FeynGrav

and Feynman rules for gravity

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Motivation

Perturbative Quantum Gravity

- Gravitons are small quantum metric perturbations
- Gravitons and particles with $s = 2$ and $m = 0$
- Flat background enforces the Poincare symmetry
- The theory is effective
- The theory is non-renormalisable
- Allows for standard calculations

Motivation

Perturbative metric expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$$

It spawns an infinite series

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa\, h^{\mu\nu} + \kappa^2\, h^{\mu\sigma} h_\sigma{}^\nu + \mathcal{O}(\kappa^3)$$

Functional integral is the way to quantise the theory

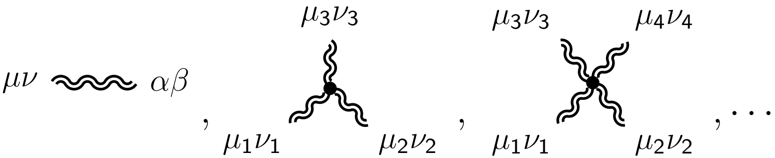
$$\mathcal{Z} = \int \mathcal{D}[g] \exp\left[i\mathcal{A}[g]\right] = \int \mathcal{D}[h] \exp\left[i\mathcal{A}[\eta + \kappa h]\right]$$

Motivation

The theory has an infinite number of interactions

$$\mathcal{A}=\int d^4x\Big[-\frac{1}{2}h_{\mu\nu}\mathcal{O}^{\mu\nu\alpha\beta}\Box h_{\alpha\beta}+\kappa\,\widehat{\mathcal{V}}^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3}_{(3)}h_{\rho_1\sigma_1}h_{\rho_2\sigma_2}h_{\rho_3\sigma_3}$$
$$+\kappa^2\,\widehat{\mathcal{V}}^{\rho_1\sigma_1\rho_2\sigma_2\rho_3\sigma_3\rho_4\sigma_4}_{(4)}h_{\rho_1\sigma_1}h_{\rho_2\sigma_2}h_{\rho_3\sigma_3}h_{\rho_4\sigma_4}+\mathcal{O}\left(\kappa^3\right)\Big]$$

The standard diagrammatic works



Motivation

The issue with the renormalisation

Infinite number of counterterms	$\Rightarrow$	Infinite amount of data

Tree level	R	1 coupling
One-loop	$R,\ R^2,\ R_{\mu\nu}^2$	3 couplings
Two-loop	$R,\ R^2,\ R_{\mu\nu}^2,\ R_{\mu\nu\alpha\beta}^3,\ \cdots$	13 couplings

Goroff, Sagnotti, Phys.Lett.B 160 (1985) 81

Kallosh, Nucl.Phys.B 78 (1974) 293

Nieuwenhuizen, Wu, J.Math.Phys. 18 (1977) 182

Motivation

The solution: the theory is effective

Perturbative expansion breaks when $h_{\mu\nu} \sim \kappa^{-1}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$$

- Perturbative expansion breaks
- Perturbative theory breaks
- The theory applicable below the Planck scale
- No need to use all order of perturbation theory

Infinite number of counterterms	$\Leftrightarrow$	Finite applicability of effective theory

Motivation

Perturbative quantum gravity
the simplest model of quantum gravity

Flat background	$\leftrightarrow$	Standard QFT Scattering amplitudes
Perturbative expansion	$\leftrightarrow$	Effective theory $\downarrow$ The lack of information on UV behaviour $\uparrow$
UV divergences	$\leftrightarrow$	Renormalisation

Motivation

The computational challenge:
interaction rules are huge!

3-graviton vertex contains 171 terms
4-graviton vertex contains 2850 terms

DeWitt, Phys.Rev. 162 (1967) 1239
Sannan, Phys.Rev.D 34 (1986) 1749

Goroff, Sagnotti, Nucl.Phys.B 266 (1986) 709
Prinz, Class.Quant.Grav. 38 (2021) 21, 215003

Motivation

$$
\begin{aligned}
V_{\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) = \text{sym}\Big[& -\frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \frac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \frac{1}{2}P_3(k_1\cdot k_2\,\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\
& + P_6(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2\,P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\
& + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2\,P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\
& + 2\,P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2\,P_3(k_1\cdot k_2\,\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})\Big]
\end{aligned}
$$

