Vacuum Viscosity: The Unruh Effect vs. String Theory Bound

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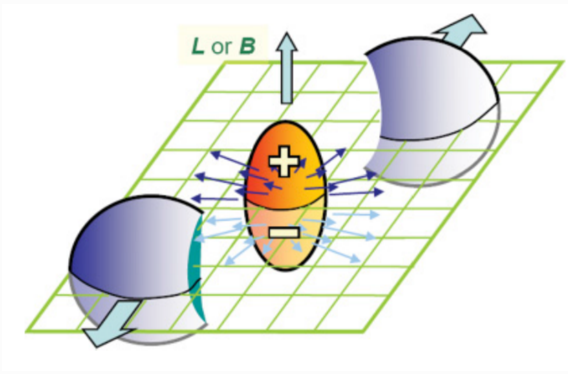
based on work:
arXiv: 2502.18199 (2025)

Contents

Part 1

Introduction and motivation

New area: vorticity, acceleration and external field effects in HICs



- **Extreme vorticity** (10^{22} sec^{-1}) and external fields observed in heavy ion collisions.

- Unusual **quantum effects** related to **vorticity** and external fields can be observed, e.g. **vortical polarization**:

  [STAR, Nature (2017), arXiv: 1701.06657]

  [Rogachevsky, Sorin, Teryaev, PRC (2010), e-Print: 1006.1331]

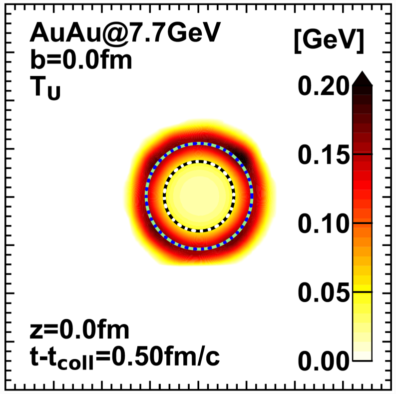
  [Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)]

- Plenty of results have been obtained related with **vorticity** and **magnetic field** effects:

  [Son, Surowka, PRL (2009), e-Print: 0906.5044]

  [Prokhorov, Teryaev, Zakharov, PRL (2022), e-Print: 2207.04449]

  [Braguta, Kotov, Kuznedelev, Roenko, PRC (2021), e-Print: 2102.05084]



Acceleration is also high!

[GP, Shohonov, Teryaev, Tsegelnik, Zakharov, 2025, 2502.10146]

Modern development: **acceleration** and **dissipation** effects

little studied

Unruh effect

[Blasone, (2018), e-Print: 1911.06002]

Formulation

The Minkowski vacuum is perceived by an accelerated observer as a medium with a finite (Unruh) temperature:

[W. G. Unruh, Notes on black hole evaporation, Phys. Rev. D14, 870 (1976)]

$$T_U = \frac{a}{2\pi}$$

Minimal viscosity bound

Bound inspired by string theory:

[Kovtun, Son, Starinets, PRL (2005), arXiv:hep-th/0405231]

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

KSS-bound

- **There are no completely ideal fluids!**
- Plenty of work about KSS Bound
- **Does not cover case of Rindler space!**

Statement of the problem

- Does **Unruh** radiation have **viscosity**?
- How is it related to the **KSS bound 1/4π**?

Part 2

Shear viscocity in Rindler space from Kubo formula

Method

Rindler coordinates and stretched horizon

- **Rindler's metric** describes the accelerated reference system:

$$ds^2 = \rho^2 d\tau^2 - d\mathrm{x}^2 - d\mathrm{y}^2 - d\rho^2$$

$$\text{Horizon :} \quad g_{00}(\rho = 0) = 0$$

$$a = \frac{1}{\rho}$$ acceleration ~ the inverse distance to the horizon.

- Fields live above the **stretched horizon**:

$$\rho \in [l_c, \infty)$$

[Parikh, Wilczek, PRD (1998), arXiv:gr-qc/9712077]

Kubo formula: Rindler space

Due to the fluctuation-dissipation theorem, dissipation coefficients can be found from fluctuations in equilibrium:

Kubo's formula for viscosity [Zubarev, Nonequilibrium statistical thermodynamics, Studies in soviet science, 1974]

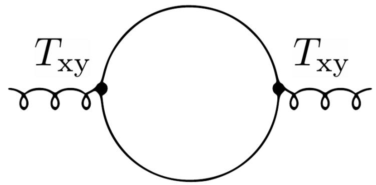
$$\eta = \lim_{\omega \to 0} \frac{1}{\omega} \int d^4 x e^{i\omega t} \theta(t) \langle [\hat{T}_{\rm xy}(x), \hat{T}_{\rm xy}(0)] \rangle$$

In the Rindler space: [Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]

$$\eta = \pi \lim_{\omega \to 0} \int_{l_c}^{\infty} \rho'\, d\rho' \int_{l_c}^{\infty} \rho\, d\rho \int_{-\infty}^{\infty} d{\rm x}\, d{\rm y}\, d\tau e^{i\omega\tau} \langle 0 | \hat{T}_{\rm xy}(\tau, {\rm x}, {\rm y}, \rho) \hat{T}_{\rm xy}(0,0,0,\rho') | 0 \rangle_{\rm M}$$

- We consider free fields:

$$\eta = \lim_{\omega \to 0} \int$$



per unit horizon area

$$\eta = \int_{l_c}^{\infty} d\rho'\, \eta_{\rm loc}(\rho')$$

Spin ½

Correlator with two EMTs

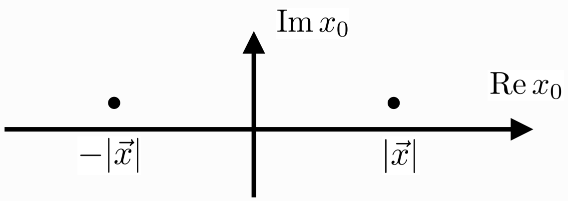
- Belinfante energy-momentum tensor for free massless Dirac fields:

$$T_{\mu\nu} = \frac{i}{4}(\bar{\psi}\gamma_\mu\partial_\nu\psi - \partial_\nu\bar{\psi}\gamma_\mu\psi + \bar{\psi}\gamma_\nu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma_\nu\partial_\nu\psi)$$

- Propagator (Wightman function)

$$S_{ab}(x) = \langle 0|\psi_a(x)\bar{\psi}_b(0)|0\rangle_M = \frac{i}{2\pi^2}\frac{(\gamma x)_{ab}}{(x^2 - i\varepsilon x_0)^2}$$

- **The poles are shifted upward relative to the real time axis:**



The result is:

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\mathrm{M}} = \frac{8}{\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y)$$

$$\mathcal{I}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{4\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{4\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{8\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{8\bar{b}^8} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{16\bar{b}^8}$$

