General Lagrangian formulations and structure of covariant cubic vertices for mixed-antisymmetric HS fields

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- I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless Integer HS Fields, PLB (2021), [arXiv:2105.12030],
- I.L. Buchbinder, A.R, Covariant Cubic Interacting Vertices for Massless and Massive Integer Higher Spin Fields, Symmetry (2023) [arXiv:2212.07097] ,
- A.R., BRST–BV approach for interacting HS fields, TMPh (2023) [arXiv:2303.02870],
- I.L. Buchbinder, A.R, Consistent Lagrangians for irreducible interacting higher-spin fields with holonomic constraints [arXiv:2304.10358] PEPAN (2023)
- A.R, Towards Lagrangian dynamics for constrained mixed-symmetric interacting HS fields, [arXiv:2505.02190] PEPAN
- **I.L. Buchbinder, Yu.Bogdanova, A.R, General Lagrangian formulations for irreducible mixed-antisymmetric tensor fields on Minkowski spaces, [arXiv:2509.xxxxx]**

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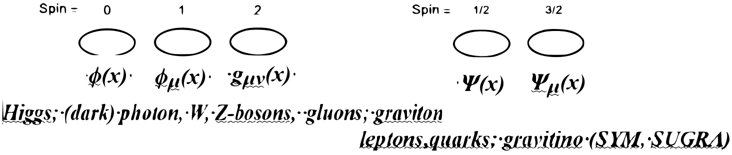
Wigner-Bargmann (1939, 1948) classification (1939, 1948) of UIRs $ISO(1, d-1)$ is characterized by $[(d+1)/2]$ Casimirs; A. Isaev (2023-2024)

1. $P^2 = m^2, W^2 = -m^2 s(s+1)$ - massive Unitary irrep (UIR) with (half)integer spin;

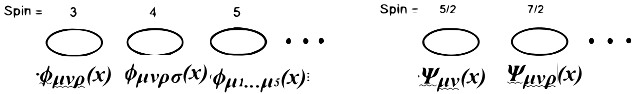
2a. $P^2 = 0, W^2 = 0,\ W^\mu = \lambda P^\mu$ - massless helicity UIR;

2b. $P^2 = 0, W^2 = \mu^2$ - massless continuous spin UIR;

Lower Spin refers to consistent classical field theories ($s \leqslant 2$)



Higher Spin (HS) stands for problematic construction ($s > 2$)



Fronsdal '78 Fang-Fronsdal '79

Interacting vertices (B,B,B), (B,B,B,B) and (F,F,B), (F,F,B, B), (F,F,F,F) in SM

Cubic vertices in SM for lower spins ($m=(\neq)0$): (1,1,1), (0,0,1), $(\frac{1}{2},\frac{1}{2},0)$, $(\frac{1}{2},\frac{1}{2},1)$

$$S_{\mathsf{SM}}=\int d^4x\,\mathcal{L}_{\mathsf{SM}}\ ,\quad \mathcal{L}_{\mathsf{SM}}=\mathcal{L}_{\text{gauge fields}}+\mathcal{L}_{\text{leptons}}+\mathcal{L}_{\text{quarks}}+\mathcal{L}_{\text{Yukawa}}+\mathcal{L}_{\text{Higgs}}\ , \qquad (1)$$

$$\mathcal{L}_{\text{leptons}}=\sum_{k=1}^{3}\left[\bar{l}^k_L i\gamma^\mu\left(\partial_\mu-i\frac{g}{2}A^{\hat{a}}_\mu\tau_{\hat{a}}+i\frac{g'}{2}A_\mu\right)l^k_L+\bar{l}^k_R i\gamma^\mu\left(\partial_\mu+ig'A_\mu\right)l^k_R\right]\ ,$$

$$\mathcal{L}_{\text{quarks}}=\sum_{k=1}^{3}\left\{\begin{bmatrix}\bar{u}_k\\ \bar{d}'_k\end{bmatrix}_L i\gamma^\mu\left[\partial_\mu-i\frac{g_{\mathrm{s}}}{2}A^{\underline{\alpha}}_\mu\lambda_{\underline{\alpha}}-i\frac{g}{2}A^{\hat{a}}_\mu\tau_{\hat{a}}-i\frac{g'}{6}A_\mu\right]\begin{bmatrix}u_k\\ d'_k\end{bmatrix}_L\right.$$

$$\left.+\,\bar{u}^k_R i\gamma^\mu\left[\partial_\mu-i\frac{g_{\mathrm{s}}}{2}A^{\underline{\alpha}}_\mu\lambda_{\underline{\alpha}}-i\frac{2g'}{3}A_\mu\right]u^k_R+\bar{d}'^k_R i\gamma^\mu\left[\partial_\mu-i\frac{g_{\mathrm{s}}}{2}A^{\underline{\alpha}}_\mu\lambda_{\underline{\alpha}}+i\frac{g'}{3}A_\mu\right]d'^k_R\right\}\ ,$$

$$d'^k=U^{kk'}_{\mathsf{CKM}}d^{k'}\ ,\quad u^k=(u,c,t)\ ,\quad d^k=(d,s,b)\ ,$$

The masses of particles are generated by the Yukawa interaction term

$$\mathcal{L}_{\text{Yukawa}}=-\frac{1}{\sqrt{2}}\sum_{k=1}^{3}\left\{f^u_k\begin{bmatrix}\bar{u}^k\\ \bar{d}^k\end{bmatrix}_L\varphi u^k_R+f^d_k\begin{bmatrix}\bar{u}^k\\ \bar{d}^k\end{bmatrix}_L\varphi d^k_R+f^l_k\bar{l}^k_L\varphi l^k_R+\text{h.c.}\right\}\ ,$$

$$\mathcal{L}_{\text{Higgs}}=\frac{1}{2}\left|\left(i\partial_\mu+(g/2)\,A^{\hat{a}}_\mu\tau_{\hat{a}}+(g'/2)\,A_\mu\right)\right|\varphi^2-\frac{\mu^2}{2}|\varphi|^2-\frac{\lambda}{4}|\varphi|^4\ ,$$

Known results on cubic vertices

- metric formalism F. Berends, J. Van Reisen, NPB164 (1980), Berends, G. Burgers, H Van Dam, Nucl. Phys. B271 (1986); A. K. H. Bengtsson, I. Bengtsson, L. Brink, NPB (1983), E.S. Fradkin, M.A. Vasiliev, NPB 291 (1987), R. Manvelyan, K. Mkrtchyan, W. Ruhl, PLB 696 (2011), [arXiv:1009.1054 [hep-th]], E. Joung, M. Taronna, NPB 861 (2012) 145, arXiv:1110.5918[hep-th], I. Buchbinder, V. Krykhtin, M. Tsulaia, Cubic Vertices for $\mathcal{N}=1$, NPB 967 (2021); NPB 859 (2012) ;
- within (with algebraic constraints, cov.) BRST approach with incomplete BRST operator (or constrained BRST approach) for integer spins -R.R. Metsaev, (2013);
- in BRST approach with (in)complete BRST operator for irreps $ISO(1,d-1)$ bosonic fields by I.Buchbinder, A.R. (2021-2023) ;
- in frame-like approach M. Vasiliev, Cubic Vertices for Symmetric higher spin Gauge Fields in (A)dS$_d$, NPB 862 (2012) 341 , arXiv:1108.5921[hep-th] arXiv:2208.02004, M. Khabarov, Yu. Zinoviev. JHEP 02 (2021);

