Active and sterile Majorana neutrinos in models with left-right chiral symmetry

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Outline

Minimal left-right model (MLRM), gauge and mass states

MLRM sterile neutrino warm dark matter

νMSM model. Seesaw type I mechanism (minimal seesaw).

Seesaw type II mechanism in the framework of MLRM

!

Inverse seesaw mechnism

Summary

$SO(10)$ **GUT** $\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ $\rightarrow SU(2)_L \times U(1)_Y$

LRSM gauge group may appear in the sequence of SO(10) Grand Unification symmetry breaking steps down to SM. $\mathcal{G}_{3221}$ corresponds to *minimai left-right model* (MLRM in the literature).

Обозначения:

$$\mathcal{G}_{51} = SU(5) \times U(1)$$
$$\mathcal{G}_{5} = SU(5)$$
$$\mathcal{G}_{PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$$
$$\mathcal{G}_{421} = SU(4)_C \times SU(2)_L \times U(1)_{B-L}$$
$$\mathcal{G}_{3221} = SU(\mathbf{3})_C \times SU(\mathbf{2})_L \times SU(\mathbf{2})_R \times U(\mathbf{1})_{B-L}$$
$$\mathcal{G}_{3211} = SU(\mathbf{3})_C \times SU(\mathbf{2})_L \times U(\mathbf{1})_R \times U(\mathbf{1})_{B-L}$$

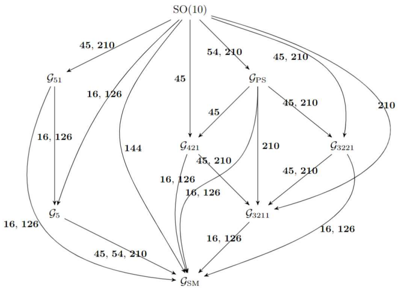


Figure: **Возможные цепочки нарушения SO(10) симметрии до $\mathcal{G}_{SM}$.** **[Fig. from M.Pernow,"Models of SO(10) Grand Unified Theories", 2021.]**

Neutrino mass problem

- Neutrinos have extremely small non-zero masses. Neutrino oscillation data:

$$m_{\text{light}} = \boxed{?} \quad \sqrt{|\Delta m^2_{21}|} \simeq 0.009 \text{ eV} \quad \sqrt{|\Delta m^2_{31}|} = 0.049 \text{ eV} \quad \sqrt{|\Delta m^2_{32|}} = 0.050 \text{ eV}$$

- Neutrino mixing matrix U_{PMNS}

$$\nu_i = \sum_{\alpha} (U_{\mathrm{PMNS}})_{\alpha i} \nu_\alpha$$

- In the Standard Model right singlet Dirac neutrino is sterile and does not mix
- More natural framework - Majorana neutrinos, mixing, **seesaw mechanism** (and its variations)
  1. Seesaw I (minimal seesaw)
  2. Seesaw II (MLRM seesaw)
  3. Inverse seesaw (ISS)
- They can be used as models with fermionic warm dark matter (WDM) candidate $m_{DM} \sim \mathcal{O}(\text{keV})$

MLRM: Спинорные и векторные поля

Калибровочная группа модели MLRM:

$$G_{LR} = SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$$

Fermions	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	New
$L_{\alpha_L} = \begin{pmatrix} \nu_{\alpha_L} \\ l_{\alpha_L} \end{pmatrix}$	**1**	**2**	**1**	-1	
$L_{\alpha_R} = \begin{pmatrix} \nu_{\alpha_R} \\ l_{\alpha_R} \end{pmatrix}$	**1**	**1**	**2**	-1	N_1, N_2, N_3
$Q_{a_L} = \begin{pmatrix} u_{a_L} \\ d_{a_L} \end{pmatrix}$	**3**	**2**	**1**	$\frac{1}{3}$	
$Q_{a_R} = \begin{pmatrix} u_{a_R} \\ d_{a_R} \end{pmatrix}$	**3**	**1**	**2**	$\frac{1}{3}$	
$W_L = \{W_L^+, W_L^-, W_L^3\}$ $W_R = \{W_R^+, W_R^-, W_R^3\}$ B	**1** **1** **1**	**3** **1** **1**	**1** **3** **1**	**0** **0** **0**	$W_2^{\pm}, Z_2$

Table: Представления для фермионных и калибровочных полей

LRSM: Скалярные поля

Higgs fields	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	New
$\Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$	**1**	**3**	**1**	2	$\begin{pmatrix} A_1^0, & A_2^0, \\ H_1^\pm, & H_2^\pm \end{pmatrix}$
$\Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$	**1**	**1**	**3**	2	$H_1^{\pm\pm}, H_2^{\pm\pm}$
$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	**1**	**2**	**2**	0	$\begin{pmatrix} H_{125}, & H_1^0 \\ H_2^0, & H_3^0 \end{pmatrix}$

Table: Представления скалярных полей в LRSM

$$\mathcal{L}_{Higgs} = tr|D_\mu \Phi|^2 + tr|D_\mu \Delta_R|^2 + tr|D_\mu \Delta_L|^2 - V(\Phi, \Delta_L, \Delta_R)$$

