On the kinetic mixing of the Weyl meson and vector bosons of the Standard Model

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1. Introduction

Hermann Weyl introduced a generalization of Riemann geometry in an attempt to combine gravity and electromagnetism in 1918.

The essence of Weyl's generalization is maintaining symmetry with respect to local scale transformations (Weyl transformations):

$$g_{\mu\nu}(x) \to \tilde{g}_{\mu\nu}(x) = \Omega^2(x) \cdot g_{\mu\nu}(x)$$

The corresponding Weyl gravitation theory is invariant if the action of the theory is invariant under the Weyl transformations: $S \to \tilde{S} = S$

It should be noted that for the value: $g \equiv Det\ g_{\mu\nu}$

$$\sqrt{-g} \to \sqrt{-\tilde{g}} = \Omega^4(x) \cdot \sqrt{-g}$$

Thus, since $S = \int L\sqrt{-g} \cdot d^4x$, for a theory with conformally invariant action, the Lagrangian should transform as

$$L \to \tilde{L} = \Omega^{-4}(x) \cdot L$$

The exponent of the function $\Omega(x)$ in the corresponding Weyl transform is called the Weyl weight.

1. Introduction

We will further denote the Weyl quantities (analogues of Riemannian) with a convex bracket at the top of the mathematical symbol, and the transformed quantities with a tilde line at the top of the mathematical symbol. The corresponding Weyl gravitation theory is invariant if the action S of the theory is invariant under Weyl transformations:

$$L \rightarrow \tilde{L} = \Omega^{-4}(x) \cdot L \qquad S = \int L\sqrt{-g} \cdot d^4x$$

If we try to classify modern approaches based on Weyl's ideas, then four options can be distinguished:

a) using a linear Lagrangian by Weyl curvature $\beta^2 \breve{R}$, leading to second-order equations of gravity with a non-minimal relation between the scalar field of β and gravity;

b) using a "square" of the Weyl curvature $\breve{R}^2$ in Lagrangian;

c) using a "square" of the Weyl tensor $C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$ in the Lagrangian, which leads to fourth-order equations of gravity;

d) using Einstein-Weil equations with variation of a special type of action.

A combination of a) -c) is also possible. In this report, we will focus on option a).

2. What is a Weyl meson

The Weyl vector **w** was considered by Lee Smolin in 1979 as a new field that, after quantization, leads to a particle with a mass close to the Planck scale. The concept of a vector meson arising in a Weil-Dirac gravity was apparently first introduced by Rosen (1982). This happened when they refused to interpret a Weyl vector **w** as an electromagnetic potential within the Weyl-Dirac gravity.

The appearance of the Standard Model made it possible to associate Weyl gravity with standard model fields, in particular, the Higgs field (Cheng; Drechler and Tann). This particle was considered by Cheng and named "weylon". Cheng's paper (1988) was the first description of the electroweak sector of SM in the context of Weyl geometry. Note that Cheng's article contains references to Dirac's work on gravity (1973).

Drechler and Tann in 1998 investigated the relationship of electroweak structure and Weyl geometry. They had the idea to associate the Higgs field with gravity and consider the appearance of mass in violation of local scale symmetry.

Nishino and Rajput in the early 2000s studied the problem of how the symmetries of the Standard Model can be complemented by the interaction of the scalar field and the Weyl meson.

A large work by Dumitru Gilenchea (2022) is devoted to the connection of non-integrable Weyl geometry with the quadratic Lagrangian $\check{R}^2$ and the Standard Model.

2. What is a Weyl meson

Our version of Weyl-Dirac theory assumes that the scalar Dirac function β is purely classical and is not quantized; wherein the Weyl vector **w** included in the geometric connection is gradient. That vector **S**, which we hereafter call Weylon or a Weyl meson or a Weyl vector boson, is included only in the Lagrangian L, but not in geometric connectivity. Therefore, unlike other authors, we distinguish between the concepts of the Weyl vector **w** and the Weyl meson **S**. The Weyl vector **w** is used in the definition of Weyl connectivity:

$$\breve{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu} w_{\nu} + \delta^{\lambda}_{\nu} w_{\mu} - g_{\mu\nu} w^{\lambda} \qquad w_{\nu} = \frac{\partial_{\nu}\beta(x)}{\beta} \equiv \frac{\beta_{\nu}(x)}{\beta} \equiv \partial_{\nu}(\ln\beta)$$

Vector S as a Weyl vector boson is used in the Lagrangian

$$L_{\beta} = \beta^2 R + 6\beta_{\lambda}\beta^{\lambda} + \alpha\left(\beta_{\mu} - S_{\mu}\cdot\beta\right)\cdot\left(\beta^{\mu} - S^{\mu}\cdot\beta\right) + 2\lambda\beta^4 + \omega^2 S^{\mu\nu} S_{\mu\nu}$$

where α, λ, ω are parameters, β is the real Dirac function, $S_{\mu\nu} = \partial_{\mu} S_{\nu} - \partial_{\nu} S_{\mu}$

P.Dirac. “Long range forces and broken symmetries”. Proceedings of Royal Society London A. 1973, № 333. P. 403–418.
Sedov S. Yu. “On Weyl-Dirac gravitation theory and its development”. Space, Time and Fundamental Interactions, 2023, no. 3-4, pp. 277–289.
Sedov S. Yu. “Weyl meson in the Weyl-Dirac theory as an extension of the Standard Model”. The International Seminar QUARKS-2024, Pereslavl-Zalessky, Russia, 20-24 May 2024.