$\bar{b}^2 = b^2 - i\varepsilon b_0$ **poles are shifted**

Fourier transform in Rindler space

Let's move on to **integration by Rindler time** $d\tau$

We move on to the Rindler coordinates in the integrand

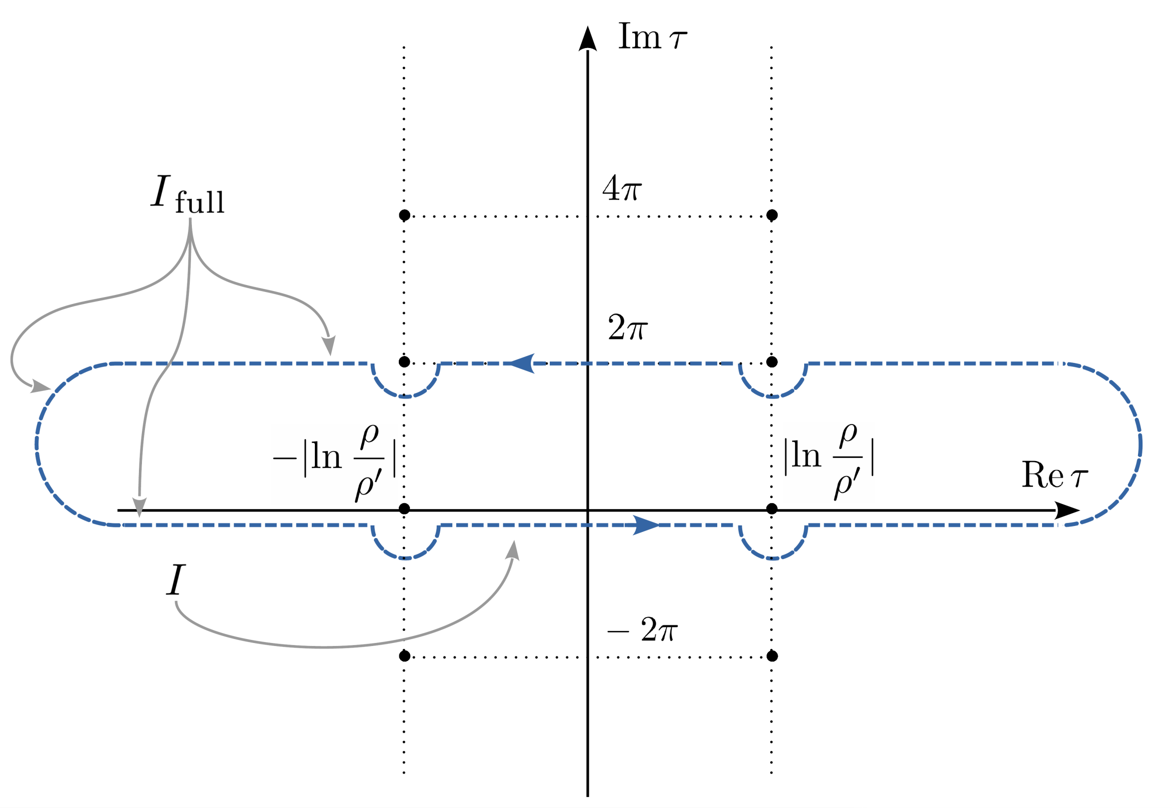
$$I = \pi \int_{-\infty}^{\infty} d\tau e^{i\tau\omega} \frac{1}{5\pi^3 \alpha^3} = \int_{-\infty}^{\infty} \frac{e^{i\tau\omega}}{5\pi^2(\rho^2 + \rho'^2 - 2\rho\rho' \cosh(\tau) + i\varepsilon\tau)^3} d\tau$$

An infinite number of **periodic poles** located parallel to the imaginary axis:

$$\tau = \pm \ln \frac{\rho}{\rho'}(1 + i\varepsilon) + 2\pi i n \quad n = 0, \pm 1, \pm 2...$$

Fourier transform in Rindler space

Using the periodicity of the integrand with respect to the shift in the direction of the imaginary axis, we can close the integral:



- The relationship between the desired integral and the integral over a closed contour:

$$I = (1 - e^{-2\pi\omega})^{-1} I_{\text{full}}$$

- Only two poles fall inside the circuit.

$$\tau = \pm \ln \frac{\rho}{\rho'}$$

Let's use **Cauchy's theorem** and find the residues at the poles:

$$I_{\text{full}} = 2\pi i \sum_{\tau_0 = \pm \ln \frac{\rho}{\rho'}} \operatorname*{Res}_{\tau \to \tau_0} \frac{e^{i\tau\omega}}{5\pi^2[\rho^2 + \rho'^2 - 2\rho\rho' \cosh(\tau)]^3}$$

Fourier transform in Rindler space

We obtain the **local viscosity:**

$$\eta_{\mathrm{loc}}^{\mathrm{Dirac}}(\rho) = \frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5l_c^4 - 4l_c^2(2\rho^2 + l_c^2)\ln\frac{\rho}{l_c}\right]}{40(\rho^2 - l_c^2)^4\pi^2}$$

By directly integrating over the distance to the horizon, we obtain the **viscosity per unit area of the horizon:**

$$\eta^{\mathrm{Dirac}} = \frac{1}{240\pi^2 l_c^2}$$

Entropy

[Page, PRD 25, 1499 (1982)]
[Prokhorov, Teryaev, Zakharov, PRD (2019), arXiv:1903.09697]
[Buzzegoli, Grossi, Becattini, JHEP (2017), arXiv:1704.02808]

The energy-momentum tensor is known:

$$\langle \hat{T}^{\text{Dirac}}_{\mu\nu} \rangle = \left(\frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

Unlike a scalar field, the quadratic acceleration term contributes to the entropy

$$s_{\text{loc}} = \frac{\partial p}{\partial T}\Big|_a \quad \Rightarrow \quad s^{\text{Dirac}}_{\text{loc}}(T, a) = \frac{7\pi^2 T^3}{45} + \frac{T|a|^2}{36}$$

- We apply approach from the **relativistic spin hydrodynamics**

[Becattini, Daher, Sheng, PLB (2024), arXiv:2309.05789]

[Obukhov, Piskareva, Class. Quantum Grav.(1989)]

Local entropy (for Minkowski vacuum):

$$s^{\text{Dirac}}_{\text{loc}}(\rho) = \frac{1}{30\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\text{Dirac}} = \frac{1}{60\pi l_c^2}$$

Shear viscosity/entropy ratio

Global viscosity and entropy:

$$\eta^{\text{Dirac}} = \frac{1}{240\pi^2 l_c^2} \qquad s^{\text{Dirac}} = \frac{1}{60\pi l_c^2}$$

- **Ratio is finite and does not depend on** l_c

$$\left.\frac{\eta}{s}\right|_{\text{Dirac}} = \frac{1}{4\pi}$$

Saturates KSS bound

The ratio of local viscosity to local entropy is described by the function:

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$

Spin 0

[Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]

Shear viscosity/entropy ratio

- Viscosity and entropy:

$$\eta^{\text{scalar}} = \frac{1}{1440\pi^2 l_c^2} \qquad s^{\text{scalar}} = \frac{1}{360\pi l_c^2}$$

- **Ratio is finite and does not depend on** l_c

$$\boxed{\left.\frac{\eta}{s}\right|_{\text{scalar}} = \frac{1}{4\pi}}$$

Saturates KSS bound

- The ratio of local viscosity to local entropy is described by a function depending on l_c :

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$

Spin 1

Shear viscosity/entropy ratio

Viscosity and entropy differ from the case of spins 0 and ½

$$\eta^{\text{photon}} = \frac{1}{120\pi^2 l_c^2} \qquad s^{\text{photon}} = \frac{1}{30\pi l_c^2}$$

The ratio satisfies the KSS bound

$$\left.\frac{\eta}{s}\right|_{\text{photon}} = \frac{1}{4\pi}$$

The ratio of local viscosity to local entropy is described by the same universal function as for spins 0 and ½:

$$\left.\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho)\right|_{\text{photon}} = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$

Discussion

Discussion

Comparison with string theory

[Buchel, Liu and Starinets, Nucl.Phys.B (2005) arXiv:hep-th/0406264]

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 + \frac{135\zeta(3)}{8(2g^2N_c)^{3/2}} + \dots]$$

From string theory: KSS-bound is saturated for strong coupling (big ’t Hooft coupling)

- In our case, the **opposite situation** – KSS-bound is saturated for **free fields**.

