On covariant quantization of the totally antisymmetric tensor-spinor field model

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August 12, 2025
AQFT-2025 Dubna

The talk is based on
A.O. Barvinsky, I.L. Buchbinder, V.A.K., D.V. Nesterov
arXiv:2507.09312 [hep-th]

The theory of totally antisymmetric tensor-spinor field model is a gauge theory with reducible generators.

The first such field theory was proposed by Ogievetsky and Polubarinov (notoph theory) [V.I. Ogievetsky, I.V. Polubarinov Yadernaya Fizika (Soviet Journal Nuclear Physics), **4** (1967) 156.]

Later a lot of various reducible gauge theories were constructed and quantized by different methods. The current state of art with the relevant references in this area is presented in detail in [S.M. Kuzenko, E.S.N. Raptakis JHEP, **09** (2024) 182, `arXiv:2406.01176 [hep-th]`.]

There exist general BV (Batalin-Vilkovisky) method of covariant quantization [I.A. Batalin and G.A. Vilkovisky Phys. Rev. D **28**(1983), 2567 [erratum: Phys. Rev. D **30** (1984), 508].]

Our purpose is to guantize a reducible gauge theory using the method analogous to the Faddeev-Popov method for irreducible gauge theories.

Plan of the talk

1. Faddeev-Popov quantization procedure for irreducible gauge theories

2. Adjustment of Faddeev-Popov quantization to reducible gauge theories

3. Example: Antisymmetric tensor fermion in AdS_d space

Faddeev-Popov procedure for irreducible gauge theories

Functional integral for non-gauge theories is

$$Z = \int D\Phi^A \, e^{iS[\Phi]}$$

Φ^A are the physical fields.

For gauge theories with fields $\phi = \phi^i$ the action $S[\phi]$ invariant under gauge transformation

$$\phi^i \to (\phi^i)^f = \phi^i + R^i{}_\alpha f^\alpha \qquad\qquad S[\phi^f] = S[\phi]$$

where $R^i{}_\alpha$ are the generators of gauge transformations and f^α are the gauge parameters.

We assume that the gauge transformations form in general a non-Abelian closed algebra, gauge field ϕ and group parameters f are bosonic, though the extension to the generic boson-fermion system is straightforward.

Faddeev-Popov procedure for irreducible gauge theories

If we consider the functional integral

$$\int D\phi^i \; e^{iS[\phi]}$$

it includes the integration over phisical gauge fields (=over orbits $\{\phi^i\}$) and integation over the gauge group (=along the orbits).

Since $S[\phi^i] = S[(\phi^i)^f] = S[\{\phi^i\}]$ the integral over the gauge group should be factorized

$$\int D\phi^i \; e^{iS[\phi]} = \int Df \int D\{\phi\} \; e^{iS[\{\phi\}]} = V \times \int D\{\phi\} \; e^{iS[\{\phi\}]}$$

The last factor is the functional integal for the gauge theory

$$Z = \int D\{\phi\} \; e^{iS[\{\phi\}]}$$

Faddeev-Popov procedure for irreducible gauge theories

In the case when the gauge generators $R^i{}_\alpha$ are independent (irreducible gauge theory) we can apply the Faddeev-Popov method.
The main idea of the Faddeev-Popov method consists in the insertion under the integration over ϕ the unity factor defined by the following integration over the group parameters

$$\Delta_\chi[\phi] \int Df\,\delta[\,\chi(\phi^f))\,] = 1.$$

Here $\chi(\phi)$ denotes the set of gauge conditions and
$\Delta_\chi[\phi]$ is the Faddeev-Popov functional determinant in this gauge, which on the surface of enforced gauge conditions reads as

$$\Delta_\chi[\phi]\Big|_{\chi(\phi)=0} = \mathrm{Det}\left(\frac{\delta\chi(\phi)}{\delta\phi}R(\phi)\right).$$

for simplicity we omit the indices of both the fields and gauge parameters

Faddeev-Popov procedure for irreducible gauge theories

Then under the assumption of the group associativity, $(\phi^f)^h = \phi^{hf}$, and invariance of the functional measure of integration over the group, $Df = D(hf)$, the Faddeev-Popov determinant turns out to be gauge invariant