2. What is a Weyl meson

Derivatives of scalars, vectors, tensors, spinors in coordinates are "extended" accordingly so that scale symmetry is observed. For a doublet of fermions ψ_L in SM, the extended covariant gauge invariant derivative is:

$$D_\mu \psi_L \to \breve{D}_\mu \psi_L = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}\left\{(1-f)w_\mu + f \cdot S_\mu\right\} + ig\vec{T}\vec{W}_\mu + \frac{i}{2} g' Y_L B_\mu \right) \psi_L$$

For singlet of fermions in SM:

$$D_\mu \psi_R \to \breve{D}_\mu \psi_R = \left(\partial_\mu + i\Gamma_\mu(x) - \frac{3}{2}\left\{(1-f)w_\mu + f \cdot S_\mu\right\} + \frac{i}{2} g' Y_R B_\mu \right) \psi_R$$

For the doublet of the Higgs field H in SM:

$$D_\mu H \to \breve{D}_\mu H = \left(\partial_\mu - \left\{(1-f)w_\mu + f \cdot S_\mu\right\} + ig\vec{T} \cdot \vec{W}_\mu + \frac{i}{2} g' B_\mu \right) H$$

Here, Y_L , Y_R are hypercharges, w_μ and B_μ are calibration fields, g and g' are coupling parameters, $\vec{T}$ is isospin matrix. In derivatives, some indices are omitted for brevity. Note that the potential of the Higgs field must include a function β so that it has a suitable Weyl weight:

$$U(H,\beta) = \lambda_H \left(H^+ H\right)^2 - \mu_H \left(H^+ H\right)\beta^2 + \xi_H \beta^4$$

3. On the interaction of the Weyl meson and fermions of the Standard Model

The interaction of Weyl mesons and fermions has been considered in a number of works, for example, see [Hayashi, Cheng, Gilencea]. The result of the consideration is as follows. The kinetic terms for the dark scalar and Higgs doublet H can provide mediated interaction with the Weyl meson, but there is no direct interaction of the Weyl mesons and spinors. The fact is that the Hermitian term in the Lagrangian

$$\frac{i}{2}\left(\bar{\psi}\cdot\gamma^{\mu}\vec{\tilde{D}}_{\mu}\psi-\bar{\psi}\cdot\gamma^{\mu}\overleftarrow{\tilde{D}}_{\mu}\psi\right)$$

does not contain a Weyl meson S_{μ} , which is simply reduced by two terms in parenthesis. The Weyl meson in the Lagrangian behaves as the imaginary part of a vector boson. This analogy helps to understand why the spinor Lagrangian should not contain the Weyl meson. Otherwise, there will be a violation of the Hermitian of the Lagrangian.
So, the Weyl meson does not interact directly with spinors.

K. Hayashi, M. Kasuya and T. Shirafuji. “Elementary Particles and Weyl’s Gauge Field”. Prog. Theor. Phys., V. 57, 1977.
Cheng, Hung. “Possible existence of Weyl’s vector meson”. Physical Review Letters N 61.P 2182–2184 (1988).
D. M. Ghilencea. “Standard Model in Weyl conformal geometry”. Eur. Phys. J. C, 82:23, 2022; arXiv:2104.15118v6 [hep-ph] 22 Nov 2022.

4. On the kinetic mixing of the Weyl meson. First mixing option. Mixing S and A

An effective Lagrangian to describe the interaction of the electromagnetic field and the Weyl meson:

$$L_{AS} = S_{\mu\nu}S^{\mu\nu} + \delta^2 F_{\mu\nu}F^{\mu\nu} - 2\varepsilon_A\delta \cdot F_{\mu\nu}S^{\mu\nu} + \alpha S_\mu S^\mu - 4J_A^\mu A_\mu - 4J_S^\mu S_\mu$$

Here, $\alpha = -2M_S^2$ J_A and J_S are electromagnetic and "weylon" currents associated with the scalar φ.

For the strengths of vectors A_μ and S_μ , the next equations take place:

$$S^{\mu\nu}{}_{;\mu} + M_S^2 S^\nu = \varepsilon_A\delta \cdot M_S^2 F^{\mu\nu}{}_{;\mu} - \xi\left[\partial^\mu\left(\varphi\varphi^*\right) - 2S^\mu\left(\varphi\varphi^*\right)\right]$$

$$\delta^2 F^{\mu\nu}{}_{;\mu} = \varepsilon_A\delta M_S^2 S^{\mu\nu}{}_{;\mu} - \xi\left[i\varphi^*\partial^\mu\varphi - i\varphi\cdot\partial^\mu\varphi^* - 2A^\mu\left(\varphi\varphi^*\right)\right]$$

Re-denote the values: $\tilde{A}_\mu = \delta A_\mu$, $\varepsilon_A = \sin\chi_A$, $M_S^2 = \cos^2\chi_A \cdot m_{DF}^2$. Then

$$\tilde{L}_{AS} = -\frac{1}{4}L_{AS} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}S_{\mu\nu}S^{\mu\nu} + \frac{1}{2}\sin\chi_A\cdot\tilde{F}_{\mu\nu}\tilde{S}^{\mu\nu} + \frac{1}{2}\cos^2\chi_A\cdot m_{DF}^2 S_\mu S^\mu + J_A^\mu\tilde{A}_\mu + J_S^\mu S_\mu$$

To diagonalize the Lagrangian, one more replacement should be made. The result is as follows:

$$\left\{A^\mu, S^\mu\right\} \to \left\{\tilde{A}^\mu = \delta\cdot A^\mu, \tilde{S}^\mu = S^\mu\right\} \to \left\{\tilde{\tilde{A}}^\mu = \cos\chi_A\cdot\tilde{A}^\mu, \tilde{\tilde{S}}^\mu = S^\mu - \sin\chi_A\cdot\tilde{A}^\mu\right\}$$

For more information, see: $C^\mu = A^\mu + iS^\mu$

S. Yu. Sedov. "Analog of Weyl Meson As Analog of the Dark Photon". Moscow University Physics Bulletin, 2024, Vol. 79, Suppl. 1, pp. S445–S446.

5. On the kinetic mixing of the Weyl meson. Second mixing option. Mixing S and B

Suppose that the kinetic mixing of the Weyl meson and the SM vector bosons takes place before the gauge symmetry is spontaneously broken. Kinetic mixing can take place between S_μ and B_μ , leading to this term in the Lagrangian of the extended SM type:

$$L \supset -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\varepsilon_B}{2} B_{\mu\nu} S^{\mu\nu} - \frac{1}{4} S_{\mu\nu} S^{\mu\nu}$$

This type of Weyl meson mixing is discussed in [D. M. Ghilencea, arXiv:2104.15118v6, 22 Nov 2022]. After breaking the Weyl symmetry, the part of the Lagrangian associated with kinetic mixing of this type is (in our notation):

$$L \supset \left\{ -\frac{M_P^2}{2} R + M_S^2 \cdot S_\mu S^\mu - \frac{1}{4} \left[S_{\mu\nu}^2 + 2 \sin\chi_B \cdot S_{\mu\nu} B^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right] \right\}$$

Here, $\varepsilon_B = \sin\chi_B$. The Weyl meson mass M_S arises from the Stückelberg mechanism when the Weyl symmetry is broken.

