On QED in the Schwarzschild spacetime

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Introduction

The problem of field quantization in the presence of black holes is widely discussed in the literature. However, many questions still remain insufficiently clarified, one of the questions being whether (or how) it is necessary to take into account the region below the black hole horizon. In papers

- V. Egorov, M. Smolyakov, I. Volobuev, "Quantization of spinor field in the Schwarzschild spacetime and spin sums for solutions of the Dirac equation," Class. Quant. Grav. **41** (2024) 045002 [arXiv:2309.06897]
- V. Egorov, M. Smolyakov, I. Volobuev, "Quantization of electromagnetic field in the Schwarzschild spacetime," [arXiv:2410.07049].

it has been shown that a consistent procedure of canonical quantization of the spinor and electromagnetic fields in the Schwarzschild spacetime can be carried out without taking into account the internal region of the black hole.

The construction of quantum theories of the spinor and electromagnetic fields in the Schwarzschild spacetime allows us to move on to quantum electrodynamics in this spacetime, which makes it possible to consider a quantum description of the fall of charged particles towards the black hole with the emission of photons. Such a description taking into account subtle quantum effects could affect the results concerning the rate of accretion of matter by the black hole or predict other observable effects.

Massive spinor field

The standard metric of the Schwarzschild spacetime in Schwarzschild coordinates looks like

$$ds^2 = \left(1 - \frac{r_0}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{r_0}{r}} - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right),$$

where $r_0 = 2M$ is the Schwarzschild radius and M is the black hole mass. We restrict ourselves to the region $r > r_0$ and consider a massive spinor field.

For a remote observer, the wave functions of physical quantum states of this field above the horizon are elements of the Hilbert space of spinor functions with finite norm, i.e., of the field configurations $\psi(t, r, \theta, \varphi)$, which satisfy the condition

$$\int\limits_{r > r_0} \frac{r^2 \sin\theta \, dr \, d\theta \, d\varphi}{\sqrt{1 - \frac{r_0}{r}}} \, \psi^\dagger(t, r, \theta, \varphi) \, \psi(t, r, \theta, \varphi) < \infty.$$

The integration goes over a hypersurface $t = \mathrm{const}$ in the Schwarzschild spacetime with respect to the volume element defined by the metric induced on this hypersurface from the Schwarzschild metric. We emphasize that the physical states of a quantum system are always normalizable. Since the normalization integral should be convergent for the wave functions of the spinor particle physical quantum states, they tend to zero at the event horizon and at spatial infinity fast enough.

The action of the massive spinor field in arbitrary curvilinear coordinates is given by

$$S = \int \sqrt{-g} \left(\frac{i}{2} \left(\bar{\psi} \gamma^{(\nu)} e^{\mu}_{(\nu)} \nabla_{\mu} \psi - \nabla_{\mu} \bar{\psi} \, e^{\mu}_{(\nu)} \gamma^{(\nu)} \psi \right) - M \bar{\psi} \psi \right) d^4 x,$$

where $e^{\mu}_{(\nu)}$ denotes the tetrad and the covariant derivative is defined as

$$\nabla_{\mu} \psi = \left(\partial_{\mu} \psi + \omega_{\mu} \psi \right), \qquad \nabla_{\mu} \bar{\psi} = \left(\partial_{\mu} \bar{\psi} - \bar{\psi} \omega_{\mu} \right),$$

$$\omega_{\mu} = \frac{1}{8} \omega_{(\nu)(\rho)\mu} \left[\gamma^{(\nu)}, \gamma^{(\rho)} \right].$$

The Dirac matrices satisfy the conditions

$$\left\{ \gamma^{(\mu)}, \gamma^{(\nu)} \right\} = 2 \eta^{(\mu)(\nu)} \quad \Leftrightarrow \quad e^{\mu}_{(\rho)} \, e^{\nu}_{(\sigma)} \left\{ \gamma^{(\rho)}, \gamma^{(\sigma)} \right\} = 2 g^{\mu\nu}.$$