$$\Delta_\chi[\phi^f] = \Delta_\chi[\phi]$$

which allows one to disentangle from the functional integral a volume of the group $\int Df = V$.
This goes by changing the order of integrations and using the invariance of the integration measure in the space of ϕ, $D(\phi^f) = D\phi$, along with the change of integration variable $\phi^f \to \phi$,

$$\begin{aligned}\int D\phi\, e^{iS[\phi]} &= \int D\phi\, e^{iS[\phi]}\Delta_\chi[\phi] \int Df\, \delta[\,\chi(\phi^f)\,] = \\ &= \int Df \int D\phi\, e^{iS[\phi]}\Delta_\chi[\phi]\,\delta[\,\chi(\phi)\,] = V \times \int D\phi\, e^{iS[\phi]}\Delta_\chi[\phi]\,\delta[\,\chi(\phi)\,].\end{aligned}$$

The last factor is a well-defined Faddeev-Popov integral which is actually independent of the choice of gauge χ.

Adjustment of FP method to reducible gauge theories

The above method of constructing the correct functional integral works well only if the gauge generators are independent.

When applied to a gauge theory with linearly dependent generators this method would break at several points.

Linear dependence implies that the gauge generators $R^i_\alpha(\phi)$ have their own right zero vectors Z^α_a enumerated by set of indices a,

$$R^i_\alpha Z^\alpha_a = 0, \qquad \alpha = 1, ...m_0, \qquad a = 1, ... m_1 < m_0$$

Z^α_a are linearly independent for stage one reducible gauge theories.

This means that the actual number of local gauge symmetries is $m_0 - m_1$ (the rank of the matrix $R^i_\alpha(\phi)$), rather than m_0.

Adjustment of FP method to reducible gauge theories

Therefore the actual number of independent gauge conditions to fix these symmetries should also be $m_0 - m_1$.

Therefore we use the redundant (or reducible) set of gauge conditions χ^μ satisfying the condition of linear dependence

$$\bar{Z}^b_\mu \chi^\mu = 0, \qquad \mu = 1, ...m_0, \qquad b = 1, \dots m_1 < m_0.$$

with some left zero vectors $\bar{Z}^b_\mu$.

The first problem is that the conventional delta function of these linearly dependent gauge conditions, $\delta^{(m_0)}[\,\chi\,] = \prod_\mu \delta[\,\chi^\mu\,] \propto \delta^{(m_1)}(0)$, becomes ill defined.

Adjustment of FP method to reducible gauge theories

The second problem is that the group integration measure $Df = \prod_\alpha Df^\alpha$ is ill defined because the actual integration should run over $(m_0 - m_1)$-dimensional group space rather than the m_0-dimensional one.

The third problem is that the Faddeev-Popov determinant

$$\Delta_\chi = \mathrm{Det}\, Q^\mu_\alpha, \qquad Q^\mu_\alpha \equiv \frac{\delta \chi^\mu}{\delta \phi^i} R^i_\alpha,$$

becomes vanishing in view of the right zero vectors Z^α_a of the Faddeev-Popov operator Q^μ_α, $Q^\mu_\alpha Z^\alpha_a = 0$.

All these three problems can be resolved by a method, which we develop in what follows. The idea of the method consists in the insertion of the unity factor, analogous to the FP procedure, and factorization of the group volume with a corrected measure $V = \int D\mu_f$.

Adjustment of FP method to reducible gauge theories

The first step is the construction of the correctly defined delta function of reducible gauge conditions. Naive delta function

$$\delta[\chi] = \int D\pi \, e^{i\pi_\mu \chi^\mu}$$

is ill defined because the integral over the Lagrange multiplier π_μ requires the factorization of the volume $V_1 = \int D\xi_1$ of the invariance group of the exponential,

$$\pi_\mu \to \pi_\mu^{\xi_1} = \pi_\mu + \bar{Z}_\mu^b \xi_{1\,b}, \qquad \pi_\mu^{\xi_1} \chi^\mu = \pi_\mu \chi^\mu.$$

This is the m_1-dimensional gauge invariance in the m_0-dimensional space of π_μ, following from the reducibility $\bar{Z}_\mu^b \chi^\mu = 0$ of gauge conditions χ^μ.

