On the interplay between the BFKL resummation and high–energy factorization in Mueller–Navelet dijet production[1][2]

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Contents

Motivation I

The DGLAP equation:

$$\frac{\partial}{\partial \ln \mu^2} O = \hat{K}_{\mathrm{DGLAP}}\, O$$

- $z = \mu^2/s$ is fixed: $z \sim 1$;
- N^{k}LO resum N^{k}LL $\sim \bar{\alpha}_s^{n+k}(\mu_R^2) \ln^n \mu^2$ in the limit $\mu^2 \to \infty$.

The **Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation:**

$$\boxed{\frac{\partial}{\partial Y} O \;=\; \hat{K}_{\mathrm{BFKL}}\, O}$$

- **Regge limit:** $z \ll 1$, while $\bar{\alpha}_s(\mu_R^2) Y \sim 1$ with $Y = \ln(1/z)$.

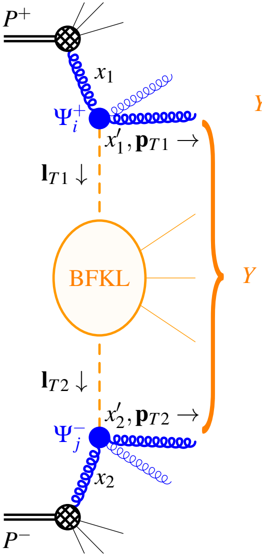
State-of-the-art *(in the QCD)*:

$$\hat{K}_{\mathrm{BFKL}} \;=\; \underbrace{\bar{\alpha}_s(\mu_R^2)\, \hat{K}^{(0)}}_{\text{[BFKL '76-78]}} \;+\; \underbrace{\bar{\alpha}_s^2(\mu_R^2)\, \hat{K}^{(1)}}_{\text{[FL '98]}} \;+\; \underbrace{\mathscr{O}(\bar{\alpha}_s^3)}_{[...]}.$$

- LO resum LL $\sim \bar{\alpha}_s^n Y^n$;
- NLO resum NLL $\sim \bar{\alpha}_s^{n+1}(\mu_R^2) Y^n$, partially includes $\bar{\alpha}_s(\mu_R^2) \ln\left(\mathbf{l}_T^2/\mu^2\right)$;
- NNLO is in progress[V. Fadin, L. Lipatov '18; V. Del Duca *et.al.* '21; E. Byrne '24; S. Abreu *et.al.* '25; ...].

Motivation II

White paper on the BFKL physics[M. Hentschinski *et.al.* '23]: *Mueller-Navelet (MN) dijets*[MN '86] and other jets topologies, Higgs, heavy quarkonia, diffraction, etc.



Regge limit:

$$Y \sim \ln\left(\frac{M^2}{\sqrt{\mathbf{p}_{T1}^2\mathbf{p}_{T2}^2}}\right), \; M^2 = x_1' x_2' P^+ P^- \gg \mathbf{p}_{T1,2}^2 \gg \Lambda_{\mathrm{QCD}}^2;$$

Hybrid approach:

- Collinear factorization & DGLAP resummation;
- The BFKL resummed coefficient function:

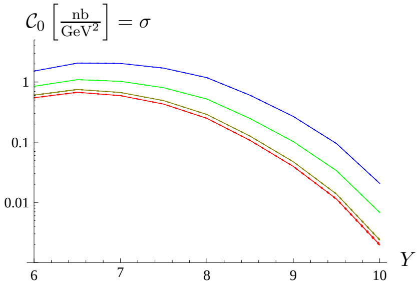
$$\frac{d}{dY} H_{ij}(x_{1,2}, \bar{\alpha}_s(\mu_R^2)) = \int\limits_{\mathbf{l}_{T1,2}} \Psi_i^+\left(\mathbf{l}_{T1}, \mathbf{p}_{T1}, x_1'\right) \times G(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y)\, \Psi_j^-\left(\mathbf{l}_{T2}, \mathbf{p}_{T2}, x_2'\right),$$

  where *NLL Green's function* in the LO basis:

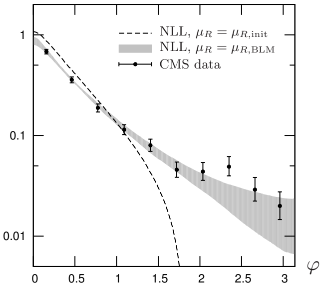
$$G(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y) = \langle f_\gamma^{(0)}(\mathbf{l}_{T2}) \,|\, e^{Y\hat{K}} \,|\, f_\gamma^{(0)}(\mathbf{l}_{T1}) \rangle;$$

- No resummation of $\ln\left(\mu_1^2/\mu_2^2\right)$ to the *impact-factors*.