Higgs potential

$$\begin{aligned}
V(\phi,\Delta_L,\Delta_R) &= -\mu_1^2\left(Tr[\phi^\dagger\phi]\right)-\mu_2^2\left(Tr[\tilde\phi\phi^\dagger]+\left(Tr[\tilde\phi^\dagger\phi]\right)\right)-\mu_3^2\left(Tr[\Delta_L\Delta_L^\dagger]+Tr[\Delta_R\Delta_R^\dagger]\right)\\
&+ \lambda_1\left(\left(Tr[\phi\phi^\dagger]\right)^2\right)+\lambda_2\left(\left(Tr[\tilde\phi\phi^\dagger]\right)^2+\left(Tr[\tilde\phi^\dagger\phi]\right)^2\right)+\lambda_3\left(Tr[\tilde\phi\phi^\dagger]Tr[\tilde\phi^\dagger\phi]\right)\\
&+ \lambda_4\left(Tr[\phi\phi^\dagger]\left(Tr[\tilde\phi\phi^\dagger]+Tr[\tilde\phi^\dagger\phi]\right)\right)\\
&+ \rho_1\left(\left(Tr[\Delta_L\Delta_L^\dagger]\right)^2+\left(Tr[\Delta_R\Delta_R^\dagger]\right)^2\right)\\
&+ \rho_2\left(Tr[\Delta_L\Delta_L]Tr[\Delta_L^\dagger\Delta_L^\dagger]+Tr[\Delta_R\Delta_R]Tr[\Delta_R^\dagger\Delta_R^\dagger]\right)\\
&+ \rho_3\left(Tr[\Delta_L\Delta_L^\dagger]Tr[\Delta_R\Delta_R^\dagger]\right)\\
&+ \rho_4\left(Tr[\Delta_L\Delta_L]Tr[\Delta_R^\dagger\Delta_R^\dagger]+Tr[\Delta_L^\dagger\Delta_L^\dagger]Tr[\Delta_R\Delta_R]\right)\\
&+ \alpha_1\left(Tr[\phi\phi^\dagger]\left(Tr[\Delta_L\Delta_L^\dagger]+Tr[\Delta_R\Delta_R^\dagger]\right)\right)\\
&+ \alpha_2\left(Tr[\phi\tilde\phi^\dagger]Tr[\Delta_R\Delta_R^\dagger]+Tr[\phi^\dagger\tilde\phi]Tr[\Delta_L\Delta_L^\dagger]\right)\\
&+ \alpha_2^*\left(Tr[\phi^\dagger\tilde\phi]Tr[\Delta_R\Delta_R^\dagger]+Tr[\tilde\phi^\dagger\phi]Tr[\Delta_L\Delta_L^\dagger]\right)\\
&+ \alpha_3\left(Tr[\phi\phi^\dagger\Delta_L\Delta_L^\dagger]+Tr[\phi^\dagger\phi\Delta_R\Delta_R^\dagger]\right)\\
&+ \beta_1\left(Tr[\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\right)+\beta_2\left(Tr[\tilde\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\tilde\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\right)\\
&+ \beta_3\left(Tr[\phi\Delta_R\tilde\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\tilde\phi\Delta_R^\dagger]\right), \qquad (1)
\end{aligned}$$

Нарушение симметрии

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

1. The initial LR symmetry is spontaneously broken

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle\Delta_R\rangle} SU(2)_L \times U(1)_Y, \quad (2)$$

$$\langle\Delta_R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \quad (3)$$

2. The bidoublet and the left handed triplet acquire VEVs as a result of spontaneous symmetry breaking

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle\phi\rangle, \langle\Delta_L\rangle} U(1)_Q, \quad (4)$$

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \qquad \langle\Delta_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \quad (5)$$

where $\sqrt{k_1^2 + k_2^2} = 246$ GeV,

Seesaw relation for Higgs triplet VEVs

β-sector of the Higgs potential:

$$V_\beta(\Phi,\Delta_L,\Delta_R) = \beta_1\Big(Tr[\phi\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+\beta_2\Big(Tr[\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger]+Tr[\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger]\Big)+ \beta_3\Big(Tr[\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger]+Tr[\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger]\Big),$$

Additional GUT motivated assumptions $\rightarrow \beta_i = 0$ or $\beta_i \simeq 0$

"Vev seesaw" relation for v_L and v_R

$$\boxed{v_L = \gamma\,\frac{(246\text{ GeV})^2}{v_R},} \qquad \text{where}\quad \gamma \equiv \frac{\beta_2 k_1^2 + \beta_1 k_1 k_2 + \beta_3 k_2^2}{(2\rho_1-\rho_3)(246\text{ GeV})^2},$$

$\beta_i = 0 :\quad (2\rho_1-\rho_3)v_R v_L = 0$

$\boxed{v_L = 0}$

General LR-condition:

$\rightarrow v_R \neq 0$

Vacuum stability

$\rightarrow (2\rho_1-\rho_3) \neq 0$

$\beta_i \rightarrow 0$

$$\boxed{v_L \simeq \frac{(246\text{ GeV})^2}{v_R}}$$

$(v_R \gg 246\text{ GeV}\quad \text{or } \gamma \ll 1)$

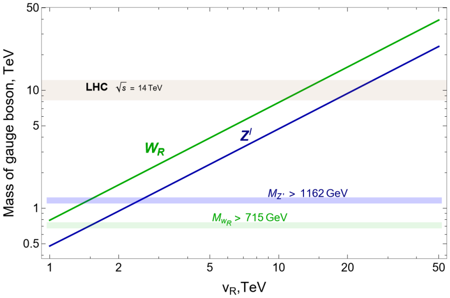


Figure: Masses of new vector gauge bosons Z_2 and W_2

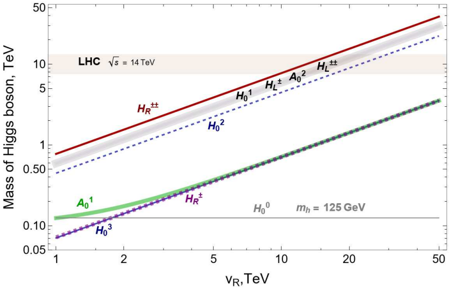


Figure: Masses of all 14 Higgs bosons in LRSM with tuning of self-interaction constants: $\alpha_3 = 0.01$ $\rho_1 = 0.1$, $\rho_2 = 0.3$, $\rho_3 = 0.9$, $\lambda_1 = \lambda_{SM} = 0.118$, $\lambda_2 = 0.01$, $\lambda_3 = 0.1$

keV sterile neutrino as warm DM, simplified

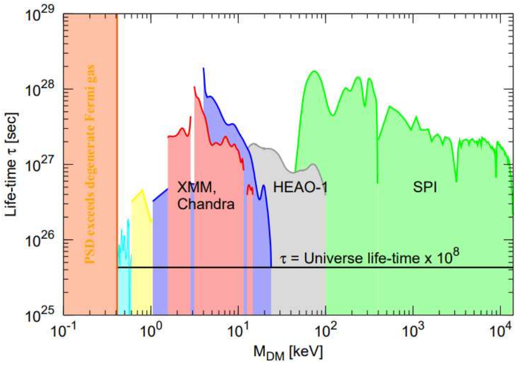
Warm Dark Matter: The lightest sterile neutrino with mass $\sim 1 - 10$ keV.