$$
\begin{aligned}
V_{\mu\alpha,\nu\beta,\sigma\gamma,\rho\lambda}(k_1,k_2,k_3,k_4) = \text{sym}\Big[& -\frac{1}{4}P_6(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}\eta_{\rho\lambda}) - \frac{1}{4}P_{12}(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}\eta_{\rho\lambda}) - \frac{1}{2}P_6(k_{1\nu}k_{2\mu}\eta_{\alpha\beta}\eta_{\sigma\gamma}\eta_{\rho\lambda}) \\
& + \frac{1}{4}P_6(k_1\cdot k_2\,\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}\eta_{\rho\lambda}) + \frac{1}{2}P_6(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\rho}\eta_{\gamma\lambda}) + \frac{1}{2}P_{12}(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\rho}\eta_{\gamma\lambda}) \\
& + P_6(k_{1\nu}k_{2\mu}\eta_{\alpha\beta}\eta_{\sigma\rho}\eta_{\gamma\lambda}) - \frac{1}{2}P_6(k_1\cdot k_2\,\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\rho}\eta_{\gamma\lambda}) + \frac{1}{2}P_{24}(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}\eta_{\rho\lambda}) \\
& + \frac{1}{2}P_{24}(k_{1\nu}k_{1\beta}\eta_{\mu\sigma}\eta_{\alpha\gamma}\eta_{\rho\lambda}) + \frac{1}{2}P_{12}(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\rho\lambda}) + P_{24}(k_{1\nu}k_{2\sigma}\eta_{\beta\mu}\eta_{\alpha\gamma}\eta_{\rho\lambda}) \\
& - P_{12}(k_1\cdot k_2\,\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu}\eta_{\rho\lambda}) + P_{12}(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}\eta_{\rho\lambda}) + P_{12}(k_{1\nu}k_{1\sigma}\eta_{\beta\gamma}\eta_{\mu\alpha}\eta_{\rho\lambda}) \\
& - P_{24}(k_1\cdot k_2\,\eta_{\mu\alpha}\eta_{\beta\sigma}\eta_{\gamma\rho}\eta_{\lambda\nu}) - 2\,P_{12}(k_{1\nu}k_{1\beta}\eta_{\alpha\sigma}\eta_{\gamma\rho}\eta_{\lambda\mu}) - 2\,P_{12}(k_{1\sigma}k_{2\gamma}\eta_{\alpha\rho}\eta_{\lambda\nu}\eta_{\beta\mu}) \\
& - 2\,P_{24}(k_{1\nu}k_{2\sigma}\eta_{\beta\rho}\eta_{\lambda\mu}\eta_{\alpha\gamma}) - 2\,P_{12}(k_{1\sigma}k_{2\rho}\eta_{\gamma\nu}\eta_{\beta\mu}\eta_{\alpha\lambda}) + 2\,P_6(k_1\cdot k_2\,\eta_{\alpha\sigma}\eta_{\gamma\nu}\eta_{\beta\rho}\eta_{\lambda\mu}) \\
& - 2\,P_{12}(k_{1\nu}k_{1\sigma}\eta_{\mu\alpha}\eta_{\beta\rho}\eta_{\lambda\gamma}) - P_{12}(k_1\cdot k_2\,\eta_{\mu\sigma}\eta_{\alpha\gamma}\eta_{\nu\rho}\eta_{\beta\lambda}) - 2\,P_{12}(k_{1\nu}k_{1\sigma}\eta_{\beta\gamma}\eta_{\mu\rho}\eta_{\alpha\lambda}) \\
& - P_{12}(k_{1\sigma}k_{2\rho}\eta_{\gamma\lambda}\eta_{\mu\nu}\eta_{\alpha\beta}) - 2\,P_{24}(k_{1\nu}k_{2\sigma}\eta_{\beta\mu}\eta_{\alpha\rho}\eta_{\lambda\gamma}) - 2\,P_{12}(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\rho}\eta_{\lambda\alpha}) \\
& + 4P_6(k_1\cdot k_2\,\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\rho}\eta_{\lambda\mu})\Big]
\end{aligned}
$$

Sannan, Phys.Rev.D 34 (1986) 1749

Motivation

"Furthermore, the graviton n-point vertex Feynman rules with $n > 2$ for (effective) Quantum General Relativity read:"

$$\mathfrak{G}_n^{\mu_1\nu_1|\cdots|\mu_n\nu_n}\left(p_1^\sigma, \cdots, p_n^\sigma\right) = \frac{i}{2^n} \sum_{\mu_i \leftrightarrow \nu_i} \sum_{s \in S_n} \mathfrak{g}_n^{\mu_{s(1)}\nu_{s(1)}|\cdots|\mu_{s(n)}\nu_{s(n)}} \left(p_{s(1)}^\sigma, \cdots, p_{s(n)}^\sigma\right)$$

$$\begin{aligned}
\mathfrak{g}_n^{\mu_1\nu_1|\cdots|\mu_n\nu_n}\left(p_1^\sigma, \cdots, p_n^\sigma\right) = \frac{(-\kappa)^{n-2}}{2} \sum_{m_1+m_2=n} \Bigg\{ & \sum_{i=0}^{m_1-1} \delta_{m_1 \neq n} \left(\hat{\delta}^\mu_{\mu_0} \hat{\delta}^\rho_{\nu_{i+1}} \prod_{a=0}^{i} \hat{\eta}^{\mu_a \nu_{a+1}}\right) \left(\hat{\delta}^\nu_{\mu_i} \hat{\delta}^\sigma_{\nu_{m_1}} \prod_{b=i}^{m_1-1} \hat{\eta}^{\mu_b \nu_{b+1}}\right) \left[p_\mu^{m_1} p_\nu^{m_1} \hat{\delta}_\rho^{\mu_{m_1}} \hat{\delta}_\sigma^{\nu_{m_1}} - p_\mu^{m_1} p_\rho^{m_1} \hat{\delta}_\nu^{\mu_{m_1}} \hat{\delta}_\sigma^{\nu_{m_1}}\right] \\
& - \sum_{j+k+l=m_1-2} \left(\hat{\delta}^\mu_{\mu_0} \hat{\delta}^\rho_{\nu_{j+1}} \prod_{a=0}^{j} \hat{\eta}^{\mu_a \nu_{a+j}}\right) \left(\hat{\delta}^\nu_{\mu_j} \hat{\delta}^\sigma_{\nu_{j+k+1}} \prod_{b=j}^{j+k} \hat{\eta}^{\mu_b \nu_{b+1}}\right) \left(\hat{\delta}^\kappa_{\mu_{j+k}} \hat{\delta}^\lambda_{\nu_{m_1-1}} \prod_{c=j+k}^{m_1-2} \hat{\eta}^{\mu_c \nu_{c+1}}\right) \left(\delta_{m_1 \neq n} \left[\left(p_\mu^{n-1} \hat{\delta}_\rho^{\mu_{n-1}} \hat{\delta}_\kappa^{\nu_{n-1}}\right) \left(\frac{1}{2} p_\lambda^n \hat{\delta}_\nu^{\mu_n} \hat{\delta}_\sigma^{\nu_n} - p_\nu^n \hat{\delta}_\lambda^{\mu_n} \hat{\delta}_\sigma^{\nu_n}\right)\right.\right. \\
& \left.\left. + \frac{1}{2} \left(p_\nu^{n-1} \hat{\delta}_\mu^{\mu_{n-1}} \hat{\delta}_\kappa^{\nu_{n-1}}\right) \left(p_\sigma^n \hat{\delta}_\rho^{\mu_n} \hat{\delta}_\lambda^{\nu_n}\right)\right] + \left(p_\kappa^{n-1} \hat{\delta}_\mu^{\mu_{n-1}} \hat{\delta}_\rho^{\nu_{n-1}}\right) \left(\frac{1}{2} p_\nu^n \hat{\delta}_\sigma^{\mu_n} \hat{\delta}_\lambda^{\nu_n} - \frac{1}{4} p_\lambda^n \hat{\delta}_\nu^{\mu_n} \hat{\delta}_\sigma^{\nu_n}\right) - \left(p_\nu^{n-1} \hat{\delta}_\mu^{\mu_{n-1}} \hat{\delta}_\kappa^{\nu_{n-1}}\right) \left(\frac{1}{2} p_\rho^n \hat{\delta}_\sigma^{\mu_n} \hat{\delta}_\lambda^{\nu_n} - \frac{1}{4} p_\sigma^n \hat{\delta}_\rho^{\mu_n} \hat{\delta}_\lambda^{\nu_n}\right)\right) \Bigg\} \\
& \times \Bigg\{ \sum_{\substack{i+j+k+l=m_2 \\ i \geq j \geq k \geq l \geq 0}} \sum_{p=0}^{j-k} \sum_{q=0}^{k-l} \sum_{r=0}^{q} \sum_{s=0}^{l} \sum_{t=0}^{s} \sum_{u=0}^{t} \sum_{v=0}^{u} \binom{\frac{1}{2}}{i} \binom{i}{j} \binom{j}{k} \binom{k}{l} \binom{j-k}{p} \binom{k-l}{q} \binom{q}{r} \binom{l}{s} \binom{s}{t} \binom{t}{u} \binom{u}{v} (-1)^{p+q-r+s-t+v} \, 2^{-j+l+r+s+2t-3u+v} \, 3^{-k+q-r+s-t+u} \\
& \times \left(\prod_{a=m_1+1}^{m_1+\boldsymbol{a}} \hat{\eta}^{\mu_a \nu_a}\right) \left(\prod_{b=m_1+\boldsymbol{a}+1}^{m_1+\boldsymbol{a}+\boldsymbol{b}} \hat{\eta}^{\mu_b \mu_{b+\boldsymbol{b}}} \hat{\eta}^{\nu_b \nu_{b+\boldsymbol{b}}}\right) \left(\prod_{c=m_1+\boldsymbol{a}+2\boldsymbol{b}+1}^{m_1+\boldsymbol{a}+2\boldsymbol{b}+\boldsymbol{c}} \hat{\eta}^{\mu_c \nu_{c+\boldsymbol{c}}} \hat{\eta}^{\mu_{c+\boldsymbol{c}} \nu_{c+2\boldsymbol{c}}} \hat{\eta}^{\mu_{c+2\boldsymbol{c}} \nu_c}\right) \left(\prod_{d=m_1+\boldsymbol{a}+2\boldsymbol{b}+3\boldsymbol{c}+1}^{m_1+\boldsymbol{a}+2\boldsymbol{b}+3\boldsymbol{c}+\boldsymbol{d}} \hat{\eta}^{\mu_d \nu_{d+\boldsymbol{d}}} \hat{\eta}^{\mu_{d+\boldsymbol{d}} \nu_{d+2\boldsymbol{d}}} \hat{\eta}^{\mu_{d+2\boldsymbol{d}} \nu_{d+3\boldsymbol{d}}} \hat{\eta}^{\mu_{d+3\boldsymbol{d}} \nu_d}\right) \Bigg\}
\end{aligned}$$

