Three-loop anomalous dimensions of fixed-charge operators in the SM

Alexander Bednyakov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research

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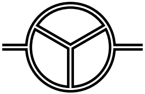
Outline

- Perturbative vs “non-perturbative”
- Large-Q expansion in ϕ^4 with global and local $U(1)$
  - Anomalous dimensions of fixed-charge operators
  - Critical indices Δ_Q as energies E_Q on $R \times S^{d-1}$
  - Non-local fixed-charge operators in scalar QED
- Fixed-hypercharge in the SM
  - Semi-classical results [Antipin et al’23]
  - Three-loop perturbative cross-check [AVB’24]
- Conclusions and outlook

Perturbative vs ‘non-perturbative”

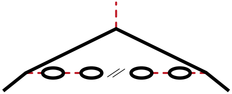
- Usual perturbative expansion in small g : $\mathcal{O} = \sum_i c_i g^i$
- Large-N_c expansion in $SU(N_c)$ gauge theories



$$, \qquad \mathcal{O} = \sum_i c_i(\mathcal{A}) N_c^{-i}, \qquad \mathcal{A} \equiv g N_c,$$

Planar diagrams dominate

- Large-N_f expansion in theories with many flavours



$$\mathcal{O} = \sum_i c_i(\mathcal{A}) N_f^{-i}, \qquad \mathcal{A} \equiv g N_f,$$

Bubble-chains dominate

- Large-charge (Q) expansion

$$\mathcal{O} = \sum_i c_i(\mathcal{A}) Q^{-i}, \qquad \mathcal{A} \equiv g Q$$

Semiclassical expansion

Fixed
’t Hooft-like
couplings

Fixed-charge operators in ϕ^4 with global $U(1)$

- Consider operators ϕ^Q in euclidean theory in $d = 4 - \varepsilon$

$$\mathcal{L} = \partial\bar{\phi}\partial\phi + \frac{\lambda}{4}(\bar{\phi}\phi)^2$$

- Compute the anomalous dimensions γ_Q in ϵ-expansion

$$[\phi^Q]_R = Z_Q \phi^Q_{bare}, \qquad \gamma_Q(\lambda) = -\frac{\partial \ln Z_Q}{\partial\lambda} \underbrace{[-\epsilon + \beta_4(\lambda)]}_{\beta(\lambda)}.$$

- At the WF fixed point $\beta(\lambda^*) = 0$, we have CFT and

$$\langle\bar{\phi}^Q(x')\phi^Q(x)\rangle_{\text{CFT}} = \frac{1}{|x - x'|^{2\Delta_Q}}, \;\; \Delta_Q = Q\left[\frac{d-2}{2}\right] + \gamma_Q(\lambda^*)$$

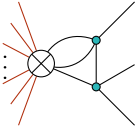
Δ_Q are physical quantities - critical (crossover) exponents

Anomalous dimensions in PT and beyond

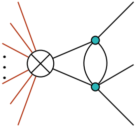
- In PT anomalous dimensions of ϕ^Q are polynomials in λ, Q

$$\gamma_Q = Q \sum_{l=1} \lambda^l P_l(Q) \sim Q \sum_{l=1} \gamma_l^{(l)} (\lambda Q)^l + \dots, \quad P_l(Q) = \sum_{n=0}^{l} \gamma_n^{(l)} Q^n.$$

For sufficently large $Q \sim 1/\lambda$, PT breaks down



$\propto Q(\lambda^* Q)^2,$



$\propto (\lambda^* Q)^2$

- Alternative expansion (“re-summation”)

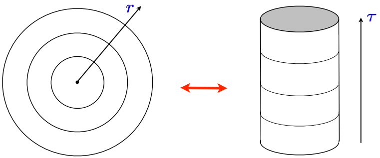
$$\gamma_Q = Q \sum_{l=0} \lambda^l F_l(\lambda Q) = Q \sum_{l=0} \tilde{F}_l(\lambda Q)/Q^l$$

How to compute?