- **Lifetime:** quasi-stable because of very small mixing with active neutrino

$$\tau_{\nu_s} = 10^{19} \left(\frac{m_s}{1\ \text{keV}}\right)^{-5} \frac{1}{\sin^2(2\theta)}\ \text{sec}\ >\ H_0^{-1} \simeq 10^{17}\ \text{sec}$$

- **Non-observation of radiative one-loop decay**

[Aliev, Vysotsky, Sov. Phys. Usp. 24 (1981)]



X-ray bound:
$N_1 \to \gamma, \nu$ with $E_\gamma \simeq M_1/2$ lead to strong lifetime limit $\tau_{\nu_s} > 10^{25}$ sec

$$\sin^2(2\theta) < 10^{-6} \left(\frac{m_s}{1\ \text{keV}}\right)^{-5}$$

[Boyarsky et al, arXiv:0811.2385v1]

DM relic density: production via oscillations

Framework of 1 $\nu_\alpha + 1\ \nu_s$ states. Mixing parameter is $\sin(\theta)$

- Boltzmann equation for DM production via $\nu_a \leftrightarrow \nu_s$

$$\frac{\partial}{\partial t} f_s(p,t) \quad - \quad H\,p\,\frac{\partial}{\partial p} f_{N_1}(p,t) = C_{\rm oscl}(p,t,T)$$

$$\text{where } C_{oscl} \approx \frac{\Gamma_\alpha(p)}{2}\sin^2(2\theta_{\rm eff})\left[1+\left(\frac{\Gamma_\alpha(p)l_m}{2}\right)^2\right]^{-1}[f_{\nu_\alpha}(p,t)-f_s(p,t)]$$

$$\sin^2(\theta_{\rm eff}) = \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\sin^2 2\theta+\left[\Delta(p)\cos 2\theta - V^D - V^T(p)\right]^2} \quad \Delta=(m_2^2-m_1^2)/2p$$

[Abazajian et al, Phys.Rev. D64 (2001) 023501]

- Gives an estimate for relic density of sterile neutrino

$$\Omega_{\nu_s} h^2 \quad = \quad K_\alpha(m_s)\left(\frac{\sin^2(2\theta)}{10^{-8}}\right)\left(\frac{m_s}{1\ \text{keV}}\right)^2$$

where $K_\alpha \sim 0.3$ with weak dependency on M_1 within the considered limits, $\alpha = e, \mu, \tau$

Seesaw type I mechanism. Three generations, MLRM with $v_R \to \infty$, $v_L = 0$: νMSM model.

Additional fields: $SU(2)_L \times U(1)_Y$ - singlets $\nu_{R,k}$, $k = \overline{1,3}$ (flavour basis) with heavy Majorana mass term $\sim M_R$. Mass states N_J, $J = \overline{1,3}$ are heavy neutral leptons (HNL).

Lagrangian: $\mathcal{L} = \mathcal{L}_{SM} + i\overline{\nu}_R \partial_\mu \gamma^\mu \nu_R - \left(Y\, \bar{l}_L \widetilde{\phi} \nu_R + \frac{1}{2} \overline{\nu^c}_R M_R \nu_R + h.c \right)$,

After SSB: $\mathcal{L} \supset (\overline{\nu_L},\ \overline{\nu_R^c}) \begin{pmatrix} \mathbb{O} & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$, $\quad \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = P_L \mathrm{U} \begin{pmatrix} \nu \\ N \end{pmatrix}$

Casas-Ibarra diagonalization: [Casas J., Ibarra A., Nucl.Phys.B 618 (2001) 171]

$$\mathrm{U} = \mathrm{WV} \qquad \mathrm{W} = \exp \begin{pmatrix} \mathbb{O} & -\theta \\ \theta^\dagger & \mathbb{O} \end{pmatrix} \simeq \begin{pmatrix} I - \frac{1}{2}\theta\theta^\dagger & -\theta \\ \theta^\dagger & I - \frac{1}{2}\theta^\dagger\theta \end{pmatrix} + \mathcal{O}(\theta^3)$$

$$\mathrm{V} = \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix} \qquad m_\nu \equiv U_\nu \hat{m} U_\nu^T \qquad M_N \equiv U_N \hat{M} U_N^T$$

here $\hat{\square} = \mathrm{diag}(\dots)$ - diagonal matrix.

$(\nu - N)$**-mixing:** $\quad \Theta \equiv \theta U_N$

PMNS: $\quad U_{\mathrm{PMNS}} = \left(I - \frac{1}{2}\theta^\dagger\theta \right) U_\nu$

Heavy neutrino mixing and νMSM

A system of matrix equations for diagonalizing transformation U: ***(leading order θ-accuracy)***

$$\begin{cases} \theta \simeq m_D M_R^{-1}, \\ m_\nu = -\theta M_R \theta^T, \\ M_N \simeq M_R \end{cases} \Rightarrow \quad \text{seesaw I equation} \quad \boxed{m_\nu = -m_D M_N^{-1} m_D^T}$$

Seesaw I equation can be rewritten

$$I = \Omega\Omega^T = \left[i\sqrt{\tilde{m}^{-1}} U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}} \right]^T \left[-i\sqrt{\tilde{m}^{-1}} U_\nu^\dagger m_D U_N \sqrt{\hat{M}^{-1}} \right],$$

$$m_D = iU^\dagger_{\mathrm{PMNS}} \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}} \quad \rightarrow \quad \Theta = iU^\dagger_{\mathrm{PMNS}} \sqrt{\hat{m}}\Omega\sqrt{\hat{M}^{-1}}$$

νMSM - model [T.Asaka, M.Shaposhnikov,Phys.Lett.B 620, 17(2005)]
3 sterile neutrino:
N_1 - WDM with $M_1 \sim \mathcal{O}(\text{keV})$
N_2 and N_3 heavy neutrinos with masses $M_2 \simeq M_3 \sim \Lambda_{EW}$, $\Delta = |M_2 - M_3| \ll M_{2,3}$ need for Resonant leptogenesis (lepton asym. $\rightarrow$ baryon asym.)