Prinz, Class.Quant.Grav. 38 (2021) 21, 215003

Motivation

Goals:

- Develop a framework
  to calculate interaction rules efficiently
- Obtain rules for
  - General relativity
  - Horndeski gravity
  - Dirac fermions
  - Vector fields
  - SU(N) Yang-Mills model
  - other theories
- Implement them in a computational package

Latosh, Class.Quant.Grav. 39 (2022) 16, 165006
Comput.Phys.Commun. 292 (2023) 108871
Comput.Phys.Commun. 310 (2025) 109508

Perturbative Expansion

Finite and infinite expansions

$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$	is a finite expansion
$g^{\mu\nu} = \eta^{\mu\nu} - \kappa\, h^{\mu\nu} + \cdots$	is an infinite expansions
$\partial_\alpha g_{\mu\nu} = \kappa\, \partial_\alpha h_{\mu\nu}$	is a finite expansion
$\partial_\alpha g^{\mu\nu} = -g^{\mu\rho} g^{\nu\sigma} \partial_\alpha g_{\rho\sigma}$	is an infinite expansion

Derivatives and infinite expansions factorise

Examples:

$$R_{\mu\nu\alpha\beta} = \partial_\mu \Gamma_{\alpha\nu\beta} - \partial_\nu \Gamma_{\alpha\mu\beta} + g^{\rho\sigma} \left\{ \Gamma_{\rho\nu\alpha} \Gamma_{\sigma\mu\beta} - \Gamma_{\rho\mu\alpha} \Gamma_{\sigma\nu\beta} \right\}$$

$$\Gamma_{\alpha\mu\nu} = \frac{\kappa}{2} \left[\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu} \right]$$

Perturbative Expansion

Factorisation

$$\mathcal{A} = \int d^4x \sqrt{-g}\mathcal{L}\left[g_{\mu\nu}, \Gamma_{\alpha\mu\nu}, g^{\mu\nu}, \Gamma^{\alpha}_{\mu\nu}, \Psi\right]$$
$$= \int d^4x \underbrace{\sqrt{-g}\,\mathcal{L}_{\text{infinite}}\left[g^{\mu\nu}\right]}_{\text{infinite part}} \underbrace{\mathcal{L}_{\text{finite}}\left[g_{\mu\nu}, \Gamma_{\alpha\mu\nu}, \Psi\right]}_{\text{finite part}}$$

- The finite part is calculated explicitly.
- The infinite part is generated on a compute for each perturbation order.

Perturbative Expansion

Notations

$$X \overset{\text{note}}{=} \sum_{n=0}^{\infty} \kappa^n \left(X\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} \; h_{\rho_1\sigma_1}\cdots h_{\rho_n\sigma_n}$$

Example

$$g^{\mu\nu} = \left(g^{\mu\nu}\right) + \kappa \left(g^{\mu\nu}\right)^{\rho_1\sigma_1} h_{\rho_1\sigma_1} + \kappa^2 \left(g^{\mu\nu}\right)^{\rho_1\sigma_1\rho_2\sigma_2} h_{\rho_1\sigma_1} h_{\rho_2\sigma_2} + \cdots$$

Perturbative expansion reduces to its coefficients.

