U(1) gauged boson stars

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Phys.Rev.D 112 (2025) 025015
Phys.Lett.B 855 (2024) 138812
Phys.Rev.D 110 (2024) 125020
Phys.Rev.D 107 (2023) 104062
Phys.Rev.D 105 (2022) 085013
Phys. Lett. B 824(2022)136811
Phys.Lett.B 812 (2021) 136027
Phys.Rev.D 103 (2021) 065009

Thanks to my collaborators:
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A. Mikhaliuk, I.Perapechka, and E Radu

AQFT´25, Dubna, 14.08.2025

Outline

- Boson Stars & Q-balls
- U(1) gauged boson stars
- Boson stars and hairy BHs
- Einstein-Maxwell BHs with scalar hairs
- Einstein-Maxwell-Fridberg-Lee-Sirlin model
- EMFLS BHs vs RN BHs
- Summary

Q-balls

G. Rosen (1968),
R. Friedberg, T.D. Lee & A. Sirlin (1976)
S. Coleman (1985)

$$L = |\partial_\mu \phi|^2 - V(|\phi|)$$

$$Q = i \int d^3x \, (\phi \partial_t \phi^* - \phi^* \partial_t \phi)$$

$$\phi = f(r) e^{i\omega t}$$

Spherically symmetric Q-ball

$$Q = 8\pi\omega \int_0^\infty dr r^2 f^2$$

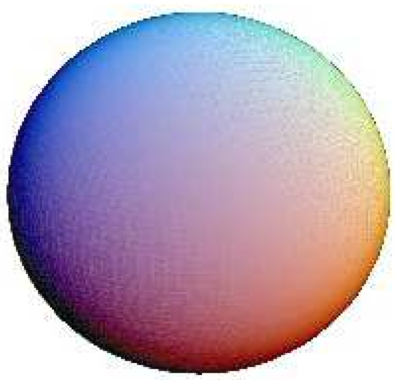
Field equation:

$$\frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr} + \omega^2 f = \frac{1}{2}\frac{dV}{df}$$

$$f \sim \frac{1}{r} e^{-\sqrt{m^2-\omega^2} r}$$

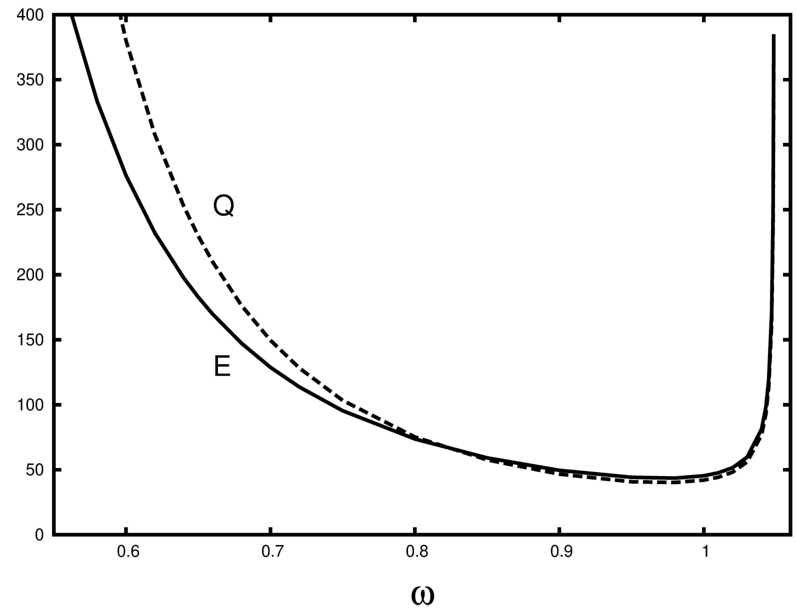
Potential:

$$V = a|\phi|^2 - b|\phi|^4 + |\phi|^6$$



Angular frequency is restricted:

$$\omega_{min} \le \omega \le \omega_{max}$$



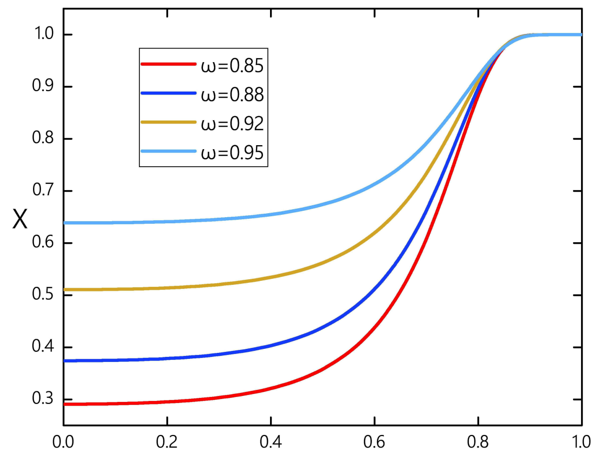
Friedberg-Lee-Sirlin Q-balls

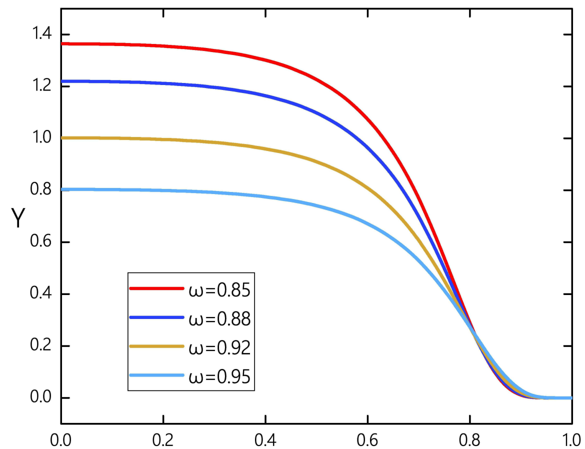
$$L = (\partial_\mu \xi)^2 + |\partial_\mu \phi|^2 - m^2 \xi^2 |\phi|^2 - \mu^2 (1-\xi^2)^2$$

$$Q = i \int d^3x \, (\phi \partial_t \phi^* - \phi^* \partial_t \phi)$$

$$\xi = X(r)\,; \qquad \phi = Y(r) e^{i\omega t}$$

Spherically symmetric FLS Q-ball



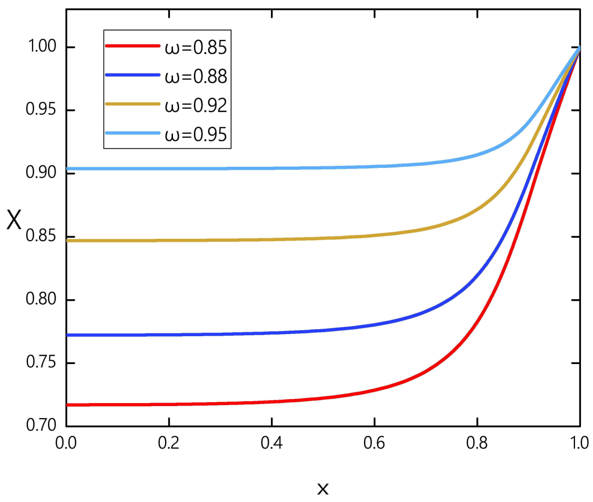


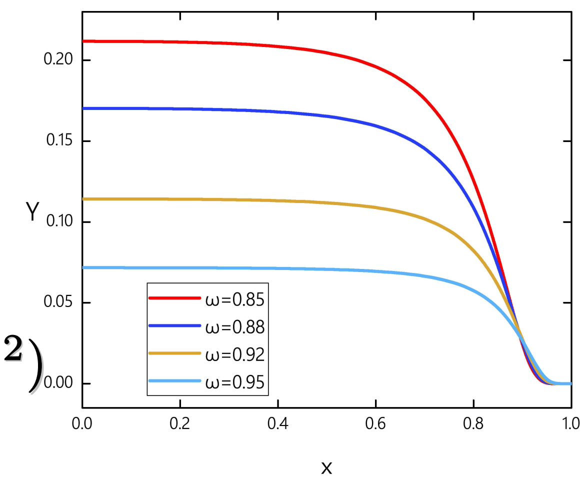
- **massive**

$$\mu^2 = 1/4$$

$$m^2 = 1$$

$$X \sim 1 - e^{-\sqrt{\mu^2-\omega^2}\; r}$$





- **massless**

$$\mu^2 = 0$$

$$m^2 = 1$$

$$X(r) \sim 1 - \frac{C}{r} + O(r^{-2})$$

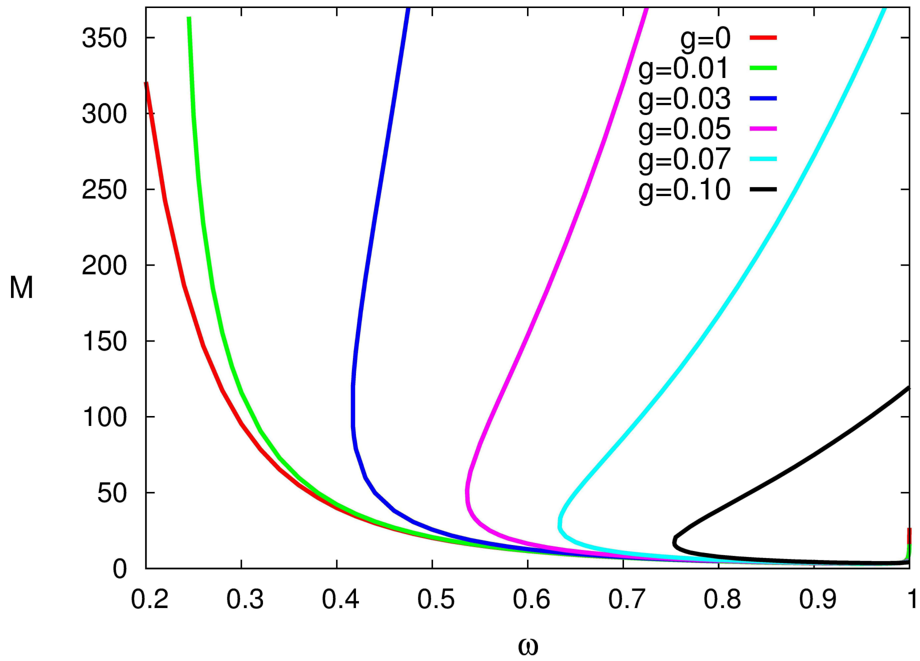
U(1) gauged Friedberg-Lee-Sirlin Q-balls

