Neutrino spin oscillations in the vicinity of a black hole

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References

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- M. Deka and M. Dvornikov, “Spin oscillations of neutrinos scattered by the supermassive black hole in the galactic center”, arXiv:2501.19404.
- M. Dvornikov, “Neutrino spin oscillations in a magnetized Polish doughnut”, JCAP 09 (2023) 039.
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Motivation

- Nonzero magnetic moment of neutrino. Experimental upper bound $\sim 10^{-11} - 10^{-12}\mu_{\mathrm{B}}$.
- Leads to interaction with the electromagnetic fields.
- If a neutrino spin precesses in an external field, i.e. its spin direction changes with respect to its momentum, a Dirac neutrino becomes right-handed.

$\Rightarrow$ Neutrino Spin Oscillations.

Fujikawa and Shrock, 1980; Giunti, et al., 2016.

- Accretion disks in SMBH in some galaxies can be sources of both photons and high energy neutrinos. Chen & Beloborodov, 2006.

- Before arriving at the observer, these neutrinos move in strong gravitational field near BH.

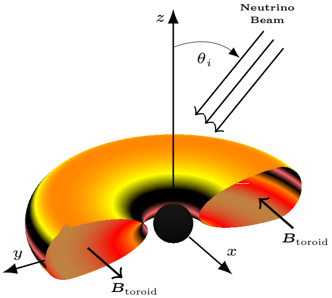
- Their spins can precess in the presence of external fields of the accretion disk.

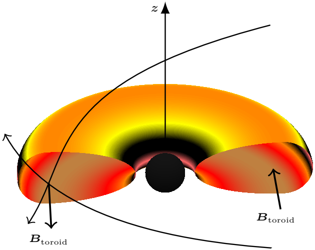
- Right-handed neutrinos are considered to be sterile in the standard model.

- We shall observe an effective reduction of the initial neutrino flux.

This Work

- We consider a uniform flux of left-polarized Dirac neutrinos approaching a BH at an angle, θ_i, w.r.t. to the BH spin, $(r, \theta, \phi)_{\text{source}} = (\infty, \theta_i, 0)$.
- They are either captured or scattered by the BH. We are interested only in the scattered neutrinos.
- We consider a thick accretion disk surrounding the BH with only the toroidal magnetic field.
- The scattered Neutrinos undergo interactions with the matter and magnetic fields in the disk resulting spin precession.
- Some of the left handed neutrinos can become right handed.
- We finally look at the probability distributions of the handedness of the neutrinos at the observer position $(\theta, \phi)_{\text{obs}}$.


Neutrino
Beam
z
θ_i
y
x
$\boldsymbol{B}_{\text{toroid}}$
$\boldsymbol{B}_{\text{toroid}}$


z
$\boldsymbol{B}_{\text{toroid}}$
$\boldsymbol{B}_{\text{toroid}}$

Kerr Metric

- We describe the spacetime of a spinning black hole in Kerr metric.
- Boyer-Lindquist coordinates, $x = (t, r, \theta, \phi)$:

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \left(1 - \frac{r r_g}{\Sigma}\right) dt^2 + 2\frac{r r_g a \sin^2\theta}{\Sigma} dt d\phi - \frac{\Sigma}{\Delta} dr^2 \\
&- \Sigma d\theta^2 - \frac{\Xi}{\Sigma} \sin^2\theta d\phi^2
\end{aligned}
$$

$$
\Delta = r^2 - r r_g + a^2, \;\; \Sigma = r^2 + a^2\cos^2\theta, \;\; \Xi = (r^2 + a^2)\Sigma + r r_g a^2 \sin^2\theta
$$

- BH mass: $M = r_g/2$.
- BH spin: $J = Ma (0 < a < M)$.

