String tension in magnetic field for HQGP

K.A. Rannu

Peoples Friendship University of Russia (RUDN)
Steklov Mathematical Institute (MI RAS)

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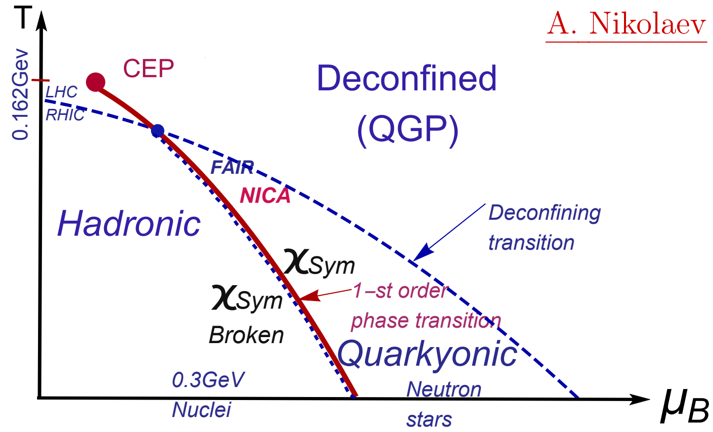
with I.Ya. Aref'eva, P. Slepov
arXiv:2505.09580 [hep-th]

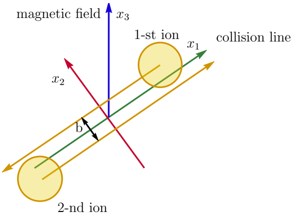
Holographic QCD phase diagram

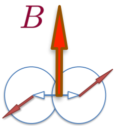
Talks by I.Ya. Aref'eva
P. Slepov
A. Golubtsova
A. Nikolaev



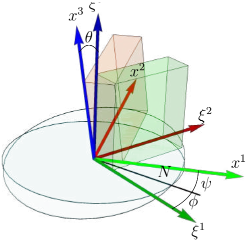
HIC

- Origins of anisotropy
  - Primary anisotropy – longitudinal and 2 transversal directions
    - Multiplicity dependencies *ALICE* $\mathcal{M}(s) \sim s^{0.155(4)}$
    - *Aref'eva, Golubtsova, JHEP (2014)* $\mathcal{M}(s) \sim s^{1/(\nu+2)} \Rightarrow \nu = 4.5$
  - Secondary anisotropy (in strong magnetic field, eB $\sim 5 - 10\ m_\pi^2$
    m_π – pion mass)

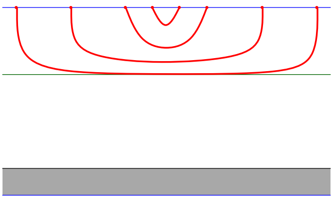




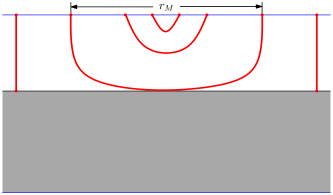
Peripheral HIC



String tension



Small black hole



Large black hole

Cornell potential

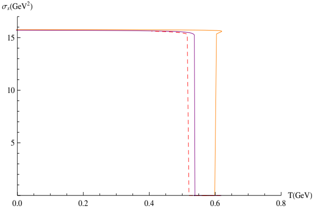
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma_s r + C$$

Aref'eva et al.

Phys.Rev.D **110** *126009 (2024)*

TMP **221** *2132 (2024)*

arXiv:2503.07521 [hep-th] (2025)



Yang, Yuan JHEP **12**, *161 (2015)*

Holographic model of anisotropic plasma in magnetic field at nonzero chemical potential

I.Aref'eva, KR'18; IA, KR, P.Slepov'21

$$S=\int d^5x\,\sqrt{-g}\left[R-\frac{f_0(\phi)}{4}\,F_{(0)}^2-\frac{f_1(\phi)}{4}\,F_{(1)}^2-\frac{f_B(\phi)}{4}\,F_{(B)}^2-\frac{1}{2}\,\partial_M\phi\partial^M\phi-V(\phi)\right]$$

$$ds^2=\frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-g(z)\,dt^2+dx^2+\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}}dy_1^2+e^{c_Bz^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}}dy_2^2+\frac{dz^2}{g(z)}\right]$$

$$A_{(1)\mu}=A_t(z)\delta_\mu^0 \qquad A_t(0)=\mu \qquad F_{(1)}=\ dy^1\wedge dy^2 \qquad F_{(B)}=\ dx\wedge dy^1$$