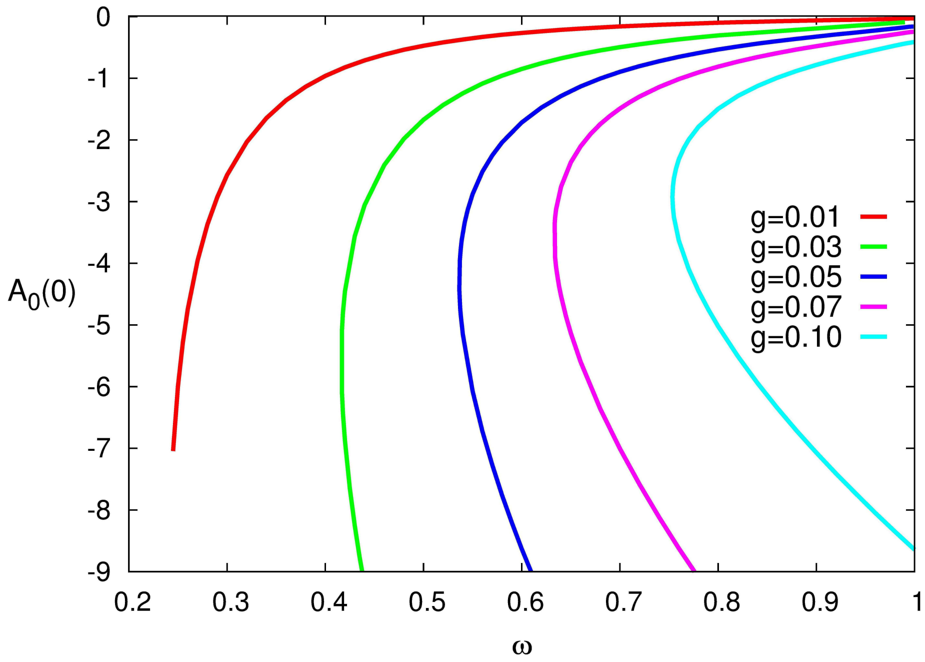
$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu \xi)^2 + |D_\mu \phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2$$

$$D_\mu = \partial_\mu + igA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- **U(1) current:** $j_\mu = i(\phi D_\mu \phi^* - \phi^* D_\mu \phi)$

$$Q = \int d^3x (gA_0 + \omega)|\phi|^2$$

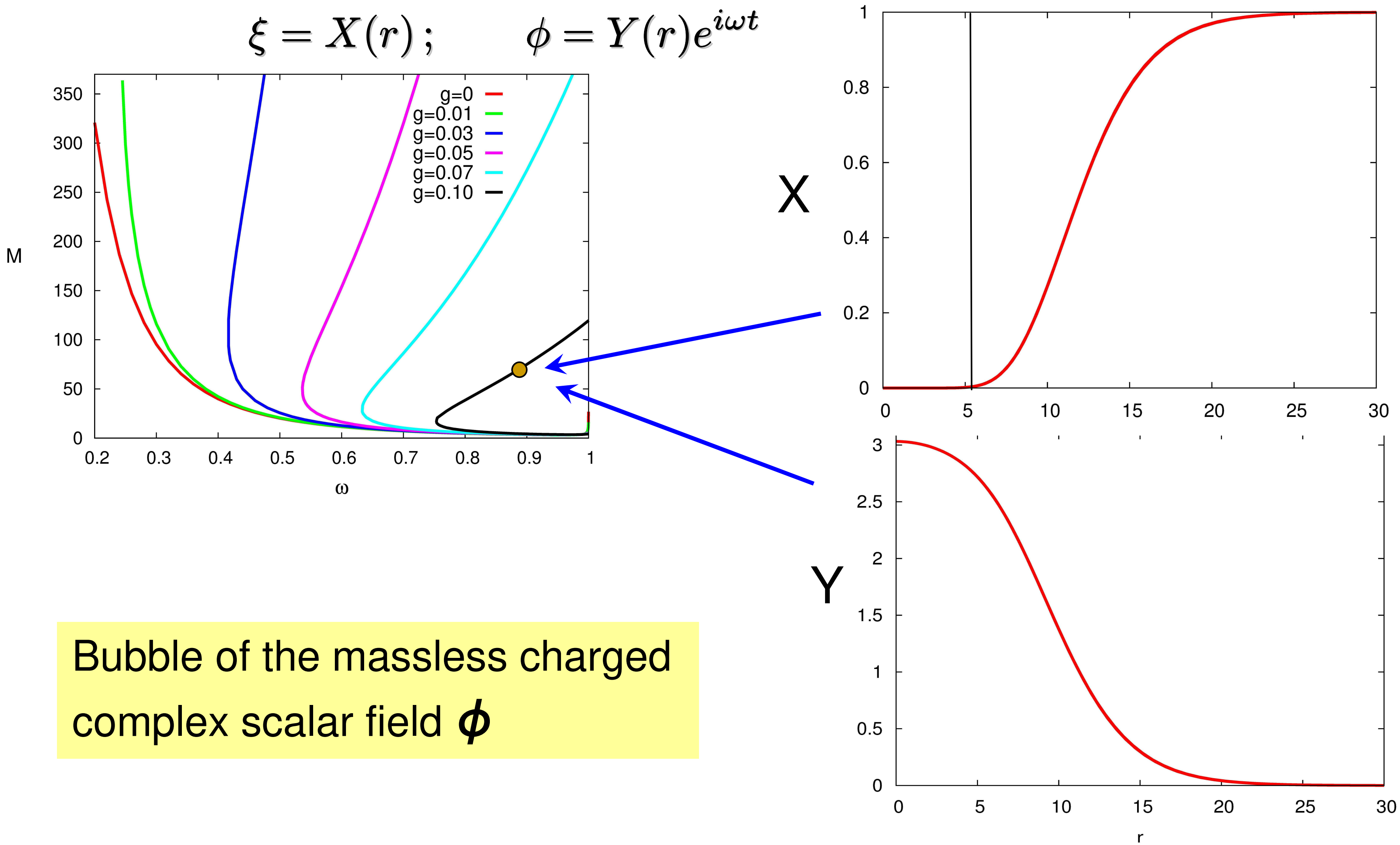




U(1) gauged Friedberg-Lee-Sirlin Q-balls

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu \xi)^2 + |D_\mu \phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2$$

$$\xi = X(r)\,; \qquad \phi = Y(r)e^{i\omega t}$$



Bubble of the massless charged complex scalar field ϕ

Localised solitons: Gravity vs Klein-Gordon

Pure gravity (attraction)

$$L = -\frac{R}{16\pi G}$$

Lichnerowicz (1955): there are black holes but there are no gravitational solitons, the only globally regular, asymptotically flat, static vacuum solution to the Einstein eqs with finite energy is Minkowski space.

Klein-Gordon massive theory

$$L = |\partial_\mu \phi|^2 + \mu^2 |\phi|^2$$

Derrek theorem: Complex Klein-Gordon theory in 3+1 dim do not admit localised soliton solutions.

Stationary spinning configurations: a way to evade Derrick's theorem

I Z Fisher (1948): Solution of the coupled Einstein-Klein-Gordon model (Thanks to ***K.Zloschastiev, gr-qc/9911008 [gr-qc]***)

Kaup (1968): Dispersion can be balanced by the gravitational attraction

The Boson Stars: $\phi(\mathbf{r}, t) = f(\mathbf{r}) e^{i\omega t}$

Boson Stars

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu \phi|^2 - \mu^2|\phi|^2\right\}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2\alpha^2 T_{\mu\nu}; \qquad \left(\Box - \mu^2\right)\phi = 0 \qquad \alpha^2 = 4\pi G$$

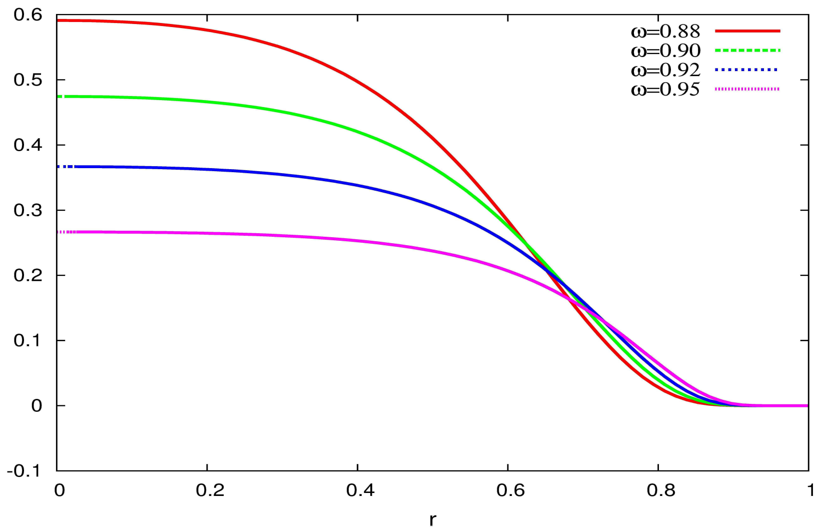
- **U(1) current:** $j_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$