Particle Trajectory

- The radial and polar potentials are given by

$$R = \left[(r^2 + a^2)E - aL\right]^2 - \Delta\left[Q + (L - aE)^2\right] \tag{1}$$

$$\Theta = Q + \cos^2\theta\left(a^2E^2 - \frac{L^2}{\sin^2\theta}\right) \tag{2}$$

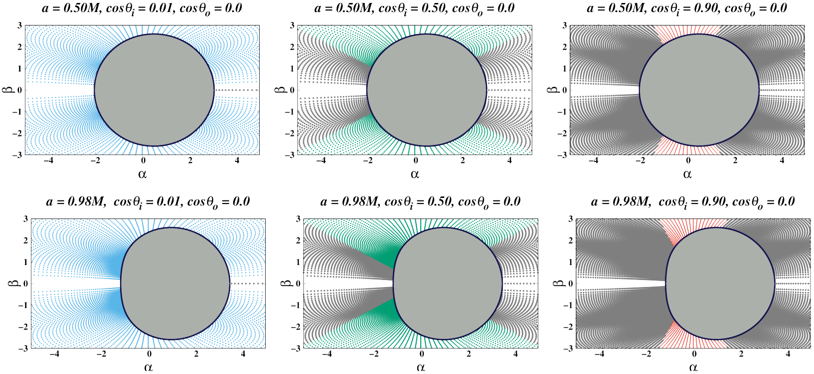
Q is Carter constant.

- Integral equations along the particle trajectories,

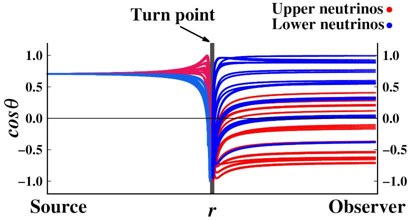
$$\int \frac{dr}{\pm\sqrt{R}} = \int \frac{d\theta}{\pm\sqrt{\Theta}} \tag{3}$$

$$\phi = a\int \frac{dr}{\Delta\sqrt{R}}\left[(r^2 + a^2)E - aL\right] + \int \frac{d\theta}{\sqrt{\Theta}}\left[\frac{L}{\sin^2\theta} - aE\right] \tag{4}$$

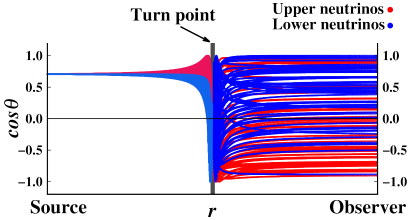
- We are interested in scattered neutrinos only.
- The edge between the scattered and captured neutrinos is given by $R(\tilde{r}) = R'(\tilde{r}) = 0 \Rightarrow$ BH Shadow curve.
- All neutrinos inside the grey area are discarded from the computation.



$a = 0.02M, \cos\theta_i = 0.707$



$a = 0.98M, \cos\theta_i = 0.707$



Neutrino spin evolution

- The covariant equation for the neutrino spin four-vector (Dvornikov, 2013; Pomeransky and Khriplovich, 1998),

$$\frac{DS^{\mu}}{D\tau} = 2\,\mu\left(F^{\mu\nu}S_{\nu} - U^{\mu}U_{\nu}F^{\nu\lambda}S_{\lambda}\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_{\nu}U_{\lambda}S_{\rho}, \frac{DU^{\mu}}{D\tau} = 0. \tag{5}$$

- We make a transformation to a local Minkowskian frame.

$$x_a = e_a^{\mu}x_{\mu}, \quad \eta_{ab} = e_a^{\mu}e_b^{\nu}g_{\mu\nu}, \quad \eta_{ab} = (1, -1, -1, -1) \tag{6}$$

- After making a boost to the particle rest frame, the neutrino invariant 3-spin vector can then be defined as

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{em}} + \boldsymbol{\Omega}_{\mathrm{matter}}. \tag{7}$$

  Dvornikov, 2023.

- $\boldsymbol{\Omega}$ can be explicitly calculated in a given metric.