Perturbative Expansion

Inverse metric

$$g^{\mu\nu} = \eta^{\mu\nu} + \sum_{n=1}^{\infty} (-\kappa)^n \left(h^n\right)^{\mu\nu}$$

$$\overset{\text{note}}{=} \sum_{n=0}^{\infty} \kappa^n \left(g^{\mu\nu}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} \cdots h_{\rho_n\sigma_n}$$

$$\left(h^n\right)^{\mu\nu} \overset{\text{note}}{=} h^{\mu}{}_{\sigma_1} h^{\sigma_1}{}_{\sigma_2} \cdots h^{\sigma_{n-1}\nu} = I_{(1+n)}^{\mu\nu\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} h_{\rho_2\sigma_2} \cdots h_{\rho_n\sigma_n}$$

Plain I-tensor of the n-th order

$$I_{(n)}^{\rho_1\sigma_1\cdots\rho_n\sigma_n} \overset{\text{def}}{=} \eta^{\sigma_1\rho_2}\eta^{\sigma_2\rho_3} \cdots \eta^{\sigma_n\rho_1}$$

Perturbative Expansion

The inverse metric expansion is completely defined.

$$g^{\mu\nu} = \sum_{n=0}^{\infty} (-\kappa)^n I_{(1+n)}^{\mu\nu\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} \cdots h_{\rho_n\sigma_n}$$

$$(g^{\mu\nu}) = I_{(1)}^{\mu\nu} = \eta^{\mu\nu}$$

$$(g^{\mu\nu})^{\rho\sigma} = -I_{(2)}^{\mu\nu\rho\sigma} = -\eta^{\nu\rho}\eta^{\sigma\mu}$$

$$(g^{\mu\nu})^{\rho_1\sigma_1\rho_2\sigma_2} = I_{(3)}^{\mu\nu\rho_1\sigma_1\rho_2\sigma_2} = \eta^{\nu\rho_1}\eta^{\sigma_1\rho_2}\eta^{\sigma_2\mu}$$

$$\downarrow$$

$$(g^{\mu\nu})^{\rho_1\sigma_1\cdots\rho_n\sigma_n} = (-1)^n I_{(1+n)}^{\mu\nu\rho_1\sigma_1\cdots\rho_n\sigma_n}$$

Perturbative Expansion

Perturbative expansion for the volume factor

$$\sqrt{-g} = \sum_{n=0}^{\infty} (-\kappa)^n \sum_{m=1}^{n} \frac{1}{m!} \left(-\frac{1}{2}\right)^m \left[\sum_{k_1+\cdots+k_m=n} \frac{\mathrm{tr}\, h^{k_1} \cdots \mathrm{tr}\, h^{k_m}}{k_1 \cdots k_m} \right]$$

$$\begin{aligned}
\sqrt{-g} &= \left[- \det \left\{ \eta_{\mu\sigma} \left(\delta^{\sigma}_{\nu} + \kappa\, h^{\sigma}{}_{\nu} \right) \right\} \right]^{1/2} = \left[\det \left\{ 1 + \kappa\, \widehat{h} \right\} \right]^{1/2} \\
&= \left[\det \exp \ln \left\{ 1 + \kappa\, \widehat{h} \right\} \right]^{1/2} = \exp \left[\frac{1}{2} \mathrm{tr} \ln \left\{ 1 + \kappa\, \widehat{h} \right\} \right] \\
&= \exp \left[-\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-\kappa)^n}{n} \mathrm{tr} \left(h^n \right) \right]
\end{aligned}$$

$$= 1 + \left(-\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-\kappa)^n}{n} \mathrm{tr} \left(h^n \right) \right) + \cdots + \frac{1}{m!} \left(-\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-\kappa)^n}{n} \mathrm{tr} \left(h^n \right) \right)^2 + \cdots$$

Perturbative Expansion

Implicit definition with the plain C-tensor

$$\sqrt{-g} = \sum_{n=0}^{\infty} \kappa^n C_{(n)}^{\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} \cdots h_{\rho_n\sigma_n}$$

The plain C-tensor admits a recurrent relation

$$C_{(n)}^{\rho_1\sigma_1\cdots\rho_n\sigma_n} = \frac{1}{2n} \sum_{k=1}^{n} (-1)^{k-1} I_{(k)}^{\rho_1\sigma_1\cdots\rho_k\sigma_k} C_{(n-k)}^{\rho_{k+1}\sigma_{k+1}\cdots\rho_n\sigma_n}$$

The relation derived with the Jacobi formula

$$\frac{d}{dz}\sqrt{-\det\left[\eta_{\mu\nu} + z\,\kappa\,h_{\mu\nu}\right]}\Bigg|_{z=1} = \frac{\kappa}{2}\sqrt{-g}\,g^{\mu\nu}\,h_{\mu\nu}$$

Perturbative Expansion

Scaled metric that is only used for calculations

$$\mathfrak{g}_{\mu\nu} \stackrel{\text{def}}{=} \eta_{\mu\nu} + z\,\kappa\,h_{\mu\nu}$$

Infinitesimal scaling action on the volume factor

$$\frac{d}{dz}\sqrt{-\det\left[\eta_{\mu\nu} + z\,\kappa\,h_{\mu\nu}\right]}\Bigg|_{z=1} = \frac{\kappa}{2}\sqrt{-g}\,g^{\mu\nu}\,h_{\mu\nu}$$

$$= \sum_{n=1}^{\infty}\kappa^n \sum_{k=1}^{n}\frac{(-1)^{k-1}}{2}\,I_{(k)}^{\rho_1\sigma_1\cdots\rho_k\sigma_k}\,C_{(n-k)}^{\rho_{k+1}\sigma_{k+1}\cdots\rho_n\sigma_n}\,h_{\rho_1\sigma_1}\cdots h_{\rho_n\sigma_n}$$