Adjustment of FP method to reducible gauge theories

Such a factorization proceeds by gauge fixing π_μ with the gauge $\sigma_b(\pi) = \pi_\mu \sigma^\mu_b$ and inserting the unity factor of the form

$$\bar{\Delta}_1 \int D\xi_1 \, \delta[\, \sigma(\pi^{\xi_1}))\,] = 1, \qquad \bar{\Delta}_1 \,\Big|_{\sigma(\pi)=0} = \mathrm{Det}\,\big(\bar{Z}^a_\mu \sigma^\mu_b\big).$$

This leads to

$$\int D\pi \, e^{i\pi_\mu \chi^\mu} \;=\; \int D\pi \, e^{i\pi_\mu \chi^\mu} \bar{\Delta}_1 \int D\xi_1 \, \delta[\, \sigma(\pi^{\xi_1})\,] =$$
$$= \int D\xi_1 \int D\pi \, e^{i\pi_\mu \chi^\mu} \mathrm{Det}\,\bar{Q}\,\delta[\,\sigma(\pi)\,] = V_1 \times \mathrm{Det}\,\bar{Q} \int Dc\,\delta[\,\chi^\mu + \sigma^\mu_b c^b\,],$$

where we expressed $\delta[\,\sigma(\pi)\,]$ as the integral over an auxiliary field c^a and took the integral over π_μ.

Adjustment of FP method to reducible gauge theories

This allows one to identify the correct expression for the delta function of reducible gauge conditions

$$\hat{\delta}[\,\chi\,] = \mathrm{Det}\,\bar{Q}\int Dc\,\delta[\,\chi^\mu + \sigma^\mu_b c^b\,], \qquad \bar{Q} \equiv \bar{Q}^a_b = \bar{Z}^a_\mu \sigma^\mu_b.$$

Here we get the extra Faddeev-Popov operator $\bar{Q}$ involving the zero vector $\bar{Z}^a_\mu$ of reducible gauge conditions χ^μ.

Note that the argument of the delta function

$$\delta[\,\chi^\mu + \sigma^\mu_b c^b\,] \equiv \prod_\mu \delta[\,\chi^\mu + \sigma^\mu_b c^b\,]$$

is no longer reducible, and this delta function is well defined.

Moreover, the gauge $\sigma_a(\pi)$ for π_μ, that is the matrix σ^μ_b above, can be freely changed without altering the "correct" delta function.

Adjustment of FP method to reducible gauge theories

The second problem is the construction of group integration measure $D\mu_f$ by the factorization of the infinite volume of the f_1^a-integration.

This type of divergent integration arises when one integrates over f^α any functional of the transformed gauge field $\Phi[\phi^f]$, because as a function of f^α it is constant on the orbit of f_1-transformation in the space of f^α,

$$f \to f^{f_1}, \quad f^\alpha \to f^\alpha + Z_a^\alpha f_1^a, \quad \phi^{f^{f_1}} = \phi^f.$$

In other words, gauge parameters themselves become gauge fields subject to f_1-transformations with the new generators Z_a^α.

Thus we need to factor out integration over the f_1-transformations.

Adjustment of FP method to reducible gauge theories

This factorization is achieved by using a new set of gauge conditions $\omega^a(f) = \omega^a_\alpha f^\alpha$ on f^α. The insertion of the new unity

$$\Delta_1 \int Df_1\, \delta[\,\omega(f^{f_1}))\,] = 1, \qquad \Delta_1\,\Big|_{\omega(f)=0} = \mathrm{Det}\,\big(\omega^a_\alpha Z^\alpha_b\big),$$

into the group integral gives by the same pattern

$$\int Df\, \Phi[\,\phi^f] \;=\; \int Df\, \Phi[\,\phi^f]\,\Delta_1 \int Df_1\, \delta[\,\omega(f^{f_1}))\,] =$$

$$= \int Df_1 \int Df\, \Delta_1\, \delta[\,\omega(f)\,]\, \Phi[\,\phi^f] = V_1 \times \int Df\, \Delta_1 \delta[\,\omega(f)\,]\, \Phi[\,\phi^f].$$

Therefore, the group integration measure can be identified with

$$D\mu_f = Df\, \mathrm{Det}\, Q_1\, \delta[\,\omega(f)\,], \qquad Q_1 \equiv Q^{\;a}_{1b} = \omega^a_\alpha Z^\alpha_b .$$

Here the new Faddeev-Popov operator Q_1 is defined in terms of zero vectors Z^α_b of the original gauge generators, and it is also assumed to be invertible by the choice of gauge functions ω^a_α.