Motivation III



LL BFKL / NLL BFKL [D. Colferai *et.al.* '10]



[B. Duclou ´, L. Szymanowski, S. Wallon '14]

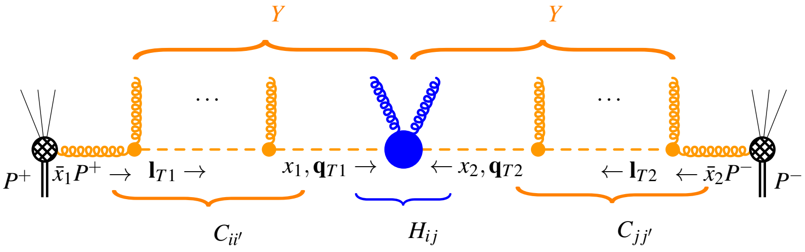
Selected results:

- Collinear improvement of the Green's function[A. Sabio-Vera, F. Schwennsen '07];
- Large corrections to the impact factors $\sim \ln(\mathbf{l}_T^2/\mu_R^2)$[D. Colferai *et.al.* '10; F. Caporale *et.al.* '13];
- Instability of the NLO BFKL computations $\Delta\phi \sim 0$[B. Duclou ´ *et.al.* '14; F. Celiberto *et.al.* '22];
- Application of the BLM procedure[S. Brodsky *et.al.* '99; B. Duclou ´ *et.al.* '14; F. Caporale *et.al.* '15];
- Sudakov resummation via TMD-factorization[A. Mueller, L. Szymanowski, S. Wallon *et.al.* '15].

The high–energy factorization

HEF

The **high–energy factorization (HEF)**[S. Catani, M. Ciafaloni, F. Hautmann '90-94; J. Collins, C. Ellis '91]:



- Resummation:

$$Y \sim \ln\left(\frac{1-z}{z}\frac{\mu^2}{\mathbf{l}_T^2}\right) \simeq \underbrace{\ln\left(\frac{1}{z}\right)}_{\text{BFKL}} + \underbrace{\ln\left(\frac{\mu^2}{\mathbf{l}_T^2}\right)}_{\text{Sudakov}}, \quad z = \frac{q^\pm}{\bar{x}P^\pm} = \frac{x}{\bar{x}};$$

- Logs Y are factorized in the Regge limit $z \ll 1$: $\int \frac{dz}{z} \int_{\mathbf{q}_T} C_{ii'}(z, \mathbf{q}_T, \mu^2)$;
- One Reggeon exchange–parton Reggeization approach[M. Nefedov, V. Saleev, A. Shipilova '13].

The HEF:

$$\sigma = \int \frac{dx_1}{x_1} \int_{\mathbf{q}_{T1}} \Phi_i(x_1, \mathbf{q}_{T1}^2, \mu^2) \int \frac{dx_2}{x_2} \int_{\mathbf{q}_{T2}} \Phi_j(x_2, \mathbf{q}_{T2}^2, \mu^2) \times H_{ij}\left(x_{1,2}, \mathbf{q}_{T1,2}, \bar{\alpha}_s(\mu_R^2)\right) + \mathcal{O}\left((\Lambda/\mu)^{\#}, \mu^2/s, \mathrm{N}^{\#}\mathrm{LL}\right),$$

where *unintegrated PDF (UPDF)*

$$\Phi_i(x, \mathbf{q}_T^2, \mu^2) = \int_x^1 \frac{dz}{z} \tilde{f}_{i'}\left(\frac{x}{z}, \mu^2\right) C_{ii'}(z, \mathbf{q}_T, \mu^2).$$

Advantages:

- Proven up to NLL[S. Catani, M. Ciafaloni, F. Hautmann '94; A. van Hameren, M. Nefedov '25];
- Coefficient function H_{ij} is **gauge invariant**;
- Resummation kernels $C_{ii'}$ are universal. The UPDFs with the **NLL Sudakov resummation**[M. Nefedov, V. Saleev '20; M. Nefedov '21].

