Decomposition of the gluon field into monopole and monopoleless components

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Plan

1. Introduction (Lattice, DSS of confinement)
2. Decomposition at $T = 0$ in $SU(2)$; $N_c = 2$ QCD
3. at $T = 0$ in $SU(3)$ gluodynamics
4. at $T > 0$ in $SU(2)$ gluodynamics
5. Conclusions and Outlook

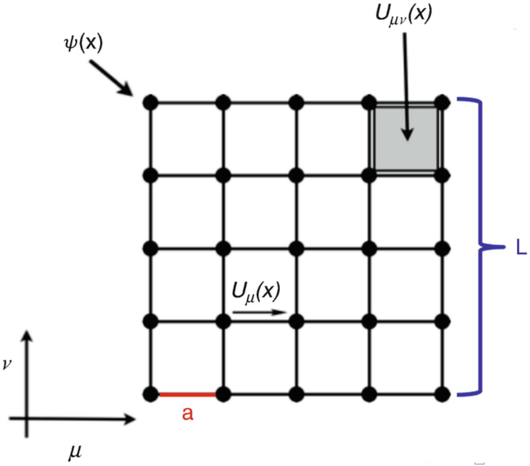
Publications :

1. Decomposition of the SU(2) gauge field in the maximal Abelian gauge
   Phys.Rev.D 105 (2022) 5, 054519
2. Decomposition of the Static Potential in SU(3) Gluodynamics
   JETP Lett. 117 (2023) 5, 328
3. Gribov copy effects in the maximal Abelian gauge
   Phys.Part.Nucl. 56 (2025) 2, 242
4. Abelian and Monopole Dominance in SU(3) Gluodynamics and Gribov Copy Effects
   Phys.Part.Nucl.Lett. 22 (2025) 1, 112
5. Decomposition of the static potential in SU(3) Gluodynamics and Gribov Copy Effects,
   in preparation

Lattice regularization

$$U_\mu(x) = P\exp\left(i \int_{C_{x,x+\hat{\mu}}} A_\mu(s)ds \right) \approx 1 + iaA_\mu(x) + O(a^2)$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N}\mathrm{ReTr}\,U_{\mu\nu}\right) = \frac{\beta}{2N_c} \sum_{x,\mu<\nu} \mathrm{Tr}[F_{\mu\nu}^2] + \mathcal{O}(a^2)\,, \quad \beta = \frac{2N_c}{g^2}$$

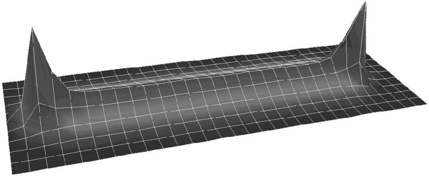


Lattice regularization, cont.

$$\langle O \rangle = \frac{1}{Z} \int \mathscr{D} U e^{-S_G(U)} O(U)$$

There is no problem of gauge fixing when $O(U)$ is gauge invariant

Examples of gauge non-invariant observables:
- Gluon, quark, ghost propagators
(needed to compare with, e.g. DSE approach)
- Various projected observables in MAG and center gauges
(needed to study respective scenarios of confinement)



Quark confinement:

- is confirmed experimentally and in lattice calculations
- linear dependence of static quarks interaction potential on a distance between them
- hasn't been proven analytically so far
- one of the approaches - to describe QCD vacuum as a dual superconductor, t'Hooft, 1976, Mandelstam, 1976

Maximal Abelian gauge in SU(3) gluodynamics

Suggested by t'Hooft, 1981 to define color-magnetic monopoles

Gauge fixing functional (breaks $SU(3)$ to $U(1)^2$)

$$F_{MAG} = \frac{1}{12V} \int d^4x \sum_{\mu=1}^{4} \sum_{a\neq 3,8} (A_\mu^a(x))^2$$

$$f^a(A) = \sum_{b\neq 3,8} (\partial_\mu \delta^{ab} - g f^{ab3} A_\mu^3 - g f^{ab8} A_\mu^8) A_\mu^b = 0\,, \quad a \neq 3,8$$

Gauge fixing functional in lattice regularization:

$$F_{MAG}^{latt} = 1 - \frac{1}{12V} \sum_{x,\mu,a=3,8} \mathrm{Tr}\{U_\mu(x)\lambda_a U_\mu^\dagger(x)\lambda_a\} \approx a^2 F_{MAG}$$