The diagonalization of kinetic terms in the Lagrangian is carried out by the transformation (Ghilencea):

$$S_\mu = \tilde{S}_\mu \sec\chi_B = \frac{\tilde{S}_\mu}{\cos\chi_B} \qquad B_\mu = \tilde{B}_\mu - \tilde{S}_\mu \tan\chi_B$$

5. On the kinetic mixing of the Weyl meson. Second mixing option. Mixing S and B

The transformed density of the Lagrangian is:

$$L \supset \left\{ -\frac{M_P^2}{2} R + M_S^2 \sec^2 \chi_B \cdot \tilde{S}_\mu \tilde{S}^\mu - \frac{1}{4} \left[\tilde{S}_{\mu\nu}^2 + \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} \right] \right\}$$

After spontaneous violation of gauge symmetry, an admixture of the original Weyl meson S_μ appears in the electromagnetic potential A_μ :

$$\tilde{A}_\mu = c_w \tilde{B}_\mu + s_w W_\mu^3 = c_w (B_\mu + S_\mu \sin \chi_B) + s_w W_\mu^3 = A_\mu + c_w S_\mu \sin \chi_B$$

Accordingly, the Z boson is represented by a linear combination:

$$\tilde{Z}_\mu = -s_w \tilde{B}_\mu + c_w W_\mu^3 = -s_w (B_\mu + S_\mu \sin \chi_B) + c_w W_\mu^3$$

Here, $c_w = \cos \theta_w$, $s_w = \sin \theta_w$, θ_w is Weinberg's angle.

5. On the kinetic mixing of the Weyl meson. Second mixing option. Mixing S and B

Due to the Higgs mechanism, the Z-boson gains mass. After that, the mass terms of vector bosons should be reduced to canonical form by orthogonal transformations. Transformations of this type are described in various publications; for example, see the preprint [Yu-Pan Zeng, Chengfeng Cai, Yu-Hang Su and Hong-Hao Zhang; arXiv: 2204.09487 v3 (hep-ph) 17 Feb 2023]:

$$\frac{1}{2}\left(W^{3\mu}, B^{\mu}, S^{\mu}\right)\begin{pmatrix} g^2v^2/4 & -gg'v^2/4 & 0 \\ -gg'v^2/4 & g'^2v^2/4 & 0 \\ 0 & 0 & M_s^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \\ S_\mu \end{pmatrix}$$

$$=\frac{1}{2}\left(\tilde{\tilde{A}}^{\mu}, \tilde{\tilde{Z}}^{\mu}, \tilde{\tilde{S}}^{\mu}\right)\begin{pmatrix} 0 & 0 & 0 \\ 0 & \tilde{\tilde{M}}_Z^2 & 0 \\ 0 & 0 & \tilde{\tilde{M}}_s^2 \end{pmatrix}\begin{pmatrix} \tilde{\tilde{A}}_\mu \\ \tilde{\tilde{Z}}_\mu \\ \tilde{\tilde{S}}_\mu \end{pmatrix}$$

Where $\tilde{\tilde{M}}_Z$ and $\tilde{\tilde{M}}_S$ depend on $\sin\chi_B$. At the same time, additional terms appear in currents.

5. On the kinetic mixing of the Weyl meson. Second mixing option. Mixing S and B

The fact is that canonical quantities contain linear combinations $W^{3\mu}, B^{\mu}, S^{\mu}$ (for example, see [K.S. Babu, Christopher Kolda and John March-Russell; arXiv:hep-ph/9710441v1 22 Oct 1997]):

$$\tilde{\tilde{A}}_\mu = c_w \tilde{B}_\mu + s_w W_\mu^3 = c_w (B_\mu + S_\mu \sin\chi_B) + s_w W_\mu^3 = A_\mu + c_w S_\mu \sin\chi_B$$

$$\tilde{\tilde{Z}}_\mu = \cos\xi_M \left(-s_w B_\mu + c_w W_\mu^3\right) + \left(-\cos\xi_M \cdot \sin\chi_B \cdot s_w + \sin\xi_M \cos\chi_B\right) S_\mu$$

$$\tilde{\tilde{S}}_\mu = \left(\cos\xi_M \cos\chi_B + \sin\xi_M \sin\chi_B \cdot s_w\right) S_\mu - \sin\xi_M \left(-s_w B_\mu + c_w W_\mu^3\right)$$

where

$$\tan 2\xi_M = \frac{-2\cos\chi_B M_Z^2 \cdot s_w \sin\chi_B}{M_S^2 - M_Z^2 \cos^2\chi_B + M_Z^2 s_w^2 \sin^2\chi_B}$$

For the case of small corrections to masses at $\chi_B \ll 1$, and in the case of $M_S > M_Z$, one can write:

$$\tilde{\tilde{M}}_Z^2 \approx M_Z^2 - \frac{M_Z^2 - M_W^2}{M_S^2 - M_Z^2} (\chi_B)^2 \qquad \tilde{\tilde{M}}_S^2 \approx M_S^2 + \frac{M_Z^2 - M_W^2}{M_S^2 - M_Z^2} (\chi_B)^2$$

5. On the kinetic mixing of the Weyl meson. Second mixing option. Mixing S and B

Despite the fact that the Weyl meson S_μ , generally speaking, has a non-zero mass, its mixing with A_μ does not lead to a non-zero photon mass. But the main problem is that charge parity S_μ and A_μ are different [D.M.Ghilencea; arXiv:2104.15118v6]. A_μ is charge odd field, S_μ is charge even field. Thus, the canonical electromagnetic field in this model does not have a certain charge parity.

