# Massive higher spins and gravity


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# Outlook

1. Singh-Hagen vs gauge invariance
2. Field redefinitions
3. (Non)-minimal intractions
4. Example: spin 5/2
5. Example: spin 3

# Singh-Hagen formalism

- Massive field with spin $s$ is described by a set of completely symmetric traceless tensors:

$$\phi^{\mu(s)}, \quad \tilde{\phi}^{\mu(k)}, \quad 0 \leq k \leq s-2$$

- Lagrangian is constructed so that from the equations of motion one obtains:

$$\tilde{\phi}^{\mu(k)} = 0, \quad 0 \leq k \leq s-2$$

$$(D^2 - m^2)\phi^{\mu(s)} = 0, \qquad (D\phi)^{\mu(s-1)} = 0$$

- An interacting theory must keep all these (appropriately modified) constraints.

# Gauge invariant formalism

Massive field from the collection of massless ones

- Fields: a set of completely symmetric double-traceless tensors

$$\Phi^{\mu(k)}, \qquad 0 \leq k \leq s$$

- Gauge parameters: a set of completely symmeric traceless tensors

$$\xi^{\mu(k)}, \qquad 0 \leq k \leq s-1$$

- Each double-traceless tensor is equivalent to two traceless ones:

$$\begin{matrix} \phi^{\mu(s)} & \phi^{\mu(s-1)} & \dots & \phi^{\mu(2)} & \phi^{\mu} & \phi \\ \tilde{\phi}^{\mu(s-2)} & \tilde{\phi}^{\mu(s-3)} & \dots & \tilde{\phi} & & \end{matrix}$$

so gauge fixing reproduces a Singh-Hagen formalism

# Types of cubic vertices

- Trivially gauge invariant ones

$$\mathcal{L}_1 \sim \mathcal{RRR} \quad \Rightarrow \quad \delta_1 \phi = 0, \qquad [\delta_1, \delta_2] = 0$$

- Abelian ones

$$\mathcal{L}_1 \sim \mathcal{RR}\Phi \quad \Rightarrow \quad \delta_1 \Phi \sim \mathcal{R}\xi, \qquad [\delta_1, \delta_2] = 0$$

- Non-abelian ones

$$\mathcal{L}_1 \sim \mathcal{R}\Phi\Phi \quad \Rightarrow \quad \delta_1 \Phi \sim \Phi\xi, \qquad [\delta_1, \delta_2] \neq 0$$

# Field redefinitions

- Field redefinitions (due to non-homogeneous transformations) can drastically change the type of the vertex.
- Any vertex with one massless and two massive fields (i.e. e/m or gravitational interactions) can be transformed into an abelian one.
- Any vertex with three massive fields can be transformed into an trivially gauge invariant one.
- In both cases the field redefinitions contain higher derivatives.
- These properties can be used for the classification of vertces.
- What about minimal ones?

# Minimal interactions

- E/m interactions
  - minimal substitution

$$D \Rightarrow D + A$$

  - for $s \geq 3/2$ we need non-minimal ones

$$\mathcal{L}_{non-min} \sim \Phi\Phi\mathcal{F}, \qquad \mathcal{F} = DA$$

- Gravitational interactions
  - minimal substituion

$$e^a \Rightarrow e^a + h^a, \qquad D \Rightarrow D + \omega^{ab} L_{ab}$$

  - for $s \geq 5/2$ we need non-minimal ones

$$\mathcal{L}_{non-min} \sim \Phi\Phi\mathcal{R}, \qquad \mathcal{R} = D\omega$$

# Massive spin 5/2

- Massive spin 5/2 $(\pm 5/2, \pm 3/2, \pm 1/2)$ requires three fields: $\Phi_\mu^{\alpha(2)\dot\alpha} + h.c.$, $\Phi_\mu^\alpha + h.c.$ and $\phi^\alpha + h.c.$. Lagrangian

$$
\begin{aligned}
\mathcal{L}_0 &= -D\Phi_{\alpha\beta\dot\alpha} e^\beta{}_{\dot\beta} \Phi^{\alpha\dot\alpha\dot\beta} + D\Phi_\alpha e^\alpha{}_{\dot\alpha} \Phi^{\dot\alpha} - 3a_0 D\phi_\alpha E^\alpha{}_{\dot\alpha} \phi^{\dot\alpha} \\
&+ \frac{M}{2}[3\Phi_{\alpha\beta\dot\alpha} E^\beta{}_\gamma \Phi^{\alpha\gamma\dot\alpha} - \Phi_{\alpha(2)\dot\alpha} E^{\dot\alpha}{}_{\dot\beta} \Phi^{\alpha(2)\dot\beta}] + 2m\Phi_{\alpha(2)\dot\alpha} E^{\alpha(2)} \Phi
\\
&-3M\Phi_\alpha E^\alpha{}_\beta \Phi^\beta + 6a_0 \Phi_\alpha E^\alpha{}_{\dot\alpha} \phi^{\dot\alpha} + 9Ma_0 E\phi_\alpha \phi^\alpha + h.c.
\end{aligned}
$$

