Higher derivatives regularization scheme for $6D$, $\mathcal{N} = (1,0)$ supersymmetric gauge theory in harmonic superspace

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Motivation

- Supersymmetric field theory is still rapidly developing field of theoretical physics, closely related to high-energy physics and mathematical physics.
- The main motivation for studying the quantum structure of such theories is their connection with the low-energy limit of superstring theory, which in principle allows the study of low-energy effects of superstring theory using quantum field theory methods.
- A broad range of problems arises related to studying the quantum structure of supersymmetric gauge theories reduced from superstring theory. Such theories require the development of methods to investigate low-energy quantum effects in superstring theory using quantum field theory techniques.

Following the paper [**I.L. Buchbinder, E. A. Ivanov, K.V.Stepanyantz and A.B. Phys. Rev. D (2025)**] we develop quantization procedure for $6D$, $\mathcal{N}=(1,0)$ supersymmetric gauge theories in harmonic superspace within a higher-derivative regularization scheme.

- We consider the higher-derivative $6D, \mathcal{N}=(1,0)$ supersymmetric gauge theory proposed in [**Ivanov, Smilga and Zupnik(2005)**] coupled to the hypermultiplet.
- This theory is formulated in $6D$, $\mathcal{N}=(1,0)$ harmonic superspace, that was originally developed in $4D$ by [**Galperin, Ivanov, Ogievetsky, Sokatchev (86')**].
- The quantization procedure is carried out in the framework of the superfield background method that ensures the manifest $6D$, $\mathcal{N}=(1,0)$ supersymmetry and the classical gauge invariance of the quantum effective action.
- In $6D, \mathcal{N}=(1,0)$ quantum theories, dimensional regularization complicates supersymmetry preservation, especially when evaluating divergent terms at higher loops.
- The higher covariant derivative regularization, ensures the manifest $6D$, $\mathcal{N}=(1,0)$ supersymmetry and the classical gauge invariance of the quantum effective action.

Key points

- The $6D, \mathcal{N} = (1,0)$ higher derivative theory interacting with the hypermultiplet in the adjoint representation seems to be renormalizable and free from the quadratic divergences.
- It is possible to suggest the existence of the all-loop NSVZ-like expression for the β-function of this theory analogous to the NSVZ equation for the pure $4D, \mathcal{N} = 1$ SYM theory.
- The NSVZ equation naturally appears (for example, NSVZ equation for the pure $4D, \mathcal{N} = 1$ SYM theory) with the higher derivative regularization because in this case the integrals giving the β-function defined in terms of the bare couplings are integrals of double total derivatives with respect to the loop momenta, and this can be seen even in the one-loop approximation.

The harmonic superspace formalism is essential, as it provides the only way to formulate extended supersymmetric theory in terms of unconstrained $\mathcal{N}=(1,0)$ off-shell superfields.

The $6D$, $\mathcal{N}=(1,0)$ harmonic superspace is parametrized by the coordinates $(x^M, \theta^a_i, u^{\pm i})$, where x^M (with $M=0,\ldots,5$) are the ordinary space-time coordinates, θ^a_i (with $a=1,\ldots,4$ and $i=1,2$) the anticommuting left-handed spinors, and the harmonic variables $u^{\pm i}$, such that

$$u^-_i = (u^{+i})^*, \qquad u^{+i}u^-_i = 1, \qquad u^\pm_i = \varepsilon_{ij}u^{\pm j}. \tag{1}$$

It contains the analytic subspace closed under supersymmetry transformations with coordinates

$$(\zeta, u) = (x^M_{\mathcal{A}}, \theta^{+a}, u^{\pm i}), \quad x^M_{\mathcal{A}} = x^M + \frac{i}{2}\theta^+\gamma^M\theta^-, \quad \theta^{\pm a} = u^\pm_k \theta^{ak}. \tag{2}$$

The gauge vector multiplet and the hypermultiplet (in the adjoint representation) are described by the analytic superfields $V^{++} = e_0 V^{++A}t^A$ and $q^+ = e_0 q^{+A}t^A$, respectively,

$$D^+_a V^{++} = 0, \qquad D^+_a q^+ = 0, \tag{3}$$

where $D^+_a = u^+_i D^i_a$.