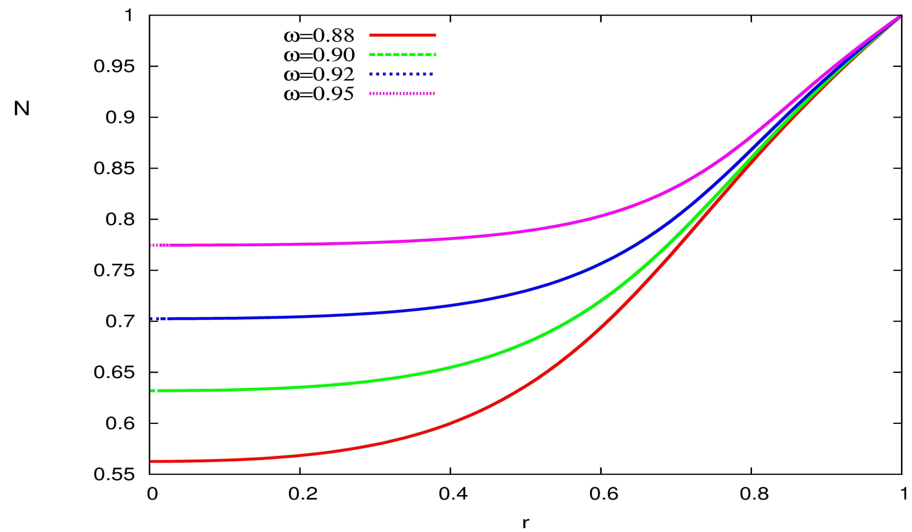
$$Q = \int \sqrt{-g}j^t d^3x$$

Spherical symmetry:

$$\phi = f(r)e^{-i\omega t}$$

$$ds^2 = -\sigma^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$





Boson Stars

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - |\partial_\mu\phi|^2 - \mu^2|\phi|^2\right\}$$

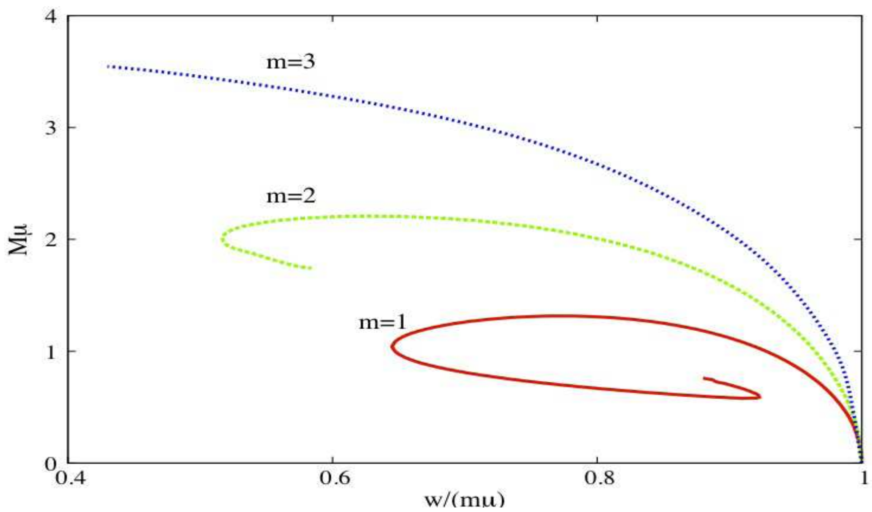
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2\alpha^2 T_{\mu\nu}; \qquad \left(\Box - \mu^2\right)\phi = 0 \qquad \alpha^2 = 4\pi G$$

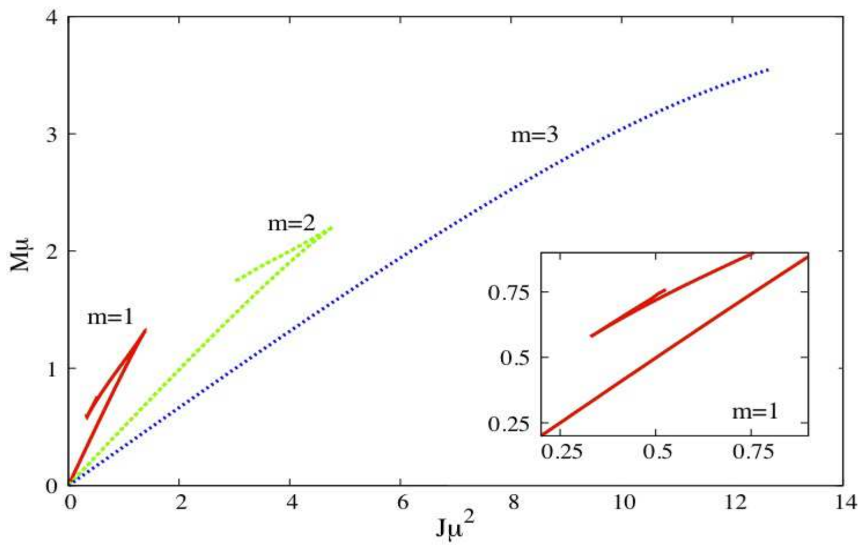
- **U(1) current:** $j_\mu = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi)$

$$Q = \int \sqrt{-g}j^t d^3x$$

Axial symmetry: $\phi = f(r,\theta)e^{i(m\varphi - \omega t)}$

Volkov & Wohnert (2002), Kleihaus, Kunz and List (2005)





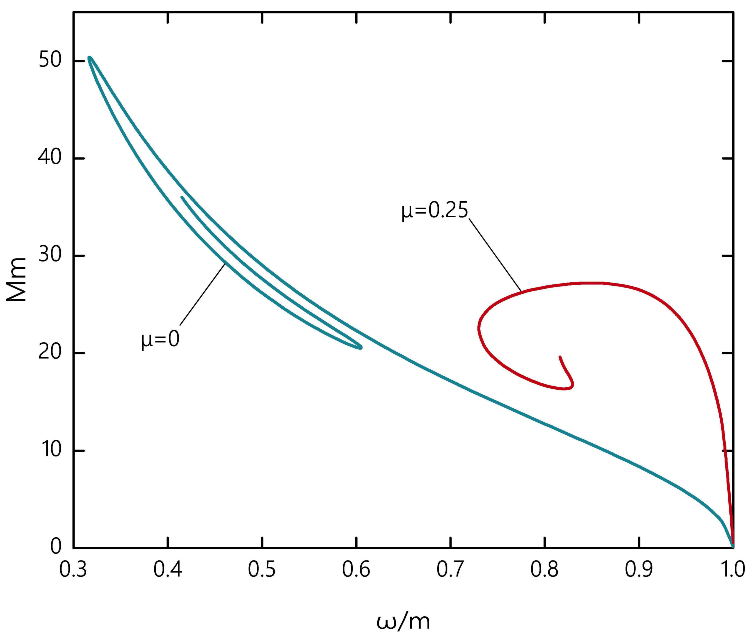
FLS Boson Stars

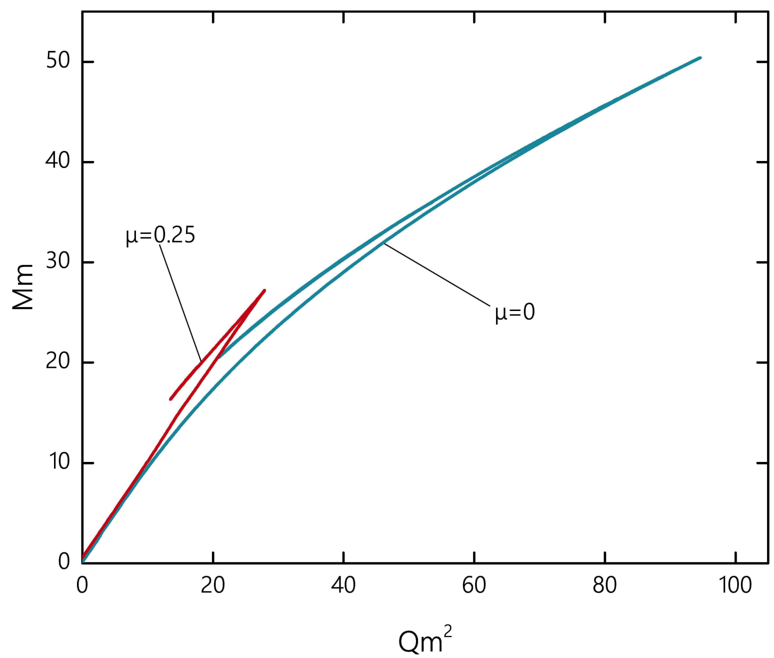
Friedberg-Lee-Sirlin model (1976):

$$\mathcal{L}_{\rm m} = \frac{1}{2}\left(\partial_\mu \xi\right)^2 + |\partial_\mu \phi|^2 + m^2 \xi^2 |\phi|^2 - \mu^2\left(1-\xi^2\right)^2$$

$$ds^2 = -F_0 dt^2 + F_1\left(dr^2 + r^2 d\theta^2\right) + r^2 \sin^2\theta F_2\left(d\varphi - \frac{W}{r}dt\right)^2$$

$$\xi = X(r,\theta), \quad \phi = Y(r,\theta)e^{i\omega t + n\varphi}$$





How large the boson stars are?

Herdeiro and Radu (arXive 2205.05395)

- **Back to physical units:** $\Box\phi = \frac{m^2c^2}{\hbar^2}\phi\,;\qquad R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta}$

The inverse Compton wavelength: $\mu = mc/\hbar$

$$M_{Pl} = \sqrt{\hbar c/G}$$

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\;R + \hbar\int d^4x\sqrt{-g}\;\left(-g^{\alpha\beta}\partial_\alpha\phi^*\partial\beta\phi - \mu^2\phi^*\phi\right)$$

$$Q = \frac{8\pi\omega}{c}\int dr\,\frac{r^2\phi^2}{N(r)\sigma(r)} \;\Rightarrow\; S_\phi \sim \hbar\int d^4x\; r^2\sin\theta\sigma(r)[-g^{00}\partial_0\phi^*\partial_0\phi] = \frac{\hbar\omega}{2}Q\Delta t$$

- **Q-ball:** a very large collection of particles in the same quantum state

Heisenberg's uncertainty principle: $\Delta x\Delta p \geq \hbar$

$$\Delta x \sim R\,,\Delta p \sim mc = \mu\hbar$$

$$\Rightarrow\quad \mu R_{min} \sim 1;\quad R_{min} \sim R_{Sch} \sim GM_{max}/c^2 \quad\Rightarrow\quad \frac{mM_{max}}{M_{Pl}^2} \sim 1$$

$$M_{max} = mQ_{max}$$

$$Q_{max} \sim \left(\frac{M_{Pl}}{m}\right)^2 \sim 10^{76}\left(\frac{M_{max}}{M_\odot}\right)^2$$

