Continuous spin fields in AdS spaces

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CS FIELDS IN MINKOWSKI SPACE

PLACE OF THE CS FIELDS IN UIRS OF THE POINCARÉ GROUP

- UIRs of $ISO(1,3)$ were described in the works of Wigner and Bargmann: **1939 - 1948**
- Algebra $\mathfrak{iso}(1,3)$ has two Casimir operators C_2 and C_4, which are defined via $\mathfrak{iso}(1,3)$-generators P_m and J_{mn} as follows

$$C_2 := P^m P_m \,, \quad C_4 := W^m W_m \,, \quad \text{where} \quad W_m = \frac{1}{2}\epsilon_{mnlr} P^n J^{lr}$$

1. Massive UIRs: $C_2 \sim \mathbf{m}^2$ and $C_4 \sim \mathbf{m}^2 j(j+1)$, where $\mathbf{m} \in \mathbb{R}_{>0}$ and $j \in \mathbb{Z}_{\geq 0} + \frac{1}{2}$
2. Massless UIRs: $C_2 \sim 0$
   2.1 **Helicity(gauge fields)**: $C_4 \sim 0$ and there is helicity operator $\hat{h} \sim \mathrm{n}$, where $\mathrm{n} \in \mathbb{Z} + \frac{1}{2}$
   2.2 **Infinite (continuous) spin**[1] : $C_4 \sim \boldsymbol{\mu}^2$, where $\boldsymbol{\mu} \in \mathbb{R}_{>0}$

- $ISO(1, D-1)$ case. Induced representations[2]. Massive: $SO(D-1)$. Helicity(gauge fields): $SO(D-2)$. Infinite (continuous): $ISO(D-2)$
- The general structure of the classification remains the same, but
  symmetric tensors $\rightarrow$ mixed-symmetry tensors
  R.Metsaev, K.Alkalaev, M.Grigoriev, M.Khabarov, Y.Zinoviev, X.Bekaert, J.Mourad

[1] X.Bekaert, E.Skvortsov, *Elementary particles with continuous spin (2017), arXiv:1708.01030*

[2] X.Bekaert, N.Boulanger, The unitary representations of the Poincaré group in any spacetime dimension (2006), arXiv:hep-th/0611263

CS fields in Minkowski space

CS equation for $\mathbb{R}^{1,3}$ in spinor formalism

- Basis
  - UIRs of $ISO(2)$ are infinite-dimensional, because this group is non-compact
  - Basis in the space of these UIRs can be take as: $|\mathrm{n}\rangle$ or $|\phi\rangle$, where $\mathrm{n} \in \mathbb{Z}$, $\phi \in [0, 2\pi)$
- Action of the $\mathfrak{iso}(2)$ algebra with generators $\{\boldsymbol{T}_{\pm}, \boldsymbol{R}\}$ on basis $|\mathrm{n}\rangle$

$$\boldsymbol{T}_{\pm}|\mathrm{n}\rangle = \boldsymbol{\mu}|\mathrm{n} \pm 1\rangle\,, \qquad \boldsymbol{R}|\mathrm{n}\rangle = \mathrm{n}|\mathrm{n}\rangle\,, \qquad \boxed{\boldsymbol{T}_{+}\boldsymbol{T}_{-}|\mathrm{n}\rangle = \boldsymbol{\mu}^2|\mathrm{n}\rangle}$$

- Wigner's wave function in momentum space $\Phi_{\mathrm{n}}(p_m)$ or $\Phi(p_m, \phi)$, where $p_m \in \mathbb{R}^{1,3}$, which is non-locally transformed under Poincaré group and has a non-relativistic form
- Converting this into a relativistic field $\Psi(x_m, y_n)$ or $\Psi(x_m, \xi^{\alpha}, \bar{\xi}^{\dot{\alpha}})$ with a local transformation law was found by **P.Schuster and N.Toro, arXiv:1302.1198** and by our group in the spinor formalism **arXiv:2303.11852**. Then we can find relativistic equations for these fields
- We can get these equations another way (which can be also applied to the **AdS** case):
  - Fix the space of representation and the realization of $\mathfrak{iso}(1,3)$
  - Take natural operator constrains (EoM), that resolve Casimir operators: $C_2 \sim 0$, $C_4 \sim \boldsymbol{\mu}^2$
  - Solve these operator constraints
  - Check that the solution includes the spectrum of the continuous spin field (decomposition in infinite sum over (all) helicity fields)