We consider higher-derivative theory which in the $6D, \mathcal{N} = (1,0)$ harmonic superspace is described by the action

$$S = \pm \frac{1}{2e_0^2} \mathrm{tr} \int d\zeta^{(-4)} (F^{++})^2 - \frac{2}{e_0^2} \mathrm{tr} \int d\zeta^{(-4)} \widetilde{q^+} \nabla^{++} q^+. \qquad (4)$$

Here e_0 is the dimensionless bare coupling constant, the gauge covariant harmonic derivatives are defined as $\nabla^{\pm\pm} = D^{\pm\pm} + iV^{\pm\pm}$, $D^{\pm\pm} = u^{\pm i}\partial/\partial u^{\mp i}$, and the integration measure is given by

$$\int d\zeta^{(-4)} \equiv \int d^6x \, d^4\theta^+ \, du. \qquad (5)$$

The harmonic superspace analog of the gauge field superstrength is defined by the equations

$$F^{++} \equiv (D^+)^4 V^{--}, \qquad (6)$$

where the (non-analytic) superfield

$$V^{--}(z,u) \equiv \sum_{n=1}^{\infty} (-i)^{n+1} \int du_1 \dots du_n \frac{V^{++}(z,u_1) V^{++}(z,u_2) \dots V^{++}(z,u_n)}{(u^+u_1^+)(u_1^+u_2^+) \dots (u_n^+u^+)} \qquad (7)$$

In components this action contains the term with higher derivatives of the gauge field

$$S = \mathrm{tr}\int d^6x\Big[\pm\frac{1}{e_0^2}(D_M F^{MN})^2 + \dots\Big] \qquad (8)$$

Due to the higher derivatives the degree of divergence does not increase with a number of loops.

The theory could contain quadratic and logarithmical divergences, but the quadratic divergences cancel each other in the one loop approximation (and presumably in all loops) due to the presence of the hypermultiplet in the adjoint representation.

The hypermultiplet and ghosts do not receive divergent quantum corrections because the corresponding parts of the action are not given by the integrals over the total harmonic superspace.

Quantization

For quantizing gauge theories it is convenient to use the background field method, because it allows constructing the manifestly gauge invariant effective action. The background-quantum splitting in harmonic superspace is carried out as follows

$$V^{++} \to \boldsymbol{V}^{++} + v^{++},\, q^+ \to q^+ \tag{9}$$

where $\boldsymbol{V}^{++}$ and v^{++} are the background and quantum gauge superfields, respectively.

In this case the original gauge invariance amounts to two different symmetries. The quantum gauge symmetry is broken by the gauge-fixing procedure

$$\boldsymbol{V}^{++} \to \boldsymbol{V}^{++},\, v^{++} \to e^{i\lambda}(v^{++}+\boldsymbol{V}^{++})e^{-i\lambda} - \boldsymbol{V}^{++} - ie^{i\lambda}D^{++}e^{-i\lambda},\, q^+ \to e^{i\lambda}q^+\,. \tag{10}$$

The second, background gauge symmetry remain the manifest symmetry of the effective action

$$\boldsymbol{V}^{++} \to e^{i\lambda}\boldsymbol{V}^{++}e^{-i\lambda} - ie^{i\lambda}D^{++}e^{-i\lambda},\, v^{++} \to e^{i\lambda}v^{++}e^{-i\lambda},\, q^+ \to e^{i\lambda}q^+\,. \tag{11}$$

Respectively, we choose the gauge-fixing term which preserves this symmetry.

We regularize the theory under consideration by higher covariant derivatives. The higher derivative term is constructed with the help of the operator

$$\Box \equiv \frac{1}{2}(D^+)^4(\nabla^{--})^2. \qquad (12)$$

which is analogous to theLaplace operator when acting on analytic superfields. The sum of the original action and the higher derivative term can be written in the form

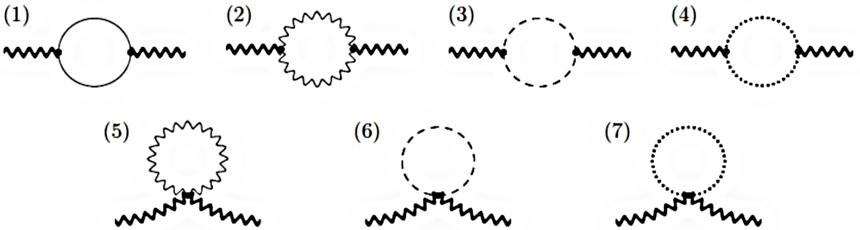
$$S_{\text{reg}} = \pm\frac{1}{2e_0^2}\text{tr}\int d\zeta^{(-4)} F^{++} R\Big(\frac{\Box}{\Lambda^2}\Big) F^{++} - \frac{2}{e_0^2}\,\text{tr}\int d\zeta^{(-4)} \widetilde{q^+}\nabla^{++} q^+, \qquad (13)$$

where $R(0) = 1$ and $R(x) \to \infty$ at $x \to \infty$. For regularizing the one-loop divergences it is also necessary to add the Pauli-Villars superfields (in order to cancel the one-loop divergences which survive after adding the higher derivative term to the classical action) with the mass $M = a\Lambda$. Then the generating functional of the regularized theory takes the form

$$Z = \int Dv^{++} D\widetilde{q^+} Dq^+ Db\, Dc\, D\varphi\, \text{Det}^{1/2}\Big[\Box^2 R\Big(\frac{\Box}{\Lambda^2}\Big)\Big]$$
$$\times\, \text{Det}(PV, M)\, \exp\Big(iS_{\text{reg}} + iS_{\text{gf}} + iS_{\text{FP}} + iS_{\text{NK}} + iS_{\text{sources}}\Big). \qquad (14)$$