- ***LF & (Non)Covariant Cubic vertex for irrep MAS integer HS fields*** $\Phi_{\mu^1[s_1],\ldots,\mu^k[s_k]}$ ***not found*** *(in BRST approach with (in)complete $Q_{(c)}$)* FOR $k>2$, $\underline{k=1}$: I.Buchbinder,V. Krykhtin (2009), Yu. Zinoviev;

  $\underline{k=2}$ (X.Bekaert, Boulanger 2004, 2005), Yu. Zinoviev, 2016 , A.R. 2016

Contents

- BRST approach with complete BRST operator Q for irreducible free integer MAS HS fields on $R^{1,d-1}$;
  1. additive conversion of operator 2=nd class constraints: Verma and Fock modules;
  2. BRST complex with spin condition and Lagrangians;
- BRST approach with incomplete BRST operator Q_c for irreducible free integer higher spins on $R^{1,d-1}$;
  1. Generating equations for superalgebra of incomplete operators: Q_c, spin σ_c and operators $\widehat{L}_{ij}, \widehat{T}_{rs}$;
  2. GI Lagrangian formulation with holonomic contraints;
- Deformation procedure with Q_c for interacting higher-spin fields;
- General solution of BRST equations for cubic vertices for constrained of helicities $(s[k_1], s[k_2], s[k_3])$ HS fields
  1. BRST-closed linear on oscillators operator $L^{(i)}$;
  2. BRST-closed cubic on oscillators operators $Z \equiv Z_{111}$;
- Summary

BRST approach with incomplete BRST operator Q_c

due to tensionless limit : $\Rightarrow$ for string BRST operator $\mathcal{Q}$ $(d = 26, 10)$ for $(\alpha' \to \infty)$: (G.Bonelli (2003), A. Sagnotti, M. Tsulaia, (2004)) now $(\forall d)$.

$\Longrightarrow \mathcal{Q} \stackrel{\alpha' \to \infty}{\longrightarrow} Q_c$:$\{\infty\}$many HS fields $\phi_\mu(x), .., \phi_{\mu(s)}(x)$ in string spectra

In BRST-BFV approach with incomplete Q_c (S. Ouvry, J. Stern, A. Bengtsson,, G. Barnich, M.Grigoriev, A.Semikhatov 2004, A.R. 2018)
instead of direct problem for generalized canonical quantization of Constrained DS by the aim inverse problem - is an construction of GI LF for HS fields with (m, s)

irrep conditions ISO(1,d-1), (SO(2,d-1)) $\xrightarrow{\text{SFT}}$ (super)algebra$\{o_I(x)\} = \{p^2 - m^2; a^+_{i\mu}p^\mu, a_{i\mu}p^\mu; \underline{o_a}, o^+_a\}$
$\{o_I(x)\} : \mathcal{H}, \quad [o_I, o_J\} = f^K_{IJ}(o)o_k$

$\xrightarrow[\text{I.B.,E.Fradkin, G.V., M.Henneaux}]{\text{BFV}}$ BRST operator $\{o_I\}$:$Q_c(x)$
$Q_c = C^A o_A + \frac{1}{2}C^A C^B F^D_{AB}\mathcal{P}_D(-1)^{\varepsilon(o_A)+\varepsilon(o_D)}$

$\xrightarrow{\text{LF}}$
$Q_c^2 = 0, \ [Q_c, \sigma_c\} = [Q_c, \mathcal{O}_a\} = 0, \ [\mathcal{O}_a, \sigma_c\} \sim \mathcal{O}_a$
mass-shell : $Q_c|\chi_c\rangle = 0, \mathrm{gh}(|\chi_c\rangle) = 0 \Rightarrow$ action : $S_c = \int d\eta_0 \langle\chi_c|Q_c|\chi_c\rangle$
spin: $(g_0 + \text{more})(|\chi_c\rangle, |\Lambda_c\rangle, ...) = (s - d/2 + ...)(|\chi\rangle, |\Lambda\rangle, ...)$
gauge symmetry: $\delta|\chi_c\rangle = Q_c|\Lambda_c\rangle, \delta|\Lambda_c\rangle = Q_c|\Lambda^1_c\rangle, ...,$ constr: $\mathcal{O}_a(|\chi\rangle, |\Lambda\rangle, ...) = 0$

Q_c - for 1-st class constraints without holonomic ones with auxiliary fields on 2 stage

Approach with complete BRST operator Q for free MAS HS fields

A. Pashnev, M. Tsulaia (1998-2004), C. Burdik, I. Buchbinder, V. Krykhtin, A.R., Takata, A. Isaev, S. Fedoruk (continuous spin in $d=4$) In BRST-BFV approach with complete Q again instead of direct problem for generalized canonical quantization (BFV 1977-1983) of Constrained DS by the aim inverse problem - is an construction of GI LF for HS field with (m, s)

$$\boxed{\begin{array}{c}\texttt{Irreps conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(Super)algebra}\{o_I(x)\}:\mathcal{H}\\ [o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)\end{array}}$$

$$\xrightarrow[\text{Burdik, Pashnev}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I = o_I + o'_I : \mathcal{H}\bigotimes\mathcal{H}'\\ [O_I, O_J\} = F^K_{IJ}(o', O)O_K\end{array}}$$

$$\xrightarrow[\text{Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}: Q'(x)\\ Q' = C^I O_I + \frac{1}{2}C^I C^J F^K_{IJ}\mathcal{P}_K(-1)^{\varepsilon(o_I)+\varepsilon(o_K)} + \text{more}\end{array}}$$

$$\xrightarrow{\texttt{LF}} \left|\begin{array}{c}Q' = Q + (g^i_0 + h^i + \text{more})C^i_g + \ldots : Q'^2 = 0 \Rightarrow Q^2 = 2B^i(g^i_0 + h^i + \text{more})\\ \texttt{mass-shell}: Q|\chi\rangle = 0, \text{gh}(\chi) = 0 \Rightarrow \texttt{action}: S = \int d\eta_0 \langle\chi|KQ|\chi\rangle = \Phi_{\mu[s]}\Box\Phi^{\mu[s]} + \ldots\\ \texttt{spin}: (g_0 + \text{more})(|\chi\rangle, |\Lambda\rangle, \ldots) = -h(|\chi\rangle, |\Lambda\rangle, \ldots)\\ \texttt{gauge transfs}: \delta|\chi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots\end{array}\right.$$

with auxiliary fields on 2,3 stages. It is the particular case of AKSZ model (1997).