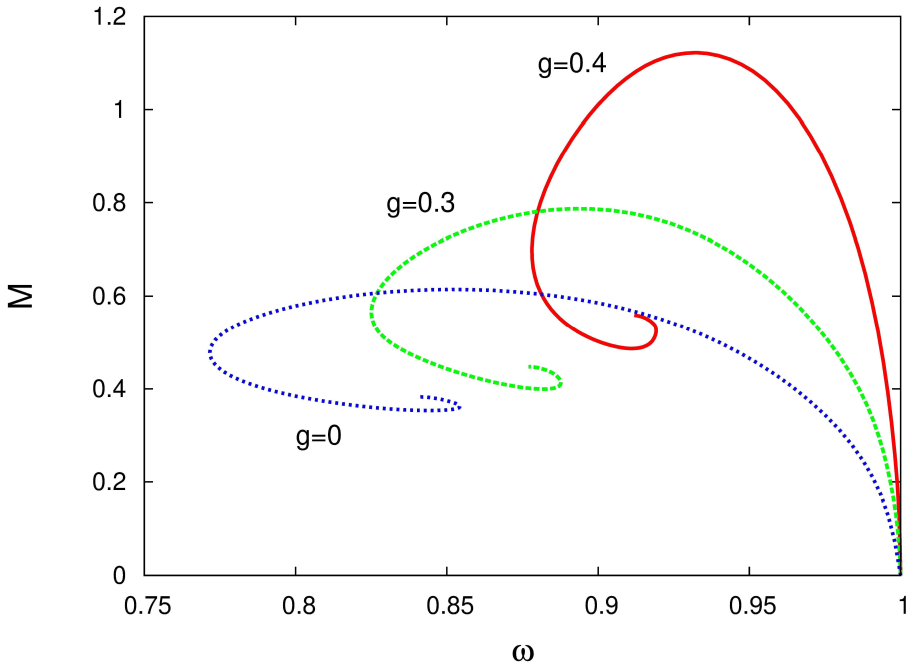
U(1) gauged Boson Stars

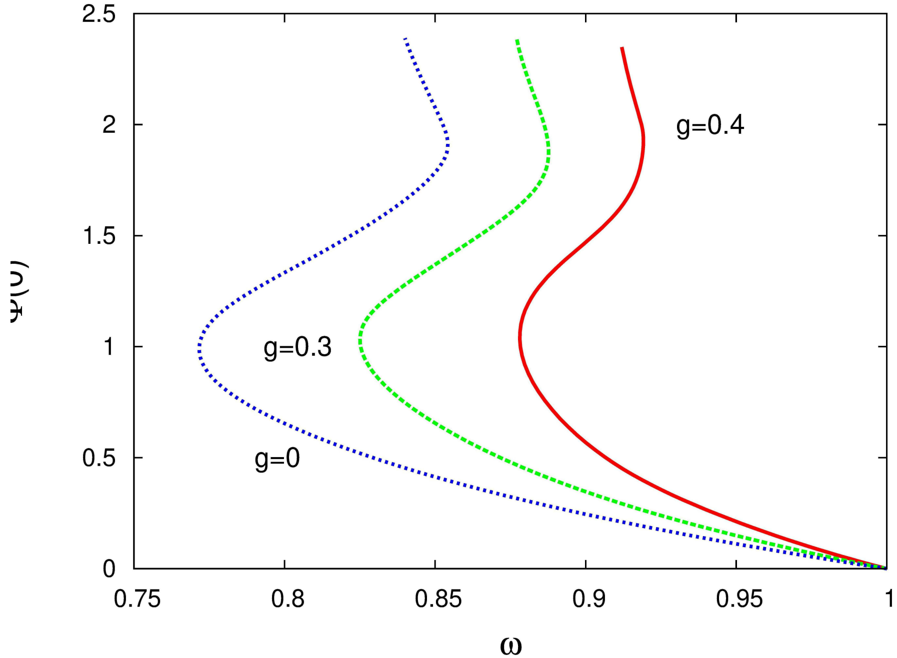
P.Jetzer and J.J.van der Bij (1989), D.Pugliese, H.Quevedo, J.Rueda and R.Ruffini (2013):
Boson stars in the Einstein-Klein-Gordon-Maxwell model:

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\phi|^2 - \mu^2|\phi|^2\right\}; \quad D_\mu\phi = \partial_\mu\phi - igA_\mu\phi$$

- **U(1) current:** $j_\mu = i(\phi D_\mu\phi^* - \phi^* D_\mu\phi)$

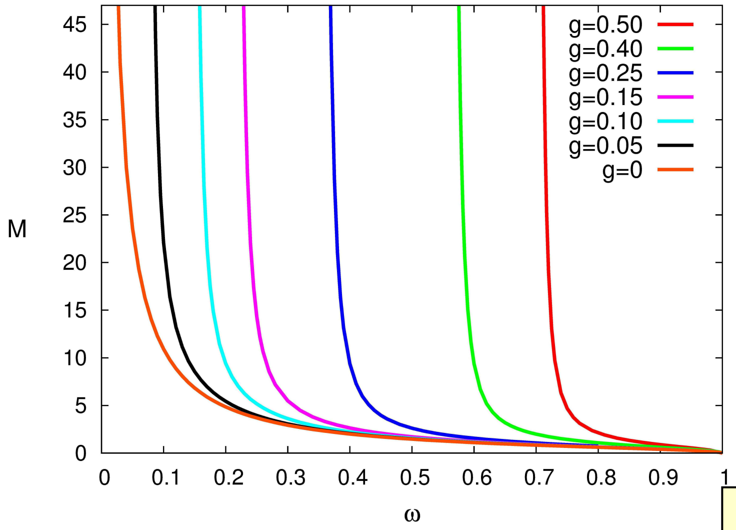
$$Q = \int d^3x(gA_0 + \omega)|\phi|^2$$





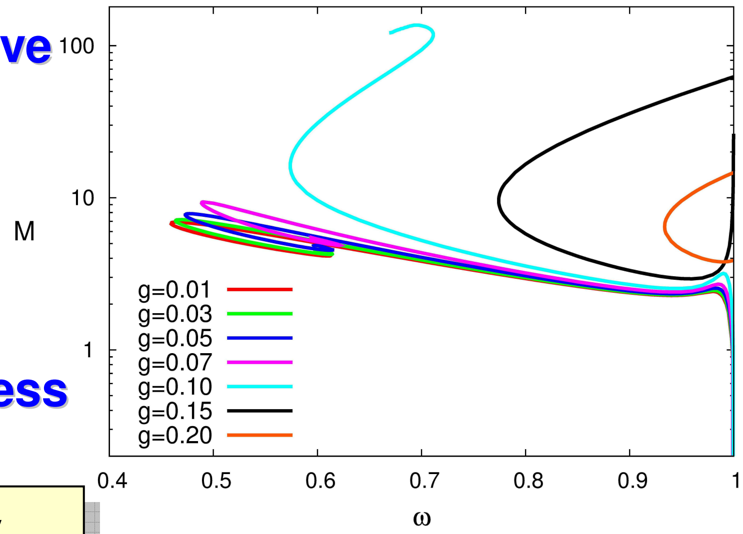
U(1) gauged FLS Boson Stars

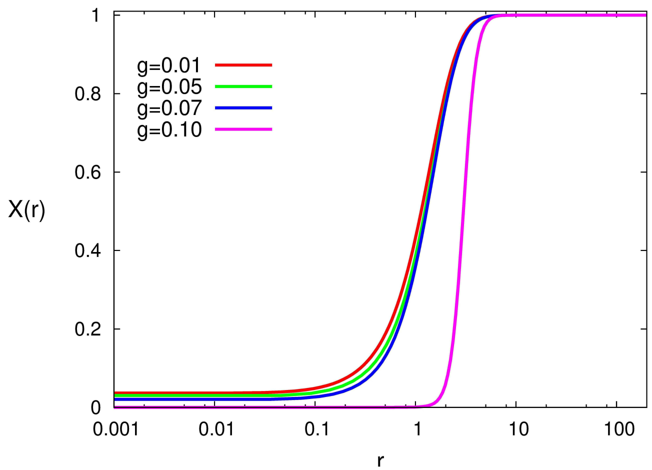
$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\xi)^2 + |D_\mu\phi|^2 - m^2\xi^2|\phi|^2 - \mu(1-\xi^2)^2\right\}$$



massive

massless

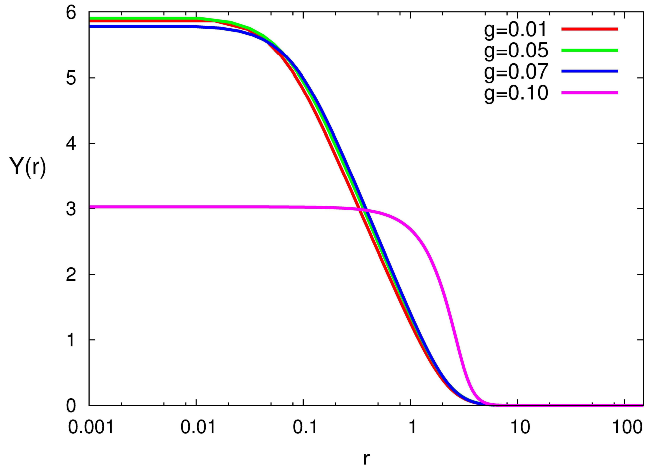




Gravity (attraction)

Electrostatic (repulsion)

2 Scalars (attraction & repulsion)



From Boson Stars to Black Holes

no-scalar-hair theorem (Pena & Sudarsky, 1997): there are no static black hole analogues of the spherically symmetric regular boson stars