Varying it with respect to the spinor field $\bar{\psi}$, we can drop the surface terms at the event horizon and infinity and obtain the corresponding equation of motion:

$$\mathrm{i}\gamma^{(\nu)} e^{\mu}_{(\nu)} \left(\partial_\mu + \omega_\mu\right) \psi - m\psi = 0.$$

It is a common knowledge that this Dirac equation can be rewritten in the Hamiltonian form. The Dirac Hamiltonian is Hermitian in the Hilbert space of the physical quantum states of the spinor particles in the Schwarzschild spacetime. However, the eigenfunctions of the Dirac Hamiltonian need not to belong to the Hilbert space of normalizable spinor functions. They may have infinite norm, i.e., they can lie in the so-called rigged Hilbert space, that is, they are generalized functions. This situation is similar to the description of a free particle in non-relativistic quantum mechanics, where the eigenfunctions of the free Hamiltonian are usually chosen as plane waves, which are not normalizable.

First we consider the Dirac equation in the isotropic coordinates, which allows us to use the standard technique for working with spherical spinors, and then return to Schwarzschild coordinates. The transition from the isotropic coordinates $\{t, x, y, z\}$ to Schwarzschild coordinates $\{t, r, \theta, \varphi\}$ is carried out according to the formulas

$$r = R + \frac{M^2}{4R} + M = R + \frac{r_0^2}{16R} + \frac{r_0}{2}, \quad R = \sqrt{x^2 + y^2 + z^2}.$$

We will look for solutions to the Dirac equation in the form

$$\psi_{Ejlm}(t, R, \theta, \varphi) = \begin{pmatrix} F_{jl}(E, R)\,\Omega_{jlm}(\theta, \varphi) \\ iG_{jl'}(E, R)\,\Omega_{jl'm}(\theta, \varphi) \end{pmatrix} \mathrm{e}^{-iEt},$$

where $l = j \pm \frac{1}{2}$, $l' = j \mp \frac{1}{2}$, and the spherical spinors are defined as follows:

$$\Omega_{jlm}(\theta, \varphi) = \begin{pmatrix} C^{jm}_{l, m-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}} Y_{l\, m-\frac{1}{2}}(\theta, \varphi) \\ C^{jm}_{l, m+\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}} Y_{l\, m+\frac{1}{2}}(\theta, \varphi) \end{pmatrix}.$$

The asymptotic behavior of the obtained radial equations for $R \to \infty$ looks like

$$\frac{\mathrm{d}\left(RF_{jl}\left(E,R\right)\right)}{\mathrm{d}R}+\frac{\kappa}{R}\left(RF_{jl}\left(E,R\right)\right)-\left(E+m+\frac{r_0}{2R}E\right)\left(RG_{jl'}\left(E,R\right)\right)=0,$$

$$\frac{\mathrm{d}\left(RG_{jl'}\left(E,R\right)\right)}{\mathrm{d}R}-\frac{\kappa}{R}\left(RG_{jl'}\left(E,R\right)\right)+\left(E-m+\frac{r_0}{2R}E\right)\left(RF_{jl}\left(E,R\right)\right)=0,$$

where $\kappa = l\left(l+1\right)-j\left(j+1\right)-\frac{1}{4}$.

These equations coincide with those for an electron in the Coulomb potential of an atomic nucleus. Comparing the signs in the last terms, we see that, as in the case of the Coulomb field, the resulting potential is attractive:

$$V\left(R\right)=-\frac{r_0}{2R}E=-\frac{M}{R}E,$$

where $M = r_0/2$ is the black hole mass. This potential reproduces the Newtonian one with the only difference that it is proportional to the total energy E of the particle instead of its mass m, which is due to the relativistic nature of the equation.

