Physical state of axion and its properties in the 3-3-1 with B-L symmetry model.

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Motivation

STANDARD MODEL

1. Extra dimesions
2. Extend the spectrum of particles
3. Extend the gauge symmetry

BEYOND STANDARD MODELDS

1. At 200GeV scale, BSM includes SM
2. Define the mixing angles of ν that fit with data from experiments
3. Explain the BAU
4. Higgs spectrum includes SMLHB
5. Exist dark matter candidates

3-3-1 MODELS

$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (2)

$$Q = T_3 + \beta T_8 + X \quad (3)$$

$$\beta = -\frac{1}{\sqrt{3}}$$

$SU(3)_{Color}$

Strong CP problem is not verified by experiments

$U(1)_{PQ}$

$SU(2)_{Left} \otimes U(1)_{Y_{weak}}$

- massless neutrino
- gravity is absent
- unlimited fermion generation
- mass of top quark 172GeV
- no darkmatter candidates

$$Q = T_3 + \frac{Y}{2} \quad (1)$$

predict the existences of Higgs 125GeV, c, t, W, Z,

$a(k) \rightarrow \gamma(p) + \gamma(q)$ (4)

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$Phys.Rev.D\mathbf{111}(2025), 115023$

3-3-1 model with axion (A331)

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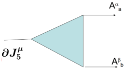
XENON1T (2019)

$Phys.Rev.D107, 095030, (2023)$

Strong CP problem

CP invariant

$$\mathcal{L}_{QCD}^{int} = -\frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu,a} + \bar{Q}i\gamma^{\mu}D_{\mu}Q \quad (5)$$



$$G\cdot\tilde{G} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\mu\nu}_{a}G^{a\alpha\beta} \quad (6)$$

-violate PT symmetry by the complicated QCD vacuum structure

- produce a neutron electric dipole moment of order $d_n \sim e\frac{m_q}{M_n^2}\theta$

angle of QCD vacuum

$$\mathcal{L}_{QCD}^{eff} = \mathcal{L}_{QCD}^{int} + \frac{\theta g_S^2}{32\pi^2}G\cdot\tilde{G}$$

EDM of neutron $d_N < 2.9\times10^{-26}e$ cm

CPV in QCD hasn't been observed (7)

$\theta \ll$ (Baluni; Crewther Di Vecchia Veneziano Witten)

Strong CP problem

$U(1)_{PQ}$

chiral symmetry drives $\theta \to \frac{a}{f_a}$

automatically satisfied

SM with $U(1)_{PQ}$ (Alfleck-Dine Model)

(SM + φ)

KSVZ model — Kim-Shifman-Vainshtein-Zakharov

(SM+ X(3) + σ^0)

DFSZ model — Dine-Fischler-Srednicki-Zhitnitsky

(SM + $\varphi + \sigma - \phi + \phi_1 + \phi_2$)

PQWW model — Peccei-Quinn-Weinberg-Wilczek.

(SM $-\phi + \phi_1 + \phi_2$)

*complicated Higgs potential with various parameters

- solve the tiny mass of neutrino
- working scale is not too large
- explain why number of fermion family is 3
- explain why mass of t quark is so heavy
- FCNC at tree-level
- avoid Landau pole (5TeV)
- exist dark matter candidate

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3-3-1 model with ALP

$SU(3)_C\otimes SU(3)_L\otimes U(1)_X\otimes Z_{11}\otimes Z_2$ (8)

$v_\phi \sim 10^{11}$ GeV

$SU(3)_C\otimes SU(3)_L\otimes U(1)_X$

$v_\chi \sim 10^5$ GeV

$SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$

$v_\eta, v_\rho \sim$ EW

$SU(3)_C\otimes U(1)_Q$.

$SU(3)_C\otimes SU(3)_L\otimes U(1)_X\otimes U(1)_{B-L}$ (9)

arXiv:2412.04269v3 [hep-ph]

A brief review of the 3-3-1 with $U(1)_{B-L}$ model

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_{B-L}$$

The particle content

Charge operator

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X \quad (10)$$

$B-L$ operator

$$B - L = -\frac{4}{\sqrt{3}} T_8 + N \quad (11)$$

one singlet $\sigma \sim (\mathbf{1}, \mathbf{1}, 0, 0)$

$$\frac{1}{\sqrt{2}}(v_\sigma + R_\sigma + iI_\sigma) \quad (16)$$

2 quark generations in anti-triplets,

$$Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, D_{\alpha L})^T \sim \left(\mathbf{3}, \mathbf{3}^*, 0, -\frac{1}{3}\right) \quad (14)$$

$\alpha = 1, 2$

$u_{aR} \sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{1}{3}\right)$, $\quad U_{3R} \sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{7}{3}\right)$

one quark generation in triplet

$$Q_{3L} = (u_{3L}, d_{3L}, U_{3L})^T \sim \left(\mathbf{3}, \mathbf{3}, \frac{1}{3}, 1\right) \quad (15)$$

$D_{\alpha R} \sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -\frac{5}{3}\right)$

$d_{aR} \sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, \frac{1}{3}\right)$,

3 Higgs triplets

$$\Phi_1 = \left(\phi_1^0, \phi_1^-, \tilde{\phi}_1^0\right)^T \sim \left(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, \frac{2}{3}\right) \quad (17)$$

$$\frac{1}{\sqrt{2}}(v_1 + R_1 + iI_1)$$

$$\Phi_2 = \left(\phi_2^+, \phi_2^0, \tilde{\phi}_2^-\right)^T \sim \left(\mathbf{1}, \mathbf{3}, \frac{2}{3}, \frac{2}{3}\right) \quad (18)$$

$$\frac{1}{\sqrt{2}}(v_2 + R_2 + iI_2)$$

$$\Phi_3 = \left(\phi_3^0, \phi_3^-, \tilde{\phi}_3^0\right)^T \sim \left(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{4}{3}\right) \quad (19)$$

$$\frac{1}{\sqrt{2}}(v_3 + R_3 + iI_3)$$

Left-handed leptons in triplets

$$\psi_{aL} = (\nu_{aL}, e_{aL}, (\nu_{aR})^c)^T \sim \left(\mathbf{3}, \mathbf{3}^*, 0, -\frac{1}{3}\right) \quad (12)$$