Zel'dovich (1971): "Generation of waves by a rotating body", JETP Lett, 14, 270

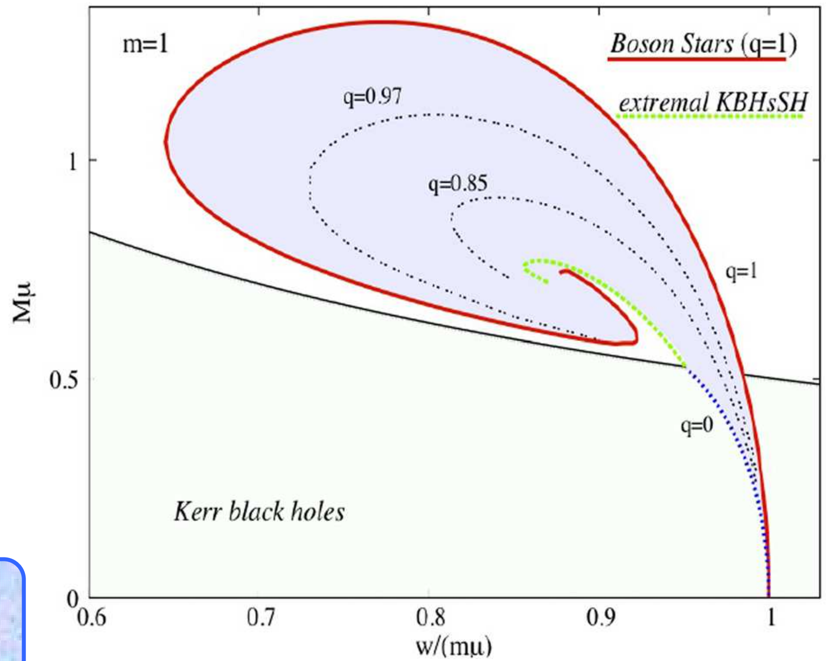
Hod (2012), Herdeiro and Radu (2014): Kerr BHs with scalar hair

Synchronisation condition: $w = m\Omega_H$

- **Two Killing vectors:c** $\zeta = \partial_\varphi; \quad \xi = \partial_t$
- **Symmetry of the solution:** $\chi = \xi + \frac{\omega}{m}\zeta$

there is no flux of scalar field into the BH: $\chi^\mu \partial_\mu \phi = 0$

Superradiant instability of the Kerr spacetime: Black hole bomb mechanism



A loophole in the no-hair theorem: *rotation*

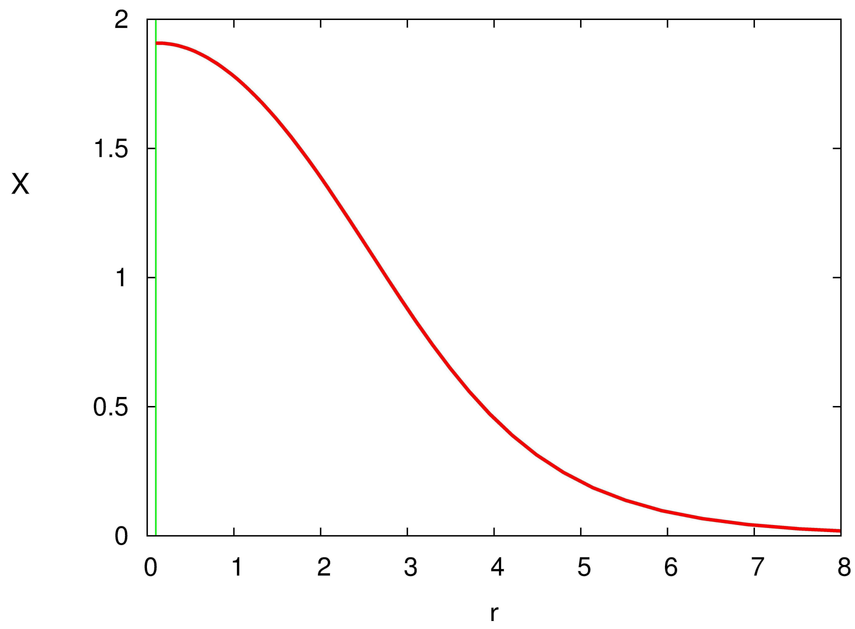
From Boson Stars to Black Holes

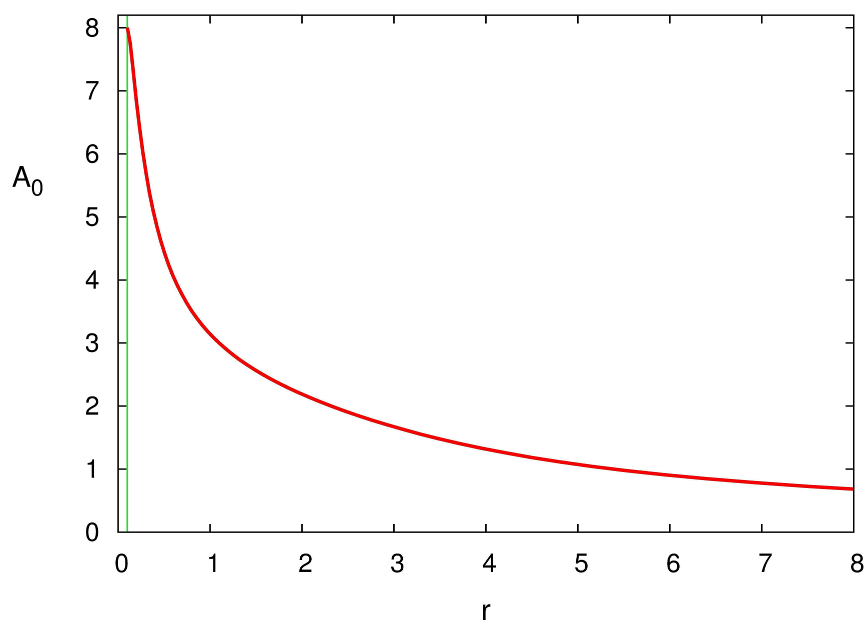
J.P. Hong et al (2020), Herdeiro and Radu (2020):* RN BHs with charged *scalar hair

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\phi|^2 - V(|\phi|)\right\}; \quad V(|\phi| = \mu^2\phi^2 - \lambda\phi^4 + \beta\phi^6$$

- **Gauge fixing:** $A_0(\infty) = 0$

Resonance condition: $gA_0(r_h) + w = 0$





Yet another loophole in the no-hair theorem: *nonlinearity*

Boson Stars and hairy BHs in the O(3) sigma-model

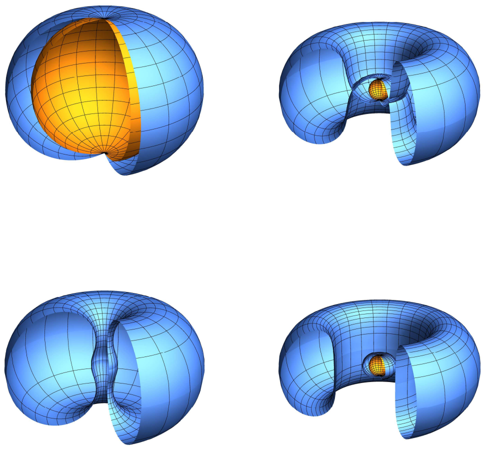
$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{8\pi G}-(\partial_\mu\phi_a)^2-\mu(1-\phi_3)\right\};\quad (\phi_a)^2=1$$

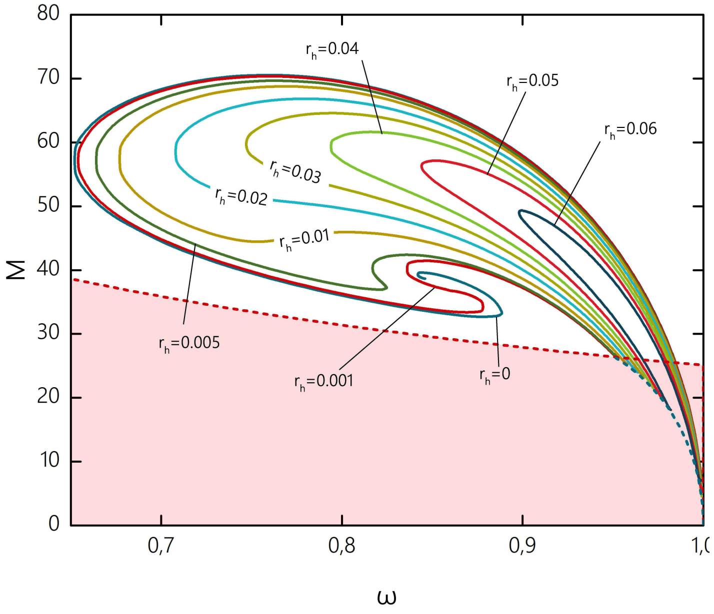
C. Herdeiro, E. Radu, I.Perapechka and Ya.Shnir, JHEP 02 (2019) 111

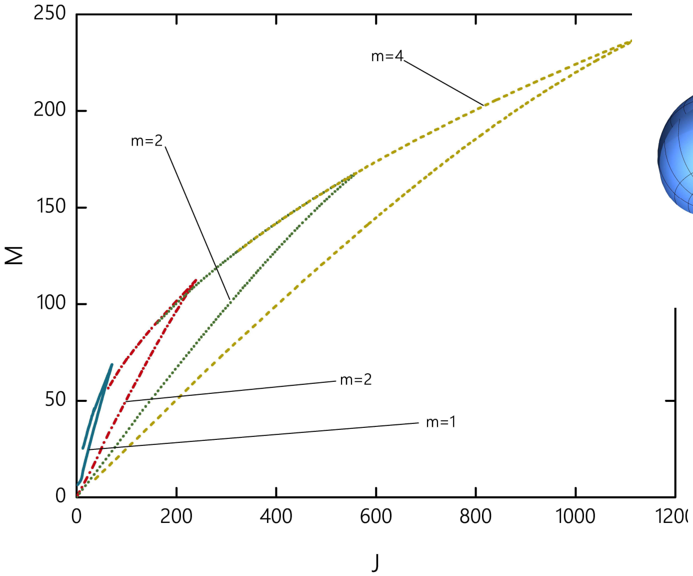
- **Spinning Q-lump:** $\phi_1=\sin f\cos(n\varphi+\omega t);\quad \phi_2=\sin f\sin(n\varphi+\omega t);\quad \phi_3=\cos f$

$$ds^2=-F_0(r,\theta)dt^2+F_1(r,\theta)\left(dr^2+r^2d\theta^2\right)+F_2(r,\theta)r^2\sin^2\theta\left[d\varphi-W(r,\theta)dt\right]^2$$