Next we return to the Schwarzschild coordinates in the radial equations for the spinor field outside the black hole horizon and make the substitution

$$F_{jl}(E,R) = \frac{f_{jl}(E,r)}{r\left(1-\frac{r_0}{r}\right)^{\frac{1}{4}}}, \qquad G_{jl'}(E,R) = \frac{g_{jl'}(E,r)}{r\left(1-\frac{r_0}{r}\right)^{\frac{1}{4}}}.$$

After such substitution, the radial equations take the simpler form

$$\sqrt{1-\frac{r_0}{r}}\left(\sigma^{(1)}\frac{\kappa}{r} - i\sigma^{(2)}\sqrt{1-\frac{r_0}{r}}\frac{d}{dr} + \sigma^{(3)}M\right)\begin{pmatrix} f_{jl} \\ g_{jl'} \end{pmatrix} = E\begin{pmatrix} f_{jl} \\ g_{jl'} \end{pmatrix}.$$

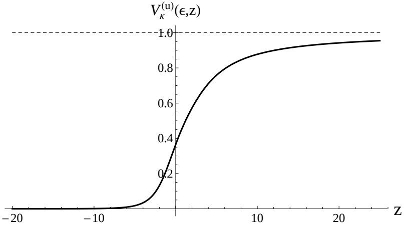
We pass to dimensionless variables $\mu = Mr_0$, $\epsilon = Er_0$, $\rho = \frac{r}{r_0}$, $z = \rho + \ln(\rho - 1)$ and obtain equations for $\varepsilon > 0$

$$-\frac{d^2 u}{dz^2} + V_\kappa^{(u)}(\varepsilon, z)\, u = \varepsilon^2 u,$$

where the original functions f_{jl} and $g_{jl'}$ can be expressed in terms of u, and the quasipotential is given by the formula

$$V_\kappa^{(u)}(\varepsilon, z) = \frac{\mu(\rho(z) - 1)^{\frac{3}{2}} - \kappa\varepsilon\,\rho(z)\sqrt{\rho(z) - 1}}{2\rho^{\frac{9}{2}}(z)\left(\varepsilon + \mu\sqrt{\frac{\rho(z)-1}{\rho(z)}}\right)}$$
$$+ \frac{\mu^2(\rho(z) - 1) - 2\mu\varepsilon\sqrt{(\rho(z) - 1)\rho(z)}}{16\rho^5(z)\left(\varepsilon + \mu\sqrt{\frac{\rho(z)-1}{\rho(z)}}\right)^2} +$$
$$+ \mu^2\frac{\rho(z) - 1}{\rho(z)} + \kappa\frac{(\rho(z) - 1)^{\frac{3}{2}}}{\rho^{\frac{7}{2}}(z)} + \kappa^2\frac{\rho(z) - 1}{\rho^3(z)}.$$

Similar equations can be derived for $\varepsilon < 0$.



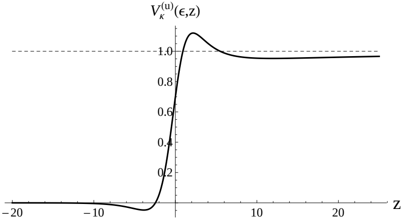


Figure 1: $V_\kappa^{(u)}(\varepsilon, z)$ for $\mu = 1$, $\kappa = -1$, $\varepsilon = 0.5$ and $\mu = 1$, $\kappa = 2$, $\varepsilon = 2$.

The solutions for $E > 0$ are given by the formulas

$$\phi_{Ejlm}(r,\theta,\varphi) = \frac{1}{r\left(1-\frac{r_0}{r}\right)^{\frac{1}{4}}} \begin{pmatrix} f_{jl}(E,r)\,\Omega_{jlm}(\theta,\varphi) \\ ig_{jl'}(E,r)\,\Omega_{jl'm}(\theta,\varphi) \end{pmatrix},$$

$$\phi^{(p)}_{Ejlm}(r,\theta,\varphi) = \frac{1}{r\left(1-\frac{r_0}{r}\right)^{\frac{1}{4}}} \begin{pmatrix} f^{(p)}_{jl}(E,r)\,\Omega_{jlm}(\theta,\varphi) \\ ig^{(p)}_{jl'}(E,r)\,\Omega_{jl'm}(\theta,\varphi) \end{pmatrix},\ p = 1,2.$$

The solutions $\chi^{(p)}_{Ejlm}(r,\theta,\varphi)$ for $E < 0$ are obtained by interchanging the functions as follows: $f_{jl} \leftrightarrow g_{jl'}$, $f^{(p)}_{jl} \leftrightarrow g^{(p)}_{jl'}$.