When kinetic mixing is taken into account, additional interaction members appear. In addition to the standard fermionic term

$$\tilde{\tilde{L}}_A = e \cdot \bar{\psi}_i \gamma^\mu Q^i \tilde{\tilde{A}}_\mu \psi_i$$

this term appears:

$$\tilde{\tilde{L}}_S = -\frac{e}{2s_w c_w} \bar{\psi}_i \gamma^\mu \left\{ \left(\tilde{\tilde{h}}_V^i - \tilde{\tilde{g}}_V^i \xi_M \right) - \left(\tilde{\tilde{h}}_A^i - \tilde{\tilde{g}}_A^i \xi_M \right) \gamma^5 \right\} \tilde{\tilde{S}}_\mu \psi_i$$

that is, the Weyl meson S_μ begins to interact with electrons. Values $\tilde{\tilde{h}}_V^i - \tilde{\tilde{g}}_V^i \xi_M$ and $\tilde{\tilde{h}}_A^i - \tilde{\tilde{g}}_A^i \xi_M$ are defined in [K.S. Babu, Christopher Kolda and John March-Russell; arXiv:hep-ph/9710441v1 22 Oct 1997]).

6.On the kinetic mixing of the Weyl meson. Third mixing option. Mixing S and Z

This option of mixing is considered by analogy with the work of [Yu-Pan Zeng, Chengfeng Cai, Yu-Hang Su and Hong-Hao Zhang; arXiv:2204.09487v3 (hep-ph) 17 Feb 2023]. In the phenomenological model "Derivative Portal Dark Matter," the Lagrangian is taken as:

$$\begin{aligned}\mathcal{L} =& -\frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} - \frac{1}{4}Z'^{\mu\nu}Z'_{\mu\nu} - \frac{\epsilon}{2}Z^{\mu\nu}Z'_{\mu\nu} \\ &+ \sum_f Z_\mu \bar{f}\gamma^\mu(g_V - g_A\gamma^5)f + g_\chi Z'_\mu \bar{\chi}\gamma^\mu\chi \\ &+ \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{1}{2}m_{Z'}^2 Z'_\mu Z'^\mu - m_\chi \bar{\chi}\chi.\end{aligned}$$

In our case, we must put $g_\chi = 0$, $m_\chi = 0$, $Z'_\mu = S_\mu$, $Z'_{\mu\nu} = S_{\mu\nu}$.

Let's enter the symbols: $k_1 = \frac{1}{\sqrt{2-2\varepsilon}}$, $k_2 = \frac{1}{\sqrt{2+2\varepsilon}}$.

After conversion to canonical form, the mass of the Z boson is:

$$m_Z^2 = \frac{1}{8k_1^2k_2^2}(m_{\hat{Z}}^2 + m_{\hat{Z}'}^2) - \sqrt{\frac{1}{64k_1^4k_2^4}(m_{\hat{Z}}^2 + m_{\hat{Z}'}^2)^2 - \frac{1}{4k_1^2k_2^2}m_{\hat{Z}}^2 m_{\hat{Z}'}^2}.$$

6.On the kinetic mixing of the Weyl meson. Third mixing option. Mixing S and Z

The interaction Lagrangians for charged current and neutral current are:

$$\mathcal{L}_{CC,Wff} = -\frac{e}{\sqrt{2}\hat{s}_w}\left(1 - \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} + \frac{\hat{c}_w^2 \alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)} + \frac{\alpha U}{8\hat{s}_w^2}\right)\sum_{ij} V_{ij}\bar{f}_i\gamma^\mu\gamma_L f_j W_\mu^\dagger + \text{c.c.}$$

$$\mathcal{L}_{NC,\hat{Z}ff} = \frac{e}{\hat{s}_w\hat{c}_w}\left(1 + \frac{\alpha T}{2}\right)\sum_f \bar{f}\gamma^\mu\left[T_f^3\frac{1-\gamma^5}{2} - Q_f\left(\hat{s}_w^2 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2\hat{s}_w^2\alpha T}{\hat{c}_w^2 - \hat{s}_w^2}\right)\right] f\hat{Z}_\mu$$

Symbols $\hat{s}_w = \sin\hat{\theta}_w$, $\hat{c}_w = \cos\hat{\theta}_w$ and next parameters are entered here:

$$\hat{s}_w\hat{c}_w m_{\hat{Z}} = s_w c_w \frac{1}{2}\sqrt{g^2 + g'^2}v = \frac{1}{2}ev = s_w c_w m_Z$$

$$\alpha T = 2\left(\frac{\hat{s}_w\hat{c}_w}{s_w c_w}(-k_2\sin\theta - k_1\cos\theta) - 1\right)$$

$$\alpha S = 4\hat{c}_w^2\hat{s}_w^2\alpha T + 4(\hat{c}_w^2 - \hat{s}_w^2)(s_w^2 - \hat{s}_w^2)$$

$$\alpha U = 8\hat{s}_w^2\left(\frac{\hat{s}_w}{s_w} - 1 + \frac{\alpha S}{4(\hat{c}_w^2 - \hat{s}_w^2)} - \frac{\hat{c}_w^2\alpha T}{2(\hat{c}_w^2 - \hat{s}_w^2)}\right).$$

See: Yu-Pan Zeng, Chengfeng Cai, Yu-Hang Su and Hong-Hao Zhang; arXiv:2204.09487v3 (hep-ph), 17 Feb 2023.

6.On the kinetic mixing of the Weyl meson. Third mixing option. Mixing S and Z

Changes in coupling parameters in comparison with the values of these parameters in CM shall be within the permissible experimental limits.

$$L_{NC,\hat{Z}ff} = \sum_f (-k_2 \sin\theta - k_1 \cos\theta) \hat{Z}_\mu \bar{f} \gamma^\mu (g_V - g_A \gamma^5) f$$

$$= \sum_f (-k_2 \sin\theta - k_1 \cos\theta) \hat{Z}_\mu \bar{f} \gamma^\mu \frac{e}{s_w c_w} (T_f^3 \frac{1-\gamma^5}{2} - Q_f s_w^2) f,$$

In this report, we do not aim to compare parameters with observed values. In our case, the emerging interaction of the Weyl meson with fermions f:

$$L_{NC,\hat{S}} = \sum_f (-k_1 \sin\theta + k_2 \cos\theta)\, \hat{S}_\mu \bar{f} \gamma^\mu \left(g_V - g_A \gamma^5\right) f$$

$$L_{NC,\hat{S}} = \sum_f (-k_1 \sin\theta + k_2 \cos\theta)\, \hat{S}_\mu \bar{f} \gamma^\mu \frac{e}{s_w c_w} \left(T_f^3 \frac{1-\gamma^5}{2} - Q_f s_w^2 \right) f$$

Here, the angle θ is given by ($M_S > M_Z$):

$$\tan 2\theta = \frac{2k_1 k_2 \left(M_S^2 - M_Z^2\right)}{\left(k_1^2 - k_2^2\right)\left(M_S^2 + M_Z^2\right)}$$

Thus, in this mixing, the electromagnetic potential A_μ remains unchanged. This option looks quite realistic.