Free fields - what is the source of viscosity?

Key question: what is the source of nontrivial viscosity for free fields?

“Entanglement” viscosity?

Indirect indication of a connection with entanglement:

- Entropy is in the denominator $s^{\,\text{scalar}} = \frac{1}{6} s^{\,\text{Dirac}} = \frac{1}{12} s^{\,\text{photon}} = \frac{1}{360\pi l_c^2}$

  is related to entanglement → viscosity in numerator is also related to entanglement

- Correlator as in **Minkowski space** $\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M$

↓

Question: why the non-trivial answer being received?

↓

Integration when taking the Fourier transform is performed only over a **part of the Minkowski space → the right Rindler wedge**

- **No final answer** → consider other systems with entanglement entropy?

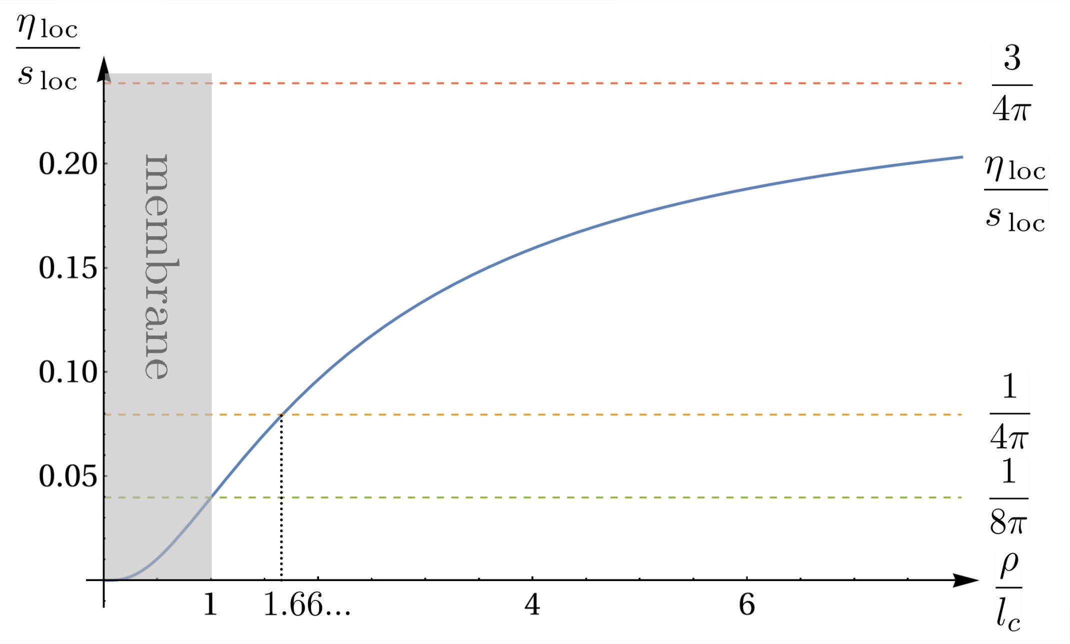
Local vs global

For all cases considered, the ratio of local shear viscosity and entropy is described by the universal function

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c)$$

where

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$



- On the surface of the membrane:

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho = l_c) = \frac{1}{8\pi}$$

- On the contrary, far away from the membrane, the ratio is **higher** than the **KSS bound**:

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho \to \infty) \to \frac{3}{4\pi}$$

Part 3

Preliminary results

Generalization: universality for conformal field theories

- For conformal field theory, the correlator of two EMTs has a universal form up to a common coefficient: [J. Erdmenger and H. Osborn, Nucl. Phys. B 483, 431-474 (1997)]

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \frac{C_T \mathcal{J}_{\mu\nu,\alpha\beta}(x-y)}{[(x-y)^2 - i\varepsilon(x-y)_0]^4}$$

Conformal central charge

- All calculations using Kubo formula are exactly the same for different fields - **universality** of function describing **local shear viscosity:**

$$\eta_{\rm loc}(\rho) = \frac{C_T\pi^2}{80}\cdot\frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5l_c^4 - 4l_c^2(2\rho^2 + l_c^2)\ln(\frac{\rho}{l_c})\right]}{(\rho^2 - l_c^2)^4\pi^2}$$

⇒

$$\eta = \frac{C_T\pi^2}{480 l_c^2}$$

global viscosity

Generalization: universality for conformal field theories

- There is technique to find perturbatively effects of small angular deficit using modular Hamiltonian (which is actually a boost operator):

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]

$$\lim_{\nu\to 1}\frac{\partial}{\partial\nu}\langle\hat{T}_{ij}\rangle = -\langle\hat{T}_{ij}\hat{K}_0\rangle = -\frac{C_T\pi^2\delta_{ij}}{120\rho^4}$$

- The derivative with respect to the angular deficit corresponds to the derivative with respect to temperature:

$$\nu = \frac{2\pi T}{a} \quad \Rightarrow \quad \lim_{\nu\to 1}\frac{\partial}{\partial\nu} \leftrightarrows \lim_{T\to T_U}\frac{\partial}{\partial T}$$

- Thus, the derivative found with respect to the angular deficit gives the entropy for an arbitrary conformal field theory:

$$s_{\,\text{loc}} = \lim_{T\to T_U}\frac{\partial p}{\partial T}\bigg|_{a=\text{const}} = \frac{C_T\pi^3}{60\rho^3} \quad \Rightarrow \quad \boxed{s = \frac{C_T\pi^3}{120 l_c^2}}$$

Generalization: universality for conformal field theories

- Thus, shear viscosity and entropy are proportional to the conformal central charge. The ratio is universal and saturates the KSS bound:

$$\frac{\eta}{s} = \frac{C_T \pi^2}{480 l_c^2} \Big/ \frac{C_T \pi^3}{120 l_c^2} = \frac{1}{4\pi}$$

- A similar answer can be obtained in a slightly different way, using the results of the work:

[M. R. Brown, A. C. Ottewill and D. N. Page, Phys. Rev. D 33, 2840-2850 (1986)]

Anomalous transport: relation to conformal anomaly

- Various new **transport phenomena** related to **quantum anomalies** (*see talk of Oleg V. Teryaev*).
- **Novel transport phenomenon** in accelerated system, associated with **conformal gravitational quantum anomaly**!

conformal anomaly:

$$\langle \hat{T}^{\mu}_{\mu} \rangle = \alpha \left(H + \frac{2}{3}\nabla^2 R \right) + bG + c\nabla^2 R$$

$$H = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$$ *term with Weyl tensor*

- It can be shown, that:

[H. Osborn and A. C. Petkou, Annals Phys. 231, 311-362 (1994)]

$$C_T = -\frac{640}{\pi^2}\alpha$$

- Then the **viscosity of the accelerated system** (curvature is zero) is **determined by the anomaly** in the curved space:

$$\eta = \frac{C_T \pi^2}{480 l_c^2} \Rightarrow \boxed{\eta = -\frac{4\alpha}{3 l_c^2}}$$

