Jet Quenching in Anisotropic Holographic QCD: Analysis of Phase Transitions and Critical Regions

Pavel Slepov
Based on arXiv:2507.19426 with I.Ya.Aref'eva, A.Hajilou and A.Nikolaev

Steklov Mathematical Institute of Russian Academy of Sciences

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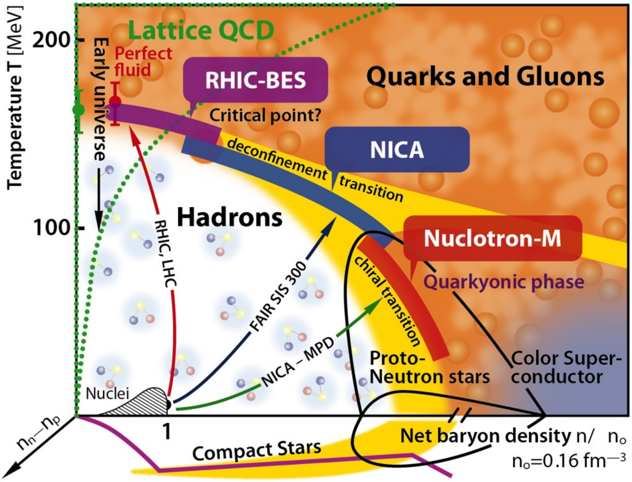
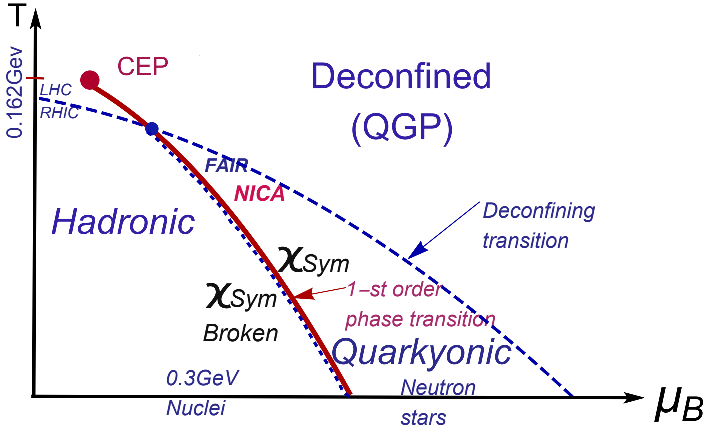
11.08.2025

Studies of QCD Phase Diagram is the main goal of new facilities



From: https://nica.jinr.ru/physics.php

Holographic QCD phase diagram for light quarks



The main question to discuss is: what directly measurable quantities indicate the presence of 1-st order phase transitions?

- Jet Quenching – this talk
- Direct photons – I.Ya. Aref'eva, A. Ermakov and P. S., "Direct photons emission rate ... with first-order phase transition," EPJC **82** (2022) 85
- Energy loss – I.Ya. Aref'eva, K. Rannu and P. S., "Energy Loss in Holographic Anisotropic Model ...," arXiv:2012.05758; TMPh **206** (2021) 400
- Cross-sections – I.Ya.Aref'eva, A. Hajilou, P. S. and M. Usova, "Running coupling for HQCD...: Isotropic case," PRD **110** (2024) 126009
  I.Ya. Aref'eva, A. Hajilou, A. Nikolaev and P. S., "HQCD running coupling ... in strong magnetic field," PRD **110** (2024) 086021

Holographic model of an anisotropic plasma in a magnetic field at a non-zero chemical potential

I.Aref'eva, K.Rannu'18; I Aref'eva, K. Rannu, P.S.'21

$$S = \int d^5x \, \sqrt{-g} \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2} \partial_M \phi \partial^M \phi - V(\phi) \right]$$

$$ds^2 = \frac{L^2}{z^2} \mathfrak{b}(z) \left[-g(z)\, dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right]$$

$$A_{(1)\mu} = A_t(z)\delta_\mu^0 \qquad A_t(0) = \mu \qquad F_{(2)} = dy^1 \wedge dy^2 \qquad F_{(B)} = dx \wedge dy^1$$