7. On the kinetic mixing of the Weyl meson. Fourth mixing option. Mixing S, B and W

In preprints [M.Bauer,P.Foldenauer,arXiv:2207.00023; P.Foldenauer, arXiv:2303.17433] the following operator is considered, invariant with respect to $U(1)_X$ and $SU(2)_L$ transformations in the case of the presence of an additional dark photon:

$$\mathcal{O}_{WX} = \frac{c_{WX}}{\Lambda^2} H^\dagger \sigma^i H \, W^i_{\mu\nu} X^{\mu\nu}$$

Here, σ^i is sigma matrices, H is Higgs doublet, and $W^i_{\mu\nu}$ is $SU(2)_L$ - invariant tensor of the strength of vector gauge fields of the Standard Model.

Thus, by analogy with this expression, we can introduce the following additive into the Lagrangian in the case of the Weyl meson S^μ with intensity $S^{\mu\nu}$:

$$L_{WS} = \frac{c_{WS}}{\beta^2} H^+ \sigma^i H \cdot W^i_{\mu\nu} S^{\mu\nu}$$

Here, β is the Dirac function. After breaking the Weyl symmetry and spontaneously breaking the symmetry, this operator can be written as:

$$L_{WS} = -\frac{1}{2} \varepsilon_W W^3_{\mu\nu} S^{\mu\nu}$$

7. On the kinetic mixing of the Weyl meson. Fourth mixing option. Mixing S, B and W

The kinetic part of the Lagrangian can be written as follows:

$$\mathcal{L} = -\frac{1}{4}(B_{\mu\nu}, W^3_{\mu\nu}, X_{\mu\nu}) \begin{pmatrix} 1 & 0 & \epsilon_B \\ 0 & 1 & \epsilon_W \\ \epsilon_B & \epsilon_W & 1 \end{pmatrix} \begin{pmatrix} B^{\mu\nu} \\ W^{3\mu\nu} \\ X^{\mu\nu} \end{pmatrix}$$
$$- g' j^Y_\nu B^\nu - g\, j^3_\nu W^{3\nu} - g_x j^x_\nu X^\nu \,,$$

Here X^ν is the dark photon, which in this case means the Weyl meson S^ν .

After spontaneous symmetry breaking, the Lagrangian is written like this:

$$\mathcal{L}_{\rm em+Z} = -\frac{1}{4}(F_{\mu\nu}, Z_{\mu\nu}, X_{\mu\nu}) \begin{pmatrix} 1 & 0 & \epsilon_A \\ 0 & 1 & \epsilon_Z \\ \epsilon_A & \epsilon_Z & 1 \end{pmatrix} \begin{pmatrix} F^{\mu\nu} \\ Z^{\mu\nu} \\ X^{\mu\nu} \end{pmatrix}$$
$$- e\, j^{\rm em}_\nu A^\nu - \frac{g}{c_w} j^Z_\nu Z^\nu - g_x j^x_\nu X^\nu \,,$$

The corresponding currents are recorded as:

$$j^{\rm em}_\nu = g'\, c_w\, j^Y_\nu + g\, s_w\, j^3_\nu \,, \qquad \epsilon_A = c_w\, \epsilon_B + s_w\, \epsilon_W \,,$$
$$j^Z_\nu = -g'\, s_w\, j^Y_\nu + g\, c_w\, j^3_\nu \,, \qquad \epsilon_Z = -s_w\, \epsilon_B + c_w\, \epsilon_W \,.$$

7. On the kinetic mixing of the Weyl meson. Fourth mixing option. Mixing S, B and W

Since the Weyl meson S_μ is C-even, its kinetic mixing with a photon A_μ is undesirable.

Therefore, we believe: $\varepsilon_A = c_w \varepsilon_B + s_w \varepsilon_W = 0$ $\quad \varepsilon_W = -\frac{c_w}{s_w}\varepsilon_B$

It follows that $\varepsilon_Z = -s_w \varepsilon_B + c_w \varepsilon_W = -\frac{\varepsilon_B}{s_w}$ $\quad$ So, $L \supset -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} S_{\mu\nu} S^{\mu\nu} - \frac{\varepsilon_Z}{2} Z_{\mu\nu} S^{\mu\nu}$

That is, we obtained the phenomenological variant of mixing S and Z discussed in the previous section.

Remark.

Let $\breve{Z}^\mu = Z^\mu + iS^\mu$ where S^μ is dual partner of the vector boson Z^μ.

Can there be global or local symmertry about the rotation of this vector? If so, could the complex vector field vector W^μ used in the complex gauge transformation be identified with the charged $W^{\pm\mu}$ -bosons of SM?

See: C. A. Z. Vasconcellos, D. Hadjimichef; CP Violation in Dual Dark Matter; arXiv:1404.0409 [hep-ph] , [Submitted on 1 Apr 2014].