$a = 1, 2, 3$

Right-handed leptons in singlets

$$e_{aR} \sim (\mathbf{1}, \mathbf{1}, -1, -1) \quad (13)$$

$$S_{aL,R} \sim (\mathbf{1}, \mathbf{1}, 0, -1)$$

vector-like neutral fermions → Type-I SeeSaw mechanism

generate neutrino masses

Physical state of axion defined via CP-odd mass mixing matrix

Scalar potential

$$V_H = \sum_{i=1}^{3} \mu_i^2 \Phi_i^\dagger \Phi_i + \lambda_i (\Phi_i^\dagger \Phi_i)^2 + \sum_{i,j=1;i<j}^{3} \lambda_{ij} (\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j)$$
$$+ \tilde{\lambda}_{ij} (\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) \quad + \mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2$$
$$+ \sum_{i=1}^{3} \lambda_{i\sigma} (\Phi_i^\dagger \Phi_i)(\sigma^* \sigma) + (\lambda_A \sigma \Phi_1 \Phi_2 \Phi_3 + H.c.) \quad (20)$$

Mass mixing matric in basis $(I_1, I_2, I_3, I_\sigma)$

$$M_I^2 = -\frac{A}{2} \begin{pmatrix} \frac{1}{v_1^2} & \frac{1}{v_1 v_2} & \frac{1}{v_1 v_3} & \frac{1}{v_1 v_\sigma} \\ & \frac{1}{v_2^2} & \frac{1}{v_2 v_3} & \frac{1}{v_2 v_\sigma} \\ & & \frac{1}{v_3^2} & \frac{1}{v_3 v_\sigma} \\ & & & \frac{1}{v_\sigma^2} \end{pmatrix} \quad (21)$$

$$A = \lambda_A v_1 v_2 v_3 v_\sigma$$

Mixing angles

$$\tan\alpha = \frac{v_1}{v_2}$$
$$\tan\alpha_1 = \frac{v_1}{v_3} |\cos\alpha|$$
$$\tan\alpha_2 = \frac{v_3}{v_\sigma} |\sin\alpha_1| \quad (22)$$

Physical states of fields in CP-odd sector

$$\begin{pmatrix} a \\ G_{Z'} \\ G_Z \\ A_5 \end{pmatrix} = \begin{pmatrix} \cos\alpha_2 & -\sin\alpha_1\sin\alpha_2 & -\sin\alpha\cos\alpha_1\sin\alpha_2 & -\cos\alpha\cos\alpha_1\sin\alpha_2 \\ 0 & \cos\alpha_1 & -\sin\alpha\sin\alpha_1 & -\cos\alpha\sin\alpha_1 \\ 0 & 0 & \cos\alpha & -\sin\alpha \\ \sin\alpha_2 & \sin\alpha_1\cos\alpha_2 & \sin\alpha\cos\alpha_1\cos\alpha_2 & \cos\alpha\cos\alpha_1\cos\alpha_2 \end{pmatrix} \begin{pmatrix} I_\sigma \\ I_3 \\ I_2 \\ I_1 \end{pmatrix} \quad (23)$$

Physical state of axion

$$a = \cos\alpha_2 I_\sigma - \sin\alpha_1 \sin\alpha_2 I_3 - \sin\alpha\cos\alpha_1\sin\alpha_2 I_2 - \cos\alpha\cos\alpha_1\sin\alpha_2 I_1 \quad (24)$$

A rule to define PQ charge of particles

1. The PQ charge of a left-handed multiplet is opposite in sign to the one of the corresponding right-handed multiplet;
2. The PQ charge of a multiplet is opposite in sign to the one of the corresponding anti-multiplet.

$$PQ_{\overline{f}_L} = -PQ_{f_L}\,, \quad PQ_{\phi^*} = -PQ_\phi\,, \quad PQ_{(S_L)^c} = -PQ_{S_L} \quad (25)$$

Field	3-3-1-1 rep	$\mathcal{B}-\mathcal{L}$	$U(1)_{PQ}$
ψ_{aL}	$(\mathbf{1},\mathbf{3},-\frac{1}{3},-\frac{1}{3})$	$(-1,-1,+1)^T$	$-\frac{PQ_\sigma}{6}$
e_{aR}	$(\mathbf{1},\mathbf{1},-1,-1)$	-1	$-\frac{PQ_\sigma}{2}$
$Q_{\alpha L}$	$(\mathbf{3},\mathbf{3}^*,0,-\frac{1}{3})$	$(\frac{1}{3},\frac{1}{3},-\frac{5}{3})^T$	$\frac{PQ_\sigma}{6}$
Q_{3L}	$(\mathbf{3},\mathbf{3},\frac{1}{3},1)$	$(\frac{1}{3},\frac{1}{3},\frac{7}{3})^T$	$-\frac{PQ_\sigma}{6}$
u_{aR}	$(\mathbf{3},\mathbf{1},\frac{2}{3},\frac{1}{3})$	$\frac{1}{3}$	$\frac{PQ_\sigma}{2}$
U_{3R}	$(\mathbf{3},\mathbf{1},\frac{2}{3},\frac{7}{3})$	$\frac{7}{3}$	$\frac{PQ_\sigma}{2}$
d_{aR}	$(\mathbf{3},\mathbf{1},-\frac{1}{3},\frac{1}{3})$	$\frac{1}{3}$	$-\frac{PQ_\sigma}{2}$
$D_{\alpha R}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3},-\frac{5}{3})$	$-\frac{5}{3}$	$-\frac{PQ_\sigma}{2}$
S_{aL}	$(\mathbf{1},\mathbf{1},0,-1)$	-1	$-\frac{PQ_\sigma}{2}$
S_{aR}	$(\mathbf{1},\mathbf{1},0,-1)$	-1	$\frac{PQ_\sigma}{2}$
Φ_1	$(\mathbf{1},\mathbf{3},-\frac{1}{3},\frac{2}{3})$	$(0,0,2)^T$	$-\frac{2}{3}PQ_\sigma$
Φ_2	$(\mathbf{1},\mathbf{3},\frac{2}{3},\frac{2}{3})$	$(0,0,2)^T$	$\frac{1}{3}PQ_\sigma$
Φ_3	$(\mathbf{1},\mathbf{3},-\frac{1}{3},-\frac{4}{3})$	$(-2,-2,0)^T$	$-\frac{2}{3}PQ_\sigma$
σ	$(\mathbf{1},\mathbf{1},0,0)$	0	PQ_σ