Giataganas'13; Aref'eva, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$\mathfrak{b}(z) = e^{2\mathcal{A}(z)} \Leftrightarrow$ quarks mass **"Bottom-up approach"**

Heavy quarks (c, b):

$\mathcal{A}(z) = -\, cz^2/4$ *Andreev, Zakharov'06*

$\mathcal{A}(z) = -\, cz^2/4 + p(c_B)z^4$ *Aref'eva, Hajilou, Rannu, P.S.' 23*

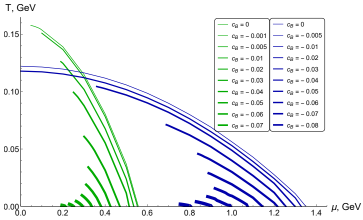
Light quarks (u, d, s)

$\mathcal{A}(z) = -\,a\ln(bz^2+1)$ *Li, Yang, Yuan'17*

$\mathcal{A}(z) = -\,a\ln((bz^2+1)(dz^4+1))$ *Zhu, Chen, Zhou, Zhang, Huang'25*

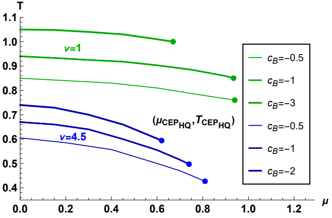
1-st order phase transition for "light" and "heavy" quarks in Holography

Light quarks



Aref'eva, Ermakov, Rannu, P.S.,EPJC'23

Heavy quarks



Aref'eva, Hajilou, Rannu, P.S.,EPJC'23

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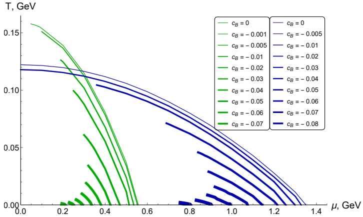
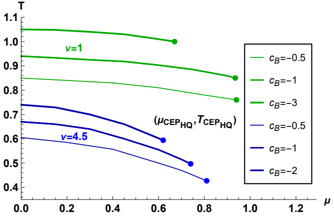
Light quarks



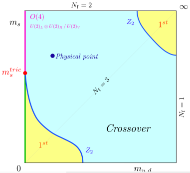
Aref'eva, Ermakov, Rannu, P.S.,EPJC'23

Heavy quarks



Aref'eva, Hajilou, Rannu, P.S.,EPJC'23

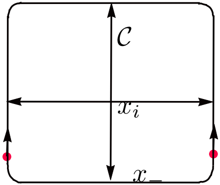
- QCD Phase Diagram from Lattice Columbia plot
  Brown et al.'90 Philipsen, Pinke'16
- **Main problem on Lattice:** $\mu \neq 0$



Jet Quenching

- The jet quenching parameter q quantifies the average transverse momentum squared that a parton transfers to the medium per unit of path length.

- **Light-like loop** $\mathcal{C} = x_- \times x_i, \quad x_- >> x_i > \ell_{QCD}$, $i = 2, 3$



$$< W_{Ad}[C] > \underset{\substack{x_- \to \infty \\ x_i \to 0}}{\sim} e^{-q\, x_- x_i^2}$$

q **- jet quenching parameter**

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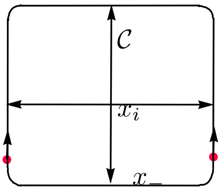

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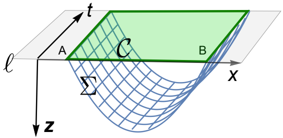
q **- jet quenching parameter**

- Wilson Loops in holographic QCD

J. Maldacena'98



- String action "on a barn": $S_{NG} = \int d\tau d\xi \; M(z(\xi)) \sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}$