Bound for bulk viscosity

- Lowest order mass corrections to **bulk viscosity** and **speed of sound**:

$$\zeta_{\text{loc}} = \frac{m^2|a|}{36\pi^2} \qquad c_s^2 = \frac{1}{3} - \frac{5m^2}{9|a|^2}$$

- **The bound for bulk viscosity** (also predicted within holographic approach) **is saturated!**

[A. Buchel, Phys. Lett. B 663, 286 (2008)]

$$\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)$$

$$\frac{\zeta_{\text{loc}}}{\eta_{\text{loc}}} = 2\left(\frac{1}{3} - c_s^2\right)$$

“Wandering” Planck constant

- In the original holographic derivation of KSS bound the **viscosity is “classical”** – Planck’s constant comes from the **“quantum”** Bekenstein-Hawking **entropy**:

$$\left.\begin{aligned} &\eta \sim \mathcal{O}(\hbar^0) \\ &s \sim \frac{A}{\hbar G} \sim \mathcal{O}(\hbar^{-1}) \end{aligned}\right\} \quad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

- In our case, the viscosity is determined by a **one-loop** diagram calculated directly within the framework of the **QFT** – it contains Planck's constant:

$$\left.\begin{aligned} &\eta_{\,\mathrm{loc}} \sim \frac{T_U^3}{\hbar^2} \\ &s_{\,\mathrm{loc}} \sim \frac{T_U^3}{\hbar^3} \end{aligned}\right\} \quad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

However, the result is the same:
“Wandering” Planck constant

Problem: higher dimensions, d=6,8...

That is, shear viscosity/entropy density ratio doesn’t depend on type of the conformal field, but can **depend on number of dimensions**:

$$\frac{\eta}{s} = \frac{g_\eta(d)}{g_s(d)}$$

In particular:

$$d = 4: \quad \frac{g_\eta(4)}{g_s(4)} = \frac{1}{4\pi}$$

$$d = 6: \quad \frac{g_\eta(6)}{g_s(6)} = \frac{1}{8\pi}$$

$$d = 8: \quad \frac{g_\eta(8)}{g_s(8)} = \frac{1}{100\pi}$$

- In higher dimensions **does not meet the expected KSS bound**.
- **Problem** also discussed in:
  [Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]
- Peculiarities of entropy calculation – why only one regularized integral?
- We assumed that the KSS bound is valid for quantities integrated over the distance from the horizon. Doesn't work in higher dimensions?

Conclusion

Conclusion

- The viscosity in the Rindler space for fields with **spins ½** and **1** is calculated directly. This viscosity is, apparently, is a manifestation of **entanglement**.
- The average values of shear viscosity and entropy are different for different fields. However, their ratio satisfies the **KSS bound** for **all** considered **fields**: $\eta/s = 1/4\pi$.
- The obtained results support the **“objective” interpretation** of the **Unruh effect** – a medium arises that has finite **temperature** $T = T_U$ and **viscosity** $\eta/s = 1/4\pi$.
- **Locally**, the viscosity-to-entropy ratio may **violate KSS bound**. On the stretched horizon $\eta_{\rm loc}/s_{\rm loc} = 1/8\pi$. In general, the ratio is described by a **universal** function that is the same for different types of fields.
- The result **is generalized to an arbitrary conformal field theory** in 4 dimensions.
- The obtained viscosity is a **new type of anomalous transport phenomenon** related with **conformal gravitational anomaly**.
- In order m^2 also another bound for **bulk viscosity** is also **saturated** (for local quantities and massive Dirac fields).
- Unlike the original duality derivation, viscosity and entropy are "quantum" – **“wandering” Planck constant**.
- **Problem:** the ratio η/s **depends on the dimension** of spacetime (but does not depend on the (conformal) field type).

Thank you for your attention!

Emergent gravity and Membrane paradigm
(general idea and very superficial overview)

1 Scenario: Emergent gravity

[Jacobson, PRL (1995), e-Print: gr-qc/9504004]
[Eling, JHEP (2008), e-Print: 0806.3165]

Принцип эквивалентности: локальный горизонт Ридлера в каждой точке

+

Horizon area is related to entropy

$$S = \frac{k_B A}{4 l_p^2}$$

equilibrium

$$\delta Q = T\delta S$$

nonequilibrium

$$\delta Q = T\delta S + \delta W$$

EMT of matter contributes to the heat flux (and entropy increase) inside horizon

$$\delta Q = \int T_{\mu\nu}\xi^{\mu} d\Sigma^{\nu}$$

Raychaudhuri equation relates horizon area (and entropy) increase to Einstein tensor

Einstein equation

Raychaudhuri equation relates horizon area (and entropy) increase to shear (constructed from tangent vectors to geodesics)

Work of shear forses in hydrodynamics

$$\delta W = 2\eta \int \sigma_{\mu\nu}\sigma^{\mu\nu} d\Sigma$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Emergent gravity and Membrane paradigm
(general idea and very superficial overview)

1 Scenario: Emergent gravity

[Jacobson, PRL (1995), e-Print: gr-qc/9504004]

[Eling, JHEP (2008), e-Print: 0806.3165]

The principle of equivalence: Ridler's local horizon at each point

+

Horizon area is related to entropy $S = \frac{k_B A}{4 l_p^2}$

equilibrium

$$\delta Q = T \delta S$$

→ Einstein equation

nonequilibrium

$$\delta Q = T \delta S + \delta W$$

→ Prediction for viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

2 Scenario: Membrane paradigm

Stretched horizon:

[Susskind, The Black Hole War, 2009]

- Due to the slowdown of the time near the horizon, the matter falling on it "stucks" at a certain distance from horizon
- "Spread" in the transverse direction.

$\rho = 0$ true horizon

$\rho = l_c$ stretched horizon

→ $\text{Membrane}: \quad 0 \leqslant \rho \leqslant l_c$

[Thorne, Price, Macdonald, Black holes: the membrane paradigm (1986)]

[Parikh, Wilczek, PRD (1998), arXiv:gr-qc/9712077]

- Membrane paradigm
- It has hydrodynamic properties
- It has viscosity $\frac{\eta}{s} = \frac{1}{4\pi}$

By integrating the action, we can obtain the Navier-Stokes equation

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}R + \frac{1}{8\pi}\int d^3x\sqrt{\pm h}K + S_{\text{matter}}$$

Correlator with two EMTs

[Birrell, Davies, Quantum Fields in Curved Space, Cambridge University Press, 1982]

- Improved stress-energy tensor of a massless real scalar field:

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\varphi\partial_\nu\varphi + (2\xi - \frac{1}{2})\eta_{\mu\nu}\partial_\alpha\varphi\partial^\alpha\varphi - 2\xi(\partial_\mu\partial_\nu\varphi)\varphi + \frac{\xi}{2}\eta_{\mu\nu}\varphi\partial^\alpha\partial_\alpha\varphi$$

- The correlator can be found in the Minkowski metric:

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\rm M} = \frac{4}{3\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y) + \frac{240(\xi-1/6)^2}{\pi^4}\widetilde{\mathcal{I}}_{\mu\nu\alpha\beta}(x-y)$$

a piece **universal for conformally symmetric theories**

deviation from conformal symmetry

[Erdmenger, Osborn, Nucl.Phys.B (1997), arXiv:hep-th/9605009]

$$\mathcal{I}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{4\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{4\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{8\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{8\bar{b}^8} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{16\bar{b}^8}$$