Факторы смешивания, измеряемые в экспериментах:

$$U^2_{\alpha I} = |\Theta_{\alpha I}|^2, \qquad U^2_i = \sum_\alpha U^2_{\alpha I}, \qquad U^2 = \sum_i U^2_i$$

$$M_1 \sum_\alpha |\Theta_{\alpha 1}|^2 \equiv m_D^{dm} = \sum_\alpha |U_{\alpha i}(\sqrt{\tilde{m}})_{ij}\Omega_{j1}|^2 = |\sqrt{\tilde{m}}|^2_{kn}\Omega_{n1}\Omega^*_{k1}.$$

Neutrino mixing in MLRM: Seesaw type II

$$-\sum_{i,j}\{\bar{L}_{iL}[(h_L)_{ij}\phi+(\tilde{h}_L)_{ij}\tilde{\phi}]L_{jR}-\overline{(L_{iR})^c}\,\Sigma_R(h_M)_{ij}L_{jR}-\overline{(L_{iL})^c}\,\Sigma_L(h_M)_{ij}L_{jL}\}+\text{h.c.},$$

где $\tilde{\phi}\equiv\tau_2\phi^*\tau_2$, $\Sigma_{L,R}=i\tau_2\Delta_{L,R}$ и h_L, $\tilde{h}_L$, h_M – 3×3 матрицы Юкавы в калибровочном базисе. Массовая матрица в калибровочном базисе

$$\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \begin{array}{c} M_D=\frac{1}{\sqrt{2}}(h_L k_1+\tilde{h}_L k_2), \\ M_L=\sqrt{2}h_M v_L, \quad M_R=\sqrt{2}h_M v_R, \end{array}$$

here $h_L,\ \tilde{h}_L,\ h_M$ are **Yukawa couplings** with left triplet Δ_L and bi-doublet Φ

Mixing matrix in Casas-Ibarra parametrization

seesaw II equation: $\boxed{m_\nu = M_L - M_D M_N^{-1} M_D^T}$

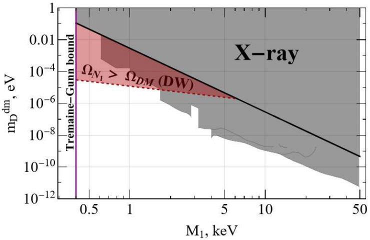
$$\Theta = iU_\nu\left(\sqrt{\tilde{m}}\right)\Omega\sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}}=\sqrt{\hat{m}-U^\dagger_{\text{PMNS}}M_L U^*_{\text{PMNS}}}$$

Assumption: $U_N = I,\ \theta^2 \ll 1$

$$h_M \simeq \frac{\hat{M}}{\sqrt{2}v_R} \Rightarrow \boxed{\tilde{m}=\hat{m}-\frac{v_L}{v_R}U^\dagger_{\text{PMNS}}\hat{M}U^*_{\text{PMNS}}}$$

DM mixing: seesaw I and seesaw II

$$m_D^{dm} = |\sqrt{\tilde{m}}|^2_{kn}\Omega_{n1}\Omega^*_{k1} \quad |\Theta_{DM}|^2 = \frac{m_D^{dm}}{M_1}$$



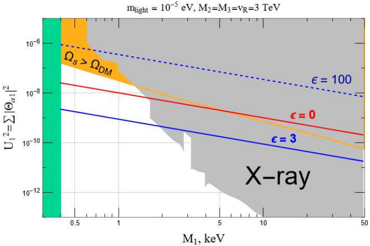
νMSM limit: pure seesaw I

$v_L = 0 \to \sqrt{\tilde{m}} = \sqrt{\hat{m}}$. Strong DM constraints and ν-oscillation data lead to a fine-tuned (FT) form of Ω

Ω_{NH} : $\Omega_{j1} \to \delta_{j1}$

Ω_{IH} : $\Omega_{j3} \to \delta_{j3}$

$$\boxed{m_D^{dm}(v_L = 0, \Omega \to \text{FT}) = m_{\text{light}}}$$



MLRM with $v_L \neq 0$: seesaw II

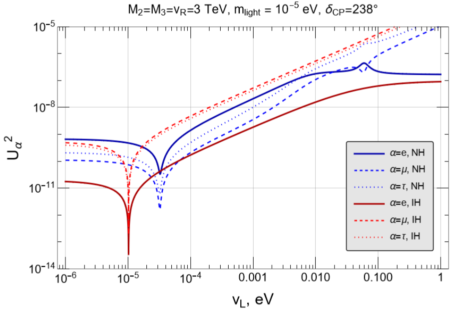
- $m_{\text{light}} \gg v_L \frac{\max M_J}{v_R}$ – νMSM-limit;
- $m_{\text{light}} \ll v_L \frac{\max M_J}{v_R}$ – **seesaw II dominance** $\to$ strong increase in mixing, inconsistent with DM constraints;
- $\hat{m} \simeq U^\dagger_{\text{PMNS}} M_L U^*_{\text{PMNS}}$ $\to$ DM mixing decreases by 1-2 order due to **seesaw I – seesaw II** cancellation for some entries of $|\sqrt{\tilde{m}}|^2_{kn}$

Cancellation effect in the seesaw II mixing

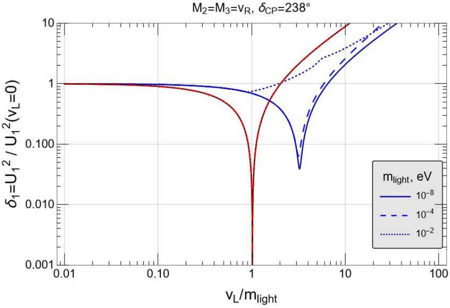
ν**MSM-benchmark for** Ω :
$$m_D^{dm} = \left| \sqrt{\hat{m} - \frac{v_L}{v_R} U^\dagger_{\rm PMNS} \hat{M} U^*_{\rm PMNS}} \right|^2_{11\ \text{(NH) or } 33\ \text{(I}}$$

$$\Omega^{(FT)}_{\rm NH} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \multicolumn{2}{c}{\Omega_{2\times2}} \\ 0 & & \end{pmatrix}, \qquad \Omega^{(FT)}_{\rm IH} = \begin{pmatrix} 0 & \multicolumn{2}{c}{\Omega_{2\times2}} \\ 0 & & \\ 1 & 0 & 0 \end{pmatrix}.$$