Plain C-tensor definition

$$\frac{d}{dz}\sqrt{-\det\left[\eta_{\mu\nu} + z\,\kappa\,h_{\mu\nu}\right]}\Bigg|_{z=1} = \frac{d}{dz}\sum_{n=1}^{\infty}\kappa^n\,C_{(n)}^{\rho_1\sigma_1\cdots\rho_n\sigma_n}\,z^n\,h_{\rho_1\sigma_1}\cdots h_{\rho_n\sigma_n}\Bigg|_{z=1}$$

Perturbative Expansion

The volume factor coefficient is completely defined.

$$\sqrt{-g} = \sum_{n=0}^{\infty} \kappa^n C_{(n)}^{\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1}\cdots h_{\rho_n\sigma_n}$$

$$(\sqrt{-g}) = C_{(0)} = 1$$

$$(\sqrt{-g})^{\rho_1\sigma_1} = C_{(1)}^{\rho_1\sigma_1} = \frac{1}{2}\eta^{\rho_1\sigma_1}$$

$$(\sqrt{-g})^{\rho_1\sigma_1\rho_2\sigma_2} = C_{(2)}^{\rho_1\sigma_1\rho_2\sigma_2} = \frac{1}{8}\eta^{\rho_1\sigma_1}\,\eta^{\rho_2\sigma_2} - \frac{1}{4}\eta^{\sigma_1\rho_2}\,\eta^{\sigma_2\rho_1}$$

$$\vdots$$

Perturbative Expansion

Vierbein is also completely defined
Prinz, Class.Quant.Grav. 38 (2021) 21, 215003

$$\mathfrak{e}^{\mu}{}_{\nu} = \sum_{n=0}^{\infty} \kappa^n \binom{\frac{1}{2}}{n} I_{\mu}{}^{\nu\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} \cdots h_{\rho_n\sigma_n}$$

$$\mathfrak{e}_{\mu}{}^{\nu} = \sum_{n=0}^{\infty} \kappa^n \binom{-\frac{1}{2}}{n} I_{\mu}{}^{\nu\rho_1\sigma_1\cdots\rho_n\sigma_n} h_{\rho_1\sigma_1} \cdots h_{\rho_n\sigma_n}$$

Here $\binom{a}{b} = \frac{\Gamma(a+1)}{\Gamma(b+1)\Gamma(a-b+1)}$ are binomial coefficients

Perturbative Expansion

The following factors mentioned are fully defined and can be generated by a computer.

$$\sqrt{-g}, \quad \sqrt{-g}\, g^{\mu\nu}, \quad \sqrt{-g}\, \mathfrak{e}_{\mu}{}^{\nu}, \quad \sqrt{-g}\, \mathfrak{e}^{\mu}{}_{\nu}$$
$$\sqrt{-g}\, g^{\mu_1\nu_1} g^{\mu_2\nu_2}, \quad \sqrt{-g}\, g^{\mu_1\nu_1} g^{\mu_2\nu_2} g^{\mu_3\nu_3}, \ldots$$

This is enough for the perturbation theory.

Feynman Rules

Scalar field

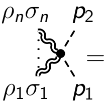
$$\mathcal{A} = \int d^4x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\,\nabla_\mu\phi\,\nabla_\mu\phi - \frac{m^2}{2}\,\phi^2\right]$$

$$= \int d^4x \left[\frac{1}{2}\sqrt{-g}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - \frac{m^2}{2}\sqrt{-g}\,\phi^2\right]$$

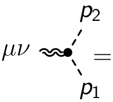
$$= \sum_{n=0}^{\infty} \int \prod_{k=1}^{n} \frac{d^4k_i}{(2\pi)^4} h_{\rho_i\sigma_i}(k_i) \prod_{j=1}^{2} \frac{d^4p_j}{(2\pi)^4} \phi(p_j)\,(2\pi)^4 \delta\Big(p_1 + p_2 + \sum k_i\Big)$$

$$\times\, \frac{-\kappa^n}{2}\left[\left(\sqrt{-g}\,g^{\mu\nu}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n}(p_1)_\mu(p_2)_\nu + m^2\left(\sqrt{-g}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n}\right]$$

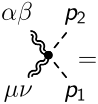
Feynman Rules



$$= -i\,\kappa^n \left[\left(\sqrt{-g}\,g^{\mu\nu}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} \frac{(p_1)_\mu (p_2)_\nu + (p_1)_\nu (p_2)_\mu}{2} + m^2 \left(\sqrt{-g}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} \right]$$



$$= i\,\frac{\kappa}{2}\Big[(p_1)_\mu (p_2)_\nu + (p_1)_\nu (p_2)_\mu - \eta_{\mu\nu}(p_1 \cdot p_2) - m^2 \eta_{\mu\nu}\Big]$$

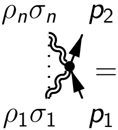


$$\begin{aligned} = i\,\frac{\kappa^2}{8}\Big[&(m^2 + p_1 \cdot p_2)\,(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}) \\ &+ \eta_{\mu\nu}\{(p_1)_\alpha (p_2)_\beta + (p_1)_\beta (p_2)_\alpha\} + \eta_{\alpha\beta}\{(p_1)_\mu (p_2)_\nu + (p_1)_\nu (p_2)_\mu\} \\ &- \eta_{\mu\alpha}\{(p_1)_\nu (p_2)_\beta + (p_1)_\beta (p_2)_\nu\} - \eta_{\mu\beta}\{(p_1)_\nu (p_2)_\alpha + (p_1)_\alpha (p_2)_\nu\} \\ &- \eta_{\nu\alpha}\{(p_1)_\mu (p_2)_\beta + (p_1)_\beta (p_2)_\mu\} - \eta_{\nu\beta}\{(p_1)_\mu (p_2)_\alpha + (p_1)_\alpha (p_2)_\mu\}\Big] \end{aligned}$$

The number of terms grows with the number of gravitons N approximately as $N!\,2^N$

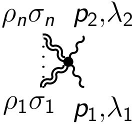
Feynman Rules

Dirac fermions



$$= i\,\kappa^n \left[\frac{1}{2} \left(\sqrt{-g}\,\mathfrak{e}_m{}^{\mu} \right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} (p_1 - p_2)_\mu \gamma^m - \left(\sqrt{-g} \right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} m \right]$$