Effective action approach

Effective action[L. Lipatov '95]:

$$S = S_{\text{kin}}[A_\pm(x)] + \sum_Y S_{\text{QCD}}[v(x)] + S_{\text{ind}}[v_\pm(x), A_\pm(x)],$$

where Reggeon fields $A_\pm(x) = A^a_\pm(x)T_a$, MRK constraints: $\partial_- A_+(x) = \partial_+ A_-(x) = 0$.

- Kinetic term:

$$\mathscr{L}_{\text{kin}}(x) = 4\,\text{tr}\left[A_+(x)\,\partial^2 A_-(x)\right];$$

- Induced term:

$$\mathscr{L}_{\text{ind}}(x) = \frac{i}{g}\,\text{tr}\left[\partial^2 A_+(x)\,\partial_- W[v_-(x)] + \partial^2 A_-(x)\,\partial_+ W[v_+(x)]\right],$$

where $W[v_\pm(x)] = P\exp\left[-\frac{ig}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp\, v_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right]$. Induced interactions:

$$\begin{aligned}\mathscr{L}_{\text{ind}}(x) \quad = \quad & \text{tr}\Big[\partial^2 A_+(x)\, v_-(x) + (-ig)\,\partial^2 A_+(x)\left(v_-(x)\,\partial_-^{-1} v_-(x)\right) \\ + \quad & (-ig)^2\,\partial^2 A_+(x)\left(v_-(x)\,\partial_-^{-1} v_-(x)\,\partial_-^{-1} v_-(x)\right) \;+\; \{+ \leftrightarrow -\}\Big] + \mathscr{O}(g^3);\end{aligned}$$

- Effective action for the Reggeized quarks[L. Lipatov, M. Vyazovsky '02].
- Hermitian form of the effective action[S. Bondarenko, M. Zubkov '18].

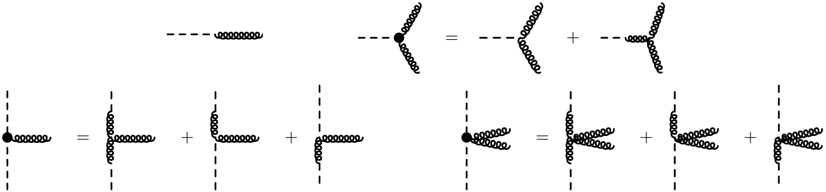
Feynman rules

Feynman rules[E. Antonov, I. Cherednikov, E. Kuraev, L. Lipatov '05]:

- Induced vertices:

$$
\begin{array}{llcl}
R_{\pm} g & \Delta^{\pm a b_1}_{\mu_1}(q, l_1) & = & i\, \mathbf{q}_T^2\, n^{\mp}_{\mu_1}\, \delta^{a b_1}, \\
R_{\pm} g g & \Delta^{\pm a b_1 b_2}_{\mu_1 \mu_2}(q, l_1, l_2) & = & 2ig\, \mathbf{q}_T^2\, n^{\mp}_{\mu_1}\, n^{\mp}_{\mu_2}\, \dfrac{\mathrm{tr}\left[T^a T^{b_1} T^{b_2} - T^{b_1} T^a T^{b_2}\right]}{l_1^{\pm}}, \\
R_{\pm} g g g & \Delta^{\pm a b_1 b_2 b_3}_{\mu_1 \mu_2 \mu_3}(q, l_1, l_2) & = & i g^2\, \mathbf{q}_T^2\, n^{\mp}_{\mu_1}\, n^{\mp}_{\mu_2}\, n^{\mp}_{\mu_3} \displaystyle\sum_{S_3} \dfrac{\mathrm{tr}\left[T^a T^{b_{i_1}} T^{b_{i_2}} T^{b_{i_3}} + (i_1 \leftrightarrow i_3)\right]}{l^{\pm}_{i_3}\left(l^{\pm}_{i_3} + l^{\pm}_{i_2}\right)}
\end{array}
$$