CS fields in Minkowski space

CS equation for $\mathbb{R}^{1,3}$ in spinor formalism

- Realization of $\mathfrak{iso}(1,3)$ on functions $\Psi(x_m, \xi^\alpha, \bar{\xi}^{\dot{\alpha}})$ is

$$P_n = \partial_n\,, \quad J_{mn} = x_m\partial_n - x_n\partial_m + \mathcal{M}_{mn}\,, \qquad \mathcal{M}_{mn} = \xi^\alpha(\sigma_{mn})_\alpha{}^\beta\partial_\beta + \bar{\xi}_{\dot{\alpha}}(\tilde{\sigma}_{mn})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{\partial}^{\dot{\beta}}$$

- Casimir operators on this realization are

$$C_2 = \partial_n\partial^n\,, \qquad C_4 = \mathcal{M}_{l(m}\mathcal{M}^l{}_{n)}\partial^m\partial^n - \frac{1}{2}\mathcal{M}_{ml}\mathcal{M}^{ml}\,\partial_n\partial^n$$

- Notation is the following

$$a^m := \xi^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}\,, \quad b^m := \partial^\alpha(\sigma^m)_{\alpha\dot{\alpha}}\bar{\partial}^{\dot{\alpha}}\,, \qquad N = \xi^\alpha\partial_\alpha\,, \quad \bar{N} = \bar{\xi}^{\dot{\alpha}}\bar{\partial}_{\dot{\alpha}}$$

- New form of C_4 is

$$C_4 = F(N,\bar{N})\partial_n\partial^n - (a^n\partial_n)(b^m\partial_m)$$

- Operator constraints are

$$\underbrace{\partial_n\partial^n}_{l_0}\Psi = 0\,, \qquad \underbrace{(i\,a^m\partial_m - \boldsymbol{\mu})}_{l_+}\Psi = 0\,, \qquad \underbrace{(i\,b^m\partial_m - \boldsymbol{\mu})}_{l}\Psi = 0 \quad \Longrightarrow \quad \boxed{C_2 \sim 0\,, \quad C_4 \sim \boldsymbol{\mu}^2}$$

- Integer spin constraint. Constraints form the closed algebra

$$U\Psi = 0\,, \quad U = (N - \bar{N})\,, \quad [l_+, l] \sim K\,l_0\,, \quad K = N + \bar{N} + 2$$

- Solutions correspond to the CS field. The BRST Lagrangian.
- **R.Metsaev, M.Khabarov, Yu. Zinoviev, A.Bengtsson, P.Schuster, N.Toro, M.Najafizadeh ...**

CS fields in Minkowski space

CS equation for $\mathbb{R}^{1,5}$ in spinor formalism

- $\mathfrak{iso}(1,5)$ has three Casimir operators: $C_2 = \ldots, C_4 = \ldots, C_6 = (1/64)\Upsilon_n\Upsilon^n, \quad \Upsilon_n = \varepsilon_{nmklrd}J^{mk}J^{lr}P^d$
- Induced form the $ISO(4)$ group. Moreover, there is a (half-)integer parameter $\boxed{s}$, which is related with $SO(3)$ in addition to the continuous parameter $\boxed{\boldsymbol{\mu}}$, **arXiv:2011.14725**
- Spinor formulation in the space $\Psi(x_m, \rho^A_\alpha)$, where $x_m \in \mathbb{R}^{1,5}$, A is $SU(2)$ index and α is $SU^*(4) = Spin(1,5)$ index, **I.B., S.F., A.I., arXiv:2108.04716** and **T.Kugo, P.Townsend, Supersymmetry and the Division Algebras (1983)**
- Realization of $\mathfrak{iso}(1,5)$ is the same, except the spin part is given by $\mathcal{M}_{mn} = -\rho^A_\alpha(\tilde{\sigma}_{mn})^\alpha{}_\beta\partial^\beta_A$
- Notation is the following