Effective Schrödinger Equation

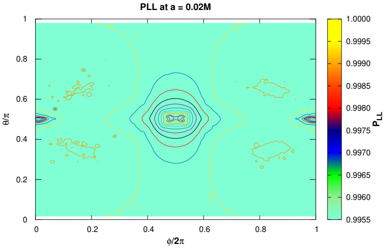
- Instead, we solve the effective Schrödinger equation for the neutrino polarization,

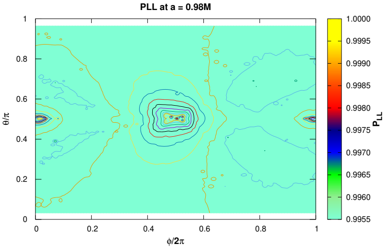
$$i\,\frac{d\psi}{dr} = H_r\,\psi \tag{8}$$

$$H_r = -\mathcal{U}_2(\boldsymbol{\sigma}.\boldsymbol{\Omega}_r)\mathcal{U}_2^\dagger, \qquad \mathcal{U}_2 = \exp(i\pi\sigma_2/4)$$

- We use four-step Adams-Bashforth and Adams-Moulton predictor-corrector method to solve for ψ.
- For an incoming left polarized neutrino, $\psi^T_{-\infty} = (1, 0)$.
- For an outgoing neutrino, it becomes, $\psi^T_{+\infty} = (\psi^{(R)}_{+\infty}, \psi^{(L)}_{+\infty})$.
- The probability of a neutrino remaining left polarized: $P_{\rm LL} = |\psi^{(L)}_{+\infty}|^2$.

$$\Omega = \Omega_g \cancel{+ \Omega_{matter} + \Omega_{em}}$$





Magnetic fields in the Accretion Disk

- Thick accretion disk surrounding the BH (Polish doughnut). Abramowicz et al., 1978.
- Only toroidal magnetic field inside the disk (Komissarov, 2006)

$$B^{\phi} = \sqrt{\frac{2p_m^{(\mathrm{tor})}}{|g_{\phi\phi} + 2l_0 g_{t\phi} + l_0^2 g_{tt}|}}, \quad B^t = l_0 B^{\phi} \tag{9}$$

$$p_m^{(\mathrm{tor})} = K_m \mathcal{L}^{\kappa-1} \left[\frac{\kappa-1}{\kappa} \frac{W_{\mathrm{in}} - W}{K + K_m \mathcal{L}^{\kappa-1}}\right]^{\frac{\kappa}{\kappa-1}}, \mathcal{L} = g_{tt} g_{\phi\phi} - g_{t\phi}^2 \tag{10}$$

- The form of the disk depends on the potential,

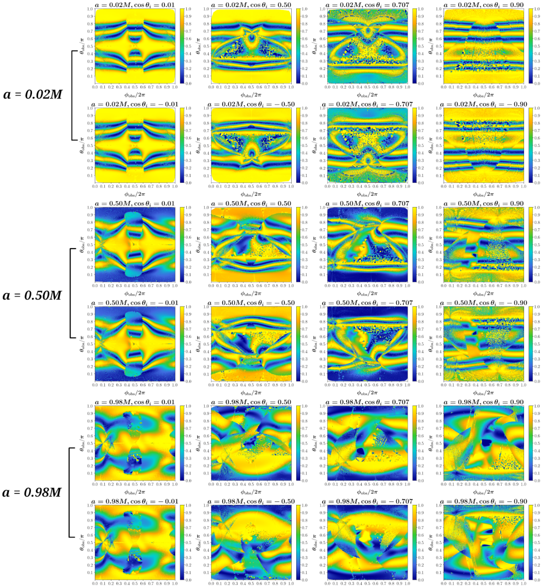
$$W(r,\theta) = \frac{1}{2} \ln \left| \frac{g_{tt} g_{\phi\phi} - g_{t\phi}^2}{g_{\phi\phi} + 2l_0 g_{t\phi} + l_0^2 g_{tt}} \right| \tag{11}$$

- We consider both co-rotating and counter-rotating disks.