Adjustment of FP method to reducible gauge theories

The third problem is to determine the overall Faddeev-Popov operator Δ from the definition of unity with the corrected delta function and the corrected group integration measure.

The equation for Δ then reads

$$\begin{aligned} 1 &= \Delta \int D\mu_f \, \hat{\delta}[\,\chi(\phi^f)\,] = \\ &= \Delta \, \mathrm{Det}\,\bar{Q}\, \mathrm{Det}\, Q_1 \int Df\, Dc\, \delta[\,\omega^a_\alpha f^\alpha\,]\, \delta[\,\chi^\mu(\phi) + Q^\mu_\alpha f^\alpha + \sigma^\mu_b c^b], \end{aligned}$$

where Q^μ_α is a naive (degenerate) Faddeev-Popov operator having right and left zero vectors

$$Q^\mu_\alpha = \frac{\delta\chi^\mu}{\delta\phi^i} R^i_\alpha, \quad Q^\mu_\alpha Z^\alpha_a = 0, \quad \bar{Z}^b_\mu Q^\mu_\alpha = 0.$$

Adjustment of FP method to reducible gauge theories

The integral of the product of two delta functions in (1) equals the inverse of the following Jacobian

$$\frac{D\big(\chi + Qf + \sigma c,\, \omega f\big)}{D\big(f,\, c\big)} = \mathrm{Det}\begin{bmatrix} Q^{\mu}_{\alpha} & \sigma^{\mu}_{b} \\ & \\ \omega^{a}_{\alpha} & 0 \end{bmatrix} = \mathrm{Det}\, F^{\mu}_{\alpha},$$

where F^{μ}_{α} is the non-degenerate operator obtained from Q^{μ}_{α} by adding the gauge-breaking term composed of the gauge matrices σ^{μ}_{b} and ω^{a}_{α}

$$F^{\mu}_{\alpha} = Q^{\mu}_{\alpha} + \sigma^{\mu}_{a}\omega^{a}_{\alpha}.$$

Thus equation for Δ takes the form $1 = \Delta\, \mathrm{Det}\, \bar{Q}\, \mathrm{Det}\, Q_1 (\mathrm{Det} F)^{-1}$, so that the full Faddeev-Popov determinant equals

$$\Delta = \frac{\mathrm{Det} F}{\mathrm{Det}\, \bar{Q}\, \mathrm{Det}\, Q_1}.$$

Adjustment of FP method to reducible gauge theories

Insertion of the unity into the functional integral by the pattern of Faddeev-Popov procedure then reads

$$\begin{aligned}\int D\phi\, e^{iS[\phi]} &= \int D\phi\, e^{iS[\phi]} \Delta \int D\mu_f\, \hat{\delta}[\,\chi(\phi^f)\,] = \\ &= \int D\mu_f \int D\phi\, e^{iS[\phi]}\, \hat{\delta}[\,\chi(\phi)\,]\, \frac{\mathrm{Det} F}{\mathrm{Det}\,\bar{Q}\,\mathrm{Det}\,Q_1} = \\ &= V \times \int D\phi\, Dc\, e^{iS[\phi]} \delta[\,\chi^\mu(\phi) + \sigma^\mu_a c^a]\, \frac{\mathrm{Det} F}{\mathrm{Det}\,Q_1}.\end{aligned}$$

As a result, after a formal factorization the functional integral for stage one reducible gauge theories equals

$$Z = \int D\phi\, Dc\; e^{iS[\phi]}\, \delta[\,\chi^\mu(\phi) + \sigma^\mu_a(\phi) c^a]\, \frac{\mathrm{Det} F[\phi]}{\mathrm{Det}\,Q_1[\phi]}$$

and coincides with that derived from a fundamental Batalin-Vilkovisky formalism, which remains valid for field-dependent $Z^\alpha_a(\phi), \sigma^\mu_a(\phi), \omega^a_\alpha(\phi)$.