Yukawa coupling of two fermions with a scalar

$$\mathcal{L}_Y^f = h\bar{f}_L\phi\psi_R + H.c. \quad (26)$$

$U(1)_{PQ}$

$$\mathcal{L}_Y^{f\,\prime} = h\bar{f}_L'\phi'\psi_R' = h\bar{f}_L e^{i\alpha PQ(\bar{f}_L)}\phi e^{-i\alpha PQ(\phi)} e^{i\alpha PQ(\psi_R)}\psi_R + H.c. \quad (27)$$

$$PQ(\phi) = PQ(\bar{f}_L) + PQ(\psi_R) \quad (28)$$

$$-\mathcal{L}_Y^l = y^e_{ab}\,\overline{\psi_{aL}}\,\Phi_2 e_{bR} + y^{\nu_1}_{ab}\,\overline{\psi_{aL}}\,\Phi_1\,S_{bR} + y^{\nu_2}_{ab}\,\overline{\psi_{aL}}\,\Phi_3\,(S_{bL})^c + y^S_{ab}\,\overline{S_{aL}}\,\sigma^*\,S_{bR} + \text{H.c.} \quad (29)$$

invariant under PQ transformation

$$-\mathcal{L}_Y^q = y^u_{\alpha a}\,\overline{Q^\alpha_L}\,\Phi_2^*\,u^a_R + y^u_{3a}\,\overline{Q^3_L}\,\Phi_1\,u^a_R + y^d_{3a}\,\overline{Q^3_L}\,\Phi_2\,d^a_R + y^d_{\alpha a}\,\overline{Q^\alpha_L}\,\Phi_1^*\,d^a_R + y^U_{33}\,\overline{Q^3_L}\,\Phi_3\,U^3_R + y^D_{\alpha\beta}\,\overline{Q^\alpha_L}\,\Phi_3^*\,D^\beta_R + \text{H.c.} \quad (30)$$

invariant under PQ transformation

$$PQ_{\Phi_1} = PQ_{\Phi_3} = -2PQ_{\Phi_2} = -\tfrac{2}{3}PQ_\sigma \quad (31)$$

PQ charges of all particles depend on only PQ_σ.
It is consistently with that σ causes $U(1)_{PQ}$ spontaneous breaking.

Physical state of axion defined via $SU(3)_L \times U(1)_X$ and PQ transformation

$$\phi = \frac{1}{\sqrt{2}}(R_\phi + iI_\phi) = \rho_\phi e^{i\theta_\phi} \quad (32)$$

$$\rho_\phi = \sqrt{\tfrac{1}{2}(R_\phi^2 + I_\phi^2)}, \quad \tan_\phi = \frac{I_\phi}{R_\phi}$$

$I_\phi \ll R_\phi$

$$\phi \approx \rho_\phi e^{i\frac{I_\phi}{R_\phi}} \quad (33)$$

expand around $R_\phi = v_\phi$

$$\phi_{v_\phi} \equiv \phi|_{R_\phi = v_\phi} = \frac{v_\phi}{\sqrt{2}} e^{i\frac{I_\phi}{v_\phi}} \quad (34)$$

$U(1)_X$ **transformation**

$$\phi_{v_\phi} \to \phi'_{v_\phi} = \frac{v_\phi}{\sqrt{2}} e^{iX_\phi \frac{I_\phi}{v_\phi}} \quad (35)$$

$$\sum_{i=1}^{3} I_{\Phi_i} \frac{X_{\Phi_i}}{v_{\Phi_i}} = 0 \Big|_{X_\sigma = 0} \quad (36)$$

invarriant under $U(1)_X$ **transformation**

$U(1)_{PQ}$ **transformation**

$$\phi'_{v_\phi} \to \phi''_{v_\phi} = \frac{v_\phi}{\sqrt{2}} e^{iX_\phi \frac{I_\phi}{v_\phi} PQ_\phi} \quad (37)$$

$$\frac{\lambda_A}{4} v_\sigma v_1 v_2 v_3\, e^{i\left[I_\sigma \frac{PQ_\sigma}{v_\sigma} + I_{\Phi_1}\frac{X_{\Phi_1}}{v_1} PQ_{\Phi_1} + I_{\Phi_2}\frac{X_{\Phi_2}}{v_2} PQ_{\Phi_2} + I_{\Phi_3}\frac{X_{\Phi_3}}{v_3} PQ_{\Phi_3}\right]} = \lambda_A\, \sigma\, \Phi_1 \Phi_2 \Phi_3 \Big|_{VEV}$$

invarriant under $U(1)_{PQ}$ **transformation**

$$I_\sigma \frac{PQ_\sigma}{v_\sigma} + I_{\Phi_1}\frac{X_{\Phi_1}}{v_1} PQ_{\Phi_1} + I_{\Phi_2}\frac{X_{\Phi_2}}{v_2} PQ_{\Phi_2} + I_{\Phi_3}\frac{X_{\Phi_3}}{v_3} PQ_{\Phi_3} = 0 \quad (38)$$

General form of axion

$$a = \frac{1}{f_{PQ}}\left(\frac{PQ_\sigma}{v_\sigma} I_\sigma + \sum_{i=1}^{3} n_i \frac{PQ_{\Phi_i}}{v_i} I_i\right) = \frac{PQ_\sigma}{v_\sigma f_{PQ}}\left[I_\sigma + \sum_{i=1}^{3} n_i \frac{v_\sigma}{v_i}\frac{PQ_{\Phi_i}}{PQ_\sigma} I_{\Phi_i}\right] \quad (39)$$

with normalization factor

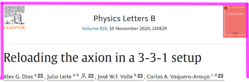
$$f_{PQ} = \left[\left(\frac{PQ_\sigma}{v_\sigma}\right)^2 + \sum_{i=1}^{3}\left(n_i \frac{PQ_{\Phi_i}}{v_i}\right)^2\right]^{1/2} \quad (40)$$

$$n_i = X_{\Phi_i}$$

Physical state of axion

$$a = \frac{\left[I_\sigma + \frac{1}{3}\frac{PQ_{\Phi_1}}{PQ_\sigma}\frac{v_\sigma}{v_1} I_{\Phi_1} - \frac{2}{3}\frac{PQ_{\Phi_2}}{PQ_\sigma}\frac{v_\sigma}{v_2} I_{\Phi_2} + \frac{1}{3}\frac{PQ_{\Phi_3}}{PQ_\sigma}\frac{v_\sigma}{v_3} I_{\Phi_3}\right]}{\left[1 + \left(\frac{1}{3}\frac{PQ_{\Phi_1}}{PQ_\sigma}\frac{v_\sigma}{v_1}\right)^2 + \left(-\frac{2}{3}\frac{PQ_{\Phi_2}}{PQ_\sigma}\frac{v_\sigma}{v_2}\right)^2 + \left(\frac{1}{3}\frac{PQ_{\Phi_3}}{PQ_\sigma}\frac{v_\sigma}{v_3}\right)^2\right]^{1/2}} \quad (41)$$