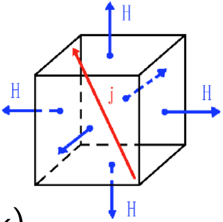
It is known that in a unitary gauge the Higgs model t'Hooft-Polyakov monopole has a form of a Dirac monopole
In $SU(N_c)$ theory without Higgs field we search for nonabelian color-magnetic monopoles making three steps (Kronfeld, Laursen, Schierholz, Wiese, 1987)

- to fix MA gauge
- to make Abelian projection

$$A_\mu(x) = \sum_{a\neq 3,8} A_\mu^a(x)\lambda_a + A_\mu^3(x)\lambda_3 + A_\mu^8(x)\lambda_8 \equiv A_\mu^{offd}(x) + A_\mu^{abel}(x)$$

- to use the Abelian component $A_\mu^{abel}(x)$ for locatation of Dirac monopoles via procedure introduced for compact $U(1)$ in DeGrand, Toussaint, 1980

Abelian lattice gauge field is defined by diagonal matrix $u_\mu(x) \in U(1) \times U(1)$



$$u_\mu^{aa}(x) = e^{i\theta_\mu^a(x)}, \quad \sum_a \theta_\mu^a(x) = 2\pi n$$

$$\theta_{\mu\nu}^a(x) = \partial_\mu \theta_\nu^a(x) - \partial_\nu \theta_\mu^a(x)$$

$$\theta_{\mu\nu}^a(x) = \bar{\theta}_{\mu\nu}^a(x) + 2\pi m_{\mu\nu}^a(x), \quad \bar{\theta}_{\mu\nu}^a(x) \in (-\pi, \pi)$$

color-magnetic current :

$$k_\mu^a(x) = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\, \partial_\nu m_{\alpha\beta}^a$$

A decomposition of a gauge field in MAG

Abelian field can be further decomposed into 'monopole' and 'photon' components (names are borrowed from compact $U(1)$)

$$A_\mu^{abel}(x) = A_\mu^{mon}(x) + A_\mu^{phot}(x)$$

$$aA_\mu^{a,mon}(x) \equiv \theta_\mu^{a,mon} = \sum_y D(x-y)\partial_\nu m_{\mu,\nu}(y)$$

The following decomposition was introduced in
Bornyakov, Polikarpov, Schierholz, Suzuki, Syritsyn, 2006

$$A_\mu(x) \;=\; \underset{(non\text{-}confining)}{A_\mu^{mod}(x)} + \underset{(confining)}{A_\mu^{mon}(x)}$$

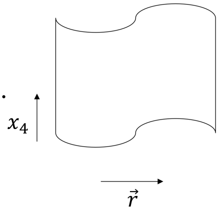
$$A_\mu^{mod}(x) = A_\mu^{offd}(x) + A_\mu^{phot}(x)$$

Static quark potential

$$W(r,t) = \frac{1}{N_c}\mathrm{Tr}\prod_{l\in C} U_l$$

$$\langle W(r,t)\rangle = C_0 e^{-tV(r)} + C_1 e^{-tE_1(r)} + \dots$$

$$aV(r) = \lim_{t\to\infty}\log\frac{\langle W(r,t)\rangle}{\langle W(r,t+a)\rangle}$$



We measure three types of $\langle W(r,t)\rangle$:

- for nonabelian gauge field $A_\mu(x)$,
- for monopole component $A_\mu^{mon}(x)$,
- for modified component $A_\mu^{mod}(x)$,

Decomposition of static potential $V(r)$ in $SU(2)$ gluodynamics and in $SU(2)$ QCD

First results on properties of this decomposition:
Bornyakov, Polikarpov, Schierhilz, Suzuki, Syritsyn, 2006

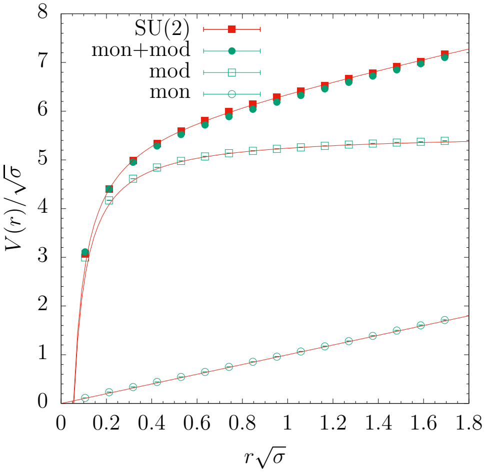
It was found that

$$V(r) \approx V_{mon}(r) + V_{mod}(r)$$

We demonstrated (Bornyakov, Kudrov, Rogalyov, 2021) that in $SU(2)$ gluodynamics the precision of this relation improves when lattice spacing a is decreasing (i.e. in the continuum limit)

Furthermore, we observed this decomposition in lattice $SU(2)$ gluodynamics with improved lattice action (universality) and in lattice $N_c = 2$ QCD.