H. Liu, K. Rajagopal, U. Wiedemann,'06 Conformal case: $q \sim T^3$

Light-like Wilson loops in a deformed metric

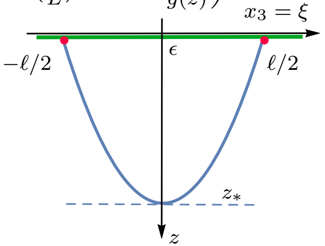
$$ds^2 = \frac{L^2 e^{2\mathcal{A}_s}}{z^2}\left(-g(z)dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-2/\nu}\left(dx_2^2 + e^{c_B z^2}dx_3^2\right) + \frac{dz^2}{g(z)}\right)$$

$$dt = \frac{dx_+ + dx_-}{\sqrt{2}},\; dx_1 = \frac{dx_+ - dx_-}{\sqrt{2}}.$$

The contour $\mathcal{C}$: “short sides” with length ℓ along the x_3(or x_2) direction and the “long sides” with length L_- along the x_- direction

$$x_- = \tau; \qquad x_3(or\, x_2) = \xi$$

$$S_{NG,3} = \frac{L^2 L_-}{\pi\alpha'}\int_0^{\frac{\ell}{2}} d\xi\, \frac{e^{2A_s(z)}}{z^2}\sqrt{\frac{1-g(z)}{2}\left(e^{c_B z^2}\left(\frac{z}{L}\right)^{2-2/\nu} + \frac{z'^2}{g(z)}\right)}$$



The integral of motion

$$P = \frac{e^{2A_s(z)}(g(z)-1)}{\sqrt{2}z^2 g(z)\sqrt{(1-g(z))\left(e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} + \frac{z'^2}{g(z)}\right)}}$$

and we get for z'

$$z' = \frac{e^{2A_s + c_B z^2\left(\frac{z}{L}\right)^{-2/\nu}}}{\sqrt{2}L^2 P}\sqrt{g(1-g) - 2gL^2P^2z^2\left(\frac{z}{L}\right)^{2/\nu} e^{-4A_s - c_B z^2}}$$

Light-like Wilson loops in a deformed metric

"Returning point":

$$g(z_*)\underbrace{\left((1-g(z_*))e^{4A_s+c_Bz_*^2}-2L^2P^2z_*^2\left(\frac{z_*}{L}\right)^{2/\nu}\right)}_{\mathcal{I}}=0 \quad (*)$$

Equation (*) has two possible solutions:

$$\begin{array}{ll} a) & g(z_*)=0, \quad \text{this hold for} \quad z_*=z_h, \\ b) & \mathcal{I}=0, \text{in our case is unstable} \end{array}$$

- **a)** $z_*=z_h$.

$$\frac{\ell}{2} = PL^2\int_0^{z_h}\frac{\sqrt{2}e^{-2\mathcal{A}_s-c_Bz^2}\left(\frac{z}{L}\right)^{2/\nu}}{\sqrt{g(1-g)}}dz+\ldots$$

$$\frac{\mathcal{S}}{2} = S_0+L^2P^2\int_0^{z_h}\frac{e^{-2\mathcal{A}_s(z)-c_Bz^2}\left(\frac{z}{L}\right)^{2/\nu}dz}{\sqrt{2g(1-g)}}+\ldots$$

Jet quenching for non-zero magnetic field and initial anisotropy. Analytical formula & Numerical results

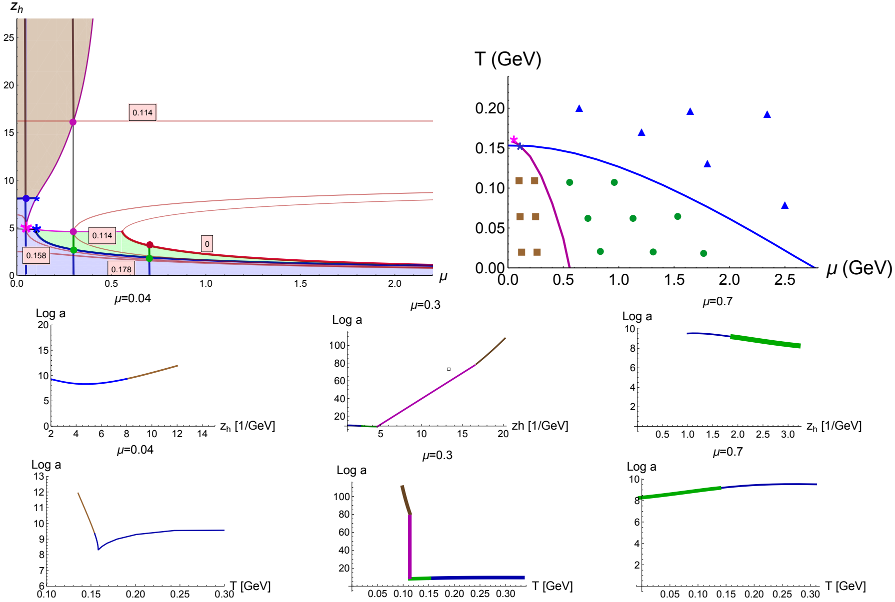
$$q_i(z_h,\mu,c_B,\nu) = \frac{L^2}{\pi\alpha' a_i} \sim \frac{1}{a_i}, \quad i = 2,3$$