Semi-classical approach: brief overview

- Tune to (perturbative) fixed point (CFT)
- Exploit conformal invariance to map from plane to cylinder

$$R^d \to R \times S^{d-1}, \qquad r = Re^{\tau/R}$$



$$d^2 s_{cyl} = d\tau^2 + R^2 d\Omega_{d-1}^2 = \frac{R^2}{r^2} ds_{flat}^2$$

Spectrum of D $\Leftrightarrow$ energy spectrum on $R \times S^{d-1}$

- State-operator correspondence $(T = \tau' - \tau)$

$$\langle \bar{\phi}^Q(x') \phi^Q(x) \rangle_{cyl} \overset{T\to\infty}{=} \mathcal{N} e^{-E_Q T}, \; E_Q = \Delta_Q / R$$

Automatically selects lowest-lying operator with charge Q

[Badel,Cuomo,Monin,Rattazzi'19]

- At the fixed point (CFT)

$$E_Q \cdot R = \Delta_Q = \sum_{l=-1}^{\infty} (\lambda^*)^l \tilde{\Delta}_l(\lambda^* Q) = \sum_{l=-1}^{\infty} \frac{\Delta_l(\lambda^* Q)}{Q^l}$$

Expansion in large charge

- Small-λ^* limit should match (predict) certain terms in PT

	1 loop		2 loop		3 loop		
$Q\Delta_{-1}(\lambda^* Q) \to$	$Q(\lambda^* Q)$	$+$	$Q(\lambda Q)^2$	$+$	$Q(\lambda Q)^3$	$+$	$\dots$
$\Delta_0(\lambda^* Q) \to$	$(\lambda^* Q)$	$+$	$(\lambda Q)^2$	$+$	$(\lambda Q)^3$	$+$	$\dots$

[Antipin et al'19, Jack&Jones'21]

- Large-Q limit should reproduce known asymptotics

$$\Delta_Q = Q^{\frac{d}{d-1}} \left[\alpha_1 + \alpha_2 Q^{-\frac{2}{d-1}} + \alpha_3 Q^{-\frac{4}{d-1}} + \dots\right] + \dots$$

EFT for goldstone bosons (phonons) [Hellerman et al, 2015]

Computing E_Q by semi-classical expansion

- For an arbitrary state $|\psi_Q\rangle$ with fixed charge Q

$$\langle\psi_Q|e^{-HT}|\psi_Q\rangle \overset{T\to\infty}{=} Ne^{-E_Q T}$$

- Path integral

$$\langle\psi_Q|e^{-HT}|\psi_Q\rangle = \mathcal{Z}^{-1}\int\mathcal{D}\Phi e^{-S_{eff}[\Phi,\lambda^*,Q]} = \mathcal{Z}^{-1}\int\mathcal{D}\Phi e^{-\frac{\tilde{S}_{eff}[\Phi,\lambda^* Q]}{Q}}$$

- evaluated semi-classically

$$S_{eff}[\Phi,Q] = \underbrace{S_{eff}[\Phi_c,Q]}_{\Rightarrow\Delta_{-1}} + \frac{1}{2}\underbrace{\Phi\cdot\frac{\delta^2 S_{eff}}{\delta\Phi_c\delta\Phi_c}\cdot\Phi}_{\Rightarrow\Delta_0} + \ldots,$$

- by expanding around a saddle point

$$\Phi_c : \frac{\delta S_{eff}}{\delta\Phi_c} = 0$$

Saddle-point approximation

$$\mathcal{L}_{eff} = \mathcal{L}\left(\phi = \frac{\rho}{\sqrt{2}} e^{i\chi}\right) + i\frac{Q}{R^{d-1}\Omega_{d-1}}\dot{\chi},$$