$$U^2_\alpha = \sum_{I=1}^{3} |\Theta_{\alpha I}|^2$$

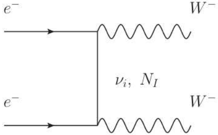


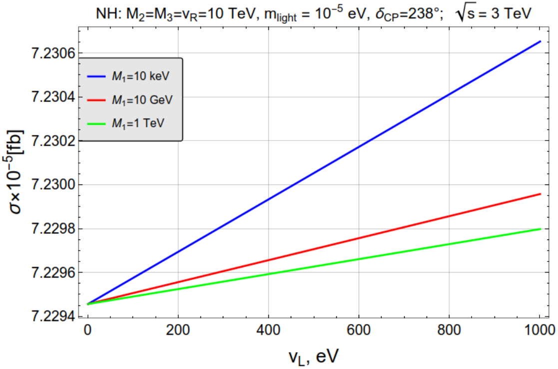
$$U^2_I = \sum_{\alpha=e,\mu,\tau} |\Theta_{\alpha I}|^2, \quad |\Theta_{\rm DM}|^2 = U^2_1$$



Inverse neutrinoless double beta decay, $i0\nu\beta\beta$

$$e^- e^- \to W^- W^-$$





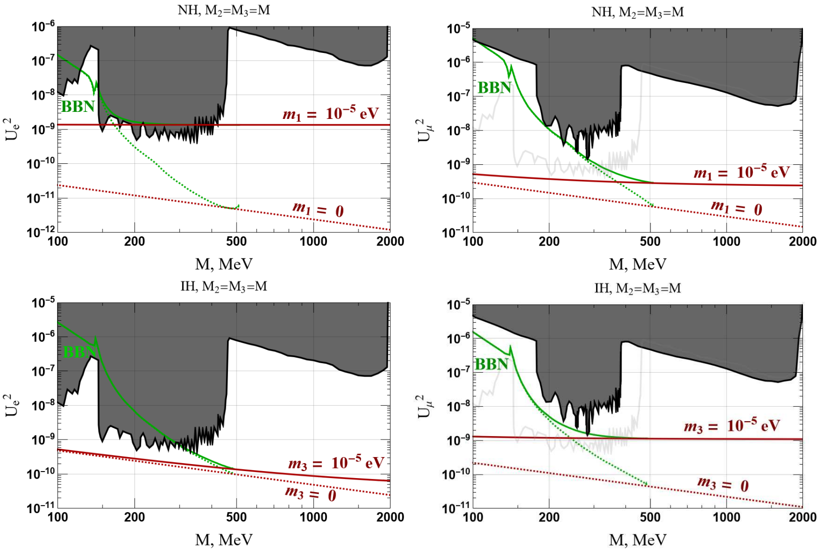
Ограничения для N_2, N_3 модели νMSM

- Ограничения сверху из ускорительных экспериментов двух типов: эксперименты с определением *недостающей энергии* (PIENU, TRIUMPH, KEK, NA62, E949) и эксперименты по определению *смещенных вершин* (PS-191, CHARM, NuTeV, DELPHI). Совокупность этих ограничений дает верхние границы для

$$U_\alpha^2 = \sum_{I=1}^{3} |\Theta_{\alpha I}|^2 = \begin{cases} \frac{m_1}{M_1}|U_{\alpha 1}|^2 + |\Theta_{\alpha 2}^{(NH)}|^2 + |\Theta_{\alpha 3}^{(NH)}|^2, & \text{NH} \\ \frac{m_3}{M_1}|U_{\alpha 3}|^2 + |\Theta_{\alpha 2}^{(IH)}|^2 + |\Theta_{\alpha 3}^{(IH)}|^2, & \text{IH} \end{cases}$$

- Неравенство для времени жизни N_2 и N_3, $\tau_N < 0.02$ секунд, при которых не возникает перепроизводства легких элементов ($^4He, ^2H$) в первичной плазме, (A. Boyarsky et al, PRD 2021) (первичный нуклеосинтез или Big Bang nucleosynthesis, **BBN**). Дает ограничение снизу на параметры U_α^2.

Ограничения для смешиваний U_e^2 и U_μ^2 в модели νMSM



Inverse seesaw: General approach $ISS(\mathrm{p},\mathrm{q})$

Field content of the $ISS(\mathrm{p},\mathrm{q})$

Consider **three types** of neutral lepton fields:

– left-handed flavour neutrino $\nu_{L\ \alpha}$, $\alpha = e, \mu, \tau$,

– right-handed neutrino $N_{R\ a}$ $a = \overline{1, \mathrm{p}}$

– right-handed sterile fermions $S_{R\ b}$ $b = \overline{1, \mathrm{q}}$

Lagrangian after SSB: **(Naturalness condition:** $\mu \ll m_D < M_R$)

$$\mathcal{L}_{ISS} = \frac{1}{2}(\overline{\nu}_L,\ \ \overline{\nu_R^c},\ \ \overline{\Sigma_R^c}) \begin{pmatrix} \mathbb{O}_{3\times 3} & m_{D\ 3\times p} & \mathbb{O}_{3\times q} \\ m_{D\ p\times 3}^T & \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ \mathbb{O}_{q\times 3} & M_{R\ q\times p}^T & \mu_{\ q\times q} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ \Sigma_R \end{pmatrix}$$

Diagonalization: step 1

Rewrite mass matrix to the **seesaw I-like** form $\tilde{m}_D \equiv (m_D,\ 0)$

$$\mathrm{M}_{9\times 9} = \begin{pmatrix} \mathbb{O}_{\ 3\times 3} & \widetilde{m}_{D\ 3\times(p+q)} \\ \widetilde{m}_{D\ (p+q)\times 3}^T & \mathcal{X}_{\ q\times q} \end{pmatrix} \quad \text{where} \quad \mathcal{X} = \begin{pmatrix} \mathbb{O}_{\ p\times p} & M_{R\ p\times q} \\ M_{R\ q\times p}^T & \mu_{\ q\times q} \end{pmatrix}$$

$$\mathrm{U}^T\mathrm{M}\mathrm{U}, \quad \mathrm{U} = \mathrm{W}\begin{pmatrix} U_{\nu\ 3\times 3} & \mathbb{O}_{3\times(p+q)} \\ \mathbb{O}_{(p+q)\times 3} & \mathcal{U}_{\ (p+q)\times(p+q)} \end{pmatrix} \quad \mathrm{W} = \exp(\omega) \simeq \mathrm{I} + \omega + \ldots$$