The expansion of the spinor field in the complete system of one-particle stationary states looks like

$$\psi\left(t,r,\theta,\varphi\right)=\sum_{j=\frac{1}{2}}^{\infty}\sum_{m=-j}^{j}\sum_{l=j\pm\frac{1}{2}}$$

$$\left(\int_{0}^{m}dE\left(\mathrm{e}^{-iEt}\phi_{Ejlm}\left(r,\theta,\varphi\right)a_{jlm}\left(E\right)+\mathrm{e}^{iEt}\chi_{Ejlm}\left(r,\theta,\varphi\right)b_{jlm}^{\dagger}\left(E\right)\right)+\right.$$

$$\left.+\sum_{p=1}^{2}\int_{m}^{\infty}dE\left(\mathrm{e}^{-iEt}\phi_{Ejlm}^{(p)}\left(r,\theta,\varphi\right)a_{jlm}^{(p)}\left(E\right)+\mathrm{e}^{iEt}\chi_{Ejlm}^{(p)}\left(r,\theta,\varphi\right)b_{jlm}^{(p)\dagger}\left(E\right)\right)\right),$$

where

$$\left\{a_{jlm}\left(E\right),a_{j'l'm'}^{\dagger}\left(E'\right)\right\}=\delta_{jj'}\,\delta_{ll'}\,\delta_{mm'}\,\delta\left(E-E'\right),$$

$$\left\{b_{jlm}\left(E\right),b_{j'l'm'}^{\dagger}\left(E'\right)\right\}=\delta_{jj'}\,\delta_{ll'}\,\delta_{mm'}\,\delta\left(E-E'\right),$$

$$\left\{a_{jlm}^{(p)}\left(E\right),a_{j'l'm'}^{(p')\dagger}\left(E'\right)\right\}=\delta_{pp'}\,\delta_{jj'}\,\delta_{ll'}\,\delta_{mm'}\,\delta\left(E-E'\right),$$

$$\left\{b_{jlm}^{(p)}\left(E\right),b_{j'l'm'}^{(p')\dagger}\left(E'\right)\right\}=\delta_{pp'}\,\delta_{jj'}\,\delta_{ll'}\,\delta_{mm'}\,\delta\left(E-E'\right).$$

One can prove that the following anticommutation relations hold

$$\left\{\psi_\alpha(t,r,\theta,\varphi),\psi^\dagger_\beta(t,r',\theta',\varphi')\right\}$$
$$=\delta_{\alpha\beta}\frac{\sqrt{1-\frac{r_0}{r}}}{r^2}\delta(r-r')\,\delta(\cos\theta-\cos\theta')\,\delta(\varphi-\varphi'),$$
$$\{\psi_\alpha(t,r,\theta,\varphi),\psi_\beta(t,r',\theta',\varphi')\}=\left\{\psi^\dagger_\alpha(t,r,\theta,\varphi),\psi^\dagger_\beta(t,r',\theta',\varphi')\right\}=0.$$

The Hamiltonian takes the form

$$H=\frac{i}{2}\int\frac{r^2\sin\theta}{\sqrt{1-\frac{r_0}{r}}}:\left(\psi^\dagger\dot{\psi}-\dot{\psi}^\dagger\psi\right):dr\,d\theta\,d\varphi$$
$$=\sum_{j=\frac{1}{2}}^{\infty}\sum_{m=-j}^{j}\sum_{l=j\pm\frac{1}{2}}\left(\int_0^m E\left(a^\dagger_{jlm}(E)\,a_{jlm}(E)+b^\dagger_{jlm}(E)\,b_{jlm}(E)\right)dE\right.$$
$$\left.+\sum_{p=1}^{2}\int_m^\infty E\left(a^{(p)\dagger}_{jlm}(E)\,a^{(p)}_{jlm}(E)+b^{(p)\dagger}_{jlm}(E)\,b^{(p)}_{jlm}(E)\right)dE\right).$$

We see that the binding energy of the states of finite motion may be of the order of the particle mass, whereas the binding energy of electrons in atoms is of the order $\alpha^2 m_e$. This means that for charged particles in the states of finite motion the electromagnetic interaction can be viewed as a perturbation that induces transitions between the energy levels of the particles in the gravitational field of a black hole. The situation looks very much like the Furry picture in QED. However, it turns out that it is impossible to use the standard QED for describing such transitions. To this end, we consider the quantization of electromagnetic field in the Schwarzschild spacetime.