Derivation of HS symmetry algebra $\mathcal{A}(Y[k],\mathbb{R}^{1,d-1})$

The m of g.asym. spin $s=[\hat{s}_1,...,\hat{s}_k]\equiv s[k]$ $ISO(1,d-1)$ group irrep with Young T. $Y[\hat{s}_1,...,\hat{s}_k]$

$$\Phi_{\mu^1[s_1],\dots,\mu^k[s_k]} \qquad \longleftrightarrow \qquad \begin{array}{|c|c|c|c|c|}\hline \mu^1_1 & \mu^2_1 & \dots & \mu^{k-1}_1 & \mu^k_1 \\ \hline \mu^1_2 & \mu^2_2 & \dots & \mu^{k-1}_2 & \mu^k_2 \\ \hline \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline \mu^1_{s_k} & \mu^2_{s_k} & \cdot & \mu^{k-1}_{s_k} & \mu^k_{s_k} \\ \hline \mu^1_{s_k+1} & \mu^2_{s_k+1} & \cdot & \mu^{k-1}_{s_k+1} \\ \cline{1-4} \cdot & \cdot & \cdot & \cdot \\ \cline{1-4} \mu^1_{s_{k-1}} & \mu^2_{s_{k-1}} & \cdot & \mu^{k-1}_{s_{k-1}} \\ \cline{1-4} \cdot & \cdot & \cdot \\ \cline{1-3} \mu^1_{s_2+1} \\ \cline{1-1} \cdots \\ \cline{1-1} \mu^1_{s_1} \\ \cline{1-1} \end{array} ,$$

$$(\partial^\mu\partial_\mu+m^2)\Phi_{\mu^1[s_1],\dots,\mu^k[s_k]}=0,$$

$$(\partial_i\Phi)_{\mu^1[s_1-\delta_{i1}],\dots,\mu^k[s_k-\delta_{ik}]}\equiv\partial^{\mu^i_{l_i}}\Phi_{\mu^1[s_1],\dots,\mu^k[s_k]}=0,\ 1\le l_i\le s_i,\ i=1,...,k,$$

$$(\mathtt{Tr}^{ij}\Phi)_{\mu^1[s_1],\dots,\mu^k[s_k]}\equiv\eta^{\mu^i_{l_i}\mu^j_{l_j}}\Phi_{\mu^1[s_1],\dots,\mu^k[s_k]}=0,\quad 1\le i<j\le k,$$

$$(Y^{ij}\Phi)_{\mu^1[s_1],\dots,[\mu^i[s_i],\dots,\mu^j_{l_j}],\hat{\mu}^j[s_j-1],\dots,\mu^k[s_k]}=0,$$

Purpose: find LF for given HS field on $\mathcal{M}_{(c)}$:

$$\mathcal{S}_{(c)s[k]}:\mathcal{M}_{(c)}=\{(\Phi_{\mu^1[s_1],\dots,\mu^k[s_k]},\Psi_{\mu^1[s_1-1],\dots,\mu^k[s_k]},\dots)\}\to\mathbb{R},$$

$$\texttt{SFT} \Longrightarrow \mathcal{H} : \{a^i_\mu, a^{j+}_\nu\} = -\eta_{\mu\nu}\delta^{ij}, \ \texttt{diag}\,\eta_{\mu\nu} = (+,-,..,-)$$

An arbitrary "string-like"vector $|\Phi\rangle \in \mathcal{H}$

$$|\Phi\rangle = \sum_{s_1=0}^{[d/2]}\sum_{s_2=0}^{s_1}\ldots\sum_{s_k=0}^{s_{k-1}} \frac{\imath^{\sum_{p=1}^k s_p}}{s_1!\ldots s_k!}\Phi_{\mu^1[s_1],\ldots,\mu^k[s_k]}\prod_{i=1}^{k}\prod_{l_i=1}^{s_i}\hat{a}_i^{+\mu^i_{l_i}}|0\rangle,$$

$$\boxed{(l_0,\, l^i,\, l^{ij},\, t^{ij})|\Phi\rangle = \left(\partial^\mu\partial_\mu + m^2,\, -i\hat{a}^i_\mu\partial^\mu,\, \tfrac{1}{2}\hat{a}^i_\mu\hat{a}^{j\mu},\, \hat{a}^{i+}_\mu\hat{a}^{j\mu}\right)|\Phi\rangle = \vec{0}} \Longleftrightarrow (1)-(4).$$

The set of $(k+1)^2$ even and k odd, l^i, primary constraints permits to realize $\Longleftrightarrow$ Eqs. as constraints on $|\Phi\rangle$ for each $s[k]$

Eqs. with number particles operators, g^i_0,

$$g^i_0|\Phi\rangle = (s_i - \frac{d}{2})|\Phi\rangle, \qquad g^i_0 = -\frac{1}{2}[\hat{a}^{i+}_\mu, \hat{a}^{\mu i}] = -\hat{a}^{i+}_\mu\hat{a}^{\mu i} - \frac{d}{2},$$

extract irrep with given $s[k]$

Construction is differed from LF with (in)complete BRST operator for MS HS field $\Phi_{\mu^1(s_1),\ldots,\mu^k(s_k)}$ (I.Buchbinder, A.R. NPB 2012)

$$\{o_I\} = \{o_\alpha, o_\alpha^+;\ l_0, g_0^i\} \equiv \{o_a, o_a^+;\ l_0,\ l^i,\ l^{i+};\ g_0^i\}.$$

$$\langle\Psi|\Phi\rangle=\int d^dx \sum_{s_1=0}^{[d/2]}\sum_{s_2=0}^{s_1}\dots\sum_{s_k=0}^{s_{k-1}} \frac{(-1)^{\sum_p s_p}}{s_1!\dots s_k!}\Psi^\star_{\mu^1[s_1],\dots,\mu^k[s_k]}(x)\Phi^{\mu^1[s_1],\dots,\mu^k[s_k]}(x$$

$$l_0 = \partial^2+m^2,\ l_{ij}^+ = \frac{1}{2}a_{j\mu}^+a_i^{+\mu},\quad l_i^+ = -\imath a_i^{+\mu}\partial_\mu,$$