$$a_2 = \int_0^{z_h} \frac{e^{-2\mathcal{A}_s(z)} \left(\frac{z}{L}\right)^{2/\nu}}{\sqrt{g(z)(1-g(z))}} dz \qquad a_3 = \int_0^{z_h} \frac{e^{-2\mathcal{A}_s(z)-c_B z^2} \left(\frac{z}{L}\right)^{2/\nu}}{\sqrt{g(z)(1-g(z))}} dz$$

$$g(z,z_h,\mu,c_B,\nu) = e^{c_B z^2}\left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2\,(2c-c_B)\,I_2(z)}{L^2\left(1-e^{(2c-c_B)z_h^2/2}\right)^2}\left(1 - \frac{I_1(z)I_2(z_h)}{I_1(z_h)I_2(z)}\right)\right]$$

$$I_1(z) = \int_0^z \left(1+b\xi^2\right)^{3a} \frac{\xi^{1+\frac{2}{\nu}}}{e^{\frac{3}{2}c_B\xi^2}} d\xi, \qquad I_2(z) = \int_0^z \left(1+b\xi^2\right)^{3a} \frac{\xi^{1+\frac{2}{\nu}}}{e^{(-c+2c_B)\xi^2}} d\xi$$

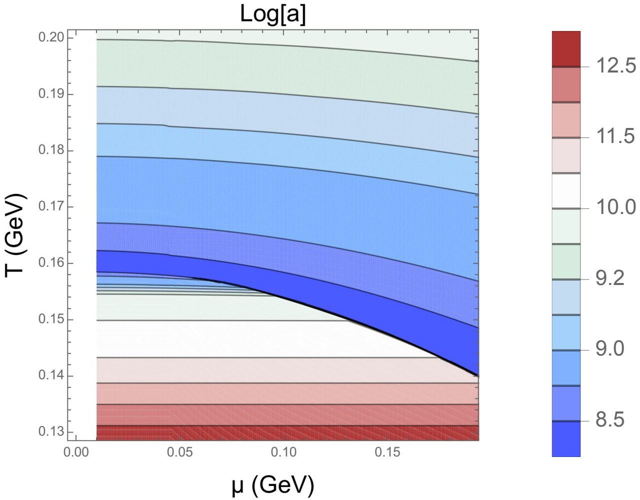
$$T = \frac{|g'|}{4\pi}\Big|_{z=z_h} \qquad s = \left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} \frac{e^{c_B z_h^2/2}\left(1+bz_h^2\right)^{-3a}}{4} \qquad F = \int_{z_h}^{z_{h_2}} s\,dT = \int_{z_h}^{z_{h_2}} s\,T' dz$$



Non-monotonic behavour of the jet quenching parameter

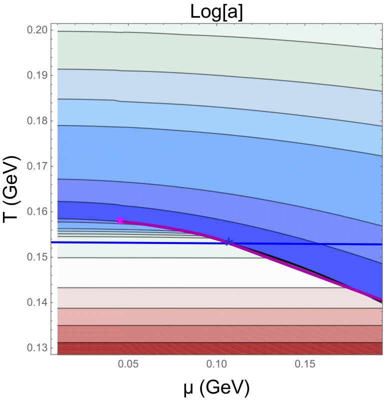
Early observed by M.Huang et al'14; Zhu, Hou'23

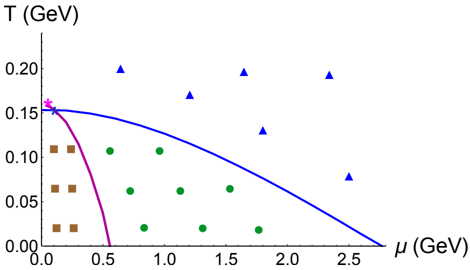
Jet quenching for zero magnetic field and $\nu = 1$ for LQ model. Numerical results