$\neq$


Physics Letters B
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Reloading the axion in a 3-3-1 setup
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$$a = \frac{1}{f_{PQ}}\left[v_1 PQ_{\Phi_1}\, a_1 + v_2 PQ_{\Phi_2}\, a_2 + w PQ_{\Phi_3}\, a_3 + v_\sigma PQ_\sigma\, a_\sigma\right] \quad (42)$$

Properties of axion in 3-3-1 model with $U(1)_{B-L}$ and PQ transformation

- Form of axion depends on ratios $\frac{PQ_{\Phi_i}}{PQ_\sigma}$ which are real number, independent from PQ charges of all particles as well as VEVs of scalar fields.
- The expression of axion physical state is inversely proportional to VEVs of scalar triplets (v_{ϕ_i}) of the model and just linearly proportional to VEV of scalar singlet (v_σ).
- The X_{Φ_i} charges of scalar triplets are always accompanied by their imaginary parts I_i in the expression of physical axion state.
- The physical state of axion must be orthogonal to Goldstone bosons G_Z and $G_{Z'}$ and a new light pseudo scalar A_5 which arise from mass mixing matrix of CP-odd sector.

Axion-fermion couplings

Axion-fermion couplings

$$-\mathcal{L}_Y^l = y^e_{ab}\,\overline{e_{aL}}\,\Phi_2 e_{bR} + \text{H.c.} = y^e_{ab}\,\overline{e_{aL}}\,\Phi_2 e_{bR} + y^e_{ab}\,\overline{e_{bR}}\,(\Phi_2)^* e_{aL} \quad (43)$$

$$\mathcal{L}_Y^{(a\,l)} = \frac{i}{\sqrt{2}} y^e_{ab} I_{\Phi_2}\,\overline{e_a}\gamma_5 e_b \quad (44)$$

$I_{\Phi_2} \supset a$

$$y^e_{ab} = \frac{\sqrt{2}}{v_2}\mathrm{diag}(\mathrm{m_e}, \mathrm{m}_\mu, \mathrm{m}_\tau)\delta_{\mathrm{ab}} \quad (45)$$

$$\mathcal{L}_Y^{(a\,l)} = -i\frac{m_l}{v_2}\sin\alpha\cos\alpha_1\sin\alpha_2\, a\,\bar{l}\gamma_5 l \quad (46)$$

$(l = e, \mu, \tau)$

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Reloading the axion in a 3-3-1 setup

Alex G. Dias [a], Julio Leite [a b], José W.F. Valle [b], Carlos A. Vaquero-Araujo [c d]

$$g_{ae} = \frac{\mathrm{diag}(m_e, m_\mu, m_\tau)}{f_a} c_{ae}$$

$$c_{ae} = \frac{C_{ae}}{C_{ag}} = \frac{PQ_{e_L} - PQ_{e_R}}{C_{ag}} = -\cos^2\delta\,\sin^2\beta,$$

$\neq$

- independent from f_a and PQ_f
- depend on mass mixing angles of CP-odd sector

$\Rightarrow$ depend on m_l, $X_{triplets}$ and PQ_{scalar} ratios and only PQ_σ

Anomaly axion-fermion couplings

depend on $f_a \equiv f_{pq}$ and $PQ_f \equiv c_f$

(as same as EM coupling of fermions and photon)

$$\mathcal{L}_f^0 = \frac{i}{2}\bar{f}\gamma^\mu \overleftrightarrow{\partial_\mu} f = \frac{i}{2}(\bar{f}\gamma^\mu\partial_\mu f - \partial_\mu\bar{f}\gamma^\mu f) \quad (47)$$

$$f \to f' = e^{i\left(\frac{c_f}{2f_{pq}}\right)\gamma_5 a} f, \quad \bar{f} \to \bar{f}' = \bar{f} e^{i\left(\frac{c_f}{2f_{pq}}\right)\gamma_5 a},$$
$$f_L \to f'_L = e^{-i\left(\frac{c_f}{2f_{pq}}\right)a} f_L, \quad \bar{f}_L \to \bar{f}'_L = \bar{f}_L e^{i\left(\frac{c_f}{2f_{pq}}\right)a},$$
$$f_R \to f'_R = e^{i\left(\frac{c_f}{2f_{pq}}\right)a} f_R, \quad \bar{f}_R \to \bar{f}'_R = \bar{f}_R e^{-i\left(\frac{c_f}{2f_{pq}}\right)a} \quad (48)$$

$$\varphi \to \varphi' = e^{i\left(\frac{c_\varphi}{2f_{pq}}\right)a}\varphi \quad (49)$$

$$\mathcal{L}_{(f-a)} = -\frac{1}{f_{pq}}\partial_\mu a\left[\bar{d}\,\mathbf{c}_d\,\gamma^\mu\gamma_5 d + \bar{u}\,\mathbf{c}_u\,\gamma^\mu\gamma_5 u + \bar{U}_3\,\mathbf{c}_{U_3}\,\gamma^\mu\gamma_5 U_3 + \bar{D}_\alpha\,\mathbf{c}_{D_\alpha}\,\gamma^\mu\gamma_5 D_\alpha + \bar{l}\,\mathbf{c}_l\gamma^\mu\,\gamma_5 l\right] \quad (50)$$

Axion-photon couplings

$$\mathcal{L}_a \supset \frac{a}{v_a}\frac{g_s^2\,N}{16\pi^2}G\tilde{G} + \frac{a}{v_a}\frac{e^2 E}{6\pi^2}F\tilde{F} + \frac{\partial^\mu a}{v_a}J_\mu^{PQ} \quad (51)$$