Proca field



$$= i\kappa^n \left[\frac{1}{2} \left(\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} (p_1)_{\mu_1} (p_2)_{\mu_2} (F_{\mu\nu})^{\mu_1\lambda_1} (F_{\alpha\beta})^{\mu_2\lambda_2} + m^2 \left(\sqrt{-g}\, g^{\lambda_1\lambda_2} \right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} \right]$$

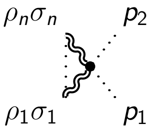
$$F_{\mu\nu} = -i\, p_\sigma \left(F_{\mu\nu} \right)^{\sigma\lambda} A_\lambda(p) \qquad \left(F_{\mu\nu} \right)^{\sigma\lambda} \overset{\text{def}}{=} \delta^\sigma_\mu\, \delta^\lambda_\nu - \delta^\sigma_\nu\, \delta^\lambda_\mu$$

Feynman Rules

Vector field

$$\mathcal{A} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} \, F_{\mu\nu} F_{\alpha\beta} + \frac{\epsilon}{2} g^{\mu\nu} g^{\alpha\beta} \, \nabla_\mu A_\nu \, \nabla_\alpha A_\beta - g^{\mu\nu} \, \nabla_\mu \bar{c} \nabla_\nu c \right]$$

Ghost vertex

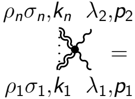


$$= i\,\kappa^n \, \left(\sqrt{-g}\, g^{\mu\nu} \right)^{\rho_1\sigma_1 \cdots \rho_n\sigma_n} \; I_{\mu\nu}{}^{\alpha\beta} (p_1)_\alpha (p_2)_\beta$$

The vertex cancels out the contribution of non-physical polarisations

Feynman Rules

Vector vertex



$$= i\,\kappa^n \Bigg[\frac{1}{2} \left(\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} (p_1)_{\sigma_1} (p_2)_{\sigma_2} (F_{\mu\nu})^{\sigma_1\lambda_1} (F_{\alpha\beta})^{\sigma_2\lambda_2}$$

$$- \epsilon \left(\sqrt{-g}\, g^{\mu_1\lambda_1} g^{\mu_2\lambda_2}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} (p_1)_{\mu_1} (p_2)_{\mu_2}$$

$$+ \epsilon \left\{ \left(\sqrt{-g}\, g^{\mu\nu} g^{\mu_1\lambda_1} g^{\mu_2\lambda_2}\right)^{\rho_2\sigma_2\cdots\rho_n\sigma_n} (k_1)_\sigma \left[(p_2)_{\mu_2} (\Gamma_{\mu_1\mu\nu})^{\sigma\rho_1\sigma_1} + (p_1)_{\mu_1} \left(\Gamma_{\mu_2\mu\nu}\right)^{\sigma\rho_1\sigma_1} \right] + \cdots \right\}$$

$$- \frac{\epsilon}{2} \Big\{ \left(\sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} g^{\mu_1\lambda_1} g^{\mu_2\lambda_2}\right)^{\rho_3\sigma_3\cdots\rho_n\sigma_n} \left[(k_1)_{\tau_1} (k_2)_{\tau_2} \left(\Gamma_{\mu_1\mu\nu}\right)^{\tau_1\rho_1\sigma_1} \left(\Gamma_{\mu_2\alpha\beta}\right)^{\tau_2\rho_2\sigma_2} \right.$$

$$\left. + (k_1)_{\tau_2} (k_2)_{\tau_1} \left(\Gamma_{\mu_2\mu\nu}\right)^{\tau_1\rho_2\sigma_2} \left(\Gamma_{\mu_1\alpha\beta}\right)^{\tau_2\rho_1\sigma_1} \right] + \cdots \Big\} \Bigg]$$

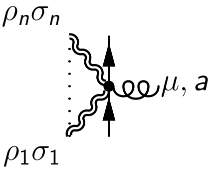
$$\Gamma_{\mu\alpha\beta} = \kappa\, (-i)\, p_\lambda\, (\Gamma_{\mu\alpha\beta})^{\lambda\rho\sigma}\, h_{\rho\sigma}(p)$$

$$(\Gamma_{\mu\alpha\beta})^{\lambda\rho\sigma} = \frac{1}{2} \left[\delta^\lambda_\alpha I_{\beta\mu}{}^{\rho\sigma} + \delta^\lambda_\beta I_{\alpha\mu}{}^{\rho\sigma} - \delta^\lambda_\mu I_{\alpha\beta}{}^{\rho\sigma} \right]$$

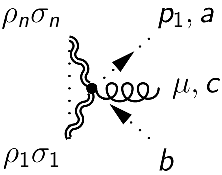
Feynman Rules

$SU(N)$ Yang-Mills

Gravity couples to all kind of energy!



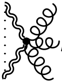
$$= i\,\kappa^n\, g_s\, \gamma^m\, T^a\, \left(\sqrt{-g}\,\mathfrak{e}_m{}^{\mu}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n}$$



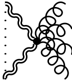
$$= -\kappa^n\, g_s\, f^{abc}\, (p_1)_\nu\, \left(\sqrt{-g}\, g^{\mu\nu}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n}$$

Feynman Rules

$SU(N)$ Yang-Mills



$$
=\kappa^n g_s f^{abc}\Big[(p_1-p_2)_\sigma\left(\sqrt{-g}\,g^{\mu_1\mu_2}g^{\mu_3\sigma}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} + (p_3-p_1)_\sigma\left(\sqrt{-g}\,g^{\mu_1\mu_3}g^{\mu_2\sigma}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} + (p_2-p_3)_\sigma\left(\sqrt{-g}\,g^{\mu_2\mu_3}g^{\mu_1\sigma}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n}\Big]
$$



$$
=-i\,g_s^2\kappa^n\Big[f^{a_1a_4s}f^{a_2a_3s}\left(\left(\sqrt{-g}\,g^{\mu_1\mu_2}g^{\mu_3\mu_4}\right)^{\rho_1\cdots\sigma_n}-\left(\sqrt{-g}\,g^{\mu_1\mu_3}g^{\mu_2\mu_4}\right)\right)^{\rho_1\cdots\sigma_n}
+ f^{a_1a_3s}f^{a_2a_4s}\left(\left(\sqrt{-g}\,g^{\mu_1\mu_2}g^{\mu_3\mu_4}\right)^{\rho_1\cdots\sigma_n}-\left(\sqrt{-g}\,g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right)\right)^{\rho_1\cdots\sigma_n}
+ f^{a_1a_2s}f^{a_3a_4s}\left(\left(\sqrt{-g}\,g^{\mu_1\mu_3}g^{\mu_2\mu_4}\right)^{\rho_1\cdots\sigma_n}-\left(\sqrt{-g}\,g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right)\right)^{\rho_1\cdots\sigma_n}\Big]
$$