$$t_{ij}^+ = a_j^{+\mu}a_{\mu i}\theta^{ji}, \theta^{ji} = 1(0), j > (<)i$$

which form together with $(l_i, l_{ij}, t_{ij}, g_0^i)$
integer HS symmetry superalgebra $\mathcal{A}(Y[k],\mathbb{R}^{1,d-1})$ w.r.t. [, }.
Subalgebra Lie of operators $S_{[k]} = \{l^{ij}, t^{ij}, g_0^i, l_{ij}^+, t_{ij}^+\} \overset{\texttt{Howe duality}}{\subset} sp(2k)$.
For $m = 0$ the only o_I from upper and lower triangular subalgebras in $S_{[k]}$ compose an invertible matrix: $\|[o_\mathbf{a}, o_\mathbf{b}]\| = \|\Delta_\mathbf{ab}(g_0^i)\| + (o_I)$,
for $m \neq 0$ its number $2k(k-1)$ increases on $2k$ items l^i, l_i^+

additive conversion of operator 2=nd class constraints: Verma and Fock modules

To convert $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$ with 2nd operator C.C. we have used the general procedure of additive conversion

$$o_I \to O_I = o_I + o'_I : [o_I, o'_J] = 0,$$

$$\texttt{so that } [O_I, O_J] \sim O_K,\ o'_I : H' \to H';\ H' \cap \mathcal{H}^f =$$

,

$\Rightarrow$ if $[o_I, o_J] = f^K_{IJ} o_K$,$\Leftrightarrow$ $[o'_I, o'_J] = f^K_{IJ} o'_K$ & $[O_I, O_J] = f^K_{IJ} O_K$.

But, it's sufficient to convert only subalgebra $S_{[k]}$ for $\{o_a, o^+_a, g^i_0\}$.
So that the algebra of O_I is the same $\mathcal{A}_c(Y[k], \mathbb{R}^{1,d-1}) = \mathcal{A}(Y[k], \mathbb{R}^{1,d-1})$ as for o_I, but for o'_I - $S_{[k]}$.

Verma module for $S_{[k]}$

Cartan decomposition

$$S_{[k]} = \Bigl\{\overbrace{l'^{ij+}, t'^{+}_{rs}}\Bigr\} \oplus \Bigl\{\overbrace{g'^{i}_{0}}\Bigr\} \oplus \Bigl\{\overbrace{l'^{ij}, t'_{rs}}\Bigr\} \equiv \mathcal{E}^{-}_{k} \oplus H_k \oplus \mathcal{E}^{+}_{k}$$

Requirement: boundary conditions for o'_I from Cartan subalgebra:

$$g^i_0 \to g'^i_0(h^i) = h^i + ...,$$

So that, following the result C.Burdik 1985 we start with highest weight vector $|0\rangle_V$ & construct following Poincare–Birkhoff–Witt theorem

$$V(S_{[k]}) = U(\mathcal{E}^{-}_{k}) \otimes |0\rangle_V : \mathcal{E}^{+}_{k}|0\rangle_V = 0\,, g'^i_0|0\rangle_V = h^i|0\rangle_V\,,$$

to find $\{o'_I\} = \{o'_I(b_{ij}, b^{+}_{ij}, \overbrace{f_i, f^{+}_i}^{m\neq 0}, d_{ln}, d^{+}_{ln})\}$, $i,j,l,n = 1,...,k; i<j, l<n$:
$[b^{+}_{ij}, f_k, f^{+}_k, d_{ln}, d^{+}_{ln}] = [o_a, o^{+}_a]$: we use results
C. Burdik, O. Navratil, A. Pashnev, for $\mathcal{A}'_b(Y(1), AdS_d)$;
A. Kuleshov, A. R. arXiv:0905.2705 for $\mathcal{A}'(Y(1), AdS_d)$;
I.Buchbinder, A.R. NPB 2012 arXiv:1110.5044 for $sp(2k)$;
A.R. arXiv:1604.00620 for $\mathcal{A}'(Y[2], R^{1,d-1})$

Verma module for $S_{[k]}$

Explicit obtaining of the $V(S_{[k]})$ meet the technical obstacle because of not commuting of t'^{+}_{ln}, l'^{+}_{ij} with each other $\mathcal{E}^{-}_{k}$. The general $V(S_{[k]})$ vector

$$|\vec{n}_{ij}, \vec{p}_{rs}\rangle_V = |n_{12}, ..., n_{1k}, n_{23}, ..., n_{2k}, ..., n_{kk+1}; p_{12}, ..., p_{1k}, p_{23}, ..., p_{2k}, ..., p_{k-1k}\rangle_V ,$$

$$|\vec{n}_{ij}, \vec{p}_{rs}\rangle_V \equiv |\vec{N}\rangle_V \equiv \prod_{i<j}^{k} \left(l'^{+}_{ij}\right)^{n_{ij}} \prod_{r,r<s}^{k} \left(t'^{+}_{rs}\right)^{p_{rs}} |0\rangle_V,$$

$$g'_{0i}|\vec{N}\rangle_V = \left(\sum_{l<i} n_{li} + \sum_{l>i} n_{il} - \sum_{s>i} p_{is} + \sum_{r<i} p_{ri} + h^i\right) \Big|\vec{N}\rangle_V ,$$

$$t'^{+}_{r's'}|\vec{N}\rangle_V = \Big|\vec{N} + \delta_{r's',rs}\rangle_V - \sum_{k'=1}^{r'-1} p_{k'r'} \Big|\vec{N} - \delta_{k'r',rs} + \delta_{k's',rs}\rangle_V - \sum_{k'=1}^{r'-1} n_{k'r'} \times$$

$$\times \Big|\vec{N} - \delta_{k'r',ij} + \delta_{k's',ij}\rangle_V + \sum_{k'=r'+1}^{s'-1} n_{r'k'} \Big|\vec{N} - \delta_{r'k',ij} + \delta_{k's',ij}\rangle_V - \sum_{k'=s'+1}^{k} n_{r'k'} \times$$

$$\times \Big|\vec{N} - \delta_{r'k',ij} + \delta_{s'k',ij}\rangle_V ,$$

$$l'^{+}_{i'j'}|\vec{N}\rangle_V \quad = \quad \Big|\vec{N} + \delta_{i'j',ij}\rangle_V \ , \qquad \texttt{for (-) root vectors} \ \in \mathcal{E}^{-}_k$$

$$\texttt{where} \ \ AB^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} B^{n-k} \mathrm{ad}^k_B A \, , \mathrm{ad}^k_B A = [[...[A, \overbrace{B\}, ...], B}^{k \text{ times}}],$$