- Effective vertices:



The BFKL equation

The BFKL kernel

The LO kernel is conformal invariant ($\Rightarrow f_\gamma^{(0)}(\mathbf{q}_T) \sim \mathbf{q}_T^{2(\gamma-1)}$)[I. Balitsky, L. Lipatov '78]:

$$\lim_{\varepsilon\to 0} \int_{\mathbf{q}_T} K_\varepsilon\,(\mathbf{l}_T,\mathbf{q}_T)\, f_\gamma^{(0)}(\mathbf{q}_T) = \bar{\alpha}_s(\mu_R^2)\,\chi^{(0)}(\gamma)\, f_\gamma^{(0)}(\mathbf{l}_T)$$

- *Lipatov's LO characteristic function:* $\chi^{(0)}(\gamma) = \chi^{(0)}(1-\gamma)$.

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Action of the NLO kernel on the LO eigenfunctions[V. Fadin, L. Lipatov '98]:

$$\lim_{\varepsilon\to 0} \int_{\mathbf{q}_T} K_\varepsilon(\mathbf{l}_T,\mathbf{q}_T)\, f_\gamma^{(0)}(\mathbf{q}_T) = \bar{\alpha}_s(\mathbf{l}_T^2)\left[\chi^{(0)}(\gamma) + \bar{\alpha}_s(\mathbf{l}_T^2)\,\frac{\delta(\gamma)}{4}\right] f_\gamma^{(0)}(\mathbf{l}_T)$$

The LO eigenfunctions are **not the basis of the NLO kernel:**

- Coupling running:

$$\bar{\alpha}_s(\mathbf{l}_T^2) = \bar{\alpha}_s(\mu_R^2)\left(1 - \bar{\alpha}_s(\mu_R^2)\,\frac{\beta_0}{4\pi}\ln\left(\frac{\mathbf{l}_T^2}{\mu_R^2}\right)\right) + \mathcal{O}(\bar{\alpha}_s^3);$$

- Structure of the $\delta(\gamma)$:

$$\frac{\delta(\gamma)}{4} = \chi^{(1)}(\gamma) \underbrace{- \frac{1}{2}\,\frac{\beta_0}{4\pi}\,\frac{\partial \chi^{(0)}(\gamma)}{\partial \gamma}}_{\text{antisym. }\gamma\leftrightarrow 1-\gamma},$$

where $\chi^{(1)}(\gamma) = \chi^{(1)}(1-\gamma)$.

The BFKL equation

General form of **the BFKL equation solution**[G. Chirilli, Y. Kovchegov '13, 14]:

- The BFKL equation for the Green's function:

$$\frac{\partial}{\partial Y} G(\mathbf{l}_{T1}, \mathbf{l}_{T2}, Y) = \int_{\mathbf{q}_T} K(\mathbf{l}_{T1}, \mathbf{q}_T)\, G(\mathbf{q}_T, \mathbf{l}_{T2}, Y),$$

  where $G(\mathbf{l}_{T1}, \mathbf{l}_{T2}, 0) = \delta(\mathbf{l}_{T1} - \mathbf{l}_{T2})$ and $K = \bar{\alpha}_s(\mu_R^2)\, K^{(0)} + \bar{\alpha}_s^2(\mu_R^2)\, K^{(1)} + \mathscr{O}(\bar{\alpha}_s^3)$;