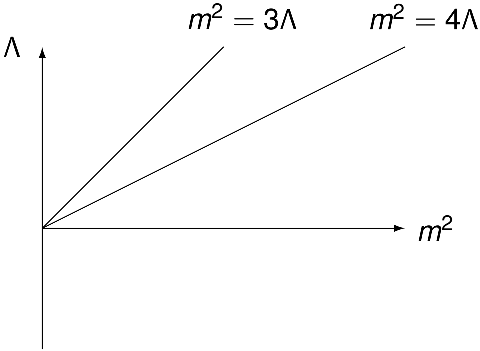
where

$$
M^2 = m^2 - 4\Lambda, \qquad a_0 = \frac{16}{3}(m^2 - 3\Lambda).
$$

- Gauge transformations

$$
\begin{aligned}
\delta\Phi^{\alpha(2)\dot\alpha} &= D\rho^{\alpha(2)\dot\alpha} + e_\beta{}^{\dot\alpha} \rho^{\alpha(2)\beta} + \frac{M}{2} e^\alpha{}_{\dot\beta} \rho^{\alpha\dot\alpha\dot\beta} + \frac{m}{3} e^{\alpha\dot\alpha} \rho^\alpha, \\
\delta\Phi^\alpha &= D\rho^\alpha + m e_{\beta\dot\alpha} \rho^{\alpha\beta\dot\alpha} + \frac{3M}{2} e^\alpha{}_{\dot\alpha} \rho^{\dot\alpha}, \\
\delta\phi^\alpha &= \rho^\alpha.
\end{aligned}
$$

# Partially massless spin 5/2




- In de Sitter space $\Lambda > 0$ there exists an unitary forbidden region $m^2 < 4\Lambda$
- Inside this region at $m^2 = 3\Lambda$ field $\phi^\alpha$ decouples leaving us with partially massless case $(\pm 5/2, \pm 3/2)$ described by $\Phi_\mu^{\alpha(2)\dot{\alpha}} + h.c.$, $\Phi_\mu^\alpha + h.c.$ only.

# Gravitational interactions

- Non-invariance of the Lagrangian after standard minimal substitutions arises due to non-commutativity of Lorentz covariant derivatives:

$$\delta\mathcal{L}_0 = 2e_\beta{}^{\dot\beta}(R^\alpha{}_\gamma\Phi_{\gamma\dot\alpha\dot\beta} + R_{\dot\alpha}{}^{\dot\gamma}\Phi_{\alpha\dot\beta\dot\gamma} + R_{\dot\beta}{}^{\dot\gamma}\Phi_{\alpha\dot\alpha\dot\gamma})\rho^{\alpha\beta\dot\alpha} + 2e_\alpha{}^{\dot\alpha}R_{\dot\alpha\dot\beta}\Phi^{\dot\beta}\rho^\alpha + h.c$$

- So we need non-minimal interactions:

$$\mathcal{L}_1 = \frac{g}{\sqrt{m^2 - 4\Lambda}} R_{\alpha\beta}\Phi^{\alpha\dot\alpha(2)}\Phi^\beta{}_{\dot\alpha(2)} + h.c.$$

- Complete variations can be compensated by the following corrections for the graviton

$$\begin{aligned} \delta h^{\alpha\dot\alpha} &= [\frac{1}{M}(\rho^{\alpha\beta(2)}\Phi_{\beta(2)}{}^{\dot\alpha} - \Phi^{\alpha\beta(2)}\rho_{\beta(2)}{}^{\dot\alpha}) + \rho^{\alpha\beta\dot\beta}\Phi_{\beta\dot\beta}{}^{\dot\alpha} \\ &\quad - \frac{2m}{3M}(\Phi^{\alpha\beta\dot\alpha}\rho_\beta - \rho^{\alpha\beta\dot\alpha}\Phi_\beta) + \Phi^{\dot\alpha}\rho^\alpha] + h.c. \end{aligned}$$

# Massive spin 3

- In massive case $(\pm 3, \pm 2, \pm 1, 0)$ each helicity requires a pair (physical field, auxiliary field)

| | | | |
|---|---|---|---|
| $\Omega_\mu^{\alpha(3)\dot\alpha} + h.c.$ | $\Omega_\mu^{\alpha(2)} + h.c.$ | $B^{\alpha(2)} + h.c.$ | $\pi^{\alpha\dot\alpha}$ |
| $f_\mu^{\alpha(2)\dot\alpha(2)}$ | $f_\mu^{\alpha\dot\alpha}$ | $A_\mu$ | $\varphi$ |