The configuration ϕ_c

$$\rho = f, \quad \chi = -i\mu\tau,$$

satisfies EOMs

$$\partial^2\rho - \rho(m^2 + (\partial\chi)^2) - \frac{\lambda}{4}\rho^3 = 0, \qquad i\partial_\mu(\rho\partial^\mu\chi) = 0$$

and the constraint that fixed the charge density

$$i\rho\dot{\chi} = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

Both f and μ (chemical potential) are fixed in terms of λ and Q:

$$\mu^2 - m^2 = \frac{\lambda}{4} f^2, \qquad f^2\mu = \frac{Q}{R^{d-1}\Omega_{d-1}}$$

Saddle-point approximation

Substituting ϕ_c into S_{eff} we get:

$$S_{eff}(\phi_c) = E_Q^0 T = T \cdot Q \left[\frac{3}{4}\mu + \frac{m^2}{4\mu}\right] = \frac{T}{R} Q \cdot \Delta_{-1}(\lambda^* Q)$$

For $d = 4$ we have $m = 1/R$, and

$$R\mu = \frac{3^{1/3} + \left[x + \sqrt{x^2 - 3}\right]^{2/3}}{3^{2/3}\left[x + \sqrt{x^2 - 3}\right]^{1/3}}, \quad x = 9 a_\lambda^* Q, \quad a_\lambda \equiv \frac{\lambda}{16\pi^2}$$

Notice that with

$$R\mu \overset{Q\to\infty}{\simeq} x^{1/3}\left[\left(\frac{2}{9}\right)^{\frac{1}{3}} + \left(\frac{1}{6}\right)^{\frac{1}{3}} x^{-2/3} + \ldots\right]$$ large-charge regime

$$R\mu \overset{\lambda\to 0}{\simeq} 1 + \frac{x}{9} - \frac{x^2}{54} + \ldots$$ small-coupling regime

perfect agreement with known PT results and large-Q asymptotics

Global $U(1)$: leading and subleading in Q

[Badel et al'19]

$$\Delta_Q = Q\Delta_{-1} + \Delta_0, \qquad \Delta_{-1} = \frac{3}{4}\mu + \frac{m^2}{4\mu}, \qquad R = 1$$

$$\Delta_0 = \frac{1}{16}\left(5 - 15\mu^4 - 6\mu^2 + \underbrace{8\sqrt{6\mu^2 - 2}}_{\omega_+(0)}\right) + \frac{1}{2}\sum_{\ell=1}^{\infty}\sigma(\ell)$$

$$\sigma(\ell) = (1+\ell)^2\left[\omega_+^*(\ell) + \omega_-^*(\ell)\right] - \left[2\ell^3 + 6\ell^2 + (2\mu^2+4)\ell + 2\mu^2\ell^0 - \frac{5(\mu^2-1)^2}{4\ell}\right]$$

$$\mu = \frac{3^{1/3} + \left[x + \sqrt{x^2-3}\right]^{2/3}}{3^{2/3}\left[x + \sqrt{x^2-3}\right]^{1/3}}, \quad x = 9a_\lambda^* Q, \quad a_\lambda \equiv \frac{\lambda}{16\pi^2}$$

$$\omega_\pm^2(l) = l(l+2) + 3\mu^2 - 1 \pm \sqrt{4l(l+2)\mu^2 + (3\mu^2-1)^2}$$

NB: Expansion in small x gives connection to perturbative results...

Large Q in local $U(1)$ theory

- **Main issue:** gauge dependence of the correlator

$$\langle \bar{\phi}^Q(x')\phi^Q(x)\rangle \qquad \text{vanish due to Elitzur's theorem}$$

- Gauged $U(1)$ on $R \times S^{d-1}$ at finite charge:

Long-range electric field (IR): non-dynamical $J_\nu^{background}$

$$\partial_\mu F_{\mu\nu} = J_\nu^{matter} + J_\nu^{background} = 0, \qquad \langle A_\nu \rangle = 0$$

  - Expect energies E_Q to be gauge-independent!
  - Operators with (lowest) scaling dimension $\Delta_Q = E_Q$?