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- Basis of the kernel (order-by-order in $\bar{\alpha}_s$):

$$\lim_{\varepsilon \to 0} \int_{\mathbf{q}_T} K_\varepsilon\left(\mathbf{l}_T, \mathbf{q}_T \mid \bar{\alpha}_s(\mu_R^2)\right) H_\gamma(\mathbf{q}_T, \mu_R^2) = \bar{\alpha}_s(\mu_R^2)\, \chi(\gamma)\, H_\gamma(\mathbf{l}_T, \mu_R^2),$$

  where

$$\begin{aligned} \chi(\gamma) &= \chi^{(0)}(\gamma) + \bar{\alpha}_s(\mu_R^2)\, \chi^{(1)}(\gamma) + \mathscr{O}(\bar{\alpha}_s^2), \\ H_\gamma(\mathbf{l}_T, \mu_R^2) &= f_\gamma^{(0)}(\mathbf{l}_T) + \bar{\alpha}_s(\mu_R^2)\, f_\gamma^{(1)}(\mathbf{l}_T, \mu_R^2) + \mathscr{O}(\bar{\alpha}_s^2); \end{aligned}$$

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  where $G\left(\mathbf{l}_{T1}, \mathbf{l}_{T2}, 0\right) = \delta\left(\mathbf{l}_{T1} - \mathbf{l}_{T2}\right)$ and $K = \bar{\alpha}_s(\mu_R^2) K^{(0)} + \bar{\alpha}_s^2(\mu_R^2) K^{(1)} + \mathcal{O}(\bar{\alpha}_s^3)$;

- Basis of the kernel (order-by-order in $\bar{\alpha}_s$):

$$\lim_{\varepsilon \to 0} \int_{\mathbf{q}_T} K_\varepsilon\left(\mathbf{l}_T, \mathbf{q}_T \mid \bar{\alpha}_s(\mu_R^2)\right) H_\gamma(\mathbf{q}_T, \mu_R^2) = \bar{\alpha}_s(\mu_R^2) \chi(\gamma) H_\gamma(\mathbf{l}_T, \mu_R^2),$$

  where

$$\begin{aligned} \chi(\gamma) &= \chi^{(0)}(\gamma) + \bar{\alpha}_s(\mu_R^2) \chi^{(1)}(\gamma) + \mathcal{O}(\bar{\alpha}_s^2), \\ H_\gamma(\mathbf{l}_T, \mu_R^2) &= f_\gamma^{(0)}(\mathbf{l}_T) + \bar{\alpha}_s(\mu_R^2) f_\gamma^{(1)}(\mathbf{l}_T, \mu_R^2) + \mathcal{O}(\bar{\alpha}_s^2); \end{aligned}$$

- Expansion over the eigenfunctions:

$$K\left(\mathbf{l}_T, \mathbf{q}_T\right) = \bar{\alpha}_s(\mu_R^2) \int \frac{d\gamma}{2\pi i} \chi(\gamma) H_{n,\gamma}(\mathbf{l}_T, \mu_R^2) H^\star_{n,\gamma}(\mathbf{q}_T, \mu_R^2).$$

The RG–invariant solution (for $n \neq 0$)**:**

$$G(\mathbf{l}_{T1},\mathbf{l}_{T2},Y) \;=\; \sum_n \int \frac{d\gamma}{2\pi i}\,\exp\left[\bar{\alpha}_s(\mu_R^2)\,\chi(n,\gamma)\,Y\right] H_{n,\gamma}(\mathbf{l}_{T1},\mu_R^2)\,H^{\star}_{n,\gamma}(\mathbf{l}_{T2},\mu_R^2).$$

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- The LO solution[I. Balitsky, L. Lipatov '78]:

$$\chi^{(0)}(n,\gamma) \;=\; 2\psi(1) - 2\,\mathrm{Re}\,\psi\left(\frac{n}{2}+\gamma\right),$$

$$f^{(0)}_{n,\gamma}(\mathbf{l}_T) \;=\; \frac{1}{\sqrt{\pi}}\,\mathbf{l}_T^{2(\gamma-1)}\, e^{in\phi_l}.$$

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- The NLO solution[G. Chirilli, Y. Kovchegov '13, 14]:

$$\chi^{(1)}(n,\gamma) = \frac{1}{2}\frac{\beta_0}{4\pi}\frac{\partial \chi^{(0)}(n,\gamma)}{\partial\gamma} + \frac{\delta(n,\gamma)}{4} = \chi^{(1)}(n, 1-\gamma),$$