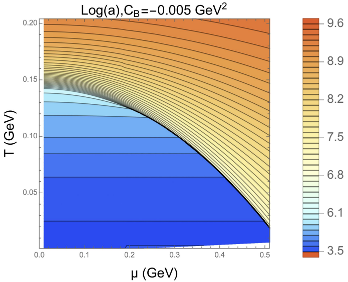
Density plots for $\log a$ for light quarks

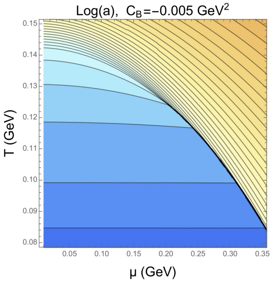
Jet quenching for zero magnetic field and $\nu = 1$ for LQ model. Numerical results



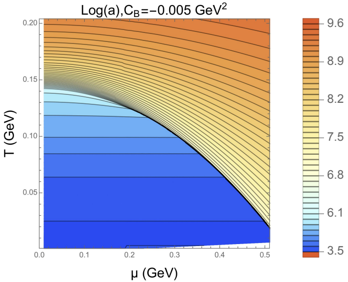


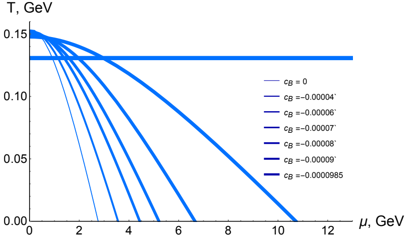
Jet quenching for non-zero magnetic field and $\nu = 1$ for LQ model. Numerical results for a_2



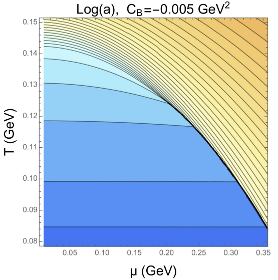


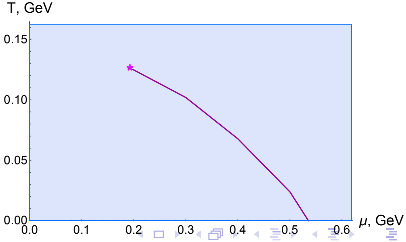
Jet quenching for non-zero magnetic field and $\nu = 1$ for LQ model. Numerical results for a_2



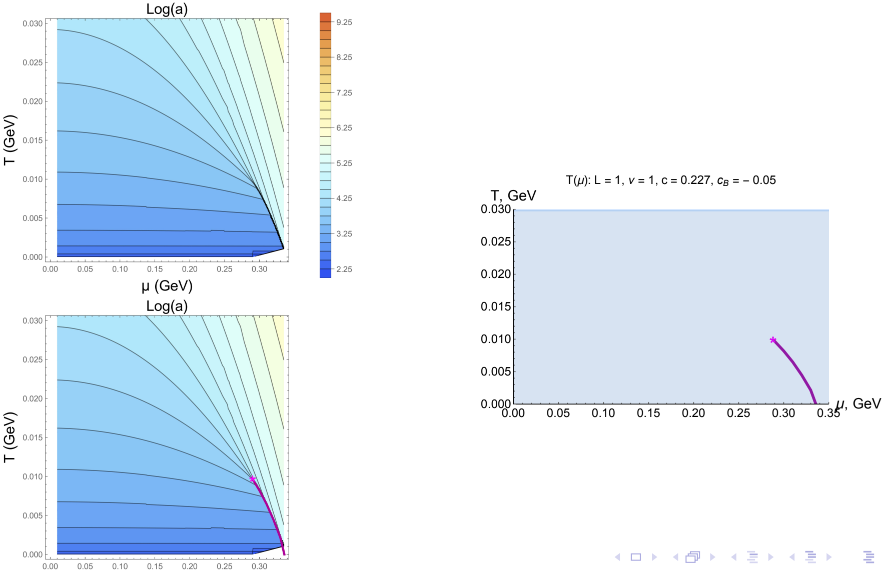


Conf/deconf. phase transition lines

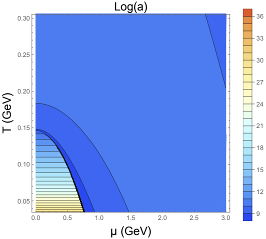




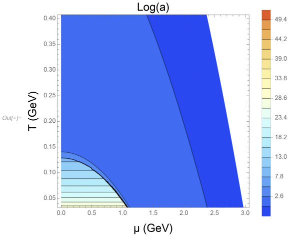
Jet quenching for non-zero magnetic field and $\nu = 1$ for LQ model. Numerical results for a_2