Interpretation of Δ_Q in PT

Two first terms in large-Q expansion are computed in [Antipin et al'23]

$$\Delta_Q = Q\Delta_{-1}(a_\lambda Q, a_e Q) + \Delta_0(a_\lambda Q, a_e Q) + \ldots \quad \left[a_e \equiv \frac{e^2}{16\pi^2}\right]$$

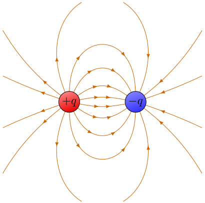
We interpret Δ_Q as

$$\langle \bar{\phi}^Q_{nl}(x')\phi^Q_{nl}(x)\rangle = \frac{1}{|x'-x|^{2\Delta_Q}},$$

with non-local gauge-invariant operator $\phi^Q_{nl}(x)$:

$$\phi_{nl}(x) \equiv e^{-ie\int d^d z J'_\mu(z-x)A^\mu(z)}\phi(x)$$

$$J'_\mu(z) = -\frac{\Gamma(d/2-1)}{4\pi^{d/2}}\partial_\mu \frac{1}{z^{d-2}}$$



In the **Landau gauge** $\partial_\mu A_\mu = 0$, $\phi_{nl}(x)$ reduces to local $\phi(x)$!

Three-loop PT computations of anomalous dimensions of $\phi^Q(x)$!

From scalar QED to the SM

Gauge group: $U(1) \to SU(3) \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{gauge} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi)$$
$$+ i \sum_j \left[\bar{Q}_j \hat{D} Q_j + \bar{L}_j \hat{D} L_j + \bar{u}^R_j \hat{D} u^R_j + \bar{d}^R_j \hat{D} d^R_j + \bar{l}^R_j \hat{D} l^R_j \right]$$

$$\mathcal{L}_{higgs+yukawa} = -\lambda \left(\Phi^\dagger \Phi \right)^2 - \sum_{i,j=1,2,3} \left[Y^{ij}_l (L^L_i \Phi) l^R_j + \text{h.c.} \right]$$
$$- \sum_{i,j=1,2,3} \left[Y^{ij}_u (Q^L_i \Phi^c) u^R_j + Y^{ij}_d (Q^L_i \Phi) d^R_j + \text{h.c.} \right], \quad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} (h + i\chi) \end{pmatrix},$$

$$D_\nu = \partial_\nu - i g_s T^a G^a_\nu - i g \frac{\tau^j}{2} W^j_\nu - i g' \frac{Y}{2} B_\nu$$

What is Q? Let us consider weak hypercharge Y

$$D_\nu \to D_\nu - i \mu Y \delta_{\nu 0}$$

Computing Δ_{-1} for $g \neq 0$ and $g = 0$

- A solution of EOMs (for cylinder $m = 1/R$ with $R = 1$) : [Antipin et al'23]

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ f \end{pmatrix}, \quad \langle B_\mu \rangle = 0, \quad \langle W_0^3 \rangle = \frac{f}{2}, \quad |\langle W_3^+ \rangle|^2 = \frac{f}{2g} \left[\mu - \frac{gf}{4} \right]$$

$$Q\Delta_1 = \frac{S_{eff}}{T} = 2\pi^2 \left\{ \frac{f^2}{2} \left[m^2 - \left(\mu - \frac{gf}{4} \right)^2 \right] + \lambda \frac{f^4}{4} \right\} + \mu Q$$

$$\frac{\partial S_{eff}}{\partial f} = 0 \Rightarrow \quad f = \frac{\sqrt{(\lambda + \frac{g^2}{64})\mu^2 - (\lambda - \frac{g^2}{8})m^2} - \frac{3g}{8}\mu}{\lambda - \frac{g^2}{8}} \quad \xrightarrow{g=0} \quad \sqrt{\frac{\mu^2 - m^2}{\lambda}}$$

$$\frac{\partial S_{eff}}{\partial \mu} = 0 \Rightarrow \quad \frac{Q}{2\pi^2} = f^2 \left(\mu - \frac{gf}{4} \right) \quad \xrightarrow{g=0} \quad \mu \cdot f^2$$