E: electromagnetic $[U(1)_Q]^2 \times U(1)_{PQ}$ anomaly coefficient

$$f_a = \frac{v_a}{2N} \quad (52)$$

N: the QCD anomaly coefficient

$$\mathcal{L}_a \supset \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G} + \frac{1}{4}g^0_{a\gamma}aF\tilde{F} \quad (53)$$

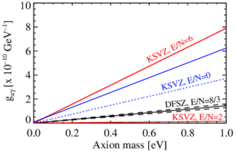
$$g^0_{a\gamma} = \frac{\alpha}{2\pi f_a}\frac{E}{N} \quad (54)$$

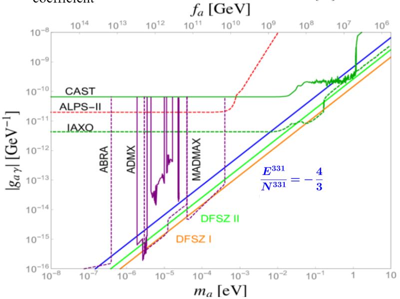
$$\begin{cases} E_{331} = -\frac{4}{3}PQ_\sigma \\ N_{331} = PQ_\sigma \end{cases} \quad (55)$$

$$g^{331}_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(-\frac{4}{3} - 1.92\right) \quad (56)$$

$$g^{DFSZ1}_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{8}{3} - 1.92\right) \quad (57)$$

$$g^{DFSZ2}_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{2}{3} - 1.92\right) \quad (58)$$



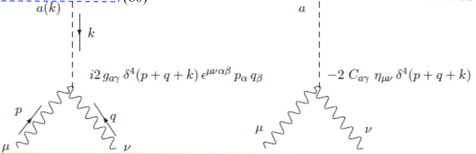


Decay of axion into a pair of photons $a \to \gamma\gamma$

$$\mathcal{L} = \mathcal{L}_{g_{a\gamma}} + \mathcal{L}_{C_{a\gamma}} = g_{a\gamma}\, aF\tilde{F} + i\, C_{a\gamma}\, aA_\mu A^\mu \quad (59)$$

$$g^{331}_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(-\frac{4}{3} - 1.92\right) \quad (60)$$

$$C_{a\gamma} = \frac{g^2}{2\,8^2}(10 + 11c_{2W})t_W^2\, s_{\alpha_2}(v_1 c_\alpha c_{\alpha_1} + 4v_2 s_\alpha c_{\alpha_1} + v_3 s_{\alpha_1}) \quad (61)$$

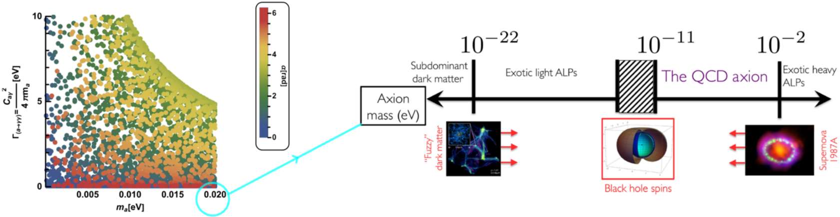


$$i2\, g_{a\gamma}\, \epsilon^{\mu\nu\alpha\beta}\, p_\alpha\, q_\beta \epsilon(p,\lambda)_\mu \epsilon(q,\sigma)_\nu - 2\, C_{a\gamma}\, \eta^{\mu\nu}\epsilon(p,\lambda)_\mu \epsilon(q,\sigma)_\nu = M_1 + M_2 = M_a \quad (62)$$

$$\Rightarrow |M_a|^2 = |M_1|^2 + |M_2|^2 = 4g^2_{a\gamma} m_a^4 + 16C^2_{a\gamma} \quad (63)$$

$$\Rightarrow \text{Decay width } \Gamma(a \to \gamma\,\gamma) = \frac{|M_a|^2}{64\pi m_a} = \frac{g^2_{a\gamma} m_a^3}{16\pi} + \frac{C^2_{a\gamma}}{4\pi m_a} \quad (64)$$

Decay of axion into a pair of photons $a \to \gamma\gamma$



$v_1, v_2 \sim$ EW scale

$v_3 \sim 10^5$ GeV

$v_\sigma \sim 10^{12}$ GeV

Which are left to work on?

1. axion-gauge boson couplings

- axion -charge gauge boson: $aW^+W^-, aY^+Y^-, aX^0X^{0*}$
- axion -neutral gauge boson: $aZZ, aZ'Z', a\gamma Z, a\gamma Z'$

2. Anomaly couplings of axion-gauge bosons

3. axion-gauge boson-charged Higgs

4. contribution of axion to $g-2$ muon via 1-loop and 2-loop corrections

Conclusions

- Applying axion theory of Peccei and Quinn to the special 3-3-1 model with $U(1)_{B-L}$ to solve Strong CP problem.
- Proposing a rule to define PQ charges of particles in the model and show that all PQ charges just depend on 1 parameter PQ_σ of σ which causes the spontaneous breaking of $U(1)_{PQ}$ symmetry.
- Physical state of axion is defined by combining the mass mixing matrix diagonalization with the invariant under $SU(3)_L \times U(1)_X$ and PQ transformation.
- The dominant contribution to the decay width of $a \to \gamma\gamma$ is from normal Higgs-gauge bosons coupling in case mass of axion is about some meV.

THANK YOU FOR YOUR ATTENTION

Thanks for the funding from National Foundation for Science and Technology Development (NAFOSTED) under grant No. 103.01-2023.50.

Backup Slide 1 - Why $a\gamma$ couplings ?