Inverse seesaw: ISS(p,q) diagonalization

Technical comment: Inverse matrix (Frobenius)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + A^{-1}B(M|A)^{-1}CA^{-1} & -A^{-1}B(M|A)^{-1} \\ -(M|A)^{-1}CA^{-1} & (M|A)^{-1} \end{pmatrix}$$

where for a matrix $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ Shur complement is $(M|A) \equiv D - CA^{-1}B$

Neutrino effective mass operator

$$m_\nu = (\mathrm{M}|\mathcal{X}) \equiv -\tilde{m}_D \mathcal{X}^{-1} \tilde{m}_D^T = -(m_D,\ \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-1} & \star \\ \star & \star \end{pmatrix} \begin{pmatrix} m_D^T \\ \mathbb{O} \end{pmatrix} = m_D \left((M_R)\mu^{-1}(M_R)^T \right)^{-1} m_D^T$$

Only if $p = q$:

$$\boxed{m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \sim \frac{\mu m_D^2}{M^2}}$$

Diagonalization: step 2 (Sterile block)

Case 1: $\boxed{p = q}$ All sterile fields form pseudo-Dirac pairs

$$\mathcal{X}' = \mathcal{U} \begin{pmatrix} \mathbb{O}_{p\times p} & M_{R\ p\times q} \\ M^T_{R\ q\times p} & \mu \end{pmatrix} \mathcal{U}^T = \begin{pmatrix} \sim -M_R + \mu & \mathcal{O}(\mu) \\ \mathcal{O}(\mu) & \sim M_R + \mu \end{pmatrix}$$

Inverse seesaw: Toy-model ISS(1,1)

Toy-model ISS(1,1): $m_\pm \equiv \frac{\mu \pm \sqrt{\mu^2 + 4M^2}}{2}$

$$\mathcal{X} = \begin{pmatrix} 0 & M \\ M & \mu \end{pmatrix} \to \begin{pmatrix} m_- & 0 \\ 0 & m_+ \end{pmatrix} \simeq \begin{pmatrix} \mu - M & 0 \\ 0 & \mu + M \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{1}{\sqrt{1+\frac{m_+^2}{M^2}}} \\ \frac{m_-}{M\sqrt{1+\frac{m_-^2}{M^2}}} & \frac{m_+}{M\sqrt{1+\frac{m_+^2}{M^2}}} \end{pmatrix} \simeq \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

S'', N'' are mass-states, N', S' - "middle" basis states (after the 1st step of diagonalization) **Pseudo-dirac** HEAVY ($\sim M$) states with small mass splitting $\sim \mu$.

$$N'' \equiv N_- \simeq \frac{1}{\sqrt{2}}(N' - S')$$
$$S'' \equiv N_+ \simeq \frac{1}{\sqrt{2}}(N' + S')$$

Warm dark matter candidate is absent in such a model

Inverse seesaw: model ISS(1,2)

ISS(1,2): $m^0_{\pm} \simeq \pm\sqrt{M_1^2 + M_2^2} + \mathcal{O}(\mu) \qquad m_3 \simeq \mathcal{O}(\mu)$

$$\mathcal{X} = \begin{pmatrix} 0 & M_1 & M_2 \\ M_1 & \mu_1 & 0 \\ M_2 & 0 & \mu_2 \end{pmatrix} = \mathcal{X}_0 + \delta\mathcal{X}(\mu) \qquad \mathcal{X}_I \to \begin{pmatrix} \boxed{\sim 0} & 0 & 0 \\ 0 & m_- & 0 \\ 0 & 0 & m_+ \end{pmatrix}$$

Mass of light sterile state: $m_3 \approx \dfrac{M_2^2\mu_1 + M_1^2\mu_2}{M_1^2 + M_2^2} \sim \mu \sim \mathcal{O}(\text{keV})$

$$\begin{pmatrix} \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_1}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{M_1^2+M_2^2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & \frac{M_2}{\sqrt{2(M_1^2+M_2^2)}} & -\frac{M_1}{\sqrt{M_1^2+M_2^2}} \end{pmatrix} \qquad \begin{pmatrix} N' \\ S_1' \\ S_2' \end{pmatrix} = U \underbrace{\begin{pmatrix} N_+ \\ N_- \\ \boxed{\mathcal{S}_3} \end{pmatrix}}_{phys.states}$$

Light sterile fermions ($q - p$ states with masses at the scale μ) are embedded in $ISS(p, q)$ model when $p < q$.

Summary

- Minimal Left-Right Model (MLRM) Higgs potential demonstrates a number of phenomenologically acceptable regimes with splitting of 16 new states which are not observed at the LHC. β-terms of the potential are important for active neutrino and sterile neutrino mixing scenarios. The ν**MSM** (neutrino minimal standard model) can be embedded into the MLRM as a limiting scenario $v_R \to \infty$, $k_2 = 0$.
- Comparisons with data are possible in the limiting scenarios and are model dependent.
- Modification of seesaw type II for MLRM yields mixing in the lepton sector defined by VEVs. Four VEVs participate in the neutrino mass and mixing matrices with untrivial (orders of magntude) cancellations dependent on v_L, v_R of scalar triplets and U_{PMNS}

$$\Theta = iU_{\text{PMNS}}\sqrt{\tilde{m}}\Omega\sqrt{\hat{M}^{-1}}, \qquad \sqrt{\tilde{m}} = \sqrt{\hat{m} - \frac{v_L}{v_R}U^{\dagger}_{\text{PMNS}}M_L U^{*}_{\text{PMNS}}}$$

- Other mass neutrino and HNL mass hierarchies are possible within the inverse seesaw ISS(2,3) or ISS$(p, p+s)$, $p, s \in \mathbb{N}$: these models can naturally provide a light warm dark matter candidate and lepton number violation scale μ of the order of keV.