Antisymmetric tensor fermion in AdS$_d$ space

The model of the rank antisymmetric fermion of tesor rank p in AdS_d spacetime is described by the field $\psi_{\mu_1\ldots\mu_p}$, $(2p < d)$ with the action

$$S_p \;=\; i\int d^d x\, g^{1/2}\bar{\psi}_{\mu_1\ldots\mu_p}\gamma^{\mu_1\ldots\mu_p\sigma\nu_1\ldots\nu_p}\, D_\sigma\, \psi_{\nu_1\ldots\nu_p}$$

$$D_\mu = \nabla_\mu \pm \frac{i\, r^{\frac{1}{2}}}{2}\,\gamma_\mu \quad R^{\mu\nu}{}_{\alpha\beta} = r(\delta^\nu_\alpha\delta^\mu_\beta - \delta^\mu_\alpha\delta^\nu_\beta)$$

This action is invariant under reducible gauge transformations with the fermionic vector parameters

$$\begin{aligned}
&\delta\psi_{\nu_1\ldots\nu_p} = p\, D_{[\nu_1}\lambda_{\nu_2\ldots\nu_p]}\\
&\ldots\\
&\delta\lambda_{\nu_1\ldots\nu_k} = k\, D_{[\nu_1}\lambda_{\nu_2\ldots\nu_k]} \qquad\qquad D_{[\nu_1}D_{\nu_2}\lambda_{\nu_3\ldots\nu_k]} \equiv 0\\
&\ldots\\
&\delta\lambda_{\nu_1\nu_2} = D_{\nu_1}\lambda_{\nu_2} - D_{\nu_2}\lambda_{\nu_1}\\
&\delta\lambda_\nu = D_\nu\lambda
\end{aligned}$$

Antisymmetric tensor fermion in AdS$_d$ space

The model of the second rank antisymmetric fermion in AdS_d spacetime is described by the field $\psi_{\mu\nu}$ with the action

$$S[\psi_{\mu\nu},\bar{\psi}_{\mu\nu}] \;=\; i\int d^dx\, g^{1/2}\bar{\psi}_{\mu_1\mu_2}\gamma^{\mu_1\mu_2\sigma\nu_1\nu_2}\, D_\sigma\, \psi_{\nu_1\nu_2},$$

This action is invariant under gauge transformations with the fermionic vector parameters λ_μ, $\bar{\lambda}_\mu$

$$\delta\psi_{\mu_1\mu_2} = D_{\mu_1}\lambda_{\mu_2} - D_{\mu_2}\lambda_{\mu_1}, \quad \delta\bar{\psi}_{\mu_1\mu_2} = \bar{D}_{\mu_1}\bar{\lambda}_{\mu_2} - \bar{D}_{\mu_2}\bar{\lambda}_{\mu_1},$$

($\bar{D}_\mu\bar{\lambda}_\nu \equiv \overline{D_\mu\lambda_\nu}$), while these transformations themselves are invariant under the first stage transformations with the spinor parameters λ and $\bar{\lambda}$,

$$\delta\lambda_\mu = D_\mu\lambda, \quad \delta\bar{\lambda}_\mu = \bar{D}_\mu\bar{\lambda} \equiv \overline{D_\mu\lambda},$$

in view of the relations $D_{[\nu_1}D_{\nu_2]}\lambda \equiv 0$, $\bar{D}_{[\nu_1}\bar{D}_{\nu_2]}\bar{\lambda} \equiv 0$ which are valid in AdS_d spacetime.

Antisymmetric tensor fermion in AdS$_d$ space

We impose on $\psi_{\mu\nu}$ the number d (per spacetime point) of spinor gauge conditions reducible to $(d-1)$-independent ones, which in their turn correspond to the parameters λ_μ reducible to a transverse vector (we count only tensor components, their spinor dimensionality $2^{[d/2]}$ being implicitly included). We choose this set of gauges $\chi^\mu \mapsto \big(\chi_\mu(\psi), \bar\chi_\mu(\bar\psi)\big)$ as

$$\chi_\mu(\psi_{\alpha\beta}) = \gamma^\nu \psi_{\mu\nu} + \frac{1}{d}\gamma_\mu \gamma^{\alpha\beta}\psi_{\alpha\beta}, \quad \gamma^\mu \chi_\mu = 0.$$

The gauge for the Dirac conjugated fermion $\bar\psi_{\mu\nu}$ is fully analogous – in what follows we will formulate everything explicitly for $\psi_{\mu\nu}$, while for $\bar\psi_{\mu\nu}$ the analogous formalism will be implicitly assumed.