Decomposition of static potential $V(r)$ in $SU(2)$ gluodynamics and in $SU(2)$ QCD



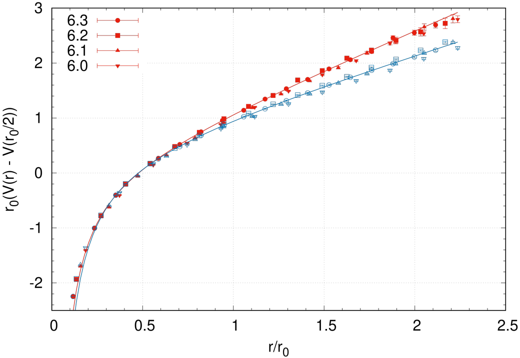
$V_{mon}(r) + V_{mod}(r)$ vs. physical static potential $V(r)$

Bornyakov, Kudrov, Rogalyov, 2021

Interpretation of this result for $V(r)$:

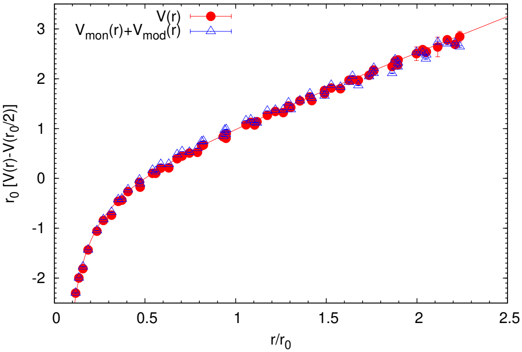
$A_\mu^{mon}(x)$ is responsible for the linear part of $V(r)$, i.e. it is a confining component,

$A_\mu^{mod}(x)$ is responsible for the perturbative part at small r and for hadron string fluctuations at large r, i.e. it is a non-confining component



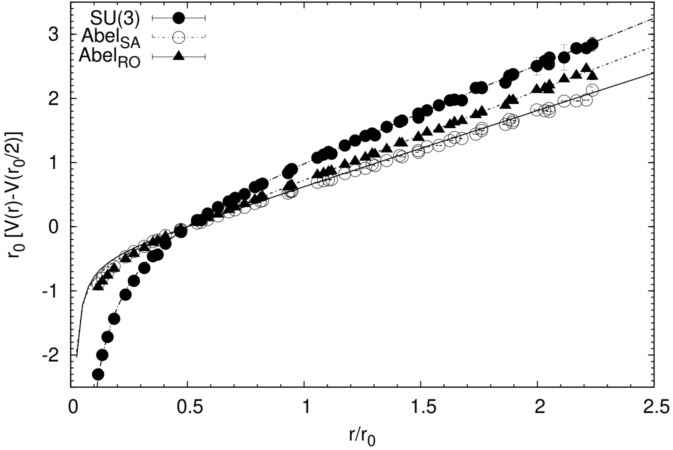
$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,
results for a few values of lattice spacing $a \in [0.06, 0.09]$ fm
With 'global' minima of F_{MAG} we find agreement at small r and disagreement at large r
Disagreement comes from low string tension in $V_{mon}(r)$

Decomposition of static potential in $SU(3)$ gluodynamics, cont.