Acknowledgments

Thank you for attention

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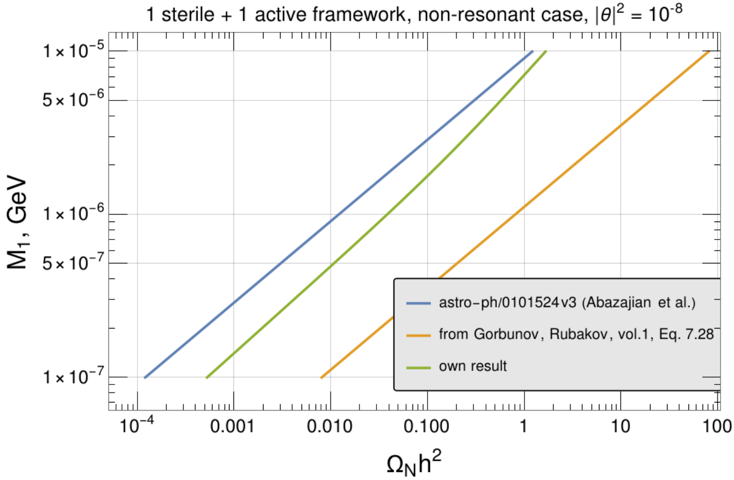
Backup slides


1 sterile + 1 active framework, non-resonant case, |θ|^2 = 10^-8
1 × 10^-5
5 × 10^-6
1 × 10^-6
5 × 10^-7
1 × 10^-7
M_1, GeV
10^-4
0.001
0.010
0.100
1
10
100
Ω_N h^2
astro-ph/0101524v3 (Abazajian et al.)
from Gorbunov, Rubakov, vol.1, Eq. 7.28
own result

Представления заряженных и нейтральных токов в общем виде

$$\mathcal{L}_{NC}^{\nu} = \frac{1}{2}\sum_{X=Z_1,Z_2,A}\overline{\nu}\hat{X}\left(U^L_{\nu\nu}a^L_X P_L + U^R_{\nu\nu}a^R_X P_R\right)\nu,$$

$$\mathcal{L}_{NC}^{N} = \frac{1}{2}\sum_{X=Z_1,Z_2,A}\overline{N}\hat{X}(U^L_{NN}a^L_X P_L + a^R_X P_R)N + \frac{1}{2}\left(\sum_{X=Z_1,Z_2,A}\overline{\nu}\hat{X}(U^L_{\nu N}a^L_X P_L - U^R_{\nu N}a^R_X P_R)N + h.c.\right),$$

$$\mathcal{L}_{CC}^{\nu} = \frac{1}{\sqrt{2}}\bar{l}\hat{W}_1^-(U^L_{l\nu}\, g_L c_\xi P_L + U^R_{l\nu} g_R s_\xi P_R)\nu + \frac{1}{\sqrt{2}}\bar{l}\hat{W}_2^-(U^L_{l\nu}\, g_L s_\xi P_L - U^R_{l\nu} g_R c_\xi P_R)\nu + h.c.,$$

$$\mathcal{L}_{CC}^{N} = \frac{1}{\sqrt{2}}\bar{l}\hat{W}_1^-(U^L_{lN} g_L c_\xi P_L - U^R_{lN} g_R s_\xi P_R)N + \frac{1}{\sqrt{2}}\bar{l}\hat{W}_2^-(U^L_{lN} g_L s_\xi P_L + U^R_{lN} g_R c_\xi P_R)N + h.c.,$$

где

$$U^L_{\nu\nu} = U^\dagger_{\mathrm{PMNS}} U_{\mathrm{PMNS}}, \qquad U^R_{\nu\nu} = U^T_\nu \theta^* \theta^T U^*_\nu,$$

$$U^L_{NN} = \Theta^\dagger\Theta, \qquad U^L_{\nu N} = U^\dagger_{\mathrm{PMNS}}\Theta, \qquad U^R_{\nu N} = U^T_\nu \theta^* U_N,$$

$$U^L_{l\nu} = (V^l_L)^\dagger U_{\mathrm{PMNS}}, \qquad U^R_{l\nu} = (V^l_R)^\dagger \theta^T U^*_\nu,$$

$$U^L_{lN} = (V^l_L)^\dagger\Theta, \qquad U^R_{lN} = (V^l_R)^\dagger U_N,$$

$$a^L_{Z_1} = g_L S_{11} - g' S_{31}, \qquad a^R_{Z_1} = g_R S_{21} - g' S_{31},$$

$$a^L_{Z_2} = g_L S_{12} - g' S_{32}, \qquad a^R_{Z_2} = g_R S_{22} - g' S_{32},$$

$$a^L_{A} = g_L S_{13} - g' S_{33}, \qquad a^R_{A} = g_R S_{23} - g' S_{33}.$$

Упрощенные представления для токов.

Обычно используются. Предположим, что секторы заряженных лептонов и стерильных нейтрино диагональны

$$V^l_{L,R} = I, \qquad U_N = I, \qquad g_L = g_R \equiv g, \qquad \theta\theta^\dagger \ll I, \text{ тогда } \quad \Theta \simeq \theta U_N^* = \theta, \quad U_{\rm PMNS} \simeq U_\nu, \tag{6}$$

то есть (здесь и ниже $U = U_{\rm PMNS}$)

$$\begin{aligned}
&U^L_{\nu\nu} = U^\dagger U, \quad U^R_{\nu\nu} = (U^T\Theta^*)(\Theta^T U^*),\\
&U^L_{NN} = \Theta^\dagger\Theta, \quad U^L_{\nu N} = U^\dagger\Theta, \quad U^R_{\nu N} = U^T\Theta^*,\\
&U^L_{l\nu} = U, \quad U^R_{l\nu} = \Theta^T U^*,\\
&U^L_{lN} = \Theta, \quad U^R_{lN} = I.
\end{aligned}$$

тогда

$$\begin{aligned}
\mathcal{L}^\nu_{NC} &= \frac{1}{2}\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{\nu}\gamma^\mu X_\mu \left[(U^\dagger U)a^L_X P_L + (U^T\Theta^*)(\Theta^T U^*)a^R_X P_R\right]\nu,\\
\mathcal{L}^N_{NC} &= \frac{1}{2}\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{N}\gamma^\mu X_\mu \left[(\Theta^\dagger\Theta)a^L_X P_L + a^R_X P_R\right] N\\
&+ \frac{1}{2}\left(\sum_{X=Z_{\mathbf{1}},Z_{\mathbf{2}},A} \overline{\nu}\gamma^\mu X_\mu \left[(U^\dagger\Theta)a^L_X P_L - (U^T\Theta^*)a^R_X P_R\right] N + h.c.\right),\\
\mathcal{L}^\nu_{CC} &= \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{1}\mu}\left[U c_\xi P_L + (\Theta^T U^*)s_\xi P_R\right]\nu + \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{2}\mu}\left[U s_\xi P_L - (\Theta^T U^*)c_\xi P_R\right]\nu + h.c.,\\
\mathcal{L}^N_{CC} &= \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{1}\mu}(\Theta c_\xi P_L - s_\xi P_R)N + \frac{g}{\sqrt{2}}\bar{l}\gamma^\mu W^-_{\mathbf{2}\mu}(\Theta s_\xi P_L + c_\xi P_R)N + h.c.,
\end{aligned}$$