- **SO(2) current:** $j_\mu=-\phi_1\partial_\mu\phi_2+\phi_2\partial_\mu\phi_1$







Boson Stars in the U(1) gauged O(3) sigma-model

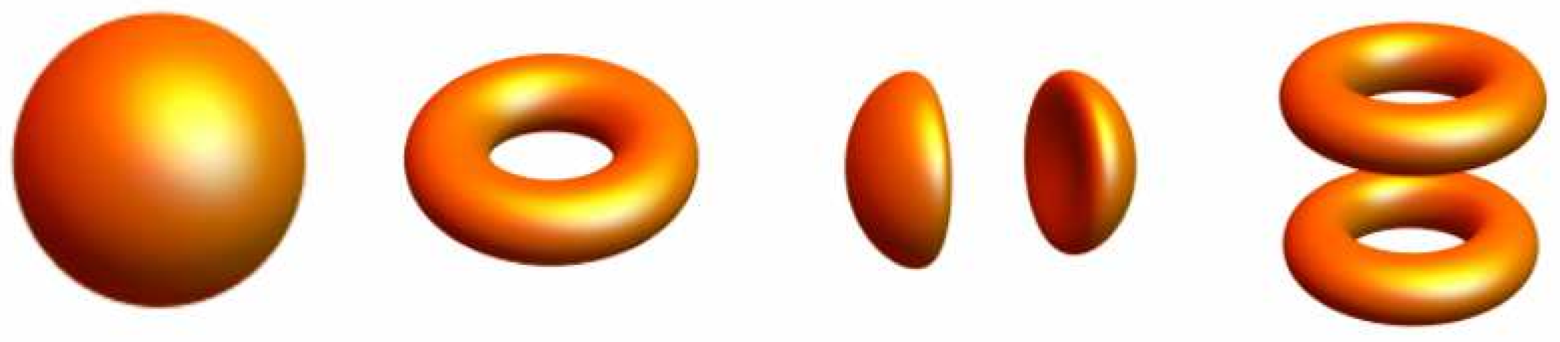
$$S = \int d^4x\sqrt{-g}\left(\frac{R}{8\pi G} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D_\mu\phi^a)^2 - \mu(1-\phi_3)\right)$$

$$D_\mu\phi^\alpha = \partial_\mu\phi^\alpha + e\,A_\mu\,\varepsilon_{\alpha\beta}\,\phi^\beta\,, \qquad D_\mu\phi^3 = \partial_\mu\phi^3, \qquad \alpha,\,\beta = 1,\,2$$

- **Noether current:** $j^\mu = -\phi^1 D^\mu\phi_2 + \phi^2 D^\mu\phi_1$

$$\phi_a = [\sin f(r,\theta)\cos(n\varphi - \omega t),\ \ \sin f(r,\theta)\sin(n\varphi - \omega t),\ \ \cos f(r,\theta)]$$

$$ds^2 = -F_0(r,\theta)dt^2 + F_1(r,\theta)\left(dr^2 + r^2 d\theta^2\right) + F_2(r,\theta)r^2\sin^2\theta\,[d\varphi - W(r,\theta)dt]^2$$



EMFLS Black holes with charged hairs

(J Kunz & Ya S 2023)

$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{16\pi G}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+(\partial_\mu\xi)^2+|D_\mu\phi|^2-m^2\xi^2|\phi|^2-\mu(1-\xi^2)^2\right\}$$

- **U(1) current:** $j_\nu=i(D_\nu\phi^*\,\phi-\phi^*D_\nu\phi)$

Spherical symmetry: $\xi=\partial_t$

- **Field equations:**

$$\begin{cases}\partial^\mu\partial_\mu\psi=2\psi(m^2|\phi|^2+2\mu^2(1-\psi^2)); & D^\mu D_\mu\phi=m^2\psi^2\phi\,. \\ R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi G\left(T^{Em}_{\mu\nu}+T^{Sc}_{\mu\nu}\right); & \partial_\mu(\sqrt{-\mathrm{g}}F^{\mu\nu})=g\sqrt{-\mathrm{g}}j^\nu\end{cases}$$

Asymptotic: $g_{00}(r)\xrightarrow[r\to\infty]{}-1+\frac{\alpha^2M}{\pi r}+O(r^{-2})\,,\qquad A_0(r)\xrightarrow[r\to\infty]{}\frac{Q}{r}+O(r^{-2})$

ADM mass: $M=M_H+M_f=-\frac{1}{2\alpha^2}\oint_\Sigma d\Sigma_{\mu\nu}\nabla^\mu\xi^\nu-\frac{1}{\alpha^2}\int_V d\Sigma_\mu(2T^\mu_\nu\xi^\nu-T\xi^\mu)$

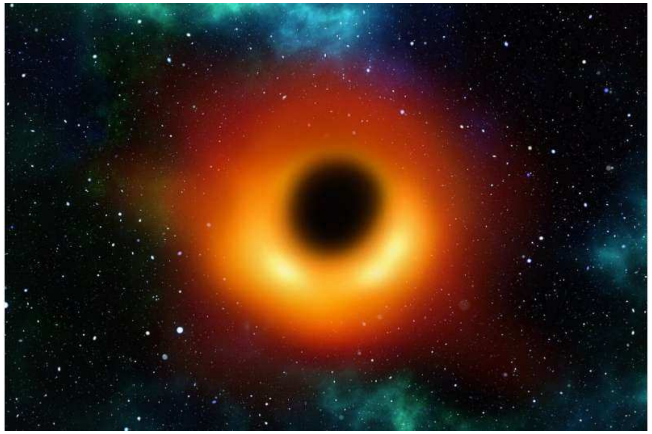
Total charge: $Q=Q_H+g\,Q_N=\frac{1}{4\pi}\oint_\Sigma d\Sigma_rF^{0r}+g\int d^3x\sqrt{-g}\,j^0$

Hairiness: $h=1-\frac{Q_H}{Q}=\frac{g\,Q_N}{Q}$

Black hole thermodynamics:

$$T_H = \frac{\kappa}{2\pi}, \quad \kappa^2 = -\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu$$