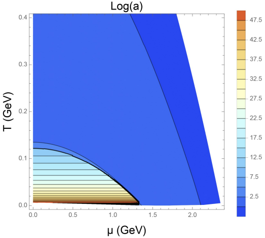
Jet quenching for non-trivial initial anisotropy and $c_B = 0$ for LQ model. Numerical results for a_2



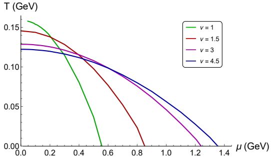
$\nu = 1.5$



$\nu = 3$



$\nu = 4.5$



$\log a_2$ (solid lines) and $\log a_3$ (dashed lines) for LQ model with $\nu = 1$ and $c_B = -0.005$

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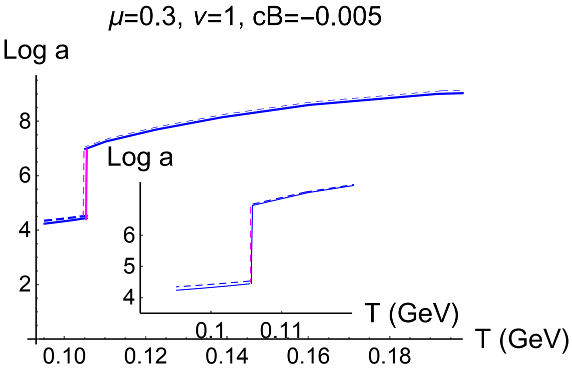
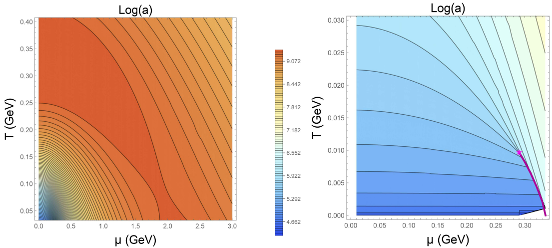
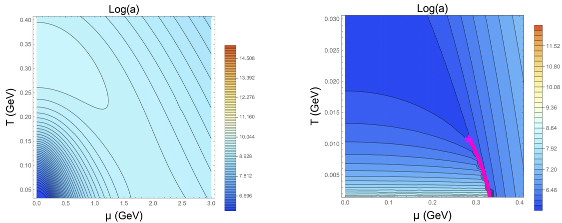


Jet quenching for $c_B = -0.05$ and $\nu = 1$ for LQ model. Numerical results for a_2 and a_3



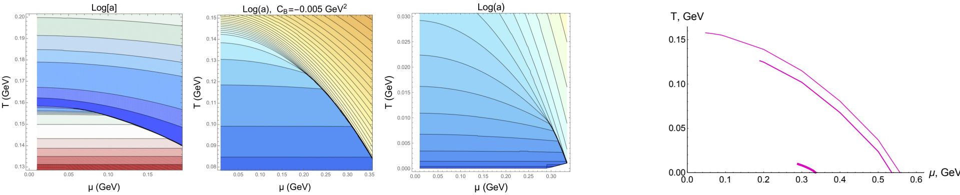
a_2 orientation



a_3 orientation

Conclusion

- Jet quenching parameter can serve as an indicator of the 1-st order phase transitions



Plots for the light quarks model

- Change in the slope of $\log(a)$ versus temperature T (at fixed μ) at the confinement/deconfinement phase transition line
- There is no strong orientation dependence near 1-st order phase transitions
- Similar behavior is observed in the heavy quark model

Open question:

- *Hybrid holographic model for light and heavy quarks*

Thank you for your attention!