где

$$a^L_{Z_{\mathbf{1}}} = gc_W c - g'(-s_W c_M c + s_M s), \qquad a^R_{Z_{\mathbf{1}}} = g(-s_W s_M c - c_M s) - g'(-s_W c_M c + s_M s),$$
$$a^L_{Z_{\mathbf{2}}} = gc_W s - g'(-s_W c_M s - s_M c), \qquad a^R_{Z_{\mathbf{2}}} = g(-s_W s_M s + c_M c) - g'(-s_W c_M s - s_M c),$$
$$a^L_A = gs_W - g' c_W c_M, \qquad a^R_A = gc_W s_M - g' c_W c_M,$$

$$c_W = \cos\theta_W, \qquad s_W = \sin\theta_W,$$
$$c_M = \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}, \qquad s_M = \tan\theta_W,$$
$$s = \sin\phi, \qquad c = \cos\phi,$$
$$g = \frac{e}{\sin\theta_W}, \qquad g' = \frac{e}{\sqrt{\cos 2\theta_W}}.$$

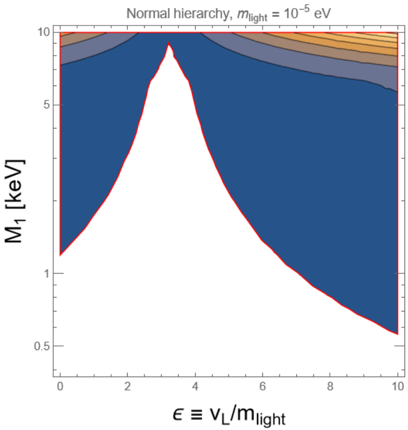
Table: Массы дополнительных бозонов MLRM для используемого параметрического набора и k_2=0

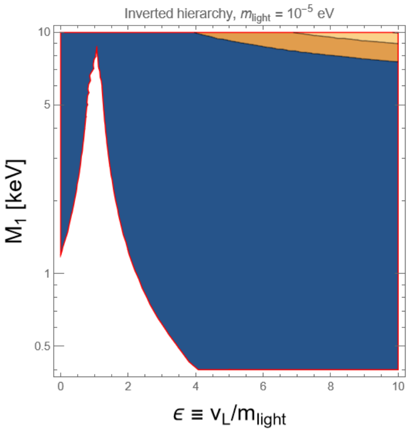
v_R, ТэВ	Массы, ГэВ									
	W_R	Z_R	H_1^0	H_2^0	H_3^0	A_1^0	A_2^0	$H_1^\pm$	$H_2^\pm$	$H_1^{\pm\pm}$
3	1412	2360	129	1342	1775	234	1775	1775	212	1775
12	5638	9437	849	5367	7099	854	7099	7099	849	7099

Ср. с

Bambhaniya G. et al. Left-right symmetry and the charged Higgs bosons at the LHC //JHEP.— 2014.— V. 33— arXiv:1311.4144 [hep-ph].

Эффект сокращения seesaw I и II





Inverse Seesaw and Casas-Ibarra Parametrization

ISS with (p=q)

Inverse Seesaw Formula

$$m_\nu = m_D (M_R^T)^{-1} \mu (M_R)^{-1} m_D^T \qquad \text{(ISS with } p = q\text{)}$$

$$\tilde{\theta} = \tilde{m}_D \mathcal{X}^{-1} = (m_D, \;\; \mathbb{O}) \begin{pmatrix} (\mathcal{X}|\mu)^{-1} & (\mathcal{X}|\mu)^{-1} M_R \mu^{-1} \\ \star & \star \end{pmatrix}$$

Casas-Ibarra Parametrization

Through matrix decomposition:

$$\Omega = \sqrt{\hat{m}}^{\,-1} U_\nu m_D (M_R^T)^{-1} \sqrt{\mu} \qquad\qquad m_D = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\,-1} M_R^T$$

where Ω is orthogonal ($\Omega\Omega^T = I$).

$\nu - N$ Mixing

$$\theta_1 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu} M_R^{-1} \sim \mathcal{O}\left(\frac{\sqrt{m_\nu \mu}}{M}\right)$$

$\nu - S$ Mixing

$$\theta_2 = U_\nu^\dagger \sqrt{\hat{m}}\, \Omega \sqrt{\mu}^{\,-1} \sim \mathcal{O}\left(\sqrt{\frac{m_\nu}{\mu}}\right)$$

Реализации MLRSM

Реализация FeynRules
Roitgrund A., Eilam G., Bar-Shalom S. Implementation of the left-right symmetric model in FeynRules, CPC 2016
Реализация LanHEP
in progress. Наблюдаются множественные несоответствия.

FeynRules: Adam Alloul et al, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput.Phys.Commun. 185 (2014) 2250-2300 (arXiv:1310.1921[hep-ph])
LanHEP: A. Semenov, LanHEP - A package for automatic generation of Feynman rules from the Lagrangian. Version 3.2, Comput.Phys.Commun. 201 (2016) 167-170
Реализация SARAH отсутствует.
CalcHEP: A. Belyaev, N. Christensen, A. Pukhov, CalcHEP 3.4 for collider physics within and beyond the Standard Model, Comput.Phys.Commun. 184 (2013) 1729 - (arXiv: 11207.6082 [hep-ph])
CompHEP: E. Boos, V. Bunichev, M. Dubinin, L. Dudko, V. Edneral, V. Ilyin, A. Kryukov, V. Savrin, A. Semenov, and A. Sherstnev [CompHEP Collaboration], CompHEP 4.4: Automatic computations from Lagrangians to events, Nucl. Instrum. Meth. A534 (2004), 250