To get the action of E^{α_i} on $|\vec{N}\rangle_V$ we get the recurrent relation

$$t'_{l'm'}|\vec{0}_{ij}, \vec{p}_{rs}\rangle_V = \Big|C^{l'm'}_{\vec{p}_{rs}}\rangle_V - \sum_{n'=1}^{l'-1} p_{n'm'} \Big|\vec{0}_{ij}, \vec{p}_{rs} - \delta_{n'm',rs} + \delta_{n'l',rs}\rangle_V$$

$$+ \sum_{k'=l'+1}^{m'-1} p_{l'k'} \Big[\prod_{r'<l', s'>r'} \ \prod_{r'=l', m'>s'>r'} (t'^{+}_{r's'})^{p_{r's'} - \delta_{l'k',r's'}} \Big] t'_{k'm'} \Big|\vec{0}_{ij}, \overset{\vec{p}_{rs}>}{\vec{p}_{q't'}}\rangle_V \ ,$$

Explicit construction of $V(S_{[k]})$

The solution of the above Eq. exists, so that the explicit form of $t'_{l'm'}$ action on the vector $|\vec{N}\rangle_V$ has the final form

$$
\begin{aligned}
t'_{l'm'}|\vec{N}\rangle_V \ &= -\sum_{k'=1}^{l'-1} n_{k'm'} \Big|\vec{N} - \delta_{k'm',ij} + \delta_{k'l',ij}\rangle_V \qquad (2)\\
&+ \sum_{k'=l'+1}^{m'-1} n_{k'm'} \Bigg|\vec{N} - \delta_{k'm',ij} + \delta_{l'k',ij}\rangle_V - \sum_{k'=m'+1}^{k} n_{m'k'} \Big|\vec{N} - \delta_{m'k',ij} + \delta_{l'k',ij}\rangle_V \\
&+ \sum_{p=0}^{m'-l'-1} \Bigg\{ \sum_{k'_1=l'+1}^{m'-1} \cdots \sum_{k'_p=l'+p}^{m'-1} \prod_{j=1}^{p} p_{k'_{j-1}k'_j} \Bigg(\Big| C^{k'_p m'}_{\vec{n}_{ij},\vec{p}_{rs} - \sum_{j=1}^{p} \delta_{k'_{j-1}k'_j,rs}} \rangle_V \\
&- \sum_{n'=!k'_{p-1}}^{k'_p-1} p_{n'm'} \Big| \vec{n}_{ij}, \vec{p}_{rs} - \sum_{j=1}^{p} \delta_{k'_{j-1}k'_j,rs} - \delta_{n'm',rs} + \delta_{n'k'_p,rs}\rangle_V \Bigg)\Bigg\}.
\end{aligned}
$$

Analogously, the action of the rest E^{α_i}: $l'_{l'm'}$ on $|\vec{N}\rangle_V$ is determined with help of the "**basic-block**"vector $\Big|C^{l'm'}_{\vec{p}_{rs}}\rangle_V$

$\Longrightarrow V(S_{[k]})$ **is explicitly found!**

Making use of the mapping (C. Burdik, 1985)

$$|\vec{n}_{ij},\vec{p}_{rs}\rangle_V \leftrightarrow |\vec{n}_{ij},\vec{n}_s\rangle = \prod_{i,j\geq i}^{k}(b^+_{ij})^{n_{ij}} \prod_{r,s,s>r}^{k}(d^+_{rs})^{p_{rs}}|0\rangle \in \mathcal{H}',$$

$$\{f_i\,, f^+_j\} \overset{m\neq 0}{=} \delta_{ij}\,, \quad [b_{ij}, b^+_{lm}] = \delta_{il}\delta_{jm}\,,\; i<j, k<l\,, \qquad [d_{r_1 s_1}\,, d^+_{r_2 s_2}] = \delta_{r_1 r_2}\delta_{s_1 s_2}\,,$$

Proposition

: The polynomial oscillator realization for the $V(S_{[k]})$ over Heisenberg-Weyl algebra $A_{k\times k}$ exists in the form of Fock module

$$C(b_{ij}, b^+_{lk}, d_{r_1 s_1}\,, d^+_{r_2 s_2}), \quad C \in \{t'_{l'm'}, t'^+_{l'm'}, l'_{i'j'}, l'^+_{i'j'} g'^i_0\}.$$

$$\langle\Psi|K' c'_{ij}|\Phi\rangle = \langle\Phi|K' c^{+\prime}_{ij}|\Psi\rangle^*, \quad c \in \{t\,, l\},\; \langle\Psi|K' g^{i\prime}_0|\Phi\rangle = \langle\Phi|K' g^{i\prime}_0|\Psi\rangle^*.$$

$$K' = Z^+Z,\; Z = \sum_{\vec{n}_{ij}=\vec{0}}^{\infty}\sum_{\vec{p}_{rs}=\vec{0}}^{\infty} |\vec{n}_{ij},\vec{p}_{rs}\rangle_V \langle 0| \prod_{(ij)=(12)} \frac{b_{ij}^{n_{ij}}}{(n_{ij})} \prod_{(rs)=(12)} \frac{d_{rs}^{p_{rs}}}{(p_{rs})},$$

$$C^{lm}(d^+, d) \equiv \Big(h^l - h^m - \sum_{n=m+1}^{k} (d^+_{ln} d_{ln} - !d^+_{mn} d_{mn}) - ! \sum_{n=l+1}^{m-1} d^+_{nm} d_{nm} - d^+_{lm} d_{lm}\Big) d_{lm}$$
$$+ \sum_{n=m+1}^{k} \Big\{ d^+_{mn} - ! \sum_{n'=l+1}^{m-1} d^+_{n'n} d_{n'm} \Big\} d_{ln}.$$

so that, f.i. for t'_{lm}:

$$t'_{lm} = \sum_{p=0}^{m-l-1} \Big[\sum_{k_1=l+1}^{m-1} \ldots \sum_{k_p=l+p}^{m-1} \Big\{ C^{k_p m}(d^+, d) - \sum_{n'=k'_{p-1}}^{k_p - 1} d^+_{n' k_p} d_{n'm} \Big\} \prod_{j=1}^{p} d_{k_{j-1} k_j} \Big]$$
$$- \sum_{n=1}^{l-1} b^+_{nl} b_{nm} + \sum_{n=l+1}^{m-1} b^+_{ln} b_{nm} - \sum_{n=m+1}^{k} b^+_{ln} b_{mn}\,, \quad k_0 \equiv l,$$

. Thus, the additive conversion of o_I into the 1st class O_I is realized! (It completely applicable for massive HS fields as well)

Complete BRST operator for Lie algebra $\mathcal{A}(Y[k], \mathbb{R}^{1,d-1})$

The BRST operator Q' for Lie algebra $\mathcal{A}_c(Y[k], \mathbb{R}^{1,d-1})$ by the standard rules of BFV-method .

$$Q' = O_I\mathcal{C}^I + \frac{1}{2}\mathcal{C}^I\mathcal{C}^J f^K_{JI}\mathcal{P}_K(-1)^{\varepsilon(o_I)+\varepsilon(o_K)}, \ {Q'}^2 = 0 \ \texttt{where} \ (\varepsilon, gh)Q' = (1,1), \quad (3)$$