$$A_m = \frac{1}{2}\varepsilon_{AB}\,\rho^B_\alpha(\tilde{\sigma}_m)^{\alpha\beta}\rho^A_\beta\,, \quad B_m = \frac{1}{2}\varepsilon^{AB}\,\partial^\alpha_B(\sigma_m)_{\alpha\beta}\partial^\beta_A\,, \qquad N := \rho^A_\alpha\partial^\alpha_A\,, \qquad \boldsymbol{N} := \rho^A_\alpha\partial^\alpha_B\,\rho^B_\beta\partial^\beta_A$$

- Constraints are

$$\underbrace{\partial_n\partial^n}_{l_0}\Psi = 0\,, \qquad \underbrace{(A^m\partial_m - \boldsymbol{\mu})}_{\tilde{l}}\Psi = 0\,, \qquad \underbrace{(B^m\partial_m - \boldsymbol{\mu})}_{l}\Psi = 0 \quad \Longrightarrow \quad \boxed{C_2 \sim 0\,, \;\; C_4 \sim \boldsymbol{\mu}^2, C_6 \sim -\boldsymbol{\mu}^2\mathrm{J}}$$

- One more constraint is required:

$$\underbrace{(\mathrm{J} - \mathrm{const})}_{\tilde{U}}\Psi = 0\,, \quad \text{where} \;\; \mathrm{J} = \frac{1}{2}(\boldsymbol{N} - \frac{1}{2}N^2) \qquad \text{and} \qquad \mathrm{const} = s(s+1)$$

  since J is the Casimir operator of $\mathfrak{so}(3)$ as shown in **arXiv:2108.04716, arXiv:2207.02640**
- The Lagrangian description of the CS field in $\mathbb{R}^{1,5}$ was given in **arXiv:2308.05622**

CS FIELD IN **AdS** SPACE

GENERAL INFORMATION AND OUR GOALS

- **$\mathbf{AdS_D}$** symmetry group is $SO(2, D-1)$
- There are no continuous spin representations in the known classification of highest weight representations $|E_0, \mathbb{Y}_0\rangle$ induced from the compact subgroup – $SO(2) \oplus SO(D-1)$, **P. Dirac, C.Fronsdal, M.Flato, N.Evans, A.Barut, N.Limić, J. Niederle, R.Raczka, V.Dobrev, E.Sezgin, M.Vasiliev, R.Metsaev, Yu.Zinoviev, . . .**
- But there are consistent models with certain parameters, which are reduced to either known Minkowski continuous spin theories or finite spin **AdS** theories, **R.Metsaev and Yu.Zinoviev, M.Khabarov, I.B., S.F., A.I., V.K.**
- Also there is some hypothesis, that these representations are induced from the non-compact subgroup – $SO(1,1) \oplus SO(1, D-2)$, **X.Bekaert, E.Skvortsov**

Our results

- Operator constraints $\boxed{l_0, l_+, l, U}$ and $\boxed{l_0, \tilde{l}, l, \tilde{U}}$ can be consistently generalized to the $\mathbf{AdS_4}$ and $\mathbf{AdS_6}$ cases, respectively
- Moreover, these constraints fix the eigenvalues of the Casimir operators of the **AdS** symmetry algebra in both cases