- The general structure follows from symmetry and dimensional considerations

$$\widetilde{\mathcal{I}}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{10\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{10\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{10\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{10\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{80\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{80\bar{b}^8} + \frac{13\eta_{\mu\nu}\eta_{\alpha\beta}}{80\bar{b}^8} - \frac{3\eta_{\mu\nu}b_\alpha b_\beta}{10\bar{b}^{10}} - \frac{3\eta_{\alpha\beta}b_\mu b_\nu}{10\bar{b}^{10}}.$$

$\bar{b}^2 = b^2 - i\varepsilon b_0$ **poles are shifted**

Fourier transform in Rindler space

The dependence on ξ goes away after integration in the horizon plane:

$$\int dx\, dy\, \langle 0|\hat{T}_{\mu\nu}(t, \mathrm{x}, \mathrm{y}, \mathrm{z})\hat{T}_{\alpha\beta}(0, 0, 0, \mathrm{z}')|0\rangle_{\mathrm{M}} = -\frac{1}{30\pi^3(t^2 - (\mathrm{z} - \mathrm{z}')^2 - i\varepsilon t)^3}$$

Local viscosity – at a certain distance from the horizon

$$\eta^{\mathrm{scalar}}_{\mathrm{loc}}(\rho) = \frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5l_c^4 - 4l_c^2(2\rho^2 + l_c^2)\ln\frac{\rho}{l_c}\right]}{240(\rho^2 - l_c^2)^4\pi^2}$$

Viscosity per unit area of the horizon:

$$\eta = \int_{l_c}^{\infty} d\rho'\, \eta_{\mathrm{loc}}(\rho')$$

$$\eta^{\mathrm{scalar}} = \frac{1}{1440\pi^2 l_c^2}$$

- Diverges in the limit $l_c \to 0$

Typical for Rindler space

[Solodukhin, Living Rev. Rel. (2011), arXiv:1104.3712]

Entropy

[Page, PRD 25, 1499 (1982)]

[Dowker, Class. Quant. Grav. (1994), arXiv:hep-th/9401159]

The energy-momentum tensor of accelerated scalar fields is well known

$$\langle \hat{T}^{\rm scalar}_{\mu\nu} \rangle = \left(\frac{\pi^2 T^4}{30} - \frac{|a|^4}{480\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

For the case $\xi = 1/6$

$$\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

Corresponding pressure:

$$p^{\rm scalar}(T, a) = \frac{1}{3} \left(\frac{\pi^2 T^4}{30} - \frac{|a|^4}{480\pi^2} \right)$$

Local entropy

$$s_{\rm loc} = \frac{\partial p}{\partial T}\bigg|_a \Rightarrow s^{\rm scalar}_{\rm loc}(T) = \frac{2\pi^2 T^3}{45} \xrightarrow[a = 1/\rho]{T = a/2\pi} s^{\rm scalar}_{\rm loc}(\rho) = \frac{1}{180\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\rm scalar} = \frac{1}{360\pi l_c^2}$$

Correlator with two EMTs

[Birrell, Davies, Quantum Fields in Curved Space, Cambridge University Press, 1982]

Let's consider electromagnetic fields in R_ξ gauge:

$$T_{\mu\nu} = T^{\mathrm{M}}_{\mu\nu} + T^{\mathrm{G}}_{\mu\nu} + T^{\mathrm{ghost}}_{\mu\nu}$$ EMT contains three contributions

$$T^{\mathrm{M}}_{\mu\nu} = -F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}\eta_{\mu\nu}F^2$$ **Maxwell's contribution**

$$T^{\mathrm{G}}_{\mu\nu} = \frac{1}{\xi}\Big\{A_\mu\partial_\nu(\partial A) + A_\nu\partial_\mu(\partial A) - \eta_{\mu\nu}\big[A^\lambda\partial_\lambda(\partial A) + \frac{1}{2}(\partial A)^2\big]\Big\}$$ **Contribution from the gauge-fixing term**

$$T^{\mathrm{ghost}}_{\mu\nu} = \partial_\mu\bar{c}\partial_\nu c + \partial_\nu\bar{c}\partial_\mu c - \eta_{\mu\nu}\partial_\rho\bar{c}\partial^\rho c$$ **Faddeev-Popov ghosts**

Propagators (Wightman function) in coordinate representation:

$$\langle 0|A_\mu(x)A_\nu(0)|0\rangle_M = \frac{1}{8\pi^2}\left(\frac{(1+\xi)\eta_{\mu\nu}}{x^2 - i\varepsilon x_0} + \frac{2(1-\xi)x_\mu x_\nu}{(x^2 - i\varepsilon x_0)^2}\right)$$

$$\langle 0|c(x)\bar{c}(0)|0\rangle_M = -\frac{1}{4\pi^2}\frac{1}{x^2 - i\varepsilon x_0}$$

Correlator with two EMTs

The logic of calculations is similar to the case with the Dirac field.

- The contributions of the ghosts and gauge-fixing terms cancel each other:

$$\langle 0|\hat{T}^{\text{ghost}}_{\mu\nu}(x)\hat{T}^{\text{ghost}}_{\alpha\beta}(y)|0\rangle_M = -\langle 0|\hat{T}^{\text{G}}_{\mu\nu}(x)\hat{T}^{\text{G}}_{\alpha\beta}(y)|0\rangle_M$$

The entire contribution is determined by the Maxwell term: the universal function

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \langle 0|\hat{T}^{\text{M}}_{\mu\nu}(x)\hat{T}^{\text{M}}_{\alpha\beta}(y)|0\rangle_M = \frac{16}{\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y)$$

Since the correlator differs only by the factor, the subsequent calculations are similar to the case of scalar and Dirac fields.

Since $\langle \hat{T}\hat{T}\rangle\Big|_{\text{photon}} = \frac{1}{2}\langle \hat{T}\hat{T}\rangle\Big|_{\text{Dirac}}$

then $\eta^{\text{photon}} = \frac{1}{2}\,\eta^{\text{Dirac}}$

We finally obtain:

$$\eta^{\text{photon}} = \frac{1}{120\pi^2 l_c^2}$$

Does not depend on the gauge-parameter ξ

The result is **gauge invariant.**

Entropy

Entropy can be found similarly to the case of spins 0 and 1/2

The energy-momentum tensor is known: [Page, PRD 25, 1499 (1982)]

$$\langle \hat{T}^{\text{photon}}_{\mu\nu} \rangle = \left(\frac{\pi^2 T^4}{15} + \frac{T^2 |a|^2}{6} - \frac{11 |a|^4}{240\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

$$s_{\text{loc}} = \left. \frac{\partial p}{\partial T} \right|_a$$

Also, the quadratic acceleration term contributes to the entropy

Local entropy (for Minkowski vacuum):

$$s^{\text{photon}}_{\text{loc}}(T = T_U, |a| = 1/\rho) = \frac{1}{15\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\text{photon}} = \frac{1}{30\pi l_c^2}$$

Fourier transform in Rindler space

Let us perform **integration in the Rindler horizon plane**:

let's move on to polar coordinates

$$\mathrm{x} = r\cos\phi\,, \qquad \mathrm{y} = r\sin\phi$$

Integration can be done explicitly (poles are shifted from the real axis).