$\mathfrak{b}(z)=e^{2\mathcal{A}(z)}$ $\Leftrightarrow$ quarks mass

"Bottom-up approach"

Heavy quarks (c, b)

$\mathcal{A}(z)=-\ cz^2/4$ — *Andreev, Zakharov'06*

$\mathcal{A}(z)=-\ cz^2/4+p(c_B)z^4$ — *IA, Hajilou, Rannu, Slepov' 23*

Light quarks (u, d, s)

$\mathcal{A}(z)=-\,a\ln(bz^2+1)$ — *Li, Yang, Yuan'17*

$\mathcal{A}(z)=-\,a\ln((bz^2+1)(dz^4+1))$ — *Zhu, Chen, Zhou, Zhang, Huang'25*

Solution for "heavy" quarks for $(p - c_B q_3) z^4$

$$g(z) = e^{c_B z^2} \left[1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2 \big(2R_{gg} + c_B(q_3 - 1)\big) I_2(z)}{L^2 \left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2} \left(1 - \frac{I_1(z)}{I_1(z_h)} \frac{I_2(z_h)}{I_2(z)}\right) \right]$$

$$I_1(z) = \int_0^z e^{(R_{gg} - \frac{3c_B}{2})\xi^2 + 3(p - c_B q_3)\xi^4} \xi^{1+\frac{2}{\nu}} \, d\xi$$

$$I_2(z) = \int_0^z e^{(R_{gg} + \frac{c_B}{2}(\frac{q_3}{2} - 2))\xi^2 + 3(p - c_B q_3)\xi^4} \xi^{1+\frac{2}{\nu}} \, d\xi$$

$$T = \Bigg| - \frac{e^{(R_{gg} - \frac{c_B}{2}) z_h^2 + 3(p - c_B q_3) z_h^4} \, z_h^{1+\frac{2}{\nu}}}{4\pi \, I_1(z_h)} \times$$

$$\times \left[1 - \frac{\mu^2 \big(2R_{gg} + c_B(q_3 - 1)\big) \left(e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2} I_1(z_h) - I_2(z_h) \right)}{L^2 \left(1 - e^{(R_{gg} + \frac{c_B(q_3-1)}{2}) z_h^2}\right)^2} \right] \Bigg|$$

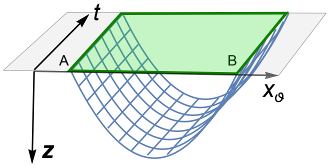
$$s = \frac{1}{4} \left(\frac{L}{z_h}\right)^{1+\frac{2}{\nu}} e^{-(R_{gg} - \frac{c_B}{2}) z_h^2 - 3(p - c_B q_3) z_h^4}$$

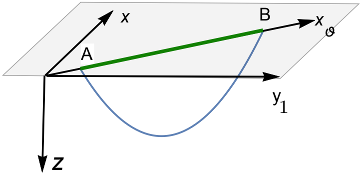
Aref'eva et al. Eur.Phys.J.C ***83*** *12 (2023) arXiv:2305.06345 [hep-th]*

Temporal Wilson loop

$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \qquad \vec{n}: \qquad n_x = \cos\vartheta, \quad n_{y_1} = \sin\vartheta, \quad n_{y_2} = 0$$

ℓ x – collision axes





Two special cases:

- $\vartheta = 0$ WL (longitudinal)
- $\vartheta = \pi/2$ WT (transversal)

$\ell \to \infty \qquad S \sim \sigma_{DW}\ \ell$

$\swarrow$

the string tension

$$\sigma_{DW} = \frac{b(z) e^{\sqrt{\frac{2}{3}}\phi(z,z_0)}}{z^2} \sqrt{g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\vartheta) + \cos^2(\vartheta)\right)}\Bigg|_{z=z_{DW}}, \qquad \frac{\partial\sigma}{\partial z}\Bigg|_{z=z_{DW}} = 0$$

*Aref'eva, K.R., Slepov PLB **792** (2019) 470 arXiv:1808.05596 [hep-th]*

Temporal Wilson Loops for $(p - c_B q_3) z^4$-term

$$\phi(z, z_0) = \int_{z_0}^{z} \frac{\sqrt{2}}{\nu\xi} \left[2(\nu - 1) + \left(6 R_{gg} \nu + (2 - 3\nu) c_B\right) \nu \xi^2 + \left(\frac{4}{3} R_{gg}^2 - c_B^2 + 60(p - c_B\, q_3) \right) \nu^2 \xi^4 + \right.$$
$$\left. + 16 R_{gg} (p - c_B\, q_3) \nu^2 \xi^6 + 48 (p - c_B\, q_3)^2 \nu^2 \xi^8 \right]^{1/2} d\xi, \qquad z_0 \neq 0$$