Feynman Rules

General relativity

$$\mathcal{A}_{\mathrm{H+gf}} = \int d^4x \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{\epsilon}{2\,\kappa^2} g_{\mu\nu} \left(g^{\alpha\beta} \Gamma^{\mu}_{\alpha\beta} \right) \left(g^{\rho\sigma} \Gamma^{\nu}_{\rho\sigma} \right) \right]$$
$$= \int d^4x \sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} g^{\rho\sigma} \left(-\frac{2}{\kappa^2} \right) \left[\Gamma_{\alpha\mu\rho} \Gamma_{\sigma\nu\beta} - \Gamma_{\alpha\mu\nu} \Gamma_{\rho\beta\sigma} - \frac{\epsilon}{4} \Gamma_{\mu\alpha\beta} \Gamma_{\nu\rho\sigma} \right]$$

Ghost sector

$$\mathcal{A}_{\mathrm{ghost}} = \int d^4x \sqrt{-g} \left[-g^{\alpha\beta} g^{\mu\nu} \nabla_{\alpha} \overline{c}_{\mu} \nabla_{\beta} c_{\nu} - 2\, \Gamma^{\mu}_{\alpha\beta} \overline{c}_{\mu} \nabla^{\alpha} c^{\beta} + R_{\mu\nu} \overline{c}^{\mu} c^{\nu} \right]$$

Feynman Rules

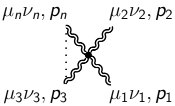
Graviton propagator

$$\mu\nu \;\sim\!\sim\!\sim\; \alpha\beta \Big|_{\epsilon=2} = \frac{i}{k^2}\frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

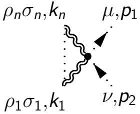
Ghost propagator

$$\mu \cdots\cdots \nu \quad = -\frac{i}{k^2}\eta_{\mu\nu}$$

Feynman Rules



$$= i\, 2\, \kappa^{n-2} \left(\sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} g^{\rho\sigma}\right)^{\mu_3\nu_3\cdots\mu_n\nu_n} (p_1)_{\lambda_1} (p_2)_{\lambda_2}$$
$$\times \left[(\Gamma_{\alpha\mu\rho})^{\lambda_1\mu_1\nu_1} (\Gamma_{\sigma\nu\beta})^{\lambda_2\mu_2\nu_2} - (\Gamma_{\alpha\mu\nu})^{\lambda_1\mu_1\nu_1} (\Gamma_{\rho\beta\sigma})^{\lambda_2\mu_2\nu_2} - \frac{\epsilon}{4} (\Gamma_{\mu\alpha\beta})^{\lambda_1\mu_1\nu_1} (\Gamma_{\nu\rho\sigma})^{\lambda_2\mu_2\nu_2} \right]$$
$$+ \text{permutations}$$



$$= i\, \kappa^n \Bigg[\left(\sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta}\right)^{\rho_1\sigma_1\cdots\rho_n\sigma_n} (p_1)_\alpha (p_2)_\beta - \left(\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} g^{\rho\sigma}\right)^{\rho_2\sigma_2\cdots\rho_n\sigma_n} (k_1)_\lambda$$
$$\times \left[(p_1)_\sigma (\Gamma_{\beta\rho\alpha})^{\lambda\rho_1\sigma_1} - (p_2)_\sigma (\Gamma_{\alpha\rho\beta})^{\lambda\rho_1\sigma_1} + (k_1)_\rho (\Gamma_{\sigma\alpha\beta})^{\lambda\rho_1\sigma_1} - (k_1)_\alpha (\Gamma_{\rho\beta\sigma})^{\lambda\rho_1\sigma_1} \right]$$
$$- \left(\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} g^{\rho\sigma} g^{\lambda\tau}\right)^{\rho_3\sigma_3\cdots\rho_n\sigma_n} (k_1)_{\lambda_1} (k_2)_{\lambda_2}$$
$$\times \left[(\Gamma_{\rho\alpha\lambda})^{\lambda_1\rho_1\sigma_1} (\Gamma_{\sigma\beta\tau})^{\lambda_2\rho_2\sigma_2} - (\Gamma_{\rho\alpha\beta})^{\lambda_1\rho_1\sigma_1} (\Gamma_{\sigma\lambda\tau})^{\lambda_2\rho_2\sigma_2} + (\Gamma_{\alpha\rho\lambda})^{\lambda_1\rho_1\sigma_1} (\Gamma_{\beta\sigma\tau})^{\lambda_2\rho_2\sigma_2} \right] \Bigg]$$
$$+ \text{permutations}$$

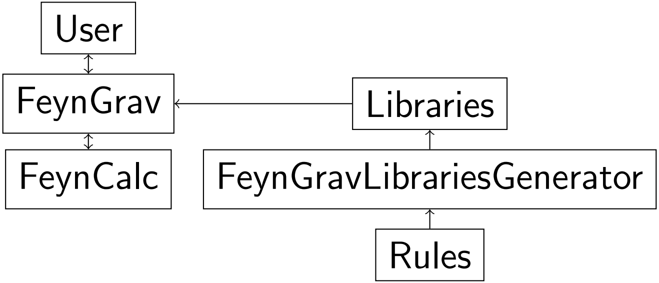
Feynman Rules

$s = 0$	[1,2]	Horndeski	[2,3]
$s = 1/2$	[1,2]	$SU(N)$ Yang-Mills	[3]
$s = 1$	[1,2]	Axion-like coupling	[3]
$s = 2,\ m = 0$	[1,2]	Quadratic gravity	[3]
$s = 2,\ m \neq 0$	[3]		