R. D. Peccei and H. Quinn, Phys.Rev. D.**16**,1791(1977)

$$\mathcal{L} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\gamma_\mu D^\mu \psi + L_{Yukawa} - V(\phi)\,, \qquad (51)$$

with transformation

$$\psi \to e^{i\sigma\gamma_5}\psi\,, \quad \bar{\psi} \to \bar{\psi}e^{i\sigma\gamma_5} \quad \phi \to e^{-2i\sigma}\phi\,.$$

then one has couplings of axion with gauge bosons

$$\mathcal{L}_{agg} = \frac{a}{v_\phi}\frac{g_s^2\,N}{16\pi^2}G\tilde{G} + \frac{a}{v_\phi}\frac{e^2 E}{6\pi^2}F\tilde{F} + \frac{\partial^\mu a}{v_\phi}J^{PQ}_\mu\,, \qquad (52)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ and

$$a(k) \to \gamma(p) + \gamma(q)$$

$$L_{eff} \supset \frac{1}{4}g_{a\gamma}aF\tilde{F} \equiv g_{a\gamma}\,a\,\mathbf{E.B} \qquad (53)$$

Backup Slide 2 - $a\gamma\gamma$ interactions

Expand scalar field around its VEV:

$$\phi' = \langle \phi' \rangle + \phi \,. \tag{1}$$

With $D_\mu = \partial^\mu + igW_\mu^a T_a + ig_X B_\mu X \equiv \left[\partial^\mu + iP_\mu^{CC} + iP_\mu^{NC}\right]$ and $T_a, (a = \overline{1,8})$ are $SU(3)_L$ generators. The kinetic term is

$$\begin{aligned}(D^\mu \phi')^\dagger D_\mu \phi &= \partial^\mu \phi \partial_\mu \phi + i\left[\langle \phi' \rangle^\dagger P_\mu^\phi \partial^\mu \phi - \partial_\mu \phi^\dagger P^{\phi\mu} \langle \phi' \rangle\right] \\ &+ i\left[\phi^\dagger P_\mu^\phi \partial^\mu \phi - \partial_\mu \phi^\dagger P^{\phi\mu} \phi\right] + \phi'^\dagger P_\mu^\phi P^{\phi\mu} \phi' \,, \end{aligned} \tag{2}$$

The last term in Eq. (2) is following

$$\begin{aligned}\phi'^\dagger P_\mu^\phi P^{\phi\mu} \phi' &= \langle \phi' \rangle^\dagger P_\mu^\phi P^{\phi\mu} \langle \phi' \rangle + \phi^\dagger P_\mu^\phi P^{\phi\mu} \phi \\ &+ i\left[\langle \phi' \rangle^\dagger P_\mu^\phi P^{\phi\mu} \phi - \phi^\dagger P_\mu^\phi P^{\phi\mu} \langle \phi' \rangle\right] . \end{aligned} \tag{3}$$

Backup Slide 2 - P_μ

$$\begin{pmatrix} W_\mu^3 + \frac{W_\mu^8}{\sqrt{3}} + \frac{g_X\sqrt{6}}{3g}XB_\mu & \sqrt{2}W_\mu^+ & \sqrt{2}X_\mu^0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{W_\mu^8}{\sqrt{3}} + \frac{g_X\sqrt{6}}{3g}XB_\mu & \sqrt{2}Y_\mu^- \\ \sqrt{2}X_\mu^{0*} & \sqrt{2}Y_\mu^+ & -\frac{2W_\mu^8}{\sqrt{3}} + \frac{g_X\sqrt{6}}{3g}XB_\mu \end{pmatrix}$$

Backup Slide 3 - The QCD anomaly coefficient

$$N = \sum_{\mathcal{Q}} N_{\mathcal{Q}} = \sum_{\mathcal{Q}} PQ(\mathcal{Q})\, n_C(\mathcal{Q})\, n_I(\mathcal{Q})\, T(\mathcal{C}_{\mathcal{Q}})\,, \qquad (5)$$

1. $n_C(\mathcal{Q})$ and $n_I(\mathcal{Q})$ - the dimension of the color and weak iso-spin representations, i.e. $(n(\mathbf{T}) = 3$ for triplet; $n(\mathbf{S}) = 1$ for singlet$)$.
2. $T(\mathcal{C}_{\mathcal{Q}})$ is the color Dynkin index, i.e. $T(3) = 1/2$.

Taking into account that $PQq_L = -\mathrm{PQ}q_R$, quantity N reads

$$N_{331} = N(Q_{\alpha L}) + N(Q_{3L}) - N(u_{aR}) - N(d_{aR}) - N(U_{3R}) - N(D_{\alpha R})\,, \qquad (6)$$

$$\begin{aligned}
N(Q_{\alpha L}) &= \frac{PQ_\sigma}{6}.3.2.\frac{1}{2} = \frac{PQ_\sigma}{2}\,, N(Q_{3L}) = -\frac{PQ_\sigma}{6}.3.1.\frac{1}{2} = -\frac{PQ_\sigma}{4}\,, \\
N(u_{aR}) &= \frac{PQ_\sigma}{2}.3.3.\frac{1}{2} = 9.\frac{PQ_\sigma}{4}\,, N(d_{aR}) = -\frac{PQ_\sigma}{2}.3.3.\frac{1}{2} = -9.\frac{PQ_\sigma}{4}\,, \\
N(U_{3R}) &= \frac{PQ_\sigma}{2}.3.1.\frac{1}{2} = 3\frac{PQ_\sigma}{4}\,, N(D_{\alpha R}) = -\frac{PQ_\sigma}{2}.3.2.\frac{1}{2} = -3\frac{PQ_\sigma}{2}
\end{aligned} \qquad (7)$$

Backup Slide 4 - The electromagnetic $[U(1)_Q]^2 \times U(1)_{PQ}$ anomaly coefficient

$$E = \sum_{i=charged} \left(PQ_{f_{iL}} - PQ_{f_{iR}}\right)\left(Q_{f_i}\right)^2 . \tag{8}$$

Number of quark colors $n_c = 3$ then

$$\begin{aligned} E_{331} &= n_c \sum_{u_a}^{a=1,2,3} 2PQ_{u_{aL}} Q_{u_a}^2 + n_c \sum_{d_a}^{a=1,2,3} 2PQ_{d_{aL}} Q_{d_a}^2 \\ &+ n_c \sum_{D_{\alpha L}}^{\alpha=1,2} 2PQ_{D_{\alpha L}} Q_{D_\alpha}^2 + n_c 2PQ_{U_{3L}} Q_{U_{3L}}^2 + \sum_{e_i}^{i=1,2,3} 2PQ_{e_{iL}} Q_{e_i}^2 \\ &= 18\left(-\frac{PQ_\sigma}{2}\right)\frac{4}{9} + 6\left(-\frac{PQ_\sigma}{2}\right)\frac{4}{9} + 18\frac{PQ_\sigma}{2}\frac{1}{9} \\ &+ 12\frac{PQ_\sigma}{2}\frac{1}{9} + 6\left(\frac{PQ_\sigma}{2}\right) = -\frac{4}{3}PQ_\sigma . \end{aligned} \tag{9}$$