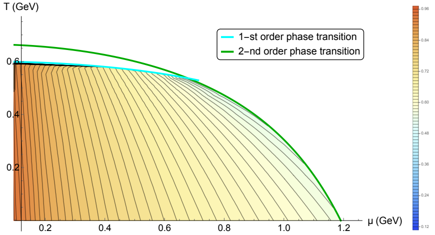
$$\mathrm{WL}x_1 : \quad - \frac{4}{3} R_{gg} z - 8(p - c_B q_3) z^3 + \sqrt{\frac{2}{3}}\, \phi'(z) + \frac{g'}{2g} - \frac{2}{z} \Big|_{z = z_{DWx_1}} = 0$$

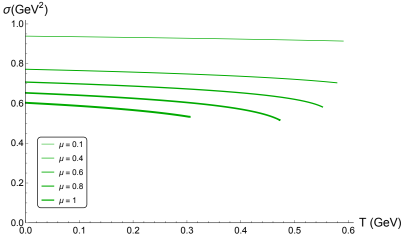
$$\mathrm{WL}x_2 : \quad - \frac{4}{3} R_{gg} z - 8(p - c_B q_3) z^3 + \sqrt{\frac{2}{3}}\, \phi'(z) + \frac{g'}{2g} - \frac{\nu + 1}{\nu z} \Big|_{z = z_{DWx_2}} = 0$$

$$\mathrm{WL}x_3 : \quad - \frac{4}{3} R_{gg} z - 8(p - c_B q_3) z^3 + \sqrt{\frac{2}{3}}\, \phi'(z) + \frac{g'}{2g} - \frac{\nu + 1}{\nu z} + c_B z \Big|_{z = z_{DWx_3}} = 0$$

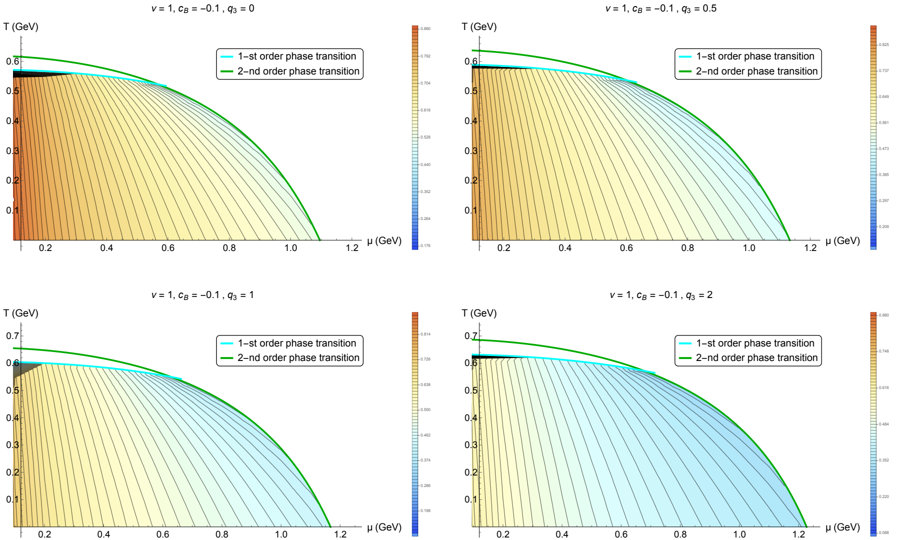
$$\sigma_{DW} = \frac{e^{-\frac{2 R_{gg} z^2}{3} - 2(p - c_B q_3 z^4)}}{z^{1 + \frac{1}{\nu}}}\; e^{\sqrt{\frac{2}{3}} \phi(z, z_0)}\; \sqrt{g(z)}$$