[1] Class.Quant.Grav. 39 (2022) 16, 165006
[2] Comput.Phys.Commun. 292 (2023) 108871
[3] Comput.Phys.Commun. 310 (2025) 109508

FeynGrav

The package is publicly available.
https://github.com/BorisNLatosh/FeynGrav



FeynGrav

Implemented rules

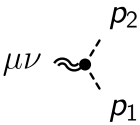
$s = 0$	Horndeski
$s = 1/2$	Dirac, SU(N) Yang-Mills
$s = 1$	Maxwell, Proca, SU(N) Yang-Mills, Axion-like
$s = 2$	General Relativity, Quadratic Gravity, Massive Gravity

FeynGrav: $\mathcal{O}\left(\kappa^2\right)$
Mendeley Data: up to $\mathcal{O}\left(\kappa^4\right)$

Latosh, Mendeley Data, "FeynGrav Libraries", (2024)

FeynGrav

Example: scalar-graviton vertex

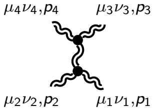


$$= \texttt{GravitonScalarVertex}[\{\mu,\nu\}, p_1, p_2, m]$$

$$= \frac{i}{2}\kappa\Big(\texttt{FVD}[p_1,\nu]\texttt{FVD}[p_2,\mu] + \texttt{FVD}[p_1,\mu]\texttt{FVD}[p_2,\nu] + m^2\texttt{MTD}[\mu,\nu] + \texttt{MTD}[\mu,\nu]\texttt{SPD}[p_1,p_2]\Big)$$

FeynGrav

Example: graviton scattering amplitude



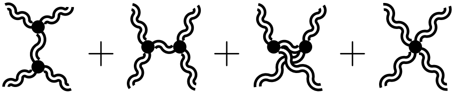
$$
\begin{aligned}
&= \texttt{GravitonVertex}[\mu_1, \nu_1, p_1, \mu_2, \nu_2, p_2, \alpha_1, \beta_1, -(p_1+p_2)] \\
&\quad \times \texttt{GravitonPropagator}[\alpha_1, \beta_1, \alpha_2, \beta_2, p_1 + p_2] \\
&\quad \times \texttt{GravitonVertex}[\mu_3, \nu_3, p_3, \mu_4, \nu_4, p_4, \alpha_2, \beta_2, p_1+p_2]
\end{aligned}
$$

Evaluation time below 6 minutes

Sannan, Phys.Rev.D 34 (1986) 1749

FeynGrav

The complete amplitude on-shell in $d = 4$



$$\mathcal{M} = i\,\frac{\kappa^2}{16}\,\frac{1}{s\,t\,u}\Big[s^4(1+h_1h_3)(1+h_2h_4)+t^4(1-h_1h_2)(1-h_3h_4)$$
$$+(2\,s^3t+3\,s^2t^2+2\,s\,t^3)\big((1+h_1h_3)(1+h_2h_4)-(h_1+h_3)(h_2+h_4)\big)\Big]$$

Evaluation time below 15 minute

FeynGrav

Example: virtual graviton exchange
Classical case

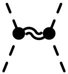
$$\mathcal{L} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{G\, m_1 m_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\frac{d\sigma}{d\Omega} = \frac{\left(G\,\mu\, m_1\, m_2\right)^2}{4\, p_{\text{cm}}^4\, \sin^4\frac{\theta}{2}}$$

Latosh, Yachmenev, Class.Quant.Grav. 40 (2023) 24, 245008

FeynGrav

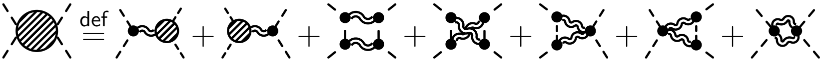
Quantum case, tree level

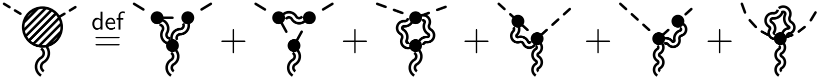


$$= -i\,\frac{\kappa^2}{4\,t}\left(s(s+t)-(2\,s+t)(m_1^2+m_2^2)+m_1^4+m_2^4\right)$$

FeynGrav

One-loop level





Latosh, Mendeley Data, doi:10.17632/zyn47cnsz3.1, (2023).

FeynGrav

Tree level + one-loop level
+ low energy limit

$$\frac{d\sigma}{d\Omega} = \frac{(G\,\mu\,m_1\,m_2)^2}{4\,p_{\rm cm}^4\,\sin^4\frac{\theta}{2}}$$

$$\times\left[\left\{1+\mathcal{O}\left(p_{\rm cm}^2\right)\right\}+G\left\{32\pi^5 p_{\rm cm}\,(m_1+m_2)\sin\frac{\theta}{2}+\mathcal{O}\left(p_{\rm cm}^2\right)\right\}+\mathcal{O}\left(G^2\right)\right]$$

Other examples

- J.Exp.Theor.Phys. 136 (2023) 5, 555
- with A. Yachmenev, Class.Quant.Grav. 40 (2023) 24, 245008
- with M. Park, Phys.Rev.D 110 (2024) 4, 046025
- Lanosa, Santillan, arXiv:2504.14434
- Bohnenblust, Ita, Kraus, Schlenk, arXiv:2505.15724
- Ma, Zeng, arXiv:2502.04332
- Cassem, Hertzberg, arXiv:2408.12118
- Ewasiuk, Profumo, Phys.Rev.D 111 (2025) 1, 015008
- Wang, Battista, Eur.Phys.J.C 85 (2025) 3, 304

Discussion

- Perturbation theory is computable for all orders
- FeynGrav is enough to calculate quantum gravitational effects in the standard model
- Further development:
  - BRST formalism
  - Fields redefinition
  - New models: massive gravity, SUSY, etc
  - Optimisation
  - Spinor helicity formalism

    and other tools

Thank you for your attention