$\mathcal{C}^I = (q; \vartheta, \eta; q^+, \vartheta^+, \eta^+), \mathcal{P}_K$ - ghost coordinates and momenta with opposite Grassmann parity to O_I with following non-vanishing C.R.

$$\{\vartheta_{rs}, \lambda^+_{tu}\} = \{\lambda_{tu}, \vartheta^+_{rs}\} = \delta_{rt}\delta_{su}, \qquad [q_i, p^+_j] = [p_j, q^+_i] = \delta_{ij} \, ,$$
$$\{\eta_{lm}, \mathcal{P}^+_{ij}\} = \{\mathcal{P}_{ij}, \eta^+_{lm}\} = \delta_{li}\delta_{jm} \, , \qquad \{\eta_0, \mathcal{P}_0\} = \imath, \ \{\eta^i_{\mathcal{G}}, \mathcal{P}^j_{\mathcal{G}}\} = \imath\delta^{ij} \, ; \quad (4)$$

and $gh(\mathcal{C}^I) = -gh(\mathcal{P}_I) = 1$.

$$Q' = Q_c + \sum_{i<j}\left(\eta_{ij}\mathcal{L}^+_{ij} + \mathcal{L}_{ij}\eta^+_{ij} + \mathcal{T}_{ij}\vartheta^+_{ij} + \vartheta_{ij}\mathcal{T}^+_{ij}\right) + \sum_i[\eta^G_i\sigma^i(G) + \imath\mathcal{B}^i\mathcal{P}^G_i], \quad (5)$$

with definite operators $\mathcal{B}^i$ and with spin $\sigma^i(G)$, complete traceless $\mathcal{L}_{ij}$ and Young constraints (and theirs h,c. ones)

$$\mathcal{L}_{ij} = \underline{L_{ij} + \frac{1}{2}q_{[i}p_{j]}} + \sum_{p<j}\vartheta^+_{ip}\mathcal{P}_{pj} + \sum_{j<p}\vartheta^+_{jp}\mathcal{P}_{ip} - \sum_{j<p}\vartheta^+_{ip}\mathcal{P}_{jp} \quad (6)$$
$$+\frac{1}{4}\Big\{\sum_{p>j}\left[\eta_{ip}\lambda^+_{jp} - \eta_{jp}\lambda^+_{ip}\right] + \sum_{p<j}\eta_{pj}\lambda^+_{ip}\Big\},$$

$$\mathcal{T}_{ij} = \underline{T_{ij} + (q_j p^+_i + q^+_i p_j)} + \sum_{p>j}\vartheta^+_{jp}\lambda_{ip} - \sum_{p<j}[\vartheta_{ip}\lambda_{pj} - \vartheta_{pj}\lambda_{ip}] \quad (7)$$
$$+\sum_{p>j}\eta_{jp}\mathcal{P}^+_{ip} - \sum_{i<p<j}\eta_{pj}\mathcal{P}^+_{ip} + \sum_{p<i}\eta_{pj}\mathcal{P}^+_{pi}.$$

$Q'^+K = KQ'$, `in` $\mathcal{H}_{tot} = \mathcal{H}\otimes\mathcal{H}'\otimes\mathcal{H}_{gh}$ `due to` $V(S_{[k]})$ `osc.realization in` $\mathcal{H}'$

The obtaining of resulting LF takes known character
As usual, we extract the spin operator from the Q':

$$\Longrightarrow Q' = Q + \eta^i_G(\sigma^i + h^i) + \mathcal{B}^i \mathcal{P}^i_G,$$
$$[Q, \sigma_i] = 0\,, \quad Q^2 = 2\mathcal{B}^i(\sigma^i + h^i).$$

The same applies to a scalar physical and gauge param. vectors

$$|\chi\rangle = \sum_{\{n\}_b=0}^{\infty} \sum_{\{n\}_f=0}^{1} \eta_0^{n_{\eta_0}} \prod_{i<j,i,j=1}^{k} \eta_{ij}^{+n_{\eta_{ij}}} \mathcal{P}_{ij}^{+n_{P_{ij}}} \prod_{r<s,r,s=1}^{k} \vartheta_{rs}^{+n_{\vartheta_{rs}}} \lambda_{rs}^{+n_{\lambda_{rs}}} \prod_{i=1}^{k} (\eta_i^G)^{n_i} q_i^{+n_{q_i}} p_i^{+n_{p_i}}$$
$$\times \prod_{i<j,i,j=1}^{k} f_j^{+n_{f_j}} b_{ij}^{+n_{b_{ij}}} \prod_{r<s,r,s=1}^{k} d_{rs}^{+n_{d_{rs}}} \Big|\chi_{n_{\eta_0} n_{\eta_{ij}} n_{\vartheta_{rs}} n_{P_{ij}} n_{\lambda_{rs}} n_i n_{q_i} n_{p_i} n_{f_j} n_{b_{ij}} n_{d_{rs}}}(\hat{a}_i^+)\rangle$$

$|\chi^0\rangle, |\chi^s\rangle \in \mathcal{H}_{tot}$ i.e. $\partial(|\chi^0\rangle)/\partial\eta^i_G = 0$: gh$(|\chi^0\rangle, |\chi^s\rangle) = (0, -s)$

$|\chi\rangle = |\Phi\rangle + |\Phi_A\rangle, |\Phi_A\rangle_{\{(b, b^+, d, d^+) = \mathcal{C} = \mathcal{P} = 0\}} = 0$ `with` $|\Phi\rangle -$ `basic HS f.`

and with the use of the BFV–BRST EQUATION $Q'|\chi^0\rangle = 0$ that determines the physical states and a sequence of reducible gauge transformations.

$$
\begin{array}{lll}
Q|\chi\rangle = 0, & (\sigma^i + h_i)|\chi\rangle = 0, & (\varepsilon, gh_H)(|\chi\rangle) = (\varepsilon_\chi, 0), \\
\delta|\chi\rangle = Q|\chi^1\rangle, & (\sigma^i + h_i)|\chi^1\rangle = 0, & (\varepsilon, gh_H)(|\chi^1\rangle) = (\varepsilon_\chi + 1, -1), \\
\delta|\chi^1\rangle = Q|\chi^2\rangle, & (\sigma^i + h_i)|\chi^2\rangle = 0, & (\varepsilon, gh_H)(|\chi^2\rangle) = (\varepsilon_\chi, -2), \\
\ldots\ldots & \ldots\ldots & \ldots\ldots \\
\delta|\chi^{n-1}\rangle = Q|\chi^n\rangle, & (\sigma^i + h_i)|\chi^n\rangle = 0, & (\varepsilon, gh_H)(|\chi^n\rangle) = (\varepsilon_\chi + n \mod 2, -n).
\end{array}
$$

the middle Eqs. determines the spectrum of spin values for $|\chi\rangle$ and gauge pars. $|\chi^i\rangle$, $i = 1, ..., \sum_i s_i + \frac{1}{2}k(k-1)$, the corresponding proper eigenvalue and eigenvectors,

$$
(\sigma^i + h_i)|\chi\rangle_{s[k]} = \left(h_i + s_i - \frac{d - 6 + \theta_{m0}}{2} - 2i\right)|\chi\rangle_{s[k]} = 0 \Longleftrightarrow
$$

$$
h_i^{s[k]} = -s^i + \frac{d - 6 + \theta_{m0} + 4i}{2}, \ (\text{for } h_i^{s[k]} = h_i(s_i)) \qquad (9)
$$

s_i from basic $|\Phi\rangle$: .