CS field in AdS space

CS fields in $\mathbf{AdS_4}$ space

- We will consider functions $\Psi(x_\mu, \xi^\alpha, \bar{\xi}^{\dot{\alpha}})$, where x_μ is a coordinate on the $\mathbf{AdS_4}$ space with the metric $g_{\mu\nu}$ and a standard definition of the frame fields: $e^m_\mu e^n_\nu \eta_{mn} = g_{\mu\nu}\,,\; e^\mu_m e^\nu_n g_{\mu\nu} = \eta_{mn}$, where η_{mn} is the flat metric $||\eta_{nm}|| = \mathrm{diag}(-,+,+,+)$
- Covariant derivative (via the spin-connection) and geometric constraints are

$$\mathcal{D}_n = e^\mu_n \left(\partial_\mu + \frac{1}{2} w_{\mu lk}\mathcal{M}^{lk}\right)\,, \quad \mathcal{T}^l_{mn} = 0\,, \quad \mathcal{R}_{mn}{}^{kl} = -\frac{1}{R^2}\left(\delta^k_m\delta^l_n - \delta^k_n\delta^l_m\right)$$

- New constraints, that are found by rules: a) change the derivative to the covariant derivative b) add a terms to make the algebra closed (check: the flat limit, the last constant U remains to be unchanged)

$$L_0 = (\mathcal{D})^2 - \frac{1}{2R^2}\mathcal{M}_{mn}\mathcal{M}^{mn} + \frac{2(1+\boldsymbol{\mu} R)}{R^2}\,, \quad L_+ = i(a^m\mathcal{D}_m) - \boldsymbol{\mu} - \frac{K^2}{4R}\,, \quad L = i(b^m\mathcal{D}_m) - \boldsymbol{\mu} - \frac{K^2}{4R}$$

- Here $K := N + \bar{N} + 2$ and spin generalization of the Laplace-Beltrami operator is

$$(\mathcal{D})^2 := \eta^{mn}(\mathcal{D}_m\mathcal{D}_n + w_{mnl}\mathcal{D}^l) = \frac{1}{\sqrt{-g}}\,\mathcal{D}_\mu\sqrt{-g}\,g^{\mu\nu}\mathcal{D}_\nu\Big|_{\mathcal{M}_{mn}=0} = \Box_{\mathbf{AdS_4}}$$

- Algebra of constraints (**I.B., S.F., A.I., V.A. Krykhtin, arXiv:2402.13879, arXiv:2403.14446**) is

$$[L_0, L_+] = [L_0, L] = 0\,, \quad [L_+, L] = K\,L_0 + R^{-1}(K-1)L_+ + R^{-1}(K+1)L$$

- These constraints were used to construct the real BRST Lagrangian (**I.B., S.F., A.I., V.K.**)

CS FIELD IN **AdS** SPACE

REALIZATION OF THE $SO(2,3)$ SYMMETRY GROUP

- **AdS$_4$** $\subset \mathbb{R}^{2,3} : X^A X^B \eta_{AB} = -R^2,\ A, B = 0, \dots 4$ and $||\eta_{AB}|| = \mathrm{diag}(-,+,+,+,-)$
- Group symmetry is $SO(2,3)$. The Lie algebra generators are $J_{AB} = -J_{BA}$. The split generators are $\{P_m = R^{-1} J_{m4}\,, J_{nl}\}$, where $m, n, l = 0, \dots 3$. The commutation relations are

$$[P_m, P_n] = R^{-2} J_{mn}\,, \quad [J_{mn}, P_l] = \eta_{nl} P_m - \eta_{ml} P_n\,, \quad [J_{mn}, J_{kl}] = \eta_{nk} J_{ml} + 3 \text{ term}$$