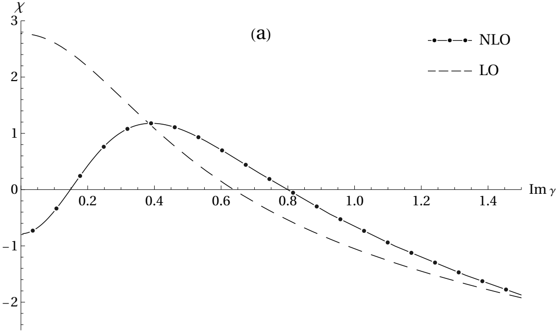
$$f^{(1)}_{n,\gamma}(\mathbf{l}_T, \mu_R^2) = \sum_m c_m(n,\gamma) \ln^m\left(\frac{\mathbf{l}_T^2}{\mu_R^2}\right) f^{(0)}_{n,\gamma}(\mathbf{l}_T),$$

see $\delta(n,\gamma)$ in[A. Kotikov, L. Lipatov '00], coefficients:

$$c_0 = 0, \quad c_1(n,\gamma) = \partial_\gamma c_2(n,\gamma), \quad c_2(n,\gamma) = \frac{1}{2}\frac{\beta_0}{4\pi}\frac{1}{\partial_\gamma \ln \chi^{(0)}(n,\gamma)}, \quad c_{m>2} = 0.$$

NOTE: the NLO eigenfunctions should be treated as a distributions due to pole of the c_2 at $\gamma = 1/2$.

The characteristic function



The NLO characteristic function for $\bar{\alpha}_s = 0.2$:

- Two complex-conjugate saddle points;
- The *"intercept"* is negative at $\gamma = 1/2$.

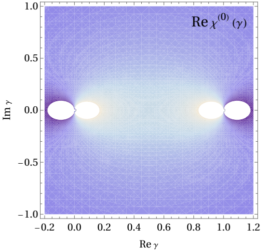
The characteristic function

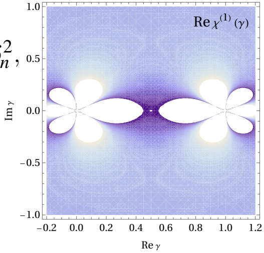
A (anti-) collinear poles:

$$\chi^{(l)}(n,\gamma) = \sum_{k=1}^{2l+1} \kappa_{l,k}\, d_0^{-k}\, \bar{\alpha}_s^l(\mu_R^2) + \mathcal{O}(\gamma) + \{\gamma \to 1-\gamma\},$$

where $d_m = d_m(n,\gamma) = m+\gamma+n/2$ and

$$\begin{aligned}
\kappa_{0,1} &= 1,\\
\kappa_{1,1} &= \mathscr{S} - \frac{\pi^2}{24} + \frac{\beta_0}{4N_c}\,\mathrm{H}_n + \frac{1}{8}\left(\psi'\left(\frac{n+1}{2}\right) - \psi'\left(\frac{n+2}{2}\right)\right)\\
&+ \frac{1}{2}\,\psi'(n+1) - \frac{1}{36}\left(67+13\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^0 - \frac{47}{1800}\left(1+\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^2,\\
-\kappa_{1,2} &= \frac{\beta_0}{8N_c} + \frac{1}{2}\,\mathrm{H}_n + \frac{1}{12}\left(11+2\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^0 + \frac{1}{60}\left(1+\frac{N_\mathrm{f}}{N_c^3}\right)\delta_n^2,\\
\kappa_{1,3} &= \frac{1}{24}\left(1+\frac{N_\mathrm{f}}{N_c^3}\right),
\end{aligned}$$





where $\mathscr{S} = (4-\pi^2+5\beta_0/N_c)/12$, $\beta_0 = (11N_c - 2N_\mathrm{f})/3$, and $\mathrm{H}_n = \psi(n+1)-\psi(1)$.

Resummation of the collinear poles

The collinear-improvement (CI): $\chi(n,\gamma) + \Delta\chi(n,\gamma)$.

- Mellin transform of the Green's function:

$$g_\omega(n,\gamma) \sim \left(\frac{\mathbf{l}_{T1}^2}{\mathbf{l}_{T2}^2}\right)^{