$V_{mon}(r) + V_{mod}(r)$ is compared with $V(r)$,
With 'proper' minima (Gribov copies) we find agreement at all distances r

Abelian dominance



Abelian static potential $V^{abel}(r)$ for 'optimal' and 'proper' Gribov copies vs. physical static potential $V(r)$

We find perfect Abelian dominance for 'proper' Gribov copies

This conclusion contradicts that of Sakumichi, Suganuma, 2014

Decomposition at $T > 0$ in $SU(2)$ gluodynamics

We extend the study of the gauge field decomposition

$$A_\mu(x) = A_\mu^{mod}(x) + A_\mu^{mon}(x)$$

to the finite temperature case.
We study the Polyakov loop correlator and respective free energy $F_{q\bar{q}}$.
Polyakov loop

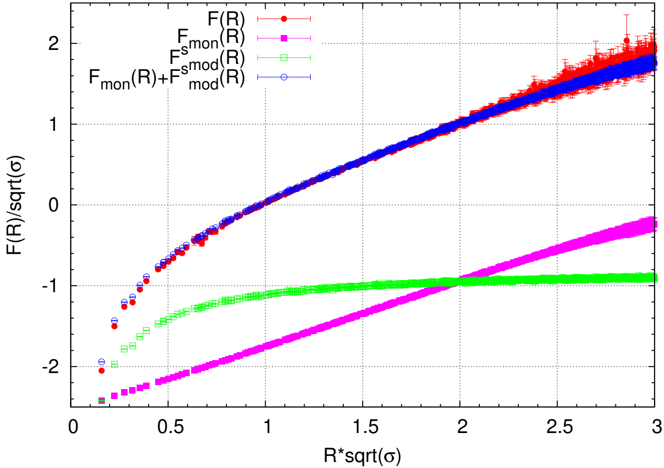
$$P_x = \frac{1}{2}\mathrm{Tr}L_x;\ \ L_x = \prod_{n=0}^{N_t-1} U_4(x+\hat{4}n);\ \ P_x^{mon,mod} = \frac{1}{2}\mathrm{Tr}\prod_{n=0}^{N_t-1} U_4^{mon,mod}(x+\hat{4}n)$$

Polyakov loop correlators C_{xy} (color averaged) and C_{xy}^s (color singlet):

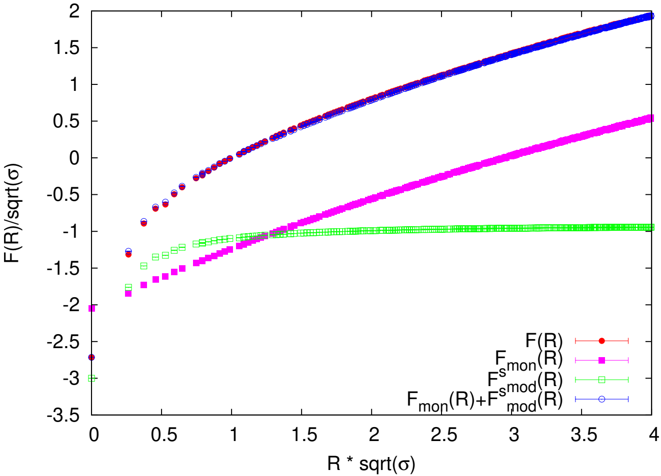
$$C_{xy} =< P_x P_y^\dagger >= \frac{1}{4} < \mathrm{Tr}L_x \mathrm{Tr}L_y^\dagger >; \qquad C_{xy}^s = \frac{1}{2} < \mathrm{Tr}L_x L_y^\dagger >$$

Free energy

$$F_{q\bar{q}}(r) = -T\ \log\ C_{xy} \qquad F_{q\bar{q}}^s(r) = -T\ \log\ C_{xy}^s$$



Free energy at $T/T_c = 0.76$



Free energy at $T/T_c = 0.90$

Conclusions

1. In our study of the gauge field decomposition
$$A_\mu(x) = A_\mu^{mon}(x) + A_\mu^{mod}(x) \tag{1}$$
in MA gauge of $SU(3)$ gluodynamics we observed that Gribov copies exist which produce a numerically precise decomposition for the static potential
$$V(r) = V_{mon}(r) + V_{mod}(r) \tag{2}$$
2. As a byproduct, we show that $\sigma_{abel} \approx \sigma$ (Abelian dominance, long standing problem) with high precision on these Gribov copies independent of the volume.
3. We found in $SU(2)$ gluodynamics that at $T > 0$ the decomposition works for the free energy $F_{q\bar{q}}$(r) of static quark-antiquark pair.

Future plans:

1. better understanding of differences between Gribov copies found in our study
2. To compute the gauge field propagators for A^{mon} and A^{mod} including mixed propagator
3. To compute the quark propagator for A^{mon} and A^{mod}
4. to study properties of this gauge field decomposition in QCD (i.e. with quarks)
5. to study decomposition for other observables, in particular, for hadron spectrum