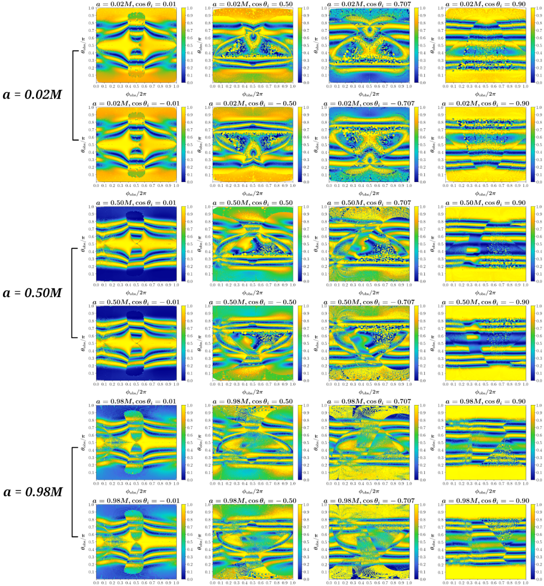
Numerical Parameters

- The mass of SMBH is $10^8 M_\odot$. The BH spin is $0 < a < 0.98M$.
- The maximal strength of the toroidal fields is 320 G. It is 1% of the Eddington limit for this BH mass Beskin, 2010.
- The maximal matter density of hydrogen plasma is 10^{18} cm^{-3}. Such density can be found in some AGN Jiang et al., 2019.
- We consider Neutrino magnetic moment, $\mu = 10^{-13}\mu_\mathrm{B}$. It is below the best astrophysical constraint Viaux et al., 2013.
- The number of scattered neutrinos for each combination of a and θ_i is more than 2 million.
- All the computations have been carried out at Govorun Supercluster of JINR. We have used more than 2000 SkyLake and IceLake processors continuously for several weeks.

Co-rotating Disk



Counter-rotating Disk



Conclusion

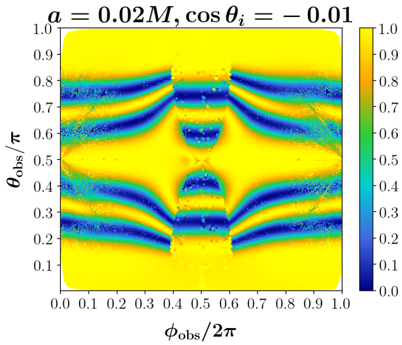
- Only toroidal magnetic field is sufficient enough for spin oscillations to occur.

- We investigate P_{LL} for a number of different θ_i's. This is important since the relative position between a neutrino source and Earth is not known during the observation.

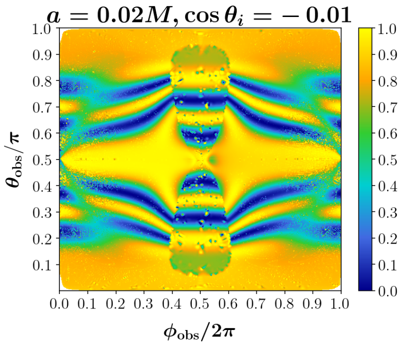
Conclusion

- There is a clear difference between P_{LL}'s for the co-rotating and counter-rotating disks even for a slowly rotating BH.

Co-rotating Disk



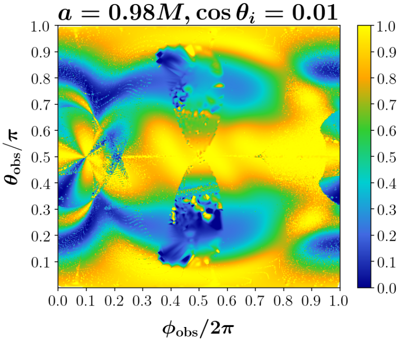
Counter-rotating Disk



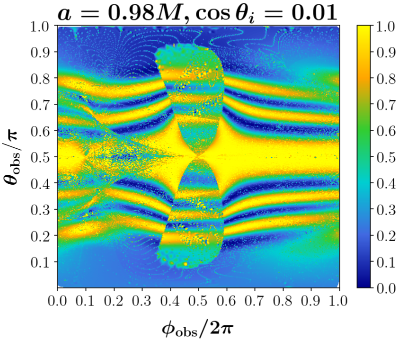
Conclusion

- No symmetric distributions of $P_{\rm LL}$ w.r.t. the $\theta_{\rm obs} = \pi/2$ plane can be seen for a rotating BH at lower $\cos\theta_i$'s.