Lagrangian formulation with complete BRST Q operator

$\Longrightarrow$ The equations of motion and the sequence of reducible gauge transformations for the field with given $\mathbf{s} = s[k]$:

$$Q_{s[k]}|\chi^0\rangle_{[s]k} = 0, \quad \delta|\chi^l\rangle_{[s]k} = Q_{[s]k}|\chi^{l+1}\rangle_{[s]k}, \quad \delta|\chi^L\rangle_{[s]k} = 0,$$
$$l = 0, ..., L = \sum_i s_i + \frac{1}{2}k(k-1),$$

for $|\chi^0\rangle \equiv |\chi\rangle$, and can be obtained from the LAGRANGIAN ACTION

$$\mathcal{S}_{[s]k} = \int d\eta_0 \;{}_{[s]k}\langle\chi^0|K_{[s]k}Q_{[s]k}|\chi^0\rangle_{[s]k}, \; K_{[s]k} = K|_{h^i = -s^i + \frac{d-6+\theta(m0)+4i}{2}},$$

The corresponding LF of a bosonic field with a specific value of spin $\mathrm{s}[k]$ subject to $Y[s_1, ..., s_k]$ is an UNCONSTRAINED REDUCIBLE GAUGE THEORY OF MAXIMALLY $L = \sum_i^k s_i + \frac{1}{2}k(k-1) - 1$-TH STAGE OF REDUCIBILITY

Corollary: the result contains as a particular case LF for bosonic HS subject to $Y[s_1], Y[s_1, s_2]$ (Buchbinder, Krycktin, 2009, A.R., 2016)

Lagrangian formulation with incomplete BRST Q operator

Corresponding incomplete BRST operator $Q'_c = Q_c + \eta_i^G \sigma_c^i$ is easily derived from complete Q' operator:

$$Q'_c = Q'\Big|_{\eta_{ij}^{(+)}=\mathcal{P}_{ij}^{(+)}=\vartheta_{rs}^{(+)}=\lambda_{rs}^{(+)}=b_{ij}^{(+)}=d_{rs}^{(+)}=h^i=0}$$

$$Q'_c = \underline{\Big\{\eta_0 l_0 + q_i^+ l_i + q_i l_i^+ + i q_i q_i^+ \mathcal{P}_0\Big\}} + \eta_i^G \sigma_c^i(g) = \underline{Q_c} + \eta_i^G \sigma_c^i(g), \quad (10)$$

where $\quad \sigma_c^i(g) = g_0^i - q_i^+ p_i - q_i p_i^+$

is incomplete spin operator. These operators as well as BRST-extended set of holonomic constraints $\widehat{L}_{ij}$, $\widehat{T}_{rs}$, are given on the incomplete Hilbert space $\mathcal{H}_c$: $\mathcal{H}_c = \mathcal{H}^f \otimes H_{gh}^{o_A}$
The algebra of $(\sigma_c^i(g), \widehat{O}_a)$ is the same as one for (g_0^i, o_a).:

$$[\widehat{L}_{ij}, \sigma_c^i(g)\} = \widehat{L}_{ij}, \quad [\widehat{T}_{rs}, \sigma_c^i(g)\} = \widehat{T}_{rs}(\delta_{si} - \delta_{ri}) : \ [Q_c, \widehat{L}_{ij}\} = [Q_c, \sigma_c^i(g)\} = 0. \quad (11)$$

the constrained gauge invariant LF of $(\sum_i s_i - 1)$ stage reducibility with the action $\mathcal{S}_{c|s[k]}$ for HS tensor field subject to Young tableaux $Y[s_1, \ldots, s_k]$ reads as,

$$\mathcal{S}_{c|s[k]}(\chi_c) = \int d\eta_0 \ {}_{s[k]}\langle \chi_c | Q_c | \chi_c \rangle_{s[k]} \ , \quad (12)$$

$$\Big(\delta;\ \widehat{L}_{ij},\ \widehat{T}_{rs}\Big)|\chi_c^l\rangle_{s[k]} = \Big(Q_c|\chi_c^{l+1}\rangle_{s[k]}\theta_{\sum_i s_i, l};\ 0,\ 0\Big),\ l = 0, 1, ..., \sum_i s_i. \quad (13)$$

For $\sum_i s_i = 0$ (which corresponds to the scalar field) the LF appears by non-gauge one.

GI Lagrangian formulation with holonomic contraints

Proposition

The set of solutions, $H_{(m,s[k])}$, for the equations extracting the Poincare group massless ($m=0$) irreps of spin $[s_1.,...,s_k]$ in terms of tensor field, $\Phi_{\mu^1[s_1],...,\mu^k[s_k]}$ is equivalent to the solutions of the Lagrangian equations of motion, for $l=-1$ in (13) subject to the reducible gauge transformations (13) for $l=0,...,\sum_i s_i$ and off-shell holonomic constraints:

$$H_{(0,s[k])} = \{|\Phi\rangle| \Big(l_0,\, l_i,\, l_{ij},\, t_{rs},\, g_0^i - [s_i - d/2]\Big)|\Phi\rangle = 0\} \quad (14)$$

$$= \Big\{|\chi_c^0\rangle\Big|\Big[Q_c,\, \Big\{\sigma_c^i - s_i + \frac{d-2}{2}\Big\}\Big]|\chi_c^0\rangle = 0,$$

$$\delta|\chi_c^l\rangle = Q_c|\chi_c^{l+1}\rangle,\; \delta|\chi_c^{\sum_i s_i}\rangle = 0$$

$$\Big(\widehat{L}_{ij}, \widehat{T}_{rs}, \Big\{\sigma_c^i - s_i + \frac{d-2}{2}\Big\}\Big)|\chi_c^l\rangle = 0\Big\}, \quad (15)$$

where, $l=0,...,\sum_i s_i$; Proof repeats result for MS HS fields (A.R. JHEP 2018)