We obtain:

$$\int_0^\infty r dr \int_0^{2\pi} d\phi\, \langle 0|\hat{T}_{\mathrm{xy}}\hat{T}_{\mathrm{xy}}|0\rangle_{\mathrm{M}} = \frac{1}{5\pi^3\alpha^3}$$

where $\alpha = -t^2 + (z - z')^2 + i\varepsilon t$

Problem: higher dimensions, d=6,8...

- Let's try to generalize to conformal field theory in an arbitrary number of dimensions:

Shear viscosity:

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \frac{C_T \mathcal{J}^{(d)}_{\mu\nu,\alpha\beta}(x-y)}{[(x-y)^2 - i\varepsilon(x-y)_0]^d}$$

$$\mathcal{J}^{(d)}_{\mu\nu,\alpha\beta} = \frac{1}{2}(I_{\mu\alpha}I_{\nu\beta} + I_{\mu\beta}I_{\nu\alpha}) - \frac{1}{d}\eta_{\mu\nu}\eta_{\alpha\beta}$$

$$\eta = C_T \frac{g_\eta(d)}{l_c^{d-2}}$$

[J. Erdmenger and H. Osborn, Nucl. Phys. B 483, 431-474 (1997)]

$$\frac{\eta}{s} = \frac{g_\eta(d)}{g_s(d)}$$

Entropy density:

$$-\lim_{\nu\to 1}\frac{\partial}{\partial\nu}\langle \hat{T}_{ij}\rangle = \frac{C_T \pi^{d/2}\Gamma(\frac{d}{2})\delta_{ij}}{\Gamma(d+2)\rho^d}$$

$$s = C_T \frac{g_s(d)}{l_c^{d-2}}$$

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]

Bound for bulk viscosity

- Let us also find the correction to the speed of sound:

$$c_s^2 = \frac{\partial p}{\partial \varepsilon}$$

- The energy and pressure of an accelerated fermion gas in the first nonzero order in mass: [G. Y. Prokhorov, O. V. Teryaev and V. I. Zakharov, JHEP 03, 137 (2020)]

$$\varepsilon = \frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2} + m^2\left(-\frac{T^2}{12} + \frac{|a|^2}{48\pi^2}\right)$$

$$p = \frac{1}{3}\left(\frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}\right) + m^2\left(-\frac{T^2}{12} + \frac{|a|^2}{48\pi^2}\right)$$

- **In the Minkowski vacuum limit the mass correction to the speed of sound:**

$$T \to T_U$$

$$c_s^2 = \frac{1}{3} - \frac{5m^2}{9|a|^2}$$

Bound for bulk viscosity

- By investigating the dispersion relation for sound waves using a holographic approach, a limit on the **bulk viscosity** is supposed:

  [A. Buchel, Phys. Lett. B 663, 286 (2008)]

$$\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)$$

- We will consider only the local viscosity in the limit of zero membrane thickness and massive Dirac fields:

  $l_c \to 0$ $\quad S = 1/2$

- We use the next **Kubo formula** for **bulk viscosity**:

$$\zeta_{\mathrm{loc}} = \frac{\pi}{9}\lim_{\omega\to 0}\int_0^{\infty}\rho\rho'\,d\rho'\int_{-\infty}^{\infty}dx\,dy\,d\tau e^{i\omega\tau}\langle 0|\hat{T}_{\mu}^{\mu}(\tau,x,y,\rho)\hat{T}_{\nu}^{\nu}(0,0,0,\rho')|0\rangle_{\mathrm{M}}$$

- **The result is** (in the order m^2):

$$\zeta_{\mathrm{loc}} = \frac{m^2|a|}{36\pi^2}$$

Anomalous transport phenomena

New **(non-dissipative)** effects are predicted at the intersection of quantum field theory and gravity *(only some of them)*:

- Chiral Magnetic Effect (**CME**):

  [K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD 78, 074033 (2008), 0808.3382]

  $$\langle \hat{j}^\mu \rangle = C e^2 \mu_A B^\mu$$

- Axial Vortical Effect (**AVE**):

  [D. T. Son, P. Surowka, PRL 103, 191601 (2009), 0906.5044]

  $$\langle \hat{j}^\mu_A \rangle = C(\mu^2 + \mu_A^2)\omega^\mu$$

- **Thermal part** of AVE:

  [K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107, 021601 (2011), 1103.5006]

  $$\langle \hat{j}^\mu_A \rangle \sim \mathcal{N} T^2 \omega^\mu$$

- **Kinematic Vortical Effect (KVE)**

  [G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, PRL 129, 151601 (2022), 2207.04449]

  $$\langle \hat{j}^A_\mu \rangle = (\lambda_1 \omega^2 + \lambda_2 a^2)\omega_\mu$$

Associated with **axial anomaly** in the electromagnetic field

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{Ce^2}{8} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Associated with an **axial anomaly** in the **gravitational field**

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\rho\sigma} R_{\alpha\beta}{}^{\rho\sigma}$$

Connection with anomaly:

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

KVE, acceleration

- The relationship of KVE to anomaly can be obtained directly from the **conservative** equations.

  The derivation is similar to:

[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]
[Shi-Zheng Yang, Jian-Hua Gao, Zuo-Tang Liang, Symmetry 14 (2022) 5, 948]
[M. Buzzegoli, Lect. Notes Phys. 987, 53-93 (2021)]

- Relation $\lambda_1 - \lambda_2 = 32\mathcal{N}$ verified by comparing KVE with anomaly:

For ***spin ½:***

[GP, Teryaev, Zakharov, JHEP, 02:146, 2019],
[V. E. Ambrus, JHEP, 08:016, 2020],
[A. Palermo, et al. JHEP 10 (2021) 077]

$$j^A_\mu = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_\mu$$

$$\nabla_\mu j^\mu_A = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

For ***spin 3/2*** *in the RSA model:*

$$j^{A(3)}_\mu = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

$$\nabla_\mu j^\mu_A = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)/32 = -\frac{19}{384\pi^2}$$

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Lett. B 840, 137839 (2023), 2211.03865]

- **The effect depends on acceleration - less studied than vorticity.**
- **New effects related to acceleration?**

Motivation: Unruh effect

[Blasone, (2018), e-Print: 1911.06002]

Formulation

The Minkowski **vacuum** is perceived by an **accelerated** observer as a medium with a finite (Unruh) **temperature**

$$T_U = \frac{a}{2\pi}$$

From the point of view of the quantum-statistical approach:

[Becattini, PRD (2018), arXiv:1712.08031]

Thus, the **mean values** of the thermodynamic quantities normalized to Minkowski vacuum should be **equal to zero** when the proper temperature, measured by comoving observer, equals to the **Unruh temperature**.

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

Example:

[Prokhorov, Teryaev, Zakharov, PRD (2019), arXiv:1903.09697]

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\mathrm{fermi}} = \left(\frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2} \right) u^\mu u^\nu - \left(\frac{7\pi^2 T^4}{180} + \frac{T^2 |a|^2}{72} - \frac{17|a|^4}{2880\pi^2} \right) \Delta^{\mu\nu}$$

- Well-known in Rindler space. But can be obtained by a statistical method without switching to Rindler coordinates
- Supports the **“objective”** interpretation of the effect of the Unruh (in contrast to the fact that it is just the effect of the detector).