Co-rotating Disk



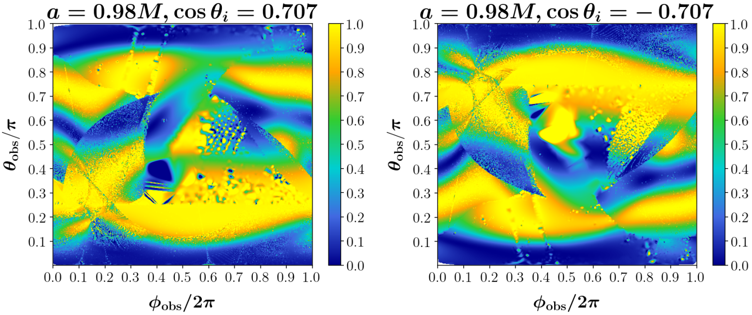
Counter-rotating Disk



Conclusion

- No inverse symmetry of $P_{\rm LL}$ for the opposite values of $\cos\theta_i$ with the same BH spin is exhibited for a rotating BH.

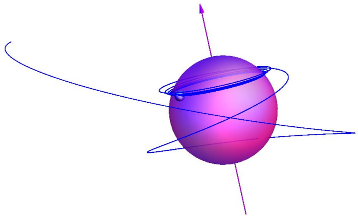
Co-rotaing Disk

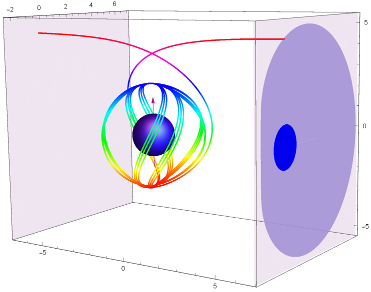


Thank you!

Extras

Dokuchaev and Nazarova, 2020





Kerr Metric

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$$
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&- \Sigma d\theta^2 - \frac{\Xi}{\Sigma} \sin^2\theta d\phi^2
\end{aligned}
$$

$$
\Delta = r^2 - rr_g + a^2, \;\; \Sigma = r^2 + a^2\cos^2\theta, \;\; \Xi = (r^2 + a^2)\Sigma + rr_g a^2 \sin^2\theta
$$

- BH mass: $M = r_g/2$.
- BH spin: $J = Ma (0 < a < M)$.

Particle Trajectory in Kerr Spacetime

- We use the Hamilton-Jacobi approach to describe the geodesic of a particle of mass, m. Later we take $m \to 0$.
- The solution of Hamilton-Jacobi equation leads to,

$$S = -\frac{1}{2}m^2\lambda - Et + L\phi + \int dr \frac{\sqrt{R}}{\Delta} + \int d\theta \sqrt{\Theta} \tag{12}$$

where,

$$\int \frac{dr}{\pm\sqrt{R}} = \int \frac{d\theta}{\pm\sqrt{\Theta}} \tag{13}$$

$$\phi = a \int \frac{dr}{\Delta\sqrt{R}} \left[(r^2 + a^2)E - aL\right] + \int \frac{d\theta}{\sqrt{\Theta}} \left[\frac{L}{\sin^2\theta} - aE\right] \tag{14}$$

$$R = \left[(r^2 + a^2)E - aL\right]^2 - \Delta\left[Q + (L - aE)^2\right] \tag{15}$$

$$\Theta = Q + \cos^2\theta \left(a^2E^2 - \frac{L^2}{\sin^2\theta}\right) \tag{16}$$