Electromagnetic field

The action has the form

$$S = -\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} \sqrt{-g}\, d^4 x,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
By varying the action with respect to the vector field and taking into account that the surface terms at the event horizon and infinity do not contribute, as well as the Schwarzschild metric being Ricci-flat, we obtain the equations of motion in the form

$$\nabla^\mu F_{\mu\nu} = \nabla^\mu \nabla_\mu A_\nu - \partial_\nu \nabla^\mu A_\mu = 0,$$

where ∇_μ is the covariant derivative, and the Greek indices take values t, r, θ, φ for the Schwarzschild coordinates.

Next we impose the condition $A_t = 0$ and consider the equations of motion with $A_t = 0$ for $\nu = t$, which reads

$$\frac{r_0}{r^2}\,\partial_t A_r + \partial_t\left(\nabla^\mu A_\mu\right) = 0$$

(note that $\nabla^t A_t \neq 0$ for $A_t = 0$). From this equation it follows that

$$\nabla^\mu A_\mu + \frac{r_0}{r^2} A_r = f\left(r, \theta, \varphi\right),$$

i.e., $\nabla^\mu A_\mu + \frac{r_0}{r^2} A_r$ does not depend on time t. The last formula suggests the gauge condition

$$\nabla^\mu A_\mu + \frac{r_0}{r^2} A_r = 0.$$

One can prove that it is possible to impose this gauge condition preserving the condition $A_t = 0$.

In what follows we use the gauge condition

$$A_0 = 0, \qquad \nabla^\mu A_\mu + \frac{r_0}{r^2} A_r = 0.$$

The physical meaning of this condition becomes clear in the isotropic coordinates $t, \vec{R}$, in which the gauge condition takes the form

$$div\vec{A} + \frac{r_0\left(\frac{r_0}{4R} - 2\right)}{2R^3\left(1 - \left(\frac{r_0}{4R}\right)^2\right)}(\vec{R}\vec{A}) = 0.$$

Thus, we see that for $R \to \infty$ we get the Coulomb gauge $div\vec{A} = 0$.
and for $R \to r_0/4$ we get the Poincaré gauge $(\vec{R}\vec{A}) = 0$.

We look for solutions to the field equations in the form

$$\vec{A}_{jm}\left(E,t,r,\theta,\varphi\right)=e^{-iEt}\sum_{\lambda=-1,0,1}F_{j}^{(\lambda)}\left(E,r\right)\vec{Y}_{jm}^{(\lambda)}\left(\theta,\varphi\right),$$

where $\vec{A}=\left(A_r,A_\theta,A_\varphi\right)$ and $\vec{Y}_{jm}^{(\lambda)}\left(\theta,\varphi\right)$ are spherical vectors

$$\vec{Y}_{jm}^{(-1)}\left(\theta,\varphi\right)=\left(1,0,0\right)Y_{jm}\left(\theta,\varphi\right),$$

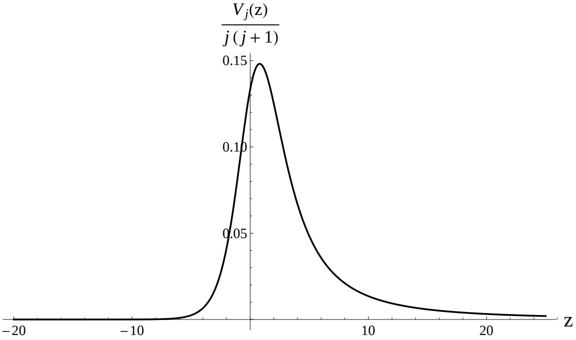
$$\vec{Y}_{jm}^{(0)}\left(\theta,\varphi\right)=\frac{i}{\sqrt{j\left(j+1\right)}}\left(0,\frac{1}{\sin\theta}\,\partial_\varphi,-\sin\theta\,\partial_\theta\right)Y_{jm}\left(\theta,\varphi\right),$$

$$\vec{Y}_{jm}^{(1)}\left(\theta,\varphi\right)=\frac{1}{\sqrt{j\left(j+1\right)}}\left(0,\partial_\theta,\partial_\varphi\right)Y_{jm}\left(\theta,\varphi\right).$$

