Gravitational effective action and anomaly driven inflation

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Outline

- Anomaly driven inflation
- Thermal effective action — calculation
- Peculiarities of thermal effective action

Inflation

- **Inflation** — cornerstone of modern classical and quantum cosmology, providing the inspiration for quantum gravity models construction and test for its validation.
- Main **observables**: amplitude and spectrum of relic photons (CMB) and gravitons.
- Models of inflation
  - Starobinsky inflation
  - Higgs inflation
  - etc
  - **Anomaly driven inflation**

Weyl anomaly

- Weyl-Invariant fields

$$S[\phi, g] \quad \mapsto \quad S[\phi^\sigma, g^\sigma] = S[\phi, g], \qquad (g^\sigma)_{\mu\nu} = e^{2\sigma} g_{\mu\nu}, \quad \sigma = \sigma(x)$$

- Example: massless non-minimally coupled scalar

$$S[\phi, g] = \frac{1}{2} \int d^D x \sqrt{g} \Big[(\nabla_\mu \phi)^2 + \xi R\, \phi^2 \Big], \qquad \xi = \tfrac{1}{4} \tfrac{D-2}{D-1}$$

- Gravitational effective action exhibits **Weyl anomaly**

$$\Gamma[g] = \tfrac{1}{2} \log \mathrm{Det} \big[-\nabla^2 + \xi R \big], \qquad \Gamma[g^\sigma] = \Gamma[g] + \Gamma_{\mathrm{a}}[\sigma, g]$$

where

$$\Gamma_{\mathrm{a}}[\sigma, g] = \tfrac{1}{16\pi^2} \int d^4 x \sqrt{g} \left\{ \left[\alpha C^2 + \beta \mathcal{E}_4 \right] \sigma + 2\beta \sigma \Delta_4 \sigma \right\}$$
$$- \tfrac{1}{32\pi^2} \left(\tfrac{\gamma}{6} + \tfrac{\beta}{9} \right) \int d^4 x \left(\sqrt{g^\sigma} (R^\sigma)^2 - \sqrt{g} R^2 \right),$$

Anomaly driven inflation — roadmap

- **Idea**: exploit R^2 terms in anomaly to mimic Starobinsky-like inflation
- Practical implementation:
  - take $\sigma = \log a(t)$, then the total action reads

$$S = S_{\rm EH}[g^\sigma] + \Gamma[g] + \Gamma_{\rm a}[\sigma, g], \qquad \sigma = \log a(t)$$

  - expand $\Gamma[g] = \Gamma[\bar{g} + h]$ about simple background $\bar{g}$
  - calculate two-point correlation function of perturbations h
- Known realizations:
  - Barvinsky, Kamenshchik 2006
  - Hawking, Hertog, Reall 2001
  - Shapiro 2006
- Choice of background $\bar{g}$:
  - R^D, $R \times S^d$
  - $S^1 \times R^d$ **(this talk)**
  - $S^1 \times S^d$

Gravitational effective action — flat spacetime

- How to calculate $\Gamma[\bar{g}+h]$?
- Usually calculated via **heat kernel** methods (Barvinsky, Vilkovisky 1990)

$$\Gamma[g] = \frac{1}{2}\ln \mathrm{Det}\, F = -\frac{1}{2}\lim_{s\to 0}\frac{\partial}{\partial s}\frac{1}{\Gamma(s)}\int_0^\infty d\tau\, \tau^{s-1}\,\mathrm{Tr}\, K(\tau)$$

- Here $\mathrm{Tr}\, K(\tau) = \mathrm{Tr}\, e^{-\tau F} =$ $\mathrm{Tr}\, K^{(0)}(\tau) + \mathrm{Tr}\, K^{(1)}(\tau) + \mathrm{Tr}\, K_1^{(2)}(\tau) + \mathrm{Tr}\, K_2^{(2)}(\tau) + \ldots$

$$\mathrm{Tr}\, K^{(1)}(\tau) = -\tau\,\mathrm{Tr}\big(e^{-\tau F_0}\delta F\big)$$

$$\mathrm{Tr}\, K_1^{(2)}(\tau) = -\frac{1}{2}\tau\,\mathrm{Tr}\big(e^{-\tau F_0}\delta^2 F\big)$$

$$\mathrm{Tr}\, K_2^{(2)}(\tau) = \tau^2\int_0^1 dx\;\mathrm{Tr}\big(e^{-x\tau F_0}\,\delta F\, e^{-(1-x)\tau F_0}\,\delta F\big)$$