- The flat limit is $R \to \infty$: $\mathfrak{so}(2,3) \to \mathfrak{iso}(1,3)$
- Realization of $\mathfrak{so}(2,3)$ on functions $\Psi(x_\mu, \xi^\alpha, \bar{\xi}^{\dot{\alpha}})$
  - From the ambient formalism. The generators are $J_{AB} = X_A \partial_B - X_B \partial_A$, the stereographic coordinates are x^μ, $g_{\mu\nu} = G(x)^{-2} \eta_{\mu\nu}$, $e^n_\mu = G(x)^{-1} \delta^n_\mu\,,\ G(x) = (1 - \eta_{\mu\nu} x^\mu x^\nu / (4R^2))$, $X^m = e^m_\mu x^\mu\,,\quad X^4 = R(2/G(x) - 1)$. Then J_{AB} splits as

$$J_{AB} \quad \to \quad \{P_n = e^\mu_n (\partial_\mu - \frac{1}{2} w_{\mu lk} \mathcal{L}^{lk})\,,\ J_{mn} = \mathcal{L}_{mn} := x_m \partial_n - x_n \partial_m\} \quad \boxed{\mathcal{L} \to \mathcal{L} + \mathcal{M}}$$

$$\boxed{\{P_n = e_n - \frac{1}{2} w_{nlk} J^{lk}\,,\ \ J_{mn} = \mathcal{L}_{mn} + \mathcal{M}_{mn}\}} \quad e_n := e^\mu_n \partial_\mu\,,\ \ w_{nlk} := e^\mu_n w_{\mu lk}$$

  - From the conformal group

$$\mathfrak{so}(2,3) \subset \mathfrak{so}(2,4) \simeq \mathfrak{conf}(\mathbb{R}^{1,3}) \quad \Longrightarrow \quad P_n \sim \mathcal{P}_n + \mathcal{K}_n \ \text{ and } \ \Delta = 0$$

  - Lie-Lorentz derivative **(Y. Kosmann (1971), W.G. Unruh (1974))**

$$P_n = \mathbb{L}_{\phi_{(n)}} := \phi^\mu_{(n)} \mathcal{D}_\mu + \frac{1}{2} g_{\lambda\nu} \left(\mathcal{D}_\mu \phi^\nu_{(n)}\right) e^\mu_a e^\lambda_b \mathcal{M}^{ab}$$

CS FIELD IN **AdS** SPACE

CASIMIR OPERATORS IN TERMS OF SPIN GENERATORS AND COVARIANT DERIVATIVE

- Constraints L_0, L_+, L, U commute with $\mathfrak{so}(2,3)$ generators. It is based on the relations

$$[P_m, \mathcal{D}_n] = w_{mnl}\mathcal{D}^l\,, \qquad [J_{mn}, \mathcal{D}_l] = \eta_{nl}\mathcal{D}_m - \eta_{ml}\mathcal{D}_n$$

- The $\mathfrak{so}(2,3)$ algebra has two Casimir operators:

$$\mathcal{C}_2 := \frac{1}{2}J_{AB}J^{AB} = -R^2 P_n P^n + \frac{1}{2}J_{mn}J^{mn}\,, \quad \mathcal{C}_4 := (1/64)\mathcal{A}^{B_1B_2B_3B_4}_{A_1A_2A_3A_4}J_{B_1B_2}J_{B_3B_4}J^{A_1A_2}J^{A_3A_4} = \mathcal{C}_4(P_n, J_{mn})$$

- Correct flat limit is $\mathcal{C}_{2,4}/R^2 \overset{R\to\infty}{=} C_{2,4}$ ($\Longleftarrow \quad \mathcal{C}_{2n} = R^2\,\widehat{C}_{2n} - \mathfrak{C}_{2n}$)
- Notation of $\mathfrak{so}(2,3)$ invariant operators

$$\mathcal{M}_{(2\mathrm{n})} := \underbrace{\mathcal{M}_{mn}\mathcal{M}^{nl}\cdots\mathcal{M}^{km}}_{\#2\mathrm{n}}\,, \qquad \boxed{(\mathcal{D}_{(\mathrm{n})})^2 := \Big(\prod_{i=1}^{\mathrm{n}}\mathcal{M}\Big)^{km}\Big(\prod_{j=1}^{\mathrm{n}}\mathcal{M}\Big)_k{}^{n}(\mathcal{D}_m\mathcal{D}_n + w_{mnl}\mathcal{D}^l)}$$