General solution of incomplete BRST equations for cubic vertices

$$S^{(m)_3}_{[1]|(\vec{s}_k)_3}[(\chi_c)_3] \; = \; \sum_{t=1}^{3} \mathcal{S}^{m_3}_{0|\vec{s}_{k_t}}[\chi^{(t)}_c] + g S^{(m)_3}_{1|(\vec{s}_k)_3}[(\chi_c)_3], \; (\chi_c)_3 = (\chi^{(1)}_c, \chi^{(2)}_c, \chi^{(3)}_c)$$

$$S^{(m)_3}_{1|(\vec{s}_k)_3}[(\chi_c)_3] = \int \prod_{j=1}^{3} d\eta^{(j)}_0 \Big({}_{\vec{s}_{k_j}}\langle \chi^{(j)}_c | V^{(3)}_c \rangle^{(m)_3}_{([\vec{s}]k)_3} + h.c. \Big), \; (\vec{s}_k)_3 \equiv ([s]k_1, [s]k_2, [s]k_3$$

with cubically deformed reducible gauge trans. (for $|\Lambda^{(t)-1}_c\rangle_{\vec{s}_{k_t}} \equiv |\chi^{(t)}_c\rangle_{\vec{s}_{k_t}}$)

$$\delta^l_1 |\Lambda^{(i)l}_c\rangle_{[s]k_i} = - \sum_{1\le i_1<i_2\le 3} \int \prod_{j=1}^{2} d\eta^{(j)}_0 \Big[{}_{\vec{s}_{k_{\{i_1}}}}\langle \chi^{(\{i_1)}_c | {}_{\vec{s}_{k_{i_2\}}}}\langle \Lambda^{(i_2\})l+1}_c \Big| V^{(3)l}_c \rangle^{(m)_{(i)_2 i}}_{(\vec{s}_k)_{(i)_2}} + h.c. \Big], \quad (16)$$

sequence of Noether identities for action and all levels of gauge trans in powers of g:

$$g^1 : \quad \delta_0 S^{(m)_p}_{1|(\vec{s}_k)_p}[(\chi_c)_p] + \delta_1 \mathcal{S}^{(m)_p}_{[0]|(\vec{s}_k)_p}[(\chi_c)_p] = 0, \quad (17)$$

$$g^1 : \quad \Big(\delta^l_1 \delta^{l-1}_0 + \delta^l_0 \delta^{l-1}_1 \Big) |\Lambda^{(t)l-1}_c\rangle_{\vec{s}_{k_t}} \big|_{\partial S^{(m)_p}_{[1]}=0} = 0, l = 0, ..., \quad (18)$$

$$\texttt{GenEq} \;\; \Big(\mathbf{Q^{tot}_c}, \; \sigma^{(\mathbf{k})}_{\mathbf{ci}} - (\mathbf{s_i} - \tfrac{\mathbf{d-2}}{\mathbf{2}}), \; \widehat{\mathbf{L}}^{(\mathbf{k})}_{\mathbf{ij}}, \widehat{\mathbf{T}}^{(\mathbf{k})}_{\mathbf{rs}} \Big) \Big| \mathbf{V}^{(\mathbf{3})}_{\mathbf{c}} \rangle_{(\mathbf{s_k})_\mathbf{3}} = \tilde{\mathbf{0}}, \quad Q^{tot}_c = \sum_{k=1} Q^{(k)}_c$$

General solution of incomplete BRST equations for cubic vertices

CV is local vertex :

$$\big|V^{(3)}_c\rangle^{(m)_3}_{([\vec{s}]k)_3} = \prod_{i=1}^{3}\delta^{(d)}\big(x-x_i\big)V^{(m)_3}_{([\vec{s}]k)_3}(x)\prod_{l=1}^{3}\eta^{(l)}_0|0\rangle, \quad |0\rangle\equiv\otimes_{t=1}^{3}|0\rangle^t. \qquad (19)$$

. ${Q^{tot}}_c$-closed solution for generating equations for $m=0$ in terms of product ${Q^{tot}}_c$-closed monomials

$$V^{(0)_3}_{([\vec{s}]k)_3}(x) = V\left(\underline{L^{(1)}_{n1};L^{(2)}_{n2};L^{(3)}_{n3};Z_{n_1n_2n_3}}\big|L^{(11)+}_{m_1n_1},L^{(22)+}_{m_2n_2},L^{(33)+}_{m_3n_3}\right).$$

. with (1,1) degrees in a^+,q^+, $p^{(i)}_\mu$ $(L^{(i)}_{n_i})$, also $(3,1)$-degrees in a^+,q^+, $p^{(i)}_\mu$ $(Z_{n_1n_2n_3})$ monomials

$$\begin{aligned}\mathbf{L^{(i)}_{n_i}} &= (\mathbf{p}^{(\mathbf{i+1})}_\mu-\mathbf{p}^{(\mathbf{i+2})}_\mu)\mathbf{a}^{(\mathbf{i})\mu+}_{\mathbf{n_i}} - \imath\big(\mathcal{P}^{(\mathbf{i+1})}_{\mathbf{0}}-\mathcal{P}^{(\mathbf{i+2})}_{\mathbf{0}}\big)\mathbf{q}^{(\mathbf{i})+}_{\mathbf{1}},\ n_i=1,...,k_i\\ \mathbf{Z_{n_1n_2n_3}} &= \mathbf{L^{(12)+}_{n_1n_2}L^{(3)}_{n_3}} + \mathbf{cycle}\big((\mathbf{1},\mathbf{n_1});(\mathbf{2},\mathbf{n_1});(\mathbf{3},\mathbf{n_3})\big).\end{aligned} \qquad (20)$$

Here we have used $p^{(i)}_\mu = -i\partial^{(i)}_\mu$ and $[i\simeq i+3]$

$$L^{(ii+1)+}_{n_1n_2} = \tfrac{1}{2}a^{(i)\mu+}_{n_1}a^{(i+1)+}_{\mu n_2} + \tfrac{1}{2}p^{(i)+}_{n_1}q^{(i+1)+}_{n_2} + \tfrac{1}{2}p^{(i+1)+}_{n_2}q^{(i)+}_{n_1}.$$

Summary

- Lagrangian formulations within approaches with complete and incomplete BRST operators for irreducible free MAS HS field on Minkowski space $R^{1,d-1}$ subject to $Y[s_1, ..., s_k]$ is firstly constructed both for massless and massive bosonic field;
- Verma and Fock modules for additive conversion subsystem of 2-nd class constraints $S_{[k]}$ in symplectic algebra $sp(2k)$ is found;
- it is shown the equivalence of respective Lagrangian dynamics within both approaches and its equivalence to solutions of Poincare group relations ;
- General structure of Lagrangian covariant cubic vertices for triple of MAS fields in approach with incomplete BRST operator are suggest and should be then studied;
- examples cubic vertices and quantization procedure?
- extension for half-integer MAS HS fields and for AdS(d)?

Thank you very much