Minimal viscosity bound

Hydrodynamics in linear gradients - corrections to EMT with **dissipation**:

$$T_{\mu\nu} = T_{\mu\nu}^{\text{ideal}} + T_{\mu\nu}^{\text{diss}} \qquad T_{\mu\nu}^{\text{ideal}} = (\varepsilon + p) u_\mu u_\nu - p g_{\mu\nu}$$

$$T_{\mu\nu}^{\text{diss}} = -\eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu - u_\mu u^\alpha \nabla_\alpha u_\nu - u_\nu u^\alpha \nabla_\alpha u_\mu) - \left(\zeta - \frac{2}{3}\eta\right) \nabla^\alpha u_\alpha (g_{\mu\nu} - u_\mu u_\nu) + \mathcal{O}(\nabla^2 u)$$

Bound inspired by string theory:

- **There are no completely ideal fluids!**
- It is believed that QGP near this limit
- **does not cover case of Rindler space!**

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

KSS-bound

[Kovtun, Son, Starinets, PRL (2005), arXiv:hep-th/0405231]

- **Some "feeling"**: according to the holographic principle, the viscosity is associated with the scattering of gravitons on black brane, and entropy with the horizon area – their ratio will be finite.

- Plenty of work about KSS Bound
- The simplest illustration: the uncertainty principle for energy

$$\eta \sim \varepsilon \tau_{\text{free}} \qquad s \sim n \quad \Longrightarrow \quad \frac{\eta}{s} \sim \frac{\varepsilon}{n}\tau_{\text{free}} = E\,\tau_{\text{free}} \gtrsim \hbar$$

[Dobado, Llanes-Estrada, Rincon, AIP Conf.Proc. (2008), e-Print: 0804.2601]

Rindler coordinates and stretched horizon

Rindler's metric describes the accelerated reference system:

$$ds^2 = \rho^2 d\tau^2 - d\mathrm{x}^2 - d\mathrm{y}^2 - d\rho^2$$

- The relationship between Rindler coordinates and Minkowski coordinates:

$$t = \rho \sinh \tau$$
$$z = \rho \cosh \tau$$

$$\text{Horizon}: \quad g_{00}(\rho = 0) = 0$$

$$a = \frac{1}{\rho}$$

Acceleration - the inverse distance to the horizon.

$$a_\mu = u^\nu \nabla_\nu u_\mu$$

- As was said, the fields are stuck at a **certain distance** from the horizon:

$$\rho \in [l_c, \infty)$$

Kubo formula: Rindler space

Due to the fluctuation-dissipation theorem, dissipation coefficients can be found from fluctuations in equilibrium:

Kubo's formula for viscosity **[Zubarev, Nonequilibrium statistical thermodynamics, Studies in soviet science, 1974]**

$$\eta = \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x e^{i\omega t}\theta(t)\langle[\hat{T}_{\rm xy}(x), \hat{T}_{\rm xy}(0)]\rangle$$

- Can be obtained from the interaction vertex with gravitons $\delta g_{\mu\nu}\hat{T}^{\mu\nu}$
- Contains a double limit $\omega, \vec{q} \to 0$

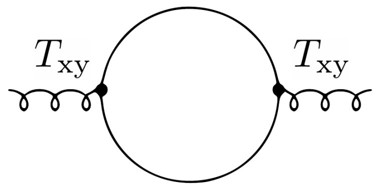
  First $\vec{q} \to 0$. *Reflects the dissipative nature.*

In the Rindler space: **[Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]**

$$\eta = \pi \lim_{\omega\to 0} \int_{l_c}^{\infty} \rho'\, d\rho' \int_{l_c}^{\infty} \rho\, d\rho \int_{-\infty}^{\infty} {\rm dx\, dy}\, d\tau e^{i\omega\tau} \langle 0|\hat{T}_{\rm xy}(\tau, {\rm x}, {\rm y}, \rho)\hat{T}_{\rm xy}(0,0,0,\rho')|0\rangle_{\rm M}$$

- In the limit $\omega \to 0$, one can pass from the retarded Green's function to the Wightman function.
- We consider free fields:

$$\eta = \lim_{\omega\to 0} \int$$



per unit horizon area

$$\eta = \int_{l_c}^{\infty} d\rho'\, \eta_{\rm loc}(\rho')$$

Entropy derivation

Thermodynamic relations are modified in a medium with spin:

$$dp = s dT + n d\mu + \frac{1}{2} S^{\mu\nu} d\omega_{\mu\nu}$$

[Becattini, Daher, Sheng, PLB (2024), arXiv:2309.05789]

[Obukhov, Piskareva, Class. Quantum Grav.(1989)]

In a state of global equilibrium, it contains the vorticity tensor.
For an accelerated medium:

$$\omega_{\mu\nu} = a_\mu u_\nu - a_\nu u_\mu$$

$$s_{\,\mathrm{loc}} = \left.\frac{\partial p}{\partial T}\right|_a$$

$$p = -\frac{1}{3} \langle \hat{T}_{\mu\nu} \rangle \Delta^{\mu\nu}$$

- Unlike viscosity case, it is necessary to move away from the Minkowski vacuum

$$T = T_U + dT$$

$$\text{Minkowsky vacuum:} \quad s_{\,\mathrm{loc}}(T = T_U, |a|) \underset{|a| \to 1/\rho}{=} s_{\,\mathrm{loc}}(\rho)$$

Entropy per unit area of the horizon: $s = \int_{l_c}^{\infty} d\rho \, s_{\,\mathrm{loc}}(\rho)$

Fourier transform in Rindler space

Finding residues at the poles and passing to the limit of zero frequency, we obtain:

$$\lim_{\omega\to 0} I = \frac{3\rho'^4 - 3\rho^4 + 2[\rho^4 + 4\rho^2\rho'^2 + \rho'^4]\ln\frac{\rho}{\rho'}}{5\pi^2(\rho^2-\rho'^2)^5}$$

Taking the last integral over the distance to the horizon in the Fourier transform, we obtain the **local viscosity:**

$$\eta^{\text{Dirac}}_{\text{loc}}(\rho) = \frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5l_c^4 - 4l_c^2(2\rho^2 + l_c^2)\ln\frac{\rho}{l_c}\right]}{40(\rho^2 - l_c^2)^4\pi^2}$$

By directly integrating over the distance to the horizon, we obtain the **viscosity per unit area of the horizon:**

$$\eta^{\text{Dirac}} = \frac{1}{240\pi^2 l_c^2}$$

Generalization: universality for conformal field theories

- For conformal field theory, the correlator of two EMTs has a universal form up to a common coefficient:

[J. Erdmenger and H. Osborn, Nucl. Phys. B 483, 431-474 (1997)]

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \frac{C_T \mathcal{J}_{\mu\nu,\alpha\beta}(x-y)}{[(x-y)^2 - i\varepsilon(x-y)_0]^4}$$

Conformal central charge

$$\mathcal{J}_{\mu\nu,\alpha\beta} = \frac{1}{2}(I_{\mu\alpha}I_{\nu\beta} + I_{\mu\beta}I_{\nu\alpha}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta}$$

$$I_{\mu\nu} = \eta_{\mu\nu} - 2\frac{(x-y)_\mu (x-y)_\nu}{(x-y)^2 - i\varepsilon(x-y)_0}$$