- New form of the Casimir operators is

$$\mathcal{C}_2 = -R^2(\mathcal{D})^2 - \frac{1}{2}\mathcal{M}_{(2)}\,, \quad \mathcal{C}_4 = \mathcal{C}_4\Big((\mathcal{D})^2, (\mathcal{D}_{(1)})^2, \mathcal{M}_{(2)}, \mathcal{M}_{(4)}\Big)$$

- Eigenvalues on the constraints L_0, L_+, L and U (L_0 is proportional to $\mathcal{C}_2$ up to a constant) are

$$\mathcal{C}_2 \sim 2(1 + \boldsymbol{\mu}R)\,, \qquad \mathcal{C}_4 \sim \boldsymbol{\mu}R + \boldsymbol{\mu}^2R^2$$

- The most degenerate continuous unitary representation of the **AdS$_4$** symmetry group, **N.Limić, J. Niederle, R. Raczka (1966)**

CS FIELD IN AdS SPACE

CS FIELD IN $\mathbf{AdS_6}$

- Symmetry algebra in this case is $\mathfrak{so}(2,5)$
- We will work in the space of functions $\Psi(x^\mu, \rho^A_\alpha)$, where $x^\mu \in \mathbf{AdS_6}$ and the operator $\mathcal{M}_{mn}$, which is included in $\mathcal{D}_\mu$ and is realized as in the flat space: $\mathcal{M}_{mn} = -\rho^A_\alpha (\tilde{\sigma}_{mn})^\alpha{}_\beta \partial^\beta_A$
- The generalization of the $\mathbb{R}^{1,5}$ continuous spin constraints $\boxed{L_0, \tilde{L}, L, \tilde{U}}$ to the $\mathbf{AdS_6}$ ones takes the following form $\left[\tilde{U} \text{ is again the same: } (1/2(\boldsymbol{N} - 1/2N^2) - s(s+1))\right]$

$$L_0 = R^{-2}\left(\mathcal{C}_2 + 2(4 + \boldsymbol{\mu} R)\right), \quad \tilde{L} = (A^m \mathcal{D}_m) - \boldsymbol{\mu} - \frac{s(s+1)}{2R} - \frac{\boldsymbol{K}^2}{4R}, \quad L = (B^m \mathcal{D}_m) - \boldsymbol{\mu} - \frac{s(s+1)}{2R} - \frac{\boldsymbol{K}^2}{4R}$$

- Here we postulate $L_0 = R^{-2}\mathcal{C}_2 + \mathbf{c}$ and $\tilde{L} = (A \cdot \mathcal{D}) - \boldsymbol{\mu} + \Delta(\boldsymbol{N}, N)$, $L = (B \cdot \mathcal{D}) - \boldsymbol{\mu} + \Delta(\boldsymbol{N}, N)$, then from the requirement of the closure of the algebra $[\tilde{L}, L] = \ldots$ we find $\mathbf{c}$ and $\Delta(\boldsymbol{N}, N)$. In this case $\boldsymbol{K} := N + 4$
- Algebra of constraints is similar to the $\mathbf{AdS_4}$ case ($L_+ \to \tilde{L}$, $U \to \tilde{U}$ and $K \to \boldsymbol{K}$)

CS FIELD IN **AdS** SPACE

CS FIELD IN **AdS$_6$**

- The second and the third Casimir operators of the $\mathfrak{so}(p+1,q)$ algebra (where $p+q=D$) in terms of covariant derivative (this was found with the help of **Cadabra**, $\mathcal{M} \sim y^{(i)}\partial^{(i)}$) are

$$\mathcal{C}_4 = (-1)^{p+1} R^2 \left((\mathcal{D}_{(1)})^2 + \frac{1}{2}\mathcal{M}_{(2)}(\mathcal{D})^2 \right) + \mathcal{O}_4(\mathcal{M})$$