Black Hole Shadow Curve

$R(\tilde{r}) = R'(\tilde{r}) = 0.$

$$\frac{L}{E} = -\frac{\tilde{r}^2(\tilde{r} - 3M) + a^2(\tilde{r} + M)}{a(\tilde{r} - M)} \tag{17}$$

$$\frac{\sqrt{Q}}{E} = \frac{\tilde{r}^{3/2}}{a(\tilde{r} - M)}\sqrt{4a^2M - \tilde{r}(\tilde{r} - 3M)^2} \tag{18}$$

$$r_{\pm} = 2M\left[1 + \cos\left(\frac{2}{3}\arccos(\pm\frac{a}{M})\right)\right] \tag{19}$$

$$r_{-} < \tilde{r} < r_{+}. \tag{20}$$

Neutrino spin evolution in curved spacetime

- We consider neutrino as a Dirac particle with nonzero magnetic moment, μ.
- Weakly interacts with the background matter.
- Four velocity of a neutrino is parallel transported along geodesics.
- The covariant equation for the neutrino spin four vector in curved spacetime (Pomeransky and Khriplovich, 1998; Dvornikov, 2013; Dvornikov, 2023),

$$\frac{DS^\mu}{D\tau} = 2\,\mu\left(F^{\mu\nu}S_\nu - U^\mu U_\nu F^{\nu\lambda}S_\lambda\right) + \sqrt{2}G_F\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}G_\nu U_\lambda S_\rho,\quad \frac{DU^\mu}{D\tau} = 0.$$

$$\begin{aligned} DS^\mu &= dS^\mu + \Gamma^\mu_{\alpha\beta}S^\alpha dx^\beta \\ G_F &= 1.17\times 10^{-5}\text{GeV}^{-2} : \text{Fermi constant} \\ G_\mu &: \text{covariant effective potential.} \end{aligned}$$

We introduce a locally Minkowskian coodinates,

$$x_a = e_a^{\mu} x_{\mu}, \tag{21}$$

where $e_a^{\mu} (a = 0 \cdots 3)$ are the vierbein vectors satisfying the relations

$$e_a^{\mu} e_b^{\nu} g_{\mu\nu} = \eta_{ab}, \;\; e_{\mu}^{a} e_{\nu}^{b} \eta_{ab} = g_{\mu\nu} \tag{22}$$

Here $e_{\mu}^{a} e_{\mu}^{a}$ are the inverse vierbein vectors ($e_a^{\mu} e_{\nu}^{a} = \delta_{\nu}^{\mu}$ and $e_a^{\mu} e_{\mu}^{b} = \delta_a^{b}$) and $\eta_{ab} = \mathrm{diag}(1, -1, -1, -1)$.

$$\begin{aligned} e_0^{\mu} &= \left(\sqrt{\frac{\Xi}{\Sigma\Delta}}, 0, 0, \frac{arr_g}{\sqrt{\Sigma\Delta\Xi}}\right), \;\; e_1^{\mu} = \left(0, \sqrt{\frac{\Delta}{\Sigma}}, 0, 0\right), \\ e_2^{\mu} &= \left(0, 0, \frac{1}{\sqrt{\Sigma}}, 0\right), \;\; e_3^{\mu} = \left(0, 0, 0, \frac{1}{\sin\theta}\sqrt{\frac{\Sigma}{\Xi}}\right) \end{aligned} \tag{23}$$

$$\frac{d\boldsymbol{\zeta}}{dt} = 2(\boldsymbol{\zeta} \times \boldsymbol{\Omega}), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{g}} + \boldsymbol{\Omega}_{\mathrm{em}} + \boldsymbol{\Omega}_{\mathrm{matt}} \tag{24}$$