8. Weyl meson and Higgs boson

The connection of the Higgs boson H and the Weyl-Dirac theory of gravity follows from the expression for the derivative after a spontaneous violation of Weyl symmetry (at β=1):

$$\breve{D}_\mu H = \left(\partial_\mu - f\cdot S_\mu + ig\vec{T}\cdot\vec{W}_\mu + \frac{i}{2}g'B_\mu\right)H \qquad H = \begin{pmatrix} 0 \\ \frac{v_H + h}{\sqrt{2}} \end{pmatrix}$$

After diagonalizing the kinetic term for the Higgs boson, the following expressions are obtained:

$$\gamma_H = \sqrt{1 - \frac{f^2 v_H^2}{\tilde{M}_S^2}} \qquad \tilde{M}_S = \sqrt{M_S^2 + f^2 v_H^2} \qquad \tilde{M}_H = \frac{v_H\sqrt{2\lambda_H}}{\gamma_H} \qquad \tilde{h} = \gamma_H h \qquad \tilde{S}_\mu = S_\mu - \partial_\mu h\cdot f\frac{v_H}{\tilde{M}_S^2}$$

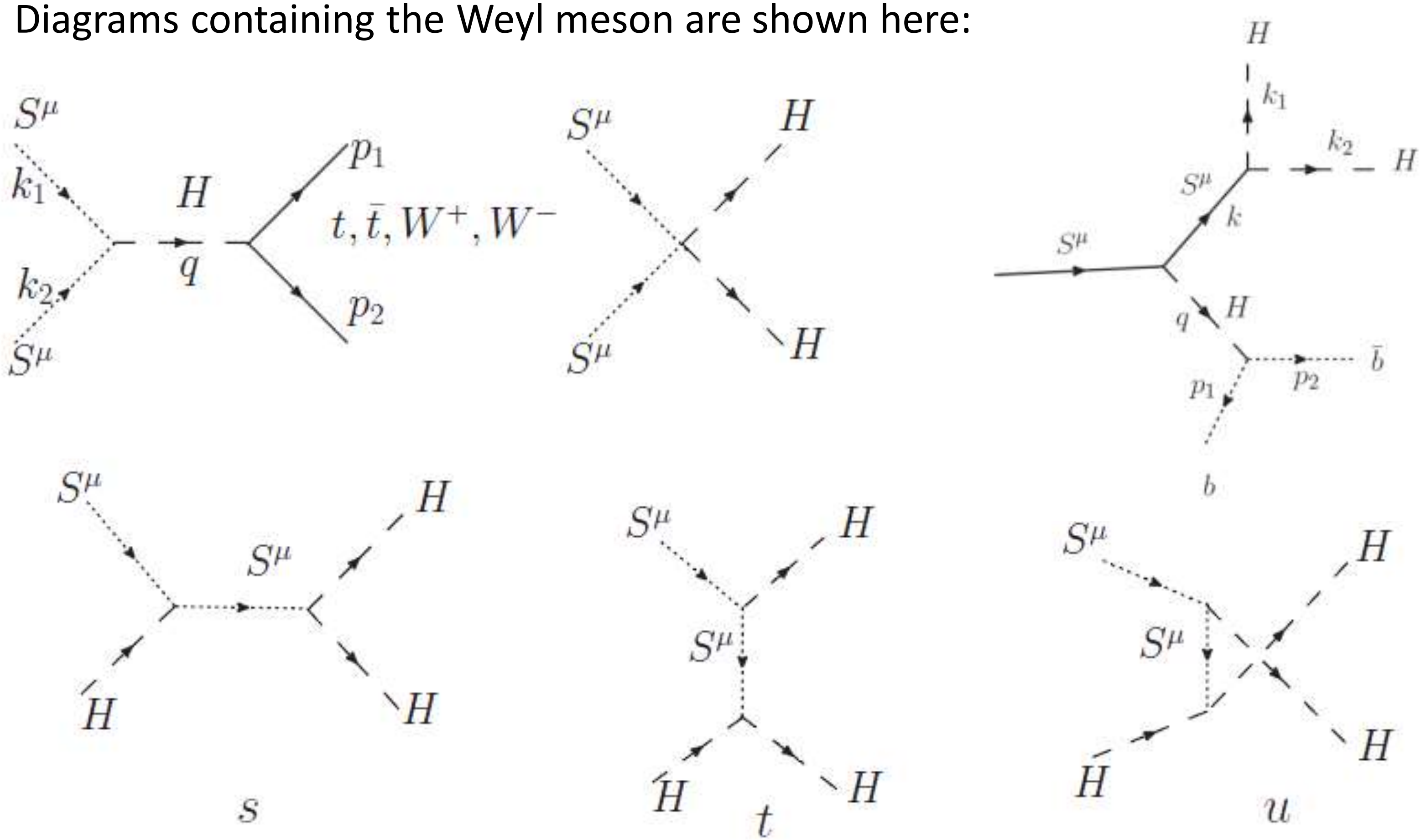
$$L \supset \frac{1}{2}\partial^\mu\tilde{h}\cdot\partial_\mu\tilde{h} - \frac{1}{2}\tilde{M}_H^2\tilde{h}^2 - \frac{1}{4}\tilde{S}_{\mu\nu}\tilde{S}^{\mu\nu} + \frac{1}{2}\tilde{M}_S^2\tilde{S}^\mu\tilde{S}_\mu + L_{\text{int}}$$

$$L_{\text{int}} = \frac{1}{2}f^2\left(\frac{2v_H\tilde{h}}{\gamma_H} + \frac{\tilde{h}^2}{\gamma_H^2}\right)\left(\tilde{S}^\mu\tilde{S}_\mu + \frac{2fv_H}{\tilde{M}_S^2\gamma_H}\tilde{h}\cdot\partial_\mu\tilde{h}\right) - \frac{\lambda_H\tilde{h}^3}{\gamma_H^3}\left(v_H + \frac{\tilde{h}}{4\gamma_H}\right)$$

See: Yong Tang and Yue-Liang Wub, “Weyl Symmetry Inspired Inflation and Dark Matter”, [hep-ph], 9 Apr. 2019.