$$S = \frac{\pi A_H}{\alpha^2} \qquad \mu_{ch} = A_0(r_h)$$

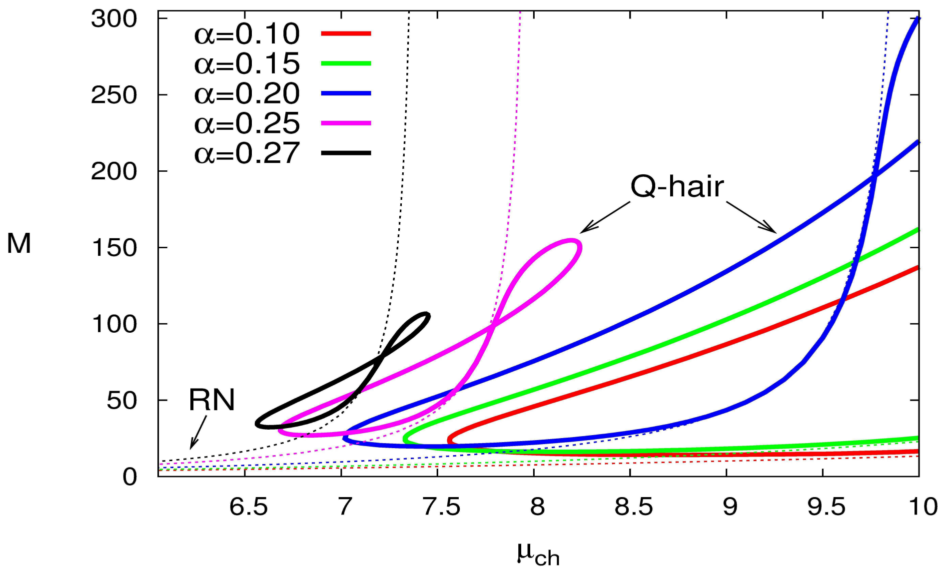


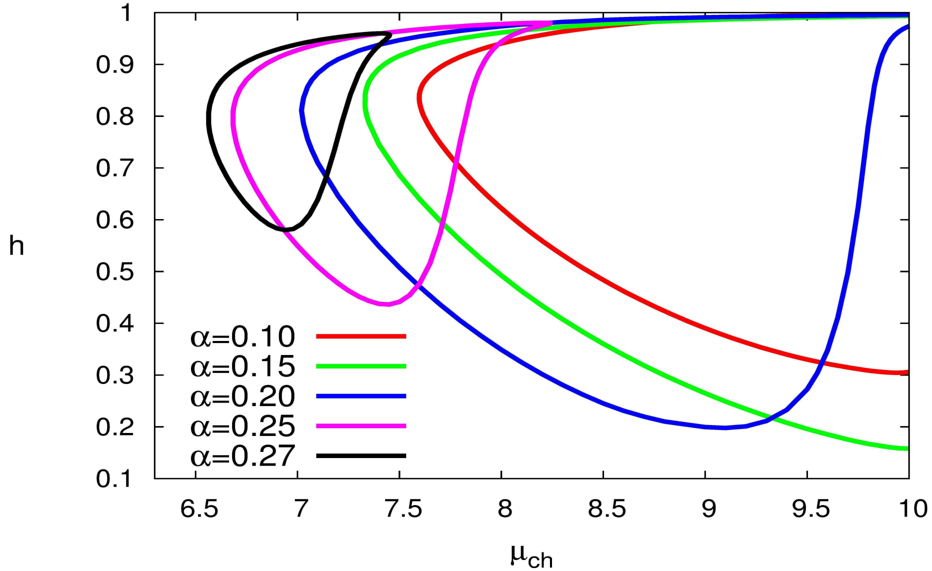
Zeroth law: $\kappa = const$

First law: $dM = TdS + \mu dQ$

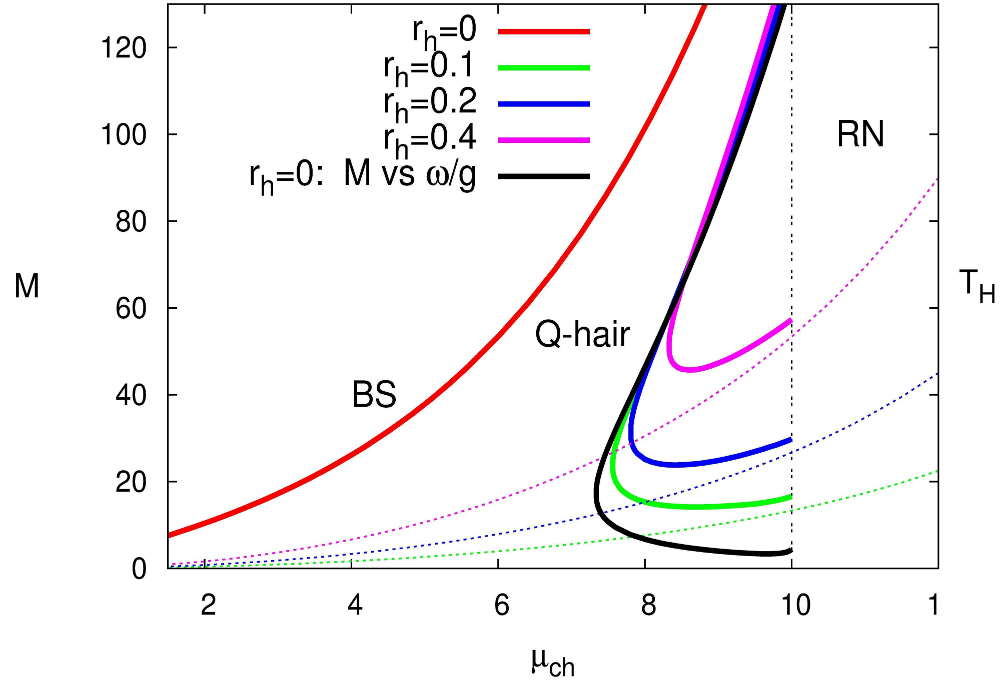
Second law: $dA_H \geq 0$

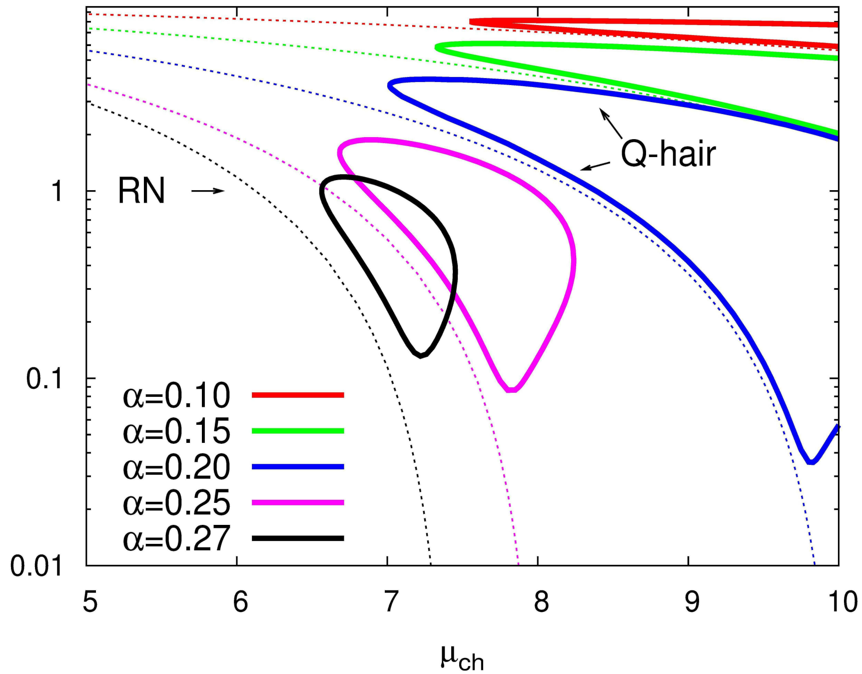
Third law: ***the limit*** $T_H \to 0$ ***does not exist***





massive scalar fields (μ^2=0.25)



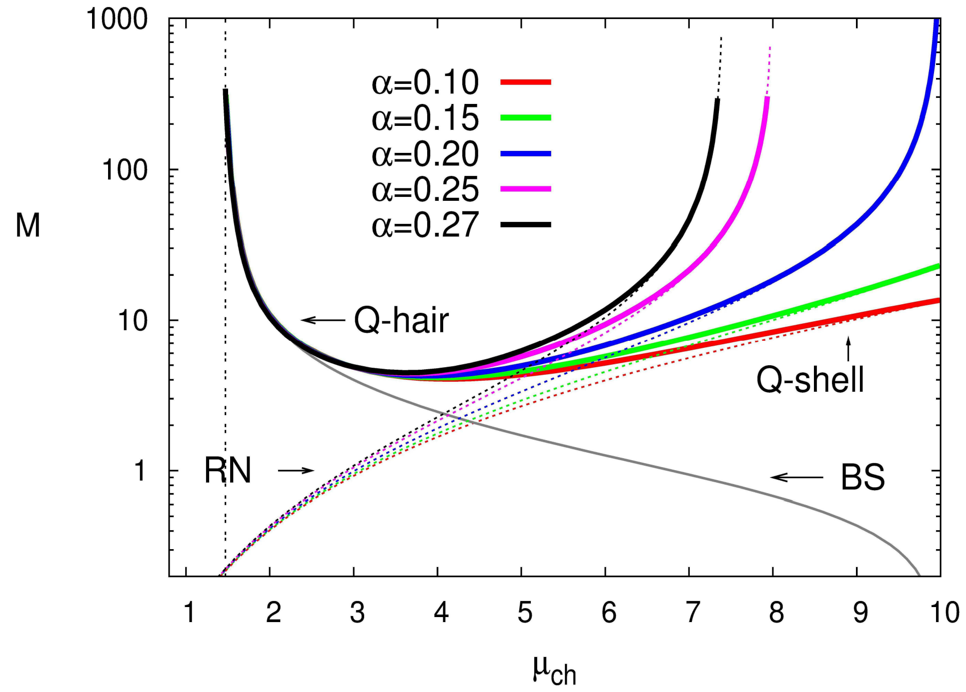


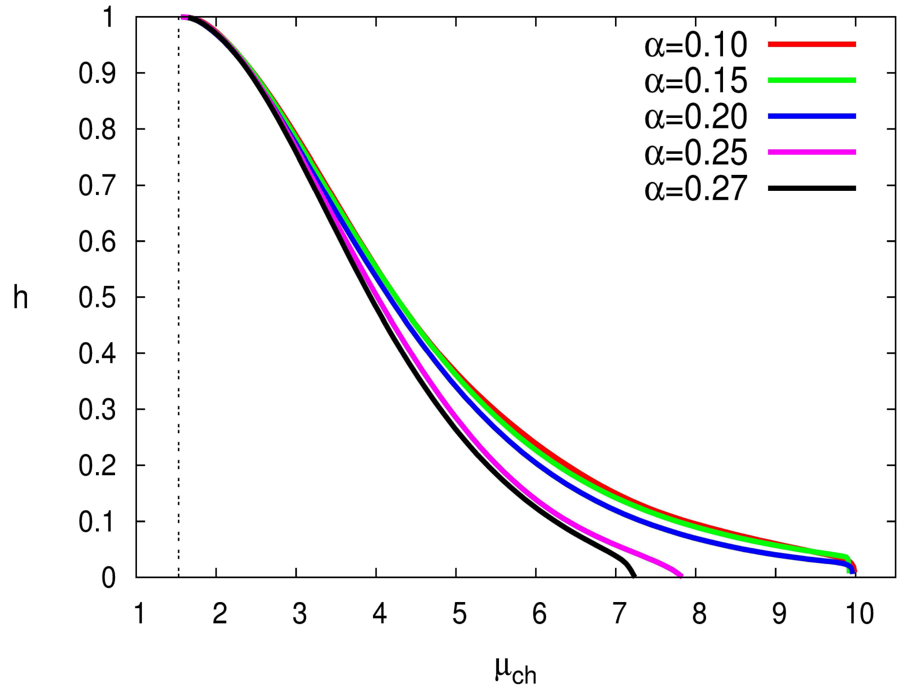
BH with charged scalar hair: a balance of the competing forces of (i) gravitational attraction, (II) scalar interactions, (III) electrostatic repulsion

BHs with massive Q-hair are disconnected from electrovacuum RN BH

BHs with massive Q-hair are disconnected from regular charged BSs

massless scalar component (μ^2=0)