One can show that the functions $F_j^{(-1)}(E,r)$ and $F_j^{(1)}(E,r)$ can be expressed in terms of the function $F_j^{(0)}(E,r) = F_j(E,r)$, which satisfies the equation

$$-\frac{d^2 F_j}{dz^2} + V_j(z)\, F_j = r_0^2 E^2\, F_j,$$

где $z = \frac{r}{r_0} + \ln\left(\frac{r}{r_0} - 1\right)$ и $V_j(z) = j(j+1)\frac{\frac{r(z)}{r_0} - 1}{\left(\frac{r(z)}{r_0}\right)^3}$.



For fixed E, j, m, and p there exist two independent solutions of the original equations of motion, which have the form

$$e^{-iEt}\vec{A}^{(a)}_{jmp}(E,\vec{r}) = e^{-iEt}\frac{\sqrt{j(j+1)}}{E}\begin{pmatrix} \frac{1}{r^2}F_{jp}(E,r)\,Y_{jm}(\theta,\varphi) \\ \frac{\left(1-\frac{r_0}{r}\right)}{j(j+1)}\partial_r F_{jp}(E,r)\,\partial_\theta Y_{jm}(\theta,\varphi) \\ \frac{\left(1-\frac{r_0}{r}\right)}{j(j+1)}\partial_r F_{jp}(E,r)\,\partial_\varphi Y_{jm}(\theta,\varphi) \end{pmatrix},$$

$$e^{-iEt}\vec{A}^{(b)}_{jmp}(E,\vec{r}) = e^{-iEt}\frac{i}{\sqrt{j(j+1)}}F_{jp}(E,r)\begin{pmatrix} 0 \\ \frac{1}{\sin\theta}\,\partial_\varphi Y_{jm}(\theta,\varphi) \\ -\sin\theta\,\partial_\theta Y_{jm}(\theta,\varphi) \end{pmatrix}.$$

The expansion of the field in the complete system of stationary sates can be written as

$$\vec{A}(t,\vec{r}) = \sum_{p=1}^{2}\sum_{j=1}^{\infty}\sum_{m=-j}^{j}\int_0^\infty \frac{dE}{\sqrt{2E}}\Big(e^{-iEt}\vec{A}^{(a)}_{jmp}(E,\vec{r})\,a_{jmp}(E) + e^{-iEt}\vec{A}^{(b)}_{jmp}(E,\vec{r})\,b_{jmp}(E) + \mathrm{h.c.}\Big),$$

где

$$\left[a_{jmp}(E),\,a^\dagger_{j'm'p'}(E')\right] = \left[b_{jmp}(E),\,b^\dagger_{j'm'p'}(E')\right] = \delta_{pp'}\,\delta_{jj'}\,\delta_{mm'}\,\delta\left(E-E'\right).$$

The Hamiltonian looks like

$$H = \frac{1}{2}\int \sum_{k} :\Big(A_k \partial_0^2 A_k - (\partial_0 A_k)^2\Big): g^{kk} g^{00} \sqrt{-g}\, d^3x$$

$$= \sum_{p=1}^{2} \sum_{j=1}^{\infty} \sum_{m=-j}^{j} \int_0^{\infty} dE\, E \left(a^{\dagger}_{jmp}(E)\, a_{jmp}(E) + b^{\dagger}_{jmp}(E)\, b_{jmp}(E) \right).$$