Gravitational effective action — flat spacetime II

- For scalar operator $F = -\nabla^2 + \xi R$

$$\mathrm{Tr}\, K(\tau) = \frac{1}{(4\pi\tau)^{D/2}} \int d^D x \sqrt{g} \Big[1 + \tau^2 \Big(R\, f_1(-\tau\nabla^2)\, R + R_{\mu\nu}\, f_2(-\tau\nabla^2)\, R_{\mu\nu} \Big) \Big] + \mathcal{O}(h^3)$$

- Here **formfactorst** $f_{1,2}(t)$ are linear combinations of

$$f^{(l)}(t) = \frac{\partial^l}{\partial t^l} \int_0^1 dx\, e^{-x(1-x)t}$$

- This gives the following answer for $\Gamma[g]$

$$\Gamma[g] = \int d^D x \Big[c_1\, R_{\mu\nu} (-\nabla^2)^{D/2-2} \log(-\nabla^2) R_{\mu\nu} + c_2\, R\, (-\nabla^2)^{D/2-2} \log(-\nabla^2) R \Big]$$

Gravitational effective action — thermal case

- Inclusion of **temperature** $R^D \mapsto S^1 \times R^d$ leads to **two modifications**
- The **first** is modification of formfactor

$$f(-\tau\nabla^2) = \int_0^1 dx\, e^{x(1-x)\tau\nabla^2} \quad \mapsto \quad f(\tau, \nabla^2, \partial_t^2)$$

$$= \int_0^1 dx\, e^{x(1-x)\tau\nabla^2} \Big[1 + 2\sum_{n=1}^{\infty} e^{-\frac{\beta^2 n^2}{4\tau}} \cosh\big(\beta\, n\, \partial_t\big)\Big] = f + f_\beta$$

- This gives the following quadratic part of effective action

$$\Gamma^{(2)}_\beta[g] = \int d^D x \sqrt{g}\Big(\gamma_1 R_{00}^2 + \gamma_2 R_{0i}^2 + \gamma_3 R_{ij}^2 + \gamma_4 R_{ii}^2 + \gamma_5 R_{00} R_{ii}\Big)$$

  where we develop **gradient expansion**

$$\gamma_i(-\partial_t^2, -\nabla^2) = \sum_{q=0}^{\infty} O_{i,q}(z, \partial_z)\, \mathrm{Re}\big[\zeta'_H(-2q, iz)\big]\big|_{2q\text{-reg}} \, (-\partial_t)^{2q}$$

  and $O_{i,q}(z, \partial_z)$ is operator of order $2q + D/2 - 2$, $z = \frac{\beta}{2\pi}\sqrt{-\nabla^2}$.
- $\Gamma^{(2)}_\beta$ is **diffeomorphism invariant**, but **not Lorentz invariant**!

Linear term — nonlocal cosmological constant?

- The **second** modification is appearance of nontrivial linear term

$$\Gamma^{(1)}_\beta[g] = \int d^D x \left(d_1\, h_{00} + d_2\, h_{ii} \right)$$

where $d_{1,2} = d_{1,2}(\beta)$.

- How to complete linear term to diffeomorphism invariant? This is manifestly non-local for $d_1 \neq d_2$!
- The particular solution for h_{00} term is

$$\int d^D x\, h_{00} \;\mapsto\; \int d^D x \left[h_{00} + \frac{1}{2} \Big(2h_{00}h_{ii} - 2h_{00} \frac{1}{\Delta} \partial_i \partial_j h_{ij} - 2h_{0i}^2 \right.$$
$$\left. + 2\partial_i h_{0i} \frac{1}{\Delta} \partial_j h_{0j} - 2\partial_t h_{0i} \frac{1}{\Delta} \partial_j h_{ij} + h_{ij} \frac{\partial_t^2}{\Delta} h_{ij} \Big) \right]$$

- This can be interpreted as **non-local cosmological constant** term.

Discussion

We have:

- Finite-temperature effective action for non-minimally coupled scalar field is constructed
- Inclusion of fermions can be done by using identity $\theta_2(0, \mathrm{e}^{-z}) = \theta_3(0, \mathrm{e}^{-z/4}) - \theta_3(0, \mathrm{e}^{-z})$
- Gradient expansion was extended beyond zero order, calculated by Gusev, Zelnikov 2000
- Strange nonlocal cosmological constant contribution was identified

What is next?

- Euclid $\mapsto$ Schwinger-Keldysh
- Compact case: role of Weyl invariance and different tensor sectors
- Gravity $\mapsto$ Yang-Mills, photon/gluon pair production?

Thank you for your attention!