Gamma matrix γ^μ plays the role of left zero vectors $\bar Z^a_\mu$, $\bar Z^a_\mu \mapsto \gamma^\mu$.

Antisymmetric tensor fermion in AdS$_d$ space

We construct the covariant delta function of gauge conditions by using the "gauge for gauge" $\sigma_a(\pi) \mapsto \sigma(\bar{\psi}^\mu) = \bar{\psi}^\mu \gamma_\mu$, $\sigma_a^\mu \mapsto \gamma_\mu$, and applying the Faddeev-Popov factorization of the group volume

$$\hat{\delta}[\chi_\mu] \;=\; \int D\bar{\psi}^\mu \exp\{i\bar{\psi}^\mu \chi_\mu(\psi_{\alpha\beta})\}\, \delta[\bar{\psi}^\mu \gamma_\mu],$$

$$\delta[\bar{\psi}^\mu \gamma_\mu] = \int D\psi \exp\{i\bar{\psi}^\mu \gamma_\mu \psi\},$$

we get with $c^a \mapsto \psi$

$$\hat{\delta}[\chi_\mu(\psi_{\alpha\beta})] = \int D\psi\, \delta[\chi_\mu(\psi_{\alpha\beta}) + \gamma_\mu \psi],$$

where we omitted the FP operator factor $(\mathrm{Det}\,\bar{Q})^{-1}$ because this operator is ultralocal in spacetime, $\bar{Q}^a_b = \bar{Z}^a_\mu \sigma^\mu_b \mapsto \gamma^\mu \gamma_\mu \delta(x,y)$ and $\mathrm{Det}\,\bar{Q} \sim \exp\big(\delta(0)(...)\big)$ can be discarded in dimensional regularization or absorbed in the irrelevant local measure.

Antisymmetric tensor fermion in AdS$_d$ space

Integration measure over the group transformation $D\mu_f \mapsto D\mu_{(\lambda)} D\mu_{(\bar{\lambda})}$, follows from the factorization procedure. It goes with $f^\mu \mapsto (\lambda_\mu, \bar{\lambda}_\mu)$ and $f_{1a} \mapsto (\lambda, \bar{\lambda})$ by using the following choice of gauge condition functions $\omega^a(f) \mapsto (\gamma^\mu \lambda_\mu, \bar{\lambda}_\mu \gamma^\mu)$ which correspond to local gauge-fixing matrices $\omega^a_\alpha \mapsto \gamma_\alpha$ acting in both λ and $\bar{\lambda}$ spinor sectors.

$$D\mu_{(\lambda)} = D\lambda_\mu \, \Delta_0^{-1} \, \delta[\gamma^\mu \lambda_\mu], \qquad D\mu_{(\bar{\lambda})} = D\bar{\lambda}_\mu \, \Delta_0^{-1} \, \delta[\bar{\lambda}_\mu \gamma^\mu].$$

Here Δ_0^{-1} is the inverse of the FP determinant, corresponding in the terminology BV to the contribution of ghosts for ghosts. The determinant $\mathrm{Det}\, Q_1{}^a_b = \mathrm{Det}\left(\omega^a_\alpha Z^\alpha_b\right) \mapsto \Delta_0$ in both $(\lambda, \bar{\lambda})$-sectors equals

$$\Delta_0 \equiv \mathrm{Det}\left[i\nabla\!\!\!/ \pm \frac{1}{2} r^{1/2} d\right], \qquad (1)$$

where the operator $\nabla\!\!\!/ = \gamma^\mu \nabla_\mu$ is acting on the tensor rank zero spinor.

Antisymmetric tensor fermion in AdS$_d$ space

The FP determinant

$$\Delta^{-1} \;=\; \int D\mu_{(\lambda)}\,\hat{\delta}\Big[\chi_\mu\big(\psi^{(\lambda)}_{\alpha\beta}\big)\Big] = \Delta_0^{-1}\Delta_1, \quad \Delta_p = \mathrm{Det}\,\big(i\nabla\!\!\!/_p\big).$$

where the operator $i\nabla\!\!\!/_p$ acting on the totally antisymmetric fermion of any rank p is defined by the equation

$$i\nabla\!\!\!/_p \equiv i\nabla\!\!\!/ \pm \frac{1}{2}\, r^{\frac{1}{2}}(d-2p).$$