Electromagnetic interaction in the Schwarzschild spacetime can be derived from the principle of local gauge invariance and has the standard form.
Electromagnetic radiation processes at the lowest order are described by the first-order S-matrix

$$S^{(1)} = i \int j^{\mu} A_{\mu}\, r^2 \sin\theta dt dr d\theta d\varphi,$$

where $j^{\mu} = e\bar{\psi}\gamma^{(\nu)} e^{\mu}_{(\nu)}\psi$ and the tetrad in the Schwarzschild coordinates is chosen to be

$$e^{\mu}_{(\nu)} = \begin{pmatrix} (1-\frac{r_0}{r})^{-1/2} & 0 & 0 & 0 \\ 0 & (1-\frac{r_0}{r})^{1/2}\sin\theta\cos\varphi & \frac{1}{r}\cos\theta\cos\varphi & -\frac{1}{r\sin\theta}\sin\varphi \\ 0 & (1-\frac{r_0}{r})^{1/2}\sin\theta\sin\varphi & \frac{1}{r}\cos\theta\sin\varphi & \frac{1}{r\sin\theta}\cos\varphi \\ 0 & (1-\frac{r_0}{r})^{1/2}\cos\theta & -\frac{1}{r}\sin\theta & 0 \end{pmatrix}.$$

It is convenient to represent the matrix $\gamma^{(\nu)} e^k_{(\nu)}$, $k = r, \theta, \varphi$, in the form

$$\gamma^{(\nu)} e^k_{(\nu)} = \begin{pmatrix} 0 & \Lambda^k \\ -\Lambda^k & 0 \end{pmatrix},$$

where

$$\begin{cases} \Lambda^r = 2\sqrt{1 - \frac{r_0}{r}}\sqrt{\frac{\pi}{3}}\left(Y_{10}\sigma_3 + \sqrt{2}Y_{1-1}\sigma_+ - \sqrt{2}Y_{11}\sigma_-\right) \\ \Lambda^\theta = \frac{2}{r}\sqrt{\frac{\pi}{3}}\frac{\partial}{\partial\theta}\left(Y_{10}\sigma_3 + \sqrt{2}Y_{1-1}\sigma_+ - \sqrt{2}Y_{11}\sigma_-\right) \\ \Lambda^\varphi = \frac{2}{r\sin^2\theta}\sqrt{\frac{2\pi}{3}}\frac{\partial}{\partial\varphi}\left(Y_{1-1}\sigma_+ - Y_{11}\sigma_-\right). \end{cases}$$

We need to calculate the probability of transition of a charged particle, which is in the state of finite motion with energy E_1 and a set of quantum numbers j_1, l_1, m_1, in the gravitational field of a Schwarzschild black hole, to a state with a lower energy E_2 and quantum numbers j_2, l_2, m_2 with the emission of a photon with energy ω, total momentum j, projection of total momentum m, extra quantum number p and polarization either a or b,

- $|i\rangle = |E_1, j_1, l_1, m_1\rangle$ - *initial state*
- $|f\rangle = |E_2, j_2, l_2, m_2; \omega, j, m, p, a\rangle$ - *final state*.

The corresponding matrix element can be written as

$$< f|S^{(1)}|i >= (2\pi i)U\delta(E_2 - E_1 + \omega),$$

where the electromagnetic radiation amplitude is given by

$$U = \int \bar{\phi}_{E_2 j_2 l_2 m_2}(r,\theta,\varphi)\,\gamma^{(\nu)} e^k_{(\nu)}(r,\theta,\varphi)\,\phi_{E_1 j_1 l_1 m_1}(r,\theta,\varphi)$$
$$A^{(a)*}_{jmp,k}(\omega, r, \theta, \varphi)\, r^2 \sin\theta d\theta d\varphi.$$

Conclusion

- Quantum electrodynamics can be consistently formulated in the Schwarzschild spacetime above the horizon.
- Selection rules for electromagnetic transitions can be found exactly.
- To calculate the amplitudes and probabilities one needs at least approximate analytic expressions for the radial wave functions.

Thank you!

The talk is based on the results of a study conducted within the scientific program of the National Center for Physics and Mathematics, section #5 "Particle Physics and Cosmology".