Homogeneous, non-isotropic - global minimum for $\mu^2 > \frac{g^2}{16} \frac{|m^2|}{\lambda}$

$$SO(3) \times SU(2)_L \times U(1)_Y \to SO(2)$$

[Gusynin, Miransky, Shovkovy'03]

Non-dynamical J^j_μ leads to inconsistencies [Watanabe,Murayama'14]

Summary of semi-classical computations

- For $g \neq 0$ only Δ_{-1} is known [$h = 1/(16\pi^2)$]: [Antipin et al'23]

$$Q\Delta_{-1} = Q + h\left[2\lambda - \frac{g^2}{16}\right]Q^2 - h^2\left[8\lambda^2 - \frac{3}{4}\lambda g^2 + \frac{g^4}{512}\right]Q^3 + \dots$$

  in the small-coupling expansion and yields correct limit $g \to 0$

- For $g = 0$, both Δ_{-1} and Δ_0 are known [Antipin et al'23]
  - Closed form for Δ_{-1} [the same as in U(1)]
  - Spectrum of fluctuation $\omega_i(l)$ and multiplicities $n_i(l)$ are computed and subleading corrections is given by

$$\Delta_0 = \Delta_0^{(bos)} - \Delta_0^{(fer)}$$
$$\Delta_0^{(bos)} = \frac{1}{2}\sum_{i=\text{bosons}}\sum_{l} n_i(l)\omega_i(l), \dots$$

  - Can be expanded in small SM couplings and matched to PT?

Lowest-lying operator identification

- In the limit $g = g' = 0$ [Antipin et al'23]

$$O_Q(x) = \Phi^{I_1} \dots \Phi^{I_Q} \qquad - \qquad \text{(Q+1) - plet of } SU(2)_L$$

- For $g' \neq 0$, but $g = 0$, only $U(1)_Y$ is gauged [c.f. scalar QED]
  - In Landau gauge $\partial_\mu B_\mu = 0$ should match local operator $O_Q(x)$
  - Can be compared to PT anomalous dimensions $\tilde{\gamma}_Q$ for local $\tilde{O}_Q$

$$\tilde{O}_Q = (\phi^+)^Q = \underbrace{\Phi^1 \dots \Phi^1}_{Q}, \qquad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}$$

extracted in $\overline{\mathrm{MS}}$ scheme from the ren. const. Z_Q [AVB'24]

$$\tilde{\gamma}_Q \equiv -\frac{d \ln Z_Q}{d \ln \mu}$$

- For $g \neq 0$ and $g' \neq 0$ the identification is to be done!

- In PT l-loop contribution to $\tilde{\gamma}_Q$ - polynomial in Q with $(l+1)$ coefficients C_{lk}:

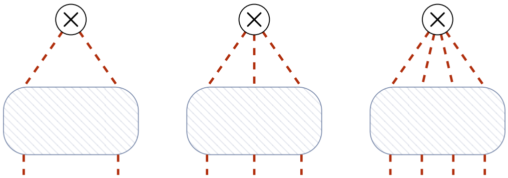
$$\tilde{\gamma}_Q \equiv -\frac{d\ln Z_Q}{d\ln\mu} = \sum_{l=1}^{\infty} \gamma_Q^{(l-\text{loop})}, \qquad \gamma_Q^{(l-\text{loop})} \equiv Q\sum_{k=0}^{l} C_{lk} Q^{l-k}$$