Backup Slide 5 - CPODD

$Phys.Rev.D$ $\mathbf{68}, 115009, (2003)$
$Phys.Lett.B$ $\mathbf{771}, 199-205, (2017)$

The neutral scalar CP-odd sector

$Phys.Rev.D\mathbf{107}, 095030, (2023)$

$(I_\phi, I_\chi^3, I_\rho, I_\eta^1)$

$$M^2_{odd} = -\frac{A}{2}\begin{pmatrix} \frac{1}{v_\phi^2} & \frac{1}{v_\phi v_\chi} & \frac{1}{v_\phi v_\rho} & \frac{1}{v_\phi v_\eta} \\ & \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} & \frac{1}{v_\chi v_\eta} \\ & & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho} \\ & & & \frac{1}{v_\eta^2} \end{pmatrix} \quad (30)$$

$A \equiv \lambda_\phi v_\phi v_\chi v_\eta\ v_\rho$

- the diagonal matrix is not unitary
- 2 Goldstone boson
- a pseudoscalar (mass is unidentified)
- an axion (combination of 2 components)

$$a = \frac{1}{\sqrt{1+\frac{v_{\chi'}^2}{v_\phi^2}}}\left(I_\phi + \frac{v_{\chi'}}{v_\phi} I_{\chi'}\right) \quad (31)$$

The 3-3-1 model with axion - the cold dark matter candidate

- the matrix which is used to diagonalize the mass matrix is unitary:

$$\begin{pmatrix} a \\ G_{Z'} \\ G_Z \\ A_5 \end{pmatrix} = \begin{pmatrix} c_{\theta_\phi} & -s_{\theta_3}s_{\theta_\phi} & -s_\alpha c_{\theta_3}s_{\theta_\phi} & -c_\alpha c_{\theta_3}s_{\theta_\phi} \\ 0 & c_{\theta_3} & -s_\alpha s_{\theta_3} & -c_\alpha s_{\theta_3} \\ 0 & 0 & c_\alpha & -s_\alpha \\ s_{\theta_\phi} & s_{\theta_3}c_{\theta_\phi} & s_\alpha c_{\theta_3}c_{\theta_\phi} & c_\alpha c_{\theta_3}c_{\theta_\phi} \end{pmatrix}\begin{pmatrix} I_\phi \\ I_\chi^3 \\ I_\rho \\ I_\eta^1 \end{pmatrix} \quad (32)$$

- mass of the pseudoscalar

$$m^2_{A_5} = -\frac{A}{2}\left(\frac{1}{v_\phi^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2}\right) \simeq -\frac{\lambda_\phi v_\phi v_\chi}{\sin 2\alpha} \quad (33)$$

$$\lambda_\phi = -\frac{m^2_{A_5}\sin 2\alpha}{v_\phi v_\chi} \qquad \lambda_\phi < 0 \quad (34)$$

- axion-like particle (combination of 4 components)

$$a = I_\phi c_{\theta_\phi} - I_\chi^3 s_{\theta_\phi}s_{\theta_3} - I_\rho c_{\theta_3}s_\alpha s_{\theta_\phi} - I_\eta^1 c_\alpha c_{\theta_3}s_{\theta_\phi} \quad (35)$$

The 3-3-1 model with axion-like particle (ALP331)

Backup Slide 6 - CPEVEN

$$\begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2} & \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2} & \lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2} \\ \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2} & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & \frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi \\ \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & 2\lambda_1 v_\chi^2 - \frac{A}{2v_\chi^2} & \frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi \\ \lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2} & \frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi & \frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi & 2\lambda_{10} v_\phi^2 - \frac{A}{2v_\phi^2} \end{pmatrix} \quad (21)$$

$$\tan 2\alpha_2 = \frac{4c_{\alpha_3} v_\eta v_\rho (A + \lambda_6 v_\eta^2 v_\rho^2)}{A c_{\alpha_3}^2 v_\eta^2 - A v_\rho^2 + 4 v_\eta^2 v_\rho^2 (\lambda_2 v_\eta^2 - \lambda_3 c_{\alpha_3}^2 v_\rho^2)} \quad (22)$$

$$\tan 2\alpha_3 = \frac{4v_\chi \left(A + 2\lambda_5 v_\rho^2 v_\chi^2\right)}{c_{\alpha_\phi} \left(A - 4\lambda_1 v_\chi^4\right)^2} \qquad \tan 2\alpha_\phi = \frac{\lambda_{11} v_\chi}{\lambda_{10} v_\phi}. \quad (23)$$

$$\begin{pmatrix} R_\eta^1 \\ R_\rho \\ R_\chi^3 \\ R_\phi \end{pmatrix} = \begin{pmatrix} c_{\alpha_2} & s_{\alpha_2} c_{\alpha_3} & s_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & -s_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ -s_{\alpha_2} & c_{\alpha_2} c_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & -c_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ 0 & -s_{\alpha_3} & c_{\alpha_3} c_{\alpha_\phi} & -c_{\alpha_3} s_{\alpha_\phi} \\ 0 & 0 & s_{\alpha_\phi} & c_{\alpha_\phi} \end{pmatrix}^{-1} \begin{pmatrix} h_5 \\ h \\ H_\chi \\ \Phi \end{pmatrix} \quad (24)$$

Backup Slide 7 - Type1SeeSaw

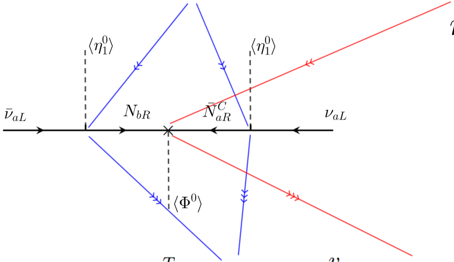
$$\mathcal{L}_Y^l = \sum_{a=1}^{3}\sum_{b=1}^{3} g_{ab}\bar{\psi}_{aL}e_{bR}\rho + \sum_{a=1}^{3}\sum_{b=1}^{3}\left(y_\nu^D\right)_{ab}\bar{\psi}_{aL}\eta N_{bR} + \sum_{a=1}^{3}\sum_{b=1}^{3}\left(y_N\right)_{ab}\phi\bar{N}_{aR}^C N_{bR} + \text{H.c.}$$

