frac{1}{771}\epsilon^2$$

At large distances, the potential of gravitational interaction has the form

$$\Phi(r) = \frac{A}{r^{1+\eta}} + \frac{e^{-r\sqrt{2\Lambda_i}}}{r}$$

где $\eta = . - \frac{2}{771}\epsilon^2$ $(\epsilon = 2)$.

$$\Lambda_i \sim \Lambda r^{0+\dots}$$

$$m_i \sim m r^{1+\dots}$$

Asymptotics of high orders of expansions?
Multi-loop calculations, resummation?

Спасибо за внимание