8. Weyl meson and Higgs boson

Diagrams containing the Weyl meson are shown here:

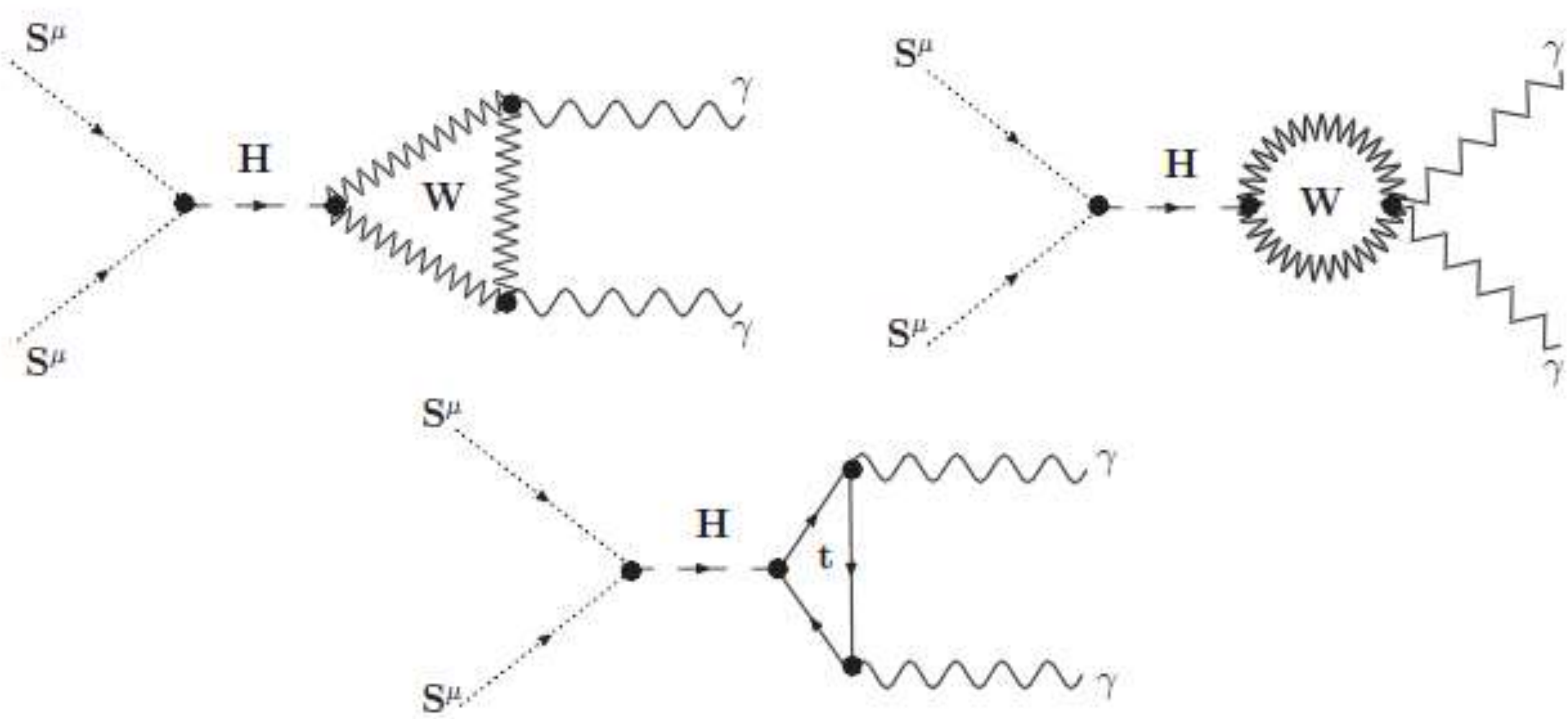


Here, we have re-designated $\tilde{h} \equiv H$, $\tilde{S}_\mu \equiv S_\mu$

See : Gopal Kashyap, "Weyl meson and its implications in collider physics and cosmology", Phys. Rev. D 87, 016018, 2013.

8. Weyl meson and Higgs boson

Diagrams of the conversion of Weyl mesons to gamma quanta are shown here.



Here, we have re-designated $\tilde{h} \equiv H$, $\tilde{S}_{\mu} \equiv S_{\mu}$

See: Gopal Kashyap, "Limits on the Weyl meson parameters due to Fermi-LAT gamma-ray observations", arXiv:1405.0679v2 [hep-ph] 13 Aug 2014.

8. Weyl meson and Higgs boson

If $\tilde{M}_H > 2\tilde{M}_S$, then decay $H \to S_\mu + S_\mu$ is possible. Here we have re-designated $\tilde{h}$ on H, $\tilde{S}_\mu$ on S_μ . The decay width Γ is written as:

$$\Gamma\left(H \to S_\mu + S_\mu\right) = \frac{f^4}{32\pi}\frac{v_H^2 \tilde{M}_H^3}{\tilde{M}_S^4}\left(1 - x_S + \frac{3}{4}x_S^2\right)\sqrt{1 - x_S}$$

where $x_S = \dfrac{4\tilde{M}_S^2}{\tilde{M}_H^2}$

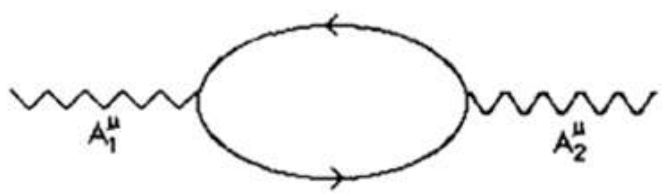
See: Yong Tang and Yue-Liang Wub, "Weyl Symmetry Inspired Inflation and Dark Matter", [hep-ph], 9 Apr. 2019.

The question of the origin of kinetic mixing of dark vector bosons and Z bosons of the Standard Model has been considered in a number of publications. The following works are mentioned here:

Pierce Giffin et al. "Higgs production in association with a dark-Z at future electron positron colliders" , J. Phys. G: Nucl. Part. Phys., 49, 015003, 2022.
Thomas G. Rizzo, "Kinetic mixing and portal matter phenomenology", Phys.Rev. D 99, 115024, 2019.
Yu-Pan Zeng, "Derivative Portal Dark Matter", arXiv:2203.09462v3 [hep-ph], 29 Apr. 2023.

8. Weyl meson and Higgs boson

Mixing dark photons and ordinary photons at the quantum correction level can be implemented in various ways (see, for example, [Tony Gherghetta, Jörn Kersten, Keith Olive and Maxim Pospelov, “Evaluating the price of tiny kinetic mixing”, Phys. Rew. D 100, 095001, 2019]. A simple option to explain the kinetic mixing of ordinary photons and dark photons, which was first considered by Bob Holdom (1985), for the Weyl meson S we consider unsuitable (see the figure below):



Indeed, in Bob Holdom's model, fermions are in a loop and must be electrically charged. But the Weyl meson is C-even and does not interact with fermions. Therefore, if the Weyl meson S interacts with a SM particle or a particle from the SM expansion, then this particle is electrically neutral and is not a fermion. Let it be dark Higgs χ.