String tension $\sigma(\mu, T)$, no magnetic field $c_B = 0$

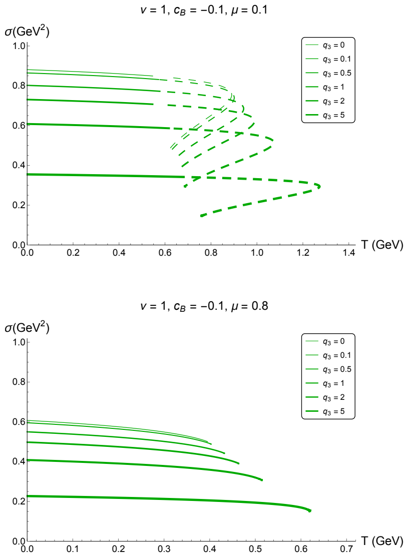




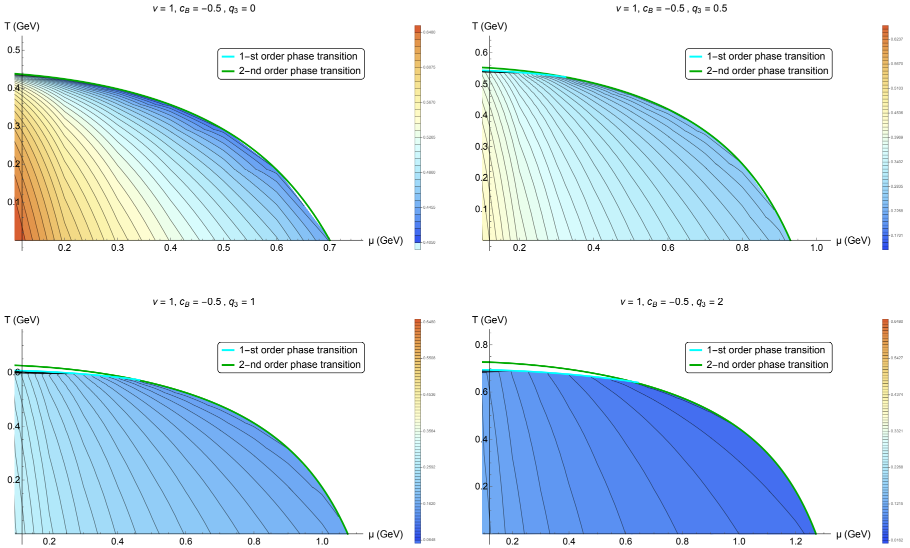
String tension $\sigma(\mu, T)$, $c_B = -0.1$



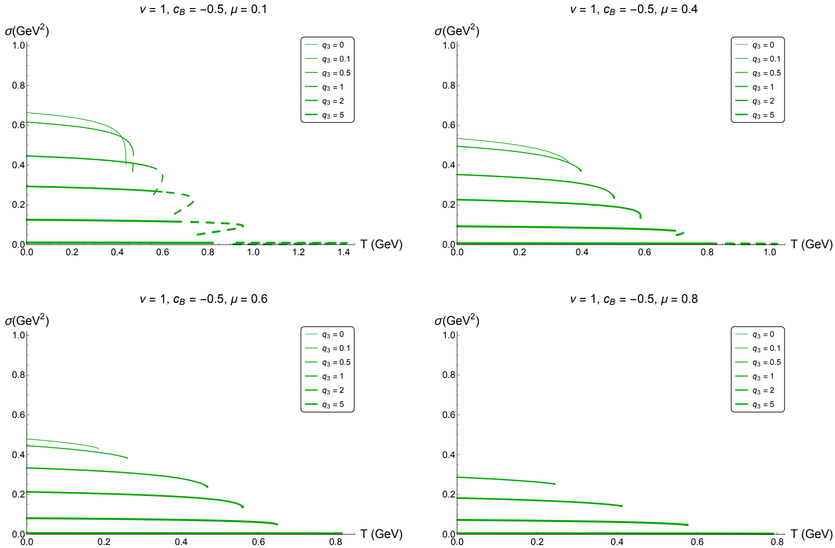
String tension $\sigma(T, \mu = \text{const})$ curves, $c_B = -0.1$



String tension $\sigma(\mu, T)$, $c_B = -0.5$



String tension $\sigma(T, \mu = \mathrm{const})$ curves, $c_B = -0.5$



Conclusion

- String tension for temporal Wilson loop is considered within HQCD model for heavy quarks with magnetic field.
- Different magnetic anisotropy parameter values in primary isotropic medium are considered.
- There exists an unstable branch "above" the 1-st order phase transition.
- String tension weakens in stronger magnetic field q_3.
- String tension weakens with magnetic anisotropy parameter absolute value $|c_B|$.

What's next?

- Anisotropy $\nu = 4.5$ should be considered.
- Cornell potential calculations.
- Fit and agree with other results.

To be continued...

**Thank you
for your attention**

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