- All calculations using Kubo formula are exactly the same for different fields - **universality of function describing local shear viscosity:**

$$\eta_{\rm loc}(\rho) = \frac{C_T \pi^2}{80} \cdot \frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5l_c^4 - 4l_c^2(2\rho^2 + l_c^2)\ln(\frac{\rho}{l_c})\right]}{(\rho^2 - l_c^2)^4 \pi^2}$$

$$\Rightarrow \quad \eta = \frac{C_T \pi^2}{480 l_c^2}$$

Species problem

- Bekenstein-Hawking: $$S_{\rm BH} = \frac{A}{4G\hbar}$$
- Entanglement entropy: $$S_{entangl} \sim A$$

BUT depends on the number and type of fields

In particular, in accordance with that, we obtain:

$$s^{\,\rm scalar} = \frac{1}{6} s^{\,\rm Dirac} = \frac{1}{12} s^{\,\rm photon} = \frac{1}{360\pi l_c^2}$$

But the same for **“entanglement” viscosity:**

$$\eta^{\,\rm scalar} = \frac{1}{6}\eta^{\,\rm Dirac} = \frac{1}{12}\eta^{\,\rm photon} = \frac{1}{1440\pi^2 l_c^2}$$

Their relation will be universal:

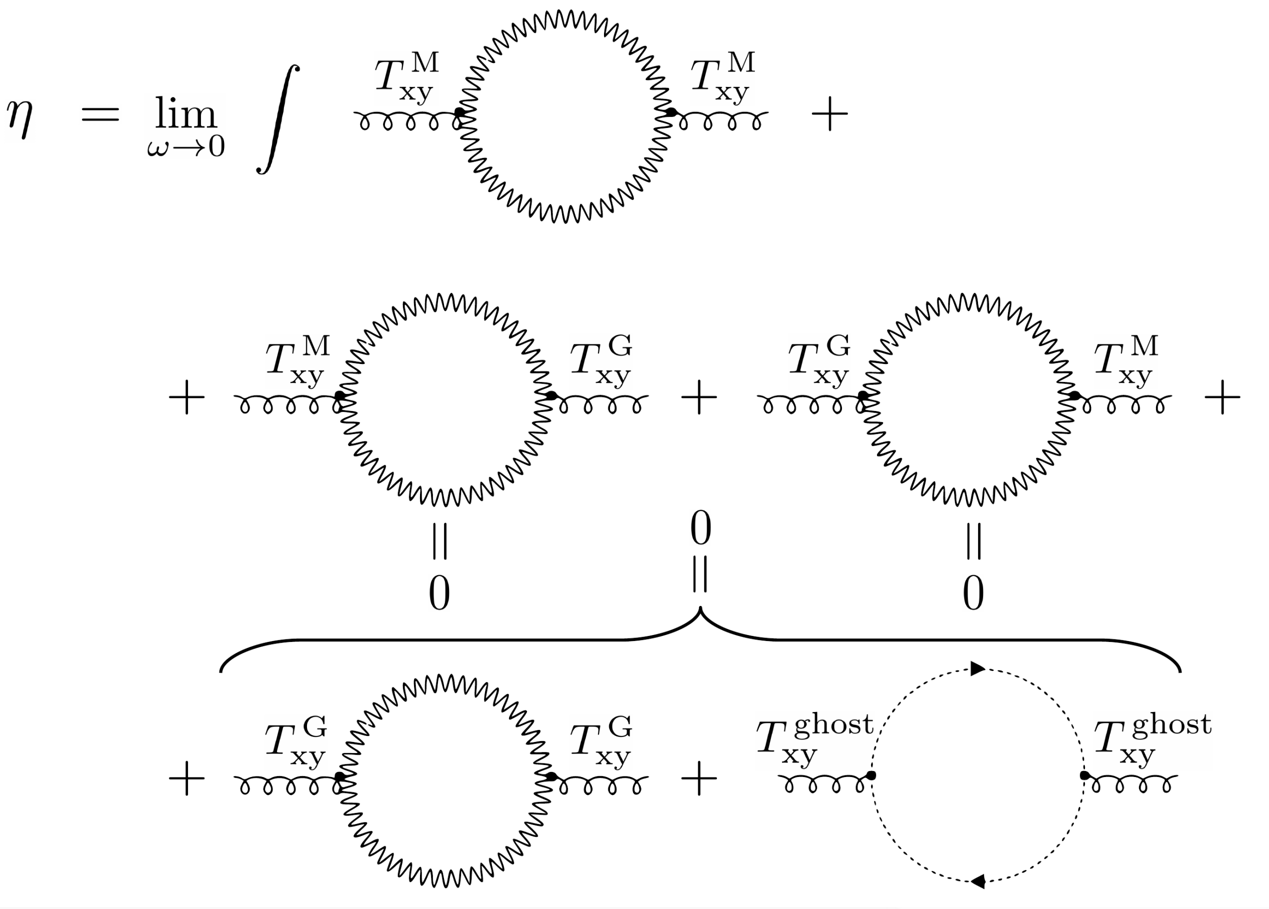
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

The “species problem” exists at the level of entropy and viscosity separately, but disappears for their ratio.

Correlator with two EMTs

Expand the two-point correlator, selecting various contributions to the EMT operator

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \langle 0|\hat{T}^{\mathrm{M}}_{\mu\nu}(x)\hat{T}^{\mathrm{M}}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\mathrm{M}}_{\mu\nu}(x)\hat{T}^{\mathrm{G}}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\mathrm{G}}_{\mu\nu}(x)\hat{T}^{\mathrm{M}}_{\alpha\beta}(y)|0\rangle_M$$
$$+\langle 0|\hat{T}^{\mathrm{G}}_{\mu\nu}(x)\hat{T}^{\mathrm{G}}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\mathrm{ghost}}_{\mu\nu}(x)\hat{T}^{\mathrm{ghost}}_{\alpha\beta}(y)|0\rangle_M \, ,$$



Discussion

Objective interpretation of the Unruh effect

- Thus, the view of the Unruh effect as an **objective effect** associated with the emergence of the media is strengthened:

  -- In an accelerated frame, the Minkowski vacuum behaves like a fluid

  Temperature of "vacuum fluid" $T = T_U$

  Viscosity of the "vacuum fluid" $\eta/s = 1/4\pi$

Comparison with string theory

[Buchel, Liu and Starinets, Nucl.Phys.B (2005) arXiv:hep-th/0406264]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{135\zeta(3)}{8(2g^2N_c)^{3/2}} + \ldots\right]$$

From string theory: KSS-bound is saturated for strong coupling (big 't Hooft coupling)

- In our case, the **opposite situation** – KSS-bound is saturated for **free fields**.

  Free fields - what is the source of viscosity?

  Naively: $\eta \sim l_{\text{free}}$

  $l_{\text{free}} \to \infty$ ⇒ $\eta \to \infty$

Entropy derivation

- Various approaches to find entropy in the space with horizon

[Solodukhin, Living Rev. Rel. (2011), arXiv:1104.3712]

Thermodynamic relations are modified in a medium with spin

$$p(a,T) = -\frac{1}{3}\langle \hat{T}_{\mu\nu}\rangle \Delta^{\mu\nu}$$

$$s_{\rm loc} = \frac{\partial p}{\partial T}\Big|_a$$

Entropy per unit area of the horizon:

$$s = \int_{l_c}^{\infty} d\rho\, s_{\rm loc}(\rho)$$