$$\mathcal{C}_6 = (-1)^{p} R^2 \left[(\mathcal{D}_{(2)})^2 + \left(1 + \frac{1}{2}\mathcal{M}_{(2)}\right)(\mathcal{D}_{(1)})^2 + \left(\beta_1 \mathcal{M}_{(2)} + \frac{1}{8}\mathcal{M}^2_{(2)} - \frac{1}{4}\mathcal{M}_{(4)}\right)(\mathcal{D})^2 \right] - \mathcal{O}_6(\mathcal{M})$$

where $\beta_1 = 1/8\left(D^2 - 5D + 10\right)$ and we define

$$\mathcal{O}_4(\mathcal{M}) = (-1)^{p+1}\left(\frac{(D-2)(D+1)}{8}\mathcal{M}_{(2)} + \frac{1}{8}\mathcal{M}^2_{(2)} - \frac{1}{4}\mathcal{M}_{(4)} \right)$$

$$O_6(\mathcal{M}) = (-1)^{p}\left(\gamma_2 \mathcal{M}_{(2)} + \gamma_{2,2}\mathcal{M}^2_{(2)} + \gamma_4 \mathcal{M}_{(4)} - \frac{1}{48}\mathcal{M}^3_{(2)} + \frac{1}{8}\mathcal{M}_{(2)}\mathcal{M}_{(4)} - \frac{1}{6}\mathcal{M}_{(6)} \right)$$

here the constants γ_i are fixed as

$$\gamma_2 = -\frac{1}{12}\left(D^4 - 6D^3 + 13D^2 - 8D - 4\right), \; \gamma_{2,2} = \frac{1}{48}\left(-3D^2 + D - 2\right), \; \gamma_4 = \frac{1}{6}\left(2D^2 - 5D + 5\right)$$

CS field in **AdS** space

CS field in **AdS$_6$**

- Eigenvalues are

$$\mathcal{C}_2 \simeq 2(4+\boldsymbol{\mu}R)\,, \quad \mathcal{C}_4 \simeq (\boldsymbol{\mu}R - 3S + 4)\,(\boldsymbol{\mu}R + S + 1)\ ,$$

$$\mathcal{C}_6 \simeq -2S\left(\boldsymbol{\mu}^2 R^2 + (2S+1)\boldsymbol{\mu}R + S(S+1)\right)$$

  where $S = s(s+1)/2$ and in the flat limit we have

$$C_2 \simeq 0\,, \quad C_4 \simeq \boldsymbol{\mu}^2\,, \quad C_6 \simeq -\boldsymbol{\mu}^2 s(s+1)\,.$$

- To resolve $\mathcal{C}_4$ we need to use the constraint $\tilde{U}$, which plays a role only for deriving C_6 in the flat space
- It is interesting, that if we can try to find the constraints directly from the Casimir operators (as the conditions for its resolution) we will see that there would be one free parameter, which can only be fixed by the condition of the closure of the algebra of these constraints
- This construction seems to be inequivalent to the description of the CS field using two additional vector variables when $s \neq 0$ (in particular, when $s = 0$, the description appears to be equivalent to the one involving a single additional vector). This can be analyzed using the $\mathfrak{so}(1,5)$ Casimir operators $\mathfrak{C}_4, \mathfrak{C}_6$

$$\mathfrak{C}_4\Big|_{\boldsymbol{N}=\frac{1}{2}N^2} = \mathfrak{C}_6\Big|_{\boldsymbol{N}=\frac{1}{2}N^2} = 0$$

Conclusions

- Done
  - The consistent deformation of the continuous spin constraints from $\mathbf{Mink_{4,6}}$ to $\mathbf{AdS_{4,6}}$
  - The identification of the CS representation in $\mathbf{AdS_{4,6}}$
- Future
  - Check unitarity and find out the group that induces this representation
  - Find an easier way and generalize to higher dimension (deformation of the Bekaert-Mourad equation for CS fields of mixed symmetry type)
  - Construction Lagrangian theory in arbitrary dimension

Thank you for your attention!