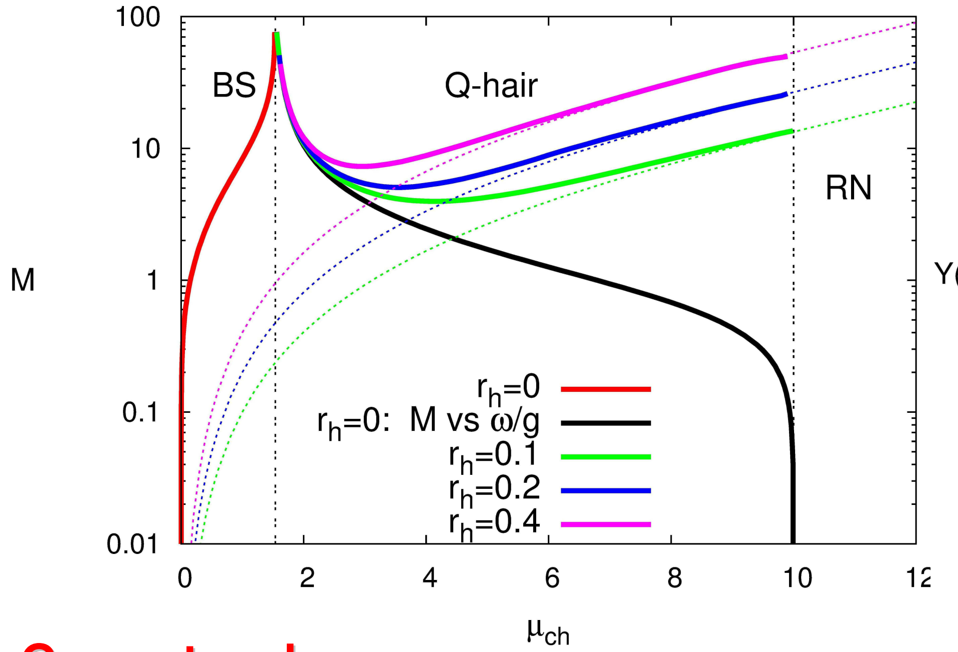


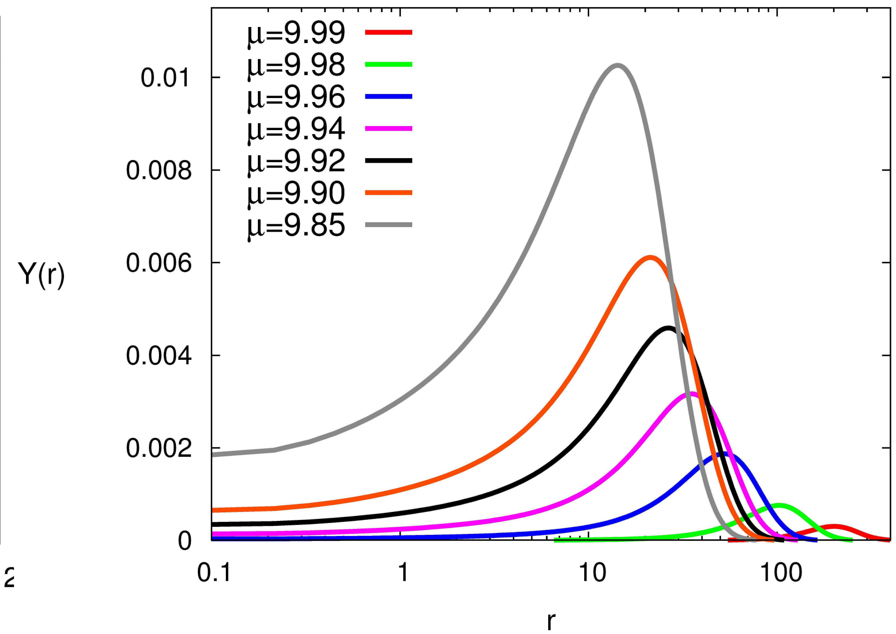


BHs with massless Q-hair bifurcate from electrovacuum RN BH

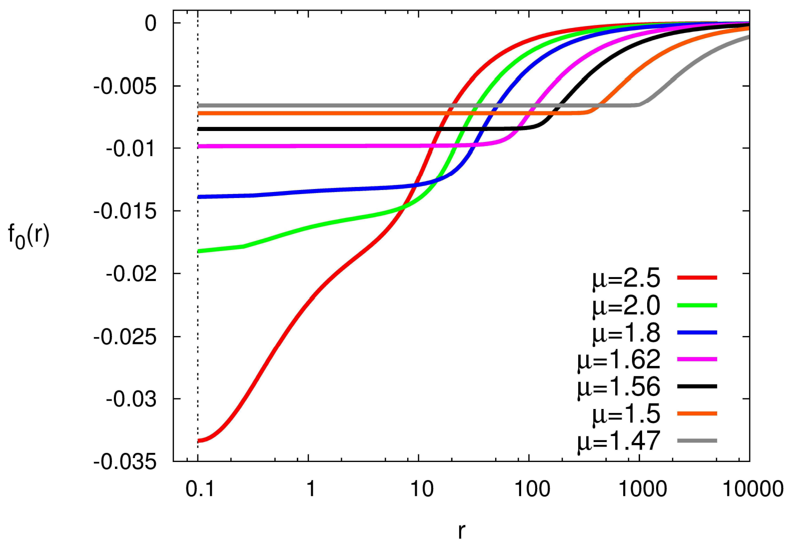
Both BHs with massless Q-hair and regular charged BSs approach the same singular limit

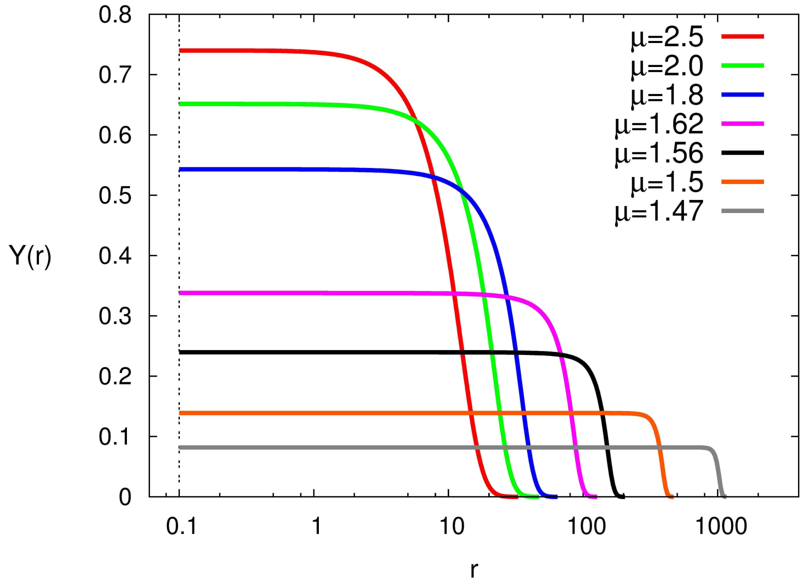
massless scalar component ($\mu^2=0$)





Geometry changes





Summary

- **There are spinning Kerr black holes with scalar hairs, the frequency must be synchronous with the rotational angular velocity of the event horizon → no spherical symmetry**
- **Black holes with charged scalar hairs are static, the condition of resonance is imposed on the horizon → spherical symmetry is allowed**
- **Massless long-range scalar component allows for bifurcations between the electrovacuum RN BH and BHs with resonance Q-hair.**
- **Boson star as light dark matter candidate?**

(e.g. L.Hui, J. Ostriker, S.Tremaine, and E. Witten *(Phys. Rev. D 95, 043541) etc)*

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Doug Bolton | @DougieBolton | Monday 18 January 2016 18:58 |

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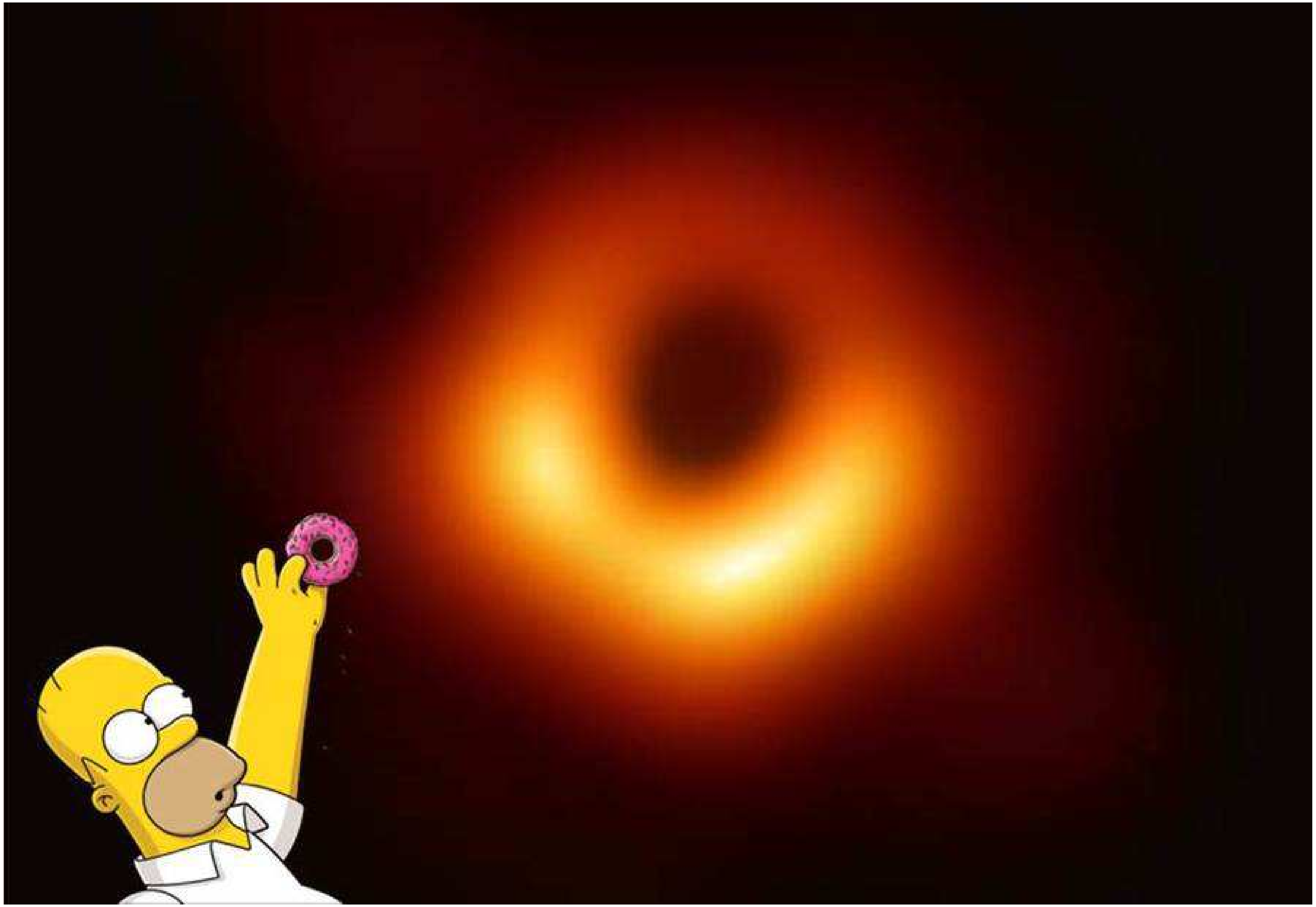

Einstein-deTurck Equations

(M. Headrick, S. Kitchen and T. Wiseman, Class. Quant. Grav. 27 (2010) 035002)

Elliptic Einstein-de Turck equations:

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 2\alpha^2 T_{\mu\nu}$$

DeTurck choice of ξ :

$$\xi^\mu = g^{\nu\rho}\left(\Gamma^\mu_{\nu\rho}(g) - \bar{\Gamma}^\mu_{\nu\rho}(\mathbf{g})\right)$$

← Reference metric

Spacetime metric:

$$ds^2 = f_1(r,\theta)\frac{dr^2}{N(r)} + S_1(r,\theta)(rd\theta + S_2(r,\theta)dr)^2$$
$$f_2(r,\theta)r^2\sin^2\theta d\phi^2 - f_0(r,\theta)N(r)dt^2$$

Reference metric:
(e.g. Schwarzschield)

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - N(r)dt^2$$

Have to verify *a posteriori* that $\xi = 0$,
to get a solution to Einstein equation