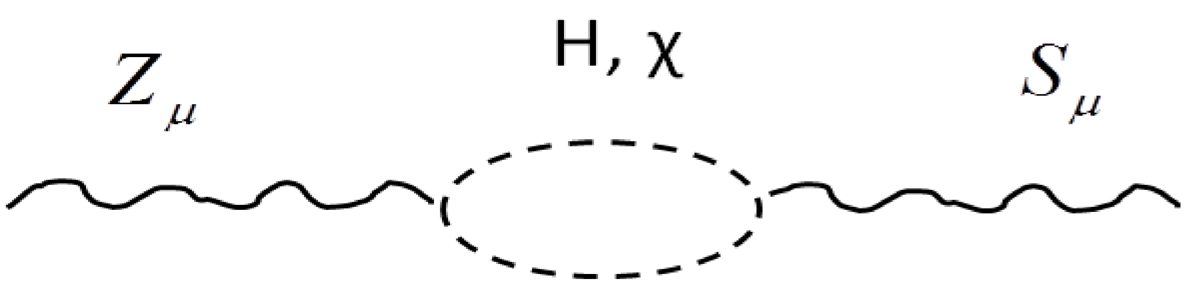
8. Weyl meson and Higgs boson

$$\breve{D}_\mu H = \left(\partial_\mu - \left\{ (1-f) w_\mu + f \cdot S_\mu \right\} + ig\vec{T} \cdot \vec{W}_\mu + \frac{i}{2} g' B_\mu \right) H$$

$$\breve{D}_\mu \chi = \left(\partial_\mu + \left\{ (1-f) w_\mu + f \cdot S_\mu \right\} + ig\vec{T} \cdot \vec{W}_\mu + \frac{i}{2} g' B_\mu \right) \chi$$

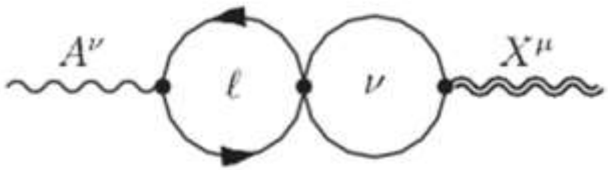
$$U(H, \chi, \beta) = \lambda_H \left(H^+ H \right)^2 - \mu_H \left(H^+ H \right) \beta^2 + \lambda_\chi \left(\chi^+ \chi \right)^2 + \lambda_{1\chi} \left(H^+ H \right) \left(\chi^+ \chi \right)$$
$$+ \lambda_{2\chi} \left(H^+ \chi \right) \left(\chi^+ H \right) - \mu_\chi \left(\chi^+ \chi \right) \beta^2 + \xi_H \beta^4$$

After breaking the Weyl symmetry and spontaneously breaking the gauge symmetry, the signs of the product of charges will be different, H and χ give a contribution with different sign $\pm f \cdot g_Z$. Mixing parameter $\varepsilon_Z \sim \frac{g_Z f}{48\pi^2} \ln \frac{M_\chi^2}{M_H^2}$.



9. Weyl meson and neutrinos

Mixing of Weyl mesons S^{α} and ordinary photons A^{α} occurs due to weak interaction. Here is a diagram from the publication [Tony Gherghetta, Jörn Kersten, Keith Olive and Maxim Pospelov, Phys. Rew. D 100, 095001, 2019)] explaining the process of mixing ordinary photons and dark photons.



Here we take the Weyl meson S^{α} as X^{μ}. The photon A^{α} decays into a pair of leptons (particle and antiparticle), which, due to weak interaction, turn into a pair (neutrino and antineutrino), and then into a Weyl meson S^{α}. This results in a phenomenological term $\sim \varepsilon F_{\mu\nu} S^{\mu\nu}$ with a mixing parameter ϵ <<1. The details of the weak interaction are insignificant in this case, so we can take the four-fermion interaction as an approximation. A rather rough assessment from [Tony Gherghetta, Jörn Kersten, Keith Olive and Maxim Pospelov for a parameter ϵ in the expression $2\varepsilon F^{\alpha\beta} E_{\alpha\beta}$, where $F^{\alpha\beta} = \partial^{\alpha} A^{\beta} - \partial^{\beta} A^{\alpha}$, $E_{\alpha\beta} = \partial_{\alpha} X_{\beta} - \partial_{\beta} X_{\alpha}$, gives the value:

$$\epsilon \sim \frac{e g_X}{(16\pi^2)^2} G_F m_X^2 \theta^2 \sim 10^{-17} \left(\frac{m_X}{1\ \mathrm{MeV}}\right)^2 \left(\frac{\theta}{0.1}\right)^2$$

Here, θ is the mixing angle of the active and sterile neutrinos, $m_X = M_S$.

10. Conclusion

The report highlights the idea that the role of the dark photon (or Z' -boson) in the extension of SM can be played by the Weyl meson **S**, whose origin is rooted in a modification of the Weyl-Dirac theory of gravity, which is a fairly simple generalization of Einstein's general theory of relativity. However, the Weyl meson **S** must be distinguished from the Weyl vector **w** in order to absolutely avoid the non-metric effects of the "second clock" type.

We have shown that the connection of the Weyl meson with fermions, absent in the Weyl gravity & Standard Model, can occur under kinetic mixing. This mixing may be implemented in various embodiments. These options are discussed in this report. In our view, S-Z mixing is phenomenologically preferred .

10. Conclusion

Note the following.

1) Our modified gravity is based on integrable Weyl geometry.

2) The Weyl meson **S** does not coincide with the Weyl vector **w**, and the role of the Weyl vector **w** is played by the gradient of the logarithm of the Dirac function β. The interaction of the Weyl meson **S** with other particles and its mass in the considered model can be chosen arbitrarily. There may be different Weyl mesons, $f \cdot S^{\mu} \Rightarrow f_1 \cdot S_1^{\mu} + f_2 \cdot S_2^{\mu} + \ldots$

3) In our view, the model where the massive Weyl meson **S** kinetically mixes with **Z-**boson looks most realistic.

4) Problems of the Weyl meson mass and its lifetime, as well as the experimental and astrophysical limitations, can be studied in a similar to the one used for a dark photon.

Thank you for your attention!