$$\begin{aligned}
\boldsymbol{\Omega}_{\mathrm{g}} &= \frac{1}{2U^t}\left[\boldsymbol{b}_g + \frac{1}{1+u^0}(\boldsymbol{e}_g \times \boldsymbol{u})\right] \\
\boldsymbol{\Omega}_{\mathrm{em}} &= \frac{\mu}{U^t}\left[u^0\boldsymbol{b} - \frac{\boldsymbol{u}(\boldsymbol{u}\boldsymbol{b})}{1+u^0} + (\boldsymbol{e} \times \boldsymbol{u})\right] \\
\boldsymbol{\Omega}_{\mathrm{matt}} &= \frac{G_{\mathrm{F}}}{\sqrt{2}U^t}\left[\boldsymbol{u}\left(g^0 - \frac{(\boldsymbol{g}\boldsymbol{u})}{1+u^0}\right) - \boldsymbol{g}\right]
\end{aligned} \tag{25}$$

Here $u^a = (u^0, \boldsymbol{u}) = e^a_\mu U^\mu$, $U^\mu = \frac{dx^\mu}{d\tau}$ is the four velocity in the world co-ordinates and τ is the proper time. $G_{ab} = (\boldsymbol{e}_g, \boldsymbol{b}_g) = \gamma_{abc}u^c$, $\gamma_{abc} = \eta_{ad}e^d_{\mu;\nu}e^\mu_b e^\nu_c$ are the Ricci rotation coefficients, the semicolon stays for the covariant derivative, and $f_{ab} = e^\mu_a e^\nu_b F_{\mu\nu} = (\boldsymbol{e}, \boldsymbol{b})$ is the electromagnetic field tensor in the locally Minkowskian frame, and $F_{\mu\nu}$ is an external electromagnetic field tensor. μ is the neutrino magnetic moment, and $G_{\mathrm{F}} = 1.17 \times 10^{-5}$ Gev^{-2} is the Fermi constant. $g^a = (g^0, \boldsymbol{g}) = e^a_\mu G^\mu$, G^μ is the contravariant effective potential of the neutrino electroweak interaction with a background matter.

Toroidal Fields

- The electromagnetic field tensor

$$F_{\mu\nu} = E_{\mu\nu\alpha\beta} U_f^\alpha B^\beta, \quad E^{\mu\nu\alpha\beta} = \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} \tag{26}$$

- The four vector fluid velocity in the disk and toroidal magnetic field are

$$U_f^\mu = (U_f^t, 0, 0, U_f^\phi), \qquad U_f^t = \sqrt{\left|\frac{\mathcal{A}}{\mathcal{L}}\right| \frac{1}{1 - l_0\Omega}}, \quad U_f^\phi = \Omega U_f^t \tag{27}$$

$$B^\mu = (B^t, 0, 0, B^\phi), \qquad B^\phi = \sqrt{\frac{2p^{(\mathrm{tor})_m}}{|\mathcal{A}|}}, \quad B^t = l_0 B^\phi \tag{28}$$

- The angular velocity in the disk

$$\Omega = -\frac{g_{t\phi} + l_0 g_{tt}}{g_{\phi\phi} + l_0 g_{t\phi}} \tag{29}$$

and

$$\mathcal{L} = g_{tt} g_{\phi\phi} - g_{t\phi}^2, \quad \mathcal{A} = g_{\phi\phi} + 2 l_0 g_{t\phi} + l_0^2 g_{tt} \tag{30}$$

Toroidal Fields

- The disk density ρ and the magnetic pressure $p_m^{(\text{tor})}$ have the form,

$$\rho = \left[\frac{\kappa-1}{\kappa}\frac{W_{\text{in}}-W}{K+K_m\mathcal{L}^{\kappa-1}}\right]^{\frac{1}{\kappa-1}}, \quad p_m^{(\text{tor})} = K_m\mathcal{L}^{\kappa-1}\left[\frac{\kappa-1}{\kappa}\frac{W_{\text{in}}-W}{K+K_m\mathcal{L}^{\kappa-1}}\right]^{\frac{\kappa}{\kappa-1}} \tag{31}$$