- Determine C_{lk} for fixed loop l by computing anomalous dimensions for $\tilde{O}_{Q=n}$ with fixed $n = 1, \ldots, l+1$ from $Z_{Q=n}$

$$\Gamma^{\mathrm{R}}_{(\phi^+)^n}(q, p_1, ..., p_n) = Z_{Q=n} \cdot Z_\Phi^{\frac{n}{2}} \cdot \Gamma^{\mathrm{bare}}_{(\phi^+)^n}(q, p_1, ..., p_n)$$

- We derived $\tilde{\gamma}_Q$ up to 3 loops by explicit computation of $\tilde{\gamma}_{Q=n}$ for $n = 1, \ldots, 4$ in the full SM with complex Yukawa matrices

[AVB'24]



NB: $\tilde{\gamma}_1$ is just an anomalous dimension of the Higgs doublet

PT results in Landau gauge

- Leading-Q coefficients (c.f. Δ_{-1})

$$C_{10} = 2a_\lambda, \qquad C_{20} = -8a_\lambda^2, \qquad C_{30} = 64a_\lambda^3$$

- Subleading coefficients (c.f. Δ_0), $[a_1 = g'^2/(16\pi^2), a_2 = g^2/(16\pi^2)]$

$$C_{11} = -\frac{3a_1}{4} - 2a_\lambda + 3\mathcal{Y}_d + 3\mathcal{Y}_u + \mathcal{Y}_l - \frac{9a_2}{4},$$

$$C_{21} = \frac{a_1^2}{16} - a_1\left(2a_\lambda - \frac{a_2}{8}\right) + \frac{3a_2^2}{16}, \qquad \left[\mathcal{Y}_f \equiv \mathrm{Tr}[Y_f^\dagger Y_f]/(16\pi^2)\right]$$
$$+ 4a_\lambda^2 - 4a_\lambda(3\mathcal{Y}_u + 3\mathcal{Y}_d + \mathcal{Y}_l) + 2(3\mathcal{Y}_{uu} + 3\mathcal{Y}_{dd} + \mathcal{Y}_{ll}),$$

$$C_{31} = -\frac{1}{16}\left(a_1^3 + 3a_1^2 a_2 + 3a_1 a_2^2 + 3a_2^3\right)(1 - 9\zeta_3)$$
$$- a_\lambda\left[a_1(a_1 + 2a_2)(1 + 3\zeta_3)\right]$$
$$+ 12a_\lambda^2 a_1(3 - 2\zeta_3) - 32a_\lambda^3(8 - 9\zeta_3) + 16a_\lambda^2(3\mathcal{Y}_u + 3\mathcal{Y}_d + \mathcal{Y}_l)$$
$$- 8a_\lambda(3\mathcal{Y}_{uu} + 3\mathcal{Y}_{dd} + \mathcal{Y}_{ll})(1 - 3\zeta_3) - 8\zeta_3\left(3\mathcal{Y}_{uuu} + 3\mathcal{Y}_{ddd} + \mathcal{Y}_{lll}\right)$$
$$+ 4a_\lambda^2 a_2(13 - 12\zeta_3) - a_\lambda a_2^2(5 + 12\zeta_3)$$

- Perfect agreement with [Antipin et al'23] result for $g = 0$!

Conclusions and outlook

Summary:

- Computed 3-loop anomalous dimensions $\tilde{\gamma}_Q$ in the SM with complex Yukawa matrices [gauge-dependent!]
- For $g = 0$ and in Landau gauge leading and subleading terms in Q of $\tilde{\gamma}_Q$ **match** small-coupling expansion of Δ_Q [Antipin et al'23]
- For $g \neq 0$ only leading-Q terms are known in Δ_Q, but in $\tilde{\gamma}_Q$ we have no dependence* on g in C_{l0} - **no match**

Outlook:

- Evaluate Δ_0 for $g \neq 0$ and its small-coupling expansion
- Identify lowest-lying gauge-independent operator with fixed Q corresponding to Δ_Q for $g \neq 0$
- Compute the anomalous dimensions of the latter in perturbation theory and **compare** with semi classical calculations.

Thank you for attention!

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