Dirac neutrino mass terms (eV scale)

Majorana neutrino mass terms 10^7 GeV

$$m_{\nu_D} = y_{ab}^{(\rho)}\left(\bar{\psi}_{aL}\right)\left(\psi_{bL}\right)^c\rho^*$$

un-invarriant under Z_2



$$M_\nu = M_\nu^D M_N^{-1}\left(M_\nu^D\right)^T, \quad M_\nu^D = y_\nu^D\frac{v_\eta}{\sqrt{2}}, \quad M_N = \sqrt{2}\,y_N v_\phi.$$

Backup Slide 8 - PLB2020

ELSEVIER

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Reloading the axion in a 3-3-1 setup

Alex G. Dias [a], Julio Leite [a b], José W.F. Valle [b], Carlos A. Vaquera-Araujo [c d]

$$\frac{PQ_{\Phi_1}}{PQ_\sigma} = -\frac{v_2^2 w^2}{v_1^2 v_2^2 + v_{EW}^2 w^2}, \quad \frac{PQ_{\Phi_2}}{PQ_\sigma} = -\frac{v_1^2 w^2}{v_1^2 v_2^2 + v_{EW}^2 w^2}, \quad \frac{PQ_{\Phi_3}}{PQ_\sigma} = -\frac{v_1^2 v_2^2}{v_1^2 v_2^2 + v_{EW}^2 w^2}.$$

$$a = \frac{1}{f_{PQ}}\left[v_1 PQ_{\Phi_1}\, a_1 + v_2 PQ_{\Phi_2}\, a_2 + w PQ_{\Phi_3}\, a_3 + v_\sigma PQ_\sigma\, a_\sigma\right]$$

$$f_{PQ} = \sqrt{PQ_\sigma^2 v_\sigma^2 + PQ_{\Phi_1}^2 v_1^2 + PQ_{\Phi_2}^2 v_2^2 + PQ_{\Phi_3}^2 w^2}$$

$$= PQ_\sigma \sqrt{v_\sigma^2 + \frac{v_1^2 v_2^2 w^2}{v_1^2 v_2^2 + v_{EW}^2 w^2}},$$

$$\frac{PQ_{\Phi_1}}{PQ_\sigma} = -(\cos\delta\cos\beta)^2, \quad \frac{PQ_{\Phi_2}}{PQ_\sigma} = -(\cos\delta\sin\beta)^2, \quad \frac{PQ_{\Phi_3}}{PQ_\sigma} = -(\sin\delta)^2$$

where δ and β are defined as

$$\tan\delta = \frac{v_1 v_2}{w v_{EW}} \quad \text{and} \quad \tan\beta = \frac{v_1}{v_2}. \quad (4$$

$$g_{ae} = \frac{\text{diag}(m_e, m_\mu, m_\tau)}{f_a} c_{ae} \quad \text{and} \quad c_{ae} = \frac{C_{ae}}{C_{ag}} = \frac{PQ_{e_L} - PQ_{e_R}}{C_{ag}} = -\cos^2\delta\, \sin^2\beta,$$

Field	3-3-1-1 rep	$B-L$	$U(1)_{PQ}$
ψ_{aL}	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{1}{3})$	$(-1, -1, +1)^T$	$\frac{1}{2}(-PQ_\sigma + PQ_{\Phi_1} + PQ_{\Phi_3})$
e_{aR}	$(\mathbf{1},\mathbf{1},-1,-1)$	-1	$\frac{1}{2}(PQ_\sigma + 3PQ_{\Phi_1} + 3PQ_{\Phi_3})$
$Q_{\alpha L}$	$(\mathbf{3}, \mathbf{3}^*, 0, -\frac{1}{3})$	$(\frac{1}{3}, \frac{1}{3}, -\frac{5}{3})^T$	$PQ_{Q_{\alpha L}}$
Q_{3L}	$(\mathbf{3}, \mathbf{3}, \frac{1}{3}, 1)$	$(\frac{1}{3}, \frac{1}{3}, \frac{7}{3})^T$	$PQ_{Q_{\alpha L}} - PQ_\sigma - PQ_{\Phi_3}$
u_{aR}	$(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{1}{3})$	$\frac{1}{3}$	$PQ_{Q_{\alpha L}} - (PQ_\sigma + PQ_{\Phi_1} + PQ_{\Phi_3})$
U_{3R}	$(\mathbf{3}, \mathbf{1}, \frac{2}{3}, \frac{7}{3})$	$\frac{7}{3}$	$PQ_{Q_{\alpha L}} - PQ_\sigma - 2PQ_{\Phi_3}$
d_{aR}	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, \frac{1}{3})$	$\frac{1}{3}$	$PQ_{Q_{\alpha L}} + PQ_{\Phi_1}$
$D_{\alpha R}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -\frac{5}{3})$	$-\frac{5}{3}$	$PQ_{Q_{\alpha L}} + PQ_{\Phi_3}$
S_{aL}	$(\mathbf{1},\mathbf{1},0,-1)$	-1	$\frac{1}{2}(PQ_\sigma - PQ_{\Phi_1} + PQ_{\Phi_3})$
S_{aR}	$(\mathbf{1},\mathbf{1},0,-1)$	-1	$\frac{1}{2}(-PQ_\sigma - PQ_{\Phi_1} + PQ_{\Phi_3})$
Φ_1	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, \frac{2}{3})$	$(0, 0, 2)^T$	PQ_{Φ_1}
Φ_2	$(\mathbf{1}, \mathbf{3}, \frac{2}{3}, \frac{2}{3})$	$(0, 0, 2)^T$	$-(PQ_\sigma + PQ_{\Phi_1} + PQ_{\Phi_3})$
Φ_3	$(\mathbf{1}, \mathbf{3}, -\frac{1}{3}, -\frac{4}{3})$	$(-2, -2, 0)^T$	PQ_{Φ_3}
σ	$(\mathbf{1},\mathbf{1},0,0)$	0	PQ_σ