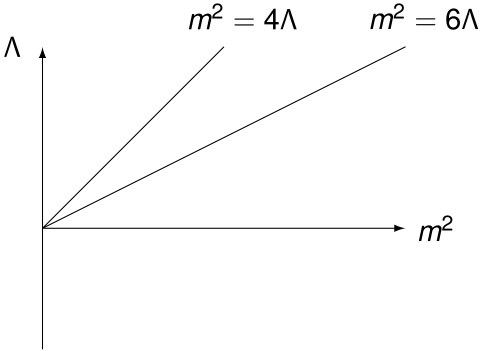
- Gauge transformations for physical fields

$$\begin{aligned}
\delta f^{\alpha(2)\dot\alpha(2)} &= D\xi^{\alpha(2)\dot\alpha(2)} + e_\beta{}^{\dot\alpha}\eta^{\alpha(2)\beta\dot\alpha} + e^\alpha{}_{\dot\beta}\eta^{\alpha\dot\alpha(2)\dot\beta} + \frac{m}{3}e^{\alpha\dot\alpha}\xi^{\alpha\dot\alpha} \\
\delta f^{\alpha\dot\alpha} &= D\xi^{\alpha\dot\alpha} + e_\beta{}^{\dot\alpha}\eta^{\alpha\beta} + e^\alpha{}_{\dot\beta}\eta^{\dot\alpha\dot\beta} + \frac{2m}{3}e_{\beta\dot\beta}\xi^{\alpha\beta\dot\alpha\dot\beta} - \frac{a_0}{4}e^{\alpha\dot\alpha}\xi \\
\delta A &= D\xi - \frac{a_0}{2}e_{\alpha\dot\alpha}\xi^{\alpha\dot\alpha}, \qquad \delta\varphi = \frac{\tilde a_0}{12}\xi
\end{aligned}$$

where

$$M^2 = m^2 - 6\Lambda, \qquad {a_0}^2 = 20[m^2 - 4\Lambda],$$

# Partially massless spin 3




- $m^2 < 6\Lambda$: unitary forbidden region
- $m^2 = 6\Lambda$: first partially massless spin 3 $(\pm 3, \pm 2, \pm 1)$
- $m^2 = 4\Lambda$: second partially massless spin 3 $(\pm 3, \pm 2)$

# Gravitational interactions

- Non-invariance of the Lagrangian after standard minimal substitutions:

$$\begin{aligned}
\delta\mathcal{L}_0 &= -2[R_\alpha{}^\gamma\Omega_{\alpha\beta\gamma\dot\alpha} + R_\beta{}^\gamma\Omega_{\alpha(2)\gamma\dot\alpha} + R_{\dot\alpha}{}^{\dot\gamma}\Omega_{\alpha(2)\beta\dot\gamma}]e^\beta{}_{\dot\beta}\xi^{\alpha(2)\dot\alpha\dot\beta} \\
&+2[R_\alpha{}^\gamma f_{\alpha\gamma\dot\alpha\dot\beta} + R_{\dot\alpha}{}^{\dot\gamma} f_{\alpha(2)\dot\beta\dot\gamma} + R_{\dot\beta}{}^{\dot\gamma} f_{\alpha(2)\dot\alpha\dot\gamma}]e_\beta{}^{\dot\beta}\eta^{\alpha(2)\beta\dot\alpha} \\
&+2[R_\alpha{}^\gamma\Omega_{\beta\gamma} + R_\beta{}^\gamma\Omega_{\alpha\gamma}]e^\beta{}_{\dot\alpha}\xi^{\alpha\dot\alpha} \\
&-2[R_\alpha{}^\gamma f_{\gamma\dot\alpha} + R_{\dot\alpha}{}^{\dot\gamma} f_{\alpha\dot\gamma}]e_\beta{}^{\dot\alpha}\eta^{\alpha\beta} + h.c.
\end{aligned}$$

- So we need non-minimal interactions:

$$\mathcal{L}_1 = gR_{\alpha\beta}[\frac{1}{m^2 - 6\Lambda}\Omega^{\alpha\dot\alpha(3)}\Omega^\beta{}_{\dot\alpha(3)} + f^{\alpha\gamma\dot\alpha(2)}f^\beta{}_{\gamma\dot\alpha(2)} - f^{\alpha\dot\alpha}f^\beta{}_{\dot\alpha}] + h.c.$$

- Now complete variations can be compensated by the appropriate corrections for the graviton.

# Final remarks

- To construct minimal vertices and solve the ambiguities related with field redefinitions, we work in down-up approach, using a minimum number of derivatives possible.
- To simplify constructions of non-minimal interactions, we start with the partially massless cases, and then turn to the general massive case.
- In both cases considered, we have a non-singular massless limit for non-zero (negative) cosmological constant, a non-singular flat limit for non-zero mass, with special points at the boundary of unitary allowed region in $dS_4$.