The divergent supergraph contributions

The divergent part of the one-loop effective action contributed by the following harmonic superdiagrams



The solid lines correspond to the hypemultiplet;
The wavy lines correspond to the gauge superfield;
The dashed lines denote propagators of the Faddeev-Popov ghosts;
The dotted lines denote propagators of the Nielsen-Kallosh ghosts.

Quadratic divergences in these superdiagrams cancel each other, while the logarithmical divergences determine the β-function $\beta(\alpha_0)$ (defined in terms of the bare coupling constant), where $\alpha_0 \equiv e_0^2/4\pi$.

The one-loop β-function defined in terms of the bare coupling constant for the theory is given by the integral of double total derivatives in the momentum space

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \mp 2\pi C_2 \frac{d}{d\ln\Lambda}\int \frac{d^6K}{(2\pi)^6}\frac{\partial^2}{\partial K_\mu \partial K^\mu}\left[\frac{1}{K^4}\ln\left(1+\frac{M^4}{K^4 R_K}\right)\right] + O(\alpha_0). \quad (15)$$

Note that, due to the presence of an arbitrary regulator function $R(x)$, this fact is highly nontrivial. Calculating the integrals we obtain the one-loop result

$$\beta(\alpha_0) = \mp\frac{C_2\alpha_0^2}{2\pi^2} + O(\alpha_0^3). \quad (16)$$

This expression agrees with the results of the calculations made with dimensional reduction by various methods if one takes into account the contribution of the hypermultiplet in the adjoint representation.

[**E. A. Ivanov, A. V. Smilga and B. M. Zupnik, Nucl. Phys. B (2005)** ;**L. Casarin and A. A. Tseytlin, JHEP (2019)** ;**I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin and K.V. Stepanyantz, JHEP (2020)**]

The resemblance in the structure of the one-loop results for $4D, \mathcal{N} = 1$ supersymmetric gauge theory and for the considered $6D, \mathcal{N} = (1,0)$ higher derivative theory allows to suggest that it may be possible to write down the all-loop exact expression for the β-function

$$\beta(\alpha_0) = \mp \frac{\alpha_0^2 C_2}{2\pi^2\Big(1 \mp \alpha_0 C_2/8\pi^2\Big)}. \qquad (17)$$

This result can be obtained with the help of a certain modification of the one-loop calculation, namely, by replacing tree propagators with the exact propagators (for $4D, \mathcal{N} = 1$ theories the similar procedure gives the NSVZ expression).

Certainly, this derivation is not rigourous and should be verified by explicit multiloop calculations. (If possible), it would be also expedient to construct its rigorous all-order proof analogous to the one for the $4D, \mathcal{N} = 1$ case.

If the expression (17) is correct and (for the β-function defined in terms of the renormalized coupling constant) is valid in the HD+MSL scheme, similarly to the pure $4D, \mathcal{N} = 1$ SYM theory, it is possible to integrate the renormalization group equation

$$\frac{d\alpha}{d\ln\mu} = \widetilde{\beta}(\alpha) = \mp\frac{\alpha^2 C_2}{2\pi^2\Big(1 \mp \alpha C_2/8\pi^2\Big)} \tag{18}$$

and obtain the expression that does not receive quantum corrections in any order of the perturbation theory

$$\left(\frac{\alpha}{\mu^4}\right)^{C_2} \exp\left(\pm\frac{8\pi^2}{\alpha}\right) = \text{RGI}. \tag{19}$$

Conclusion

- The higher covariant derivative regularization, ensures the manifest $6D$, $\mathcal{N} = (1,0)$ supersymmetry and the classical gauge invariance of the quantum effective action and plays significant role for higher-loop calculations.
- The $6D, \mathcal{N} = (1,0)$ higher derivative theory interacting with the hypermultiplet in the adjoint representation is seems to be renormalizable and free from the quadratic divergences.
- It is possible to suggest the existence of the all-loop NSVZ-like expression for the β-function of this theory analogous to the NSVZ equation for the pure $4D, \mathcal{N} = 1$ SYM theory.
- The proposed expression should be verified in higher orders and proved (or rejected) by rigorous methods.

Thank you for your attention!