It is important that the functional determinant Δ_1 should be calculated here on the functional space of γ^μ-irreducible vector fermions Ψ_μ satisfying the irreducibility constraint $\gamma^\mu\Psi_\mu = 0$. The same result holds for the contribution of the $\bar{\psi}_{\alpha\beta}$-sector of the theory.
Therefore, the determinant factor equals

$$\frac{\mathrm{Det}\,Q_1}{\mathrm{Det}F} = \Delta^{-2} = \frac{\Delta_1^2}{\Delta_0^2}.$$

Antisymmetric tensor fermion in AdS$_d$ space

The functional integral

$$Z = \frac{\Delta_0^2}{\Delta_1^2} \int D\psi_{\mu\nu} D\bar{\psi}_{\mu\nu} D\psi D\bar{\psi}$$
$$\times\, \delta[\chi_\mu(\psi_{\mu\nu}) + \gamma_\mu \psi]\, \delta[\bar{\chi}_\mu(\bar{\psi}_{\mu\nu}) + \bar{\psi}\gamma_\mu] \exp\Big\{ iS[\psi_{\mu\nu}, \bar{\psi}_{\mu\nu}] \Big\}$$

is easy to calculate in terms of the γ^μ-irreducible components of all integration variables $\psi_{\mu\nu}$ and $\bar{\psi}_{\mu\nu}$. Decomposing them into these components $\Psi_{\mu\nu}$, Ψ_μ and Ψ according to the equations

$$\psi_{\mu\nu} = \Psi_{\mu\nu} + \frac{1}{d-2}\Big(\gamma_\mu \Psi_\nu - \gamma_\nu \Psi_\mu\Big) + \gamma_{\mu\nu}\, \Psi, \quad \gamma^\mu \Psi_{\mu\nu} = 0, \quad \gamma^\mu \Psi_\mu = 0,$$

(similarly for $\bar{\psi}_{\mu\nu}$) one gets

$$Z = \frac{\Delta_0^2}{\Delta_1^2} \int D\Psi_{\mu\nu} D\bar{\Psi}_{\mu\nu} D\Psi_\mu D\bar{\Psi}_\mu D\Psi D\bar{\Psi} D\psi D\bar{\psi}\ \times$$
$$\times\, \delta[\Psi_\mu + \gamma_\mu \psi]\, \delta[\bar{\Psi}_\mu + \bar{\psi}\gamma_\mu] \exp\Big\{ i\tilde{S}[\,\Psi_{\mu\nu}, \Psi_\mu, \Psi, \bar{\Psi}_{\mu\nu}, \bar{\Psi}_\mu, \bar{\Psi}\,] \Big\},$$

Antisymmetric tensor fermion in AdS$_d$ space

Taking the Gaussian integral we finally get the one-loop contribution to the functional integral for second rank antisymmetric fermion fields on the AdS_d background

$$Z_{p=2} = \frac{\Delta_0^2}{\Delta_1^2} \times \Delta_2 \Delta_0 = \frac{\Delta_2 \, \Delta_0^3}{\Delta_1^2}, \qquad \Delta_p = \mathrm{Det}\left(i\nabla\!\!\!/_p\right)$$

and the operators $i\nabla\!\!\!/_p$, acting on γ-ireducible antisymmetric spinor p-forms, $p = 0, 1, 2$, are defined

$$i\nabla\!\!\!/_p \equiv i\nabla\!\!\!/ \pm \frac{1}{2}\, r^{\frac{1}{2}} (d - 2p).$$

This is a final result for effective action in the theory under consideration. The calculation of the functional determinants $\Delta_2, \Delta_1, \Delta_0$ on irreducible spinor forms is a separate problem.

Antisymmetric tensor fermion in AdS_d space

$$Z_p = \frac{\Delta_p \, \Delta_{p-2}^3 \, \Delta_{p-4}^5 \cdots}{\Delta_{p-1}^2 \, \Delta_{p-3}^4 \cdots}, \qquad \Delta_p = \mathrm{Det}\left(i\nabla\!\!\!/_p\right)$$

and the operators $i\nabla\!\!\!/_p$, acting on γ-ireducible antisymmetric spinor p-forms, are defined

$$i\nabla\!\!\!/_p \equiv i\nabla\!\!\!/ \pm \frac{1}{2}\, r^{\frac{1}{2}} (d - 2p).$$

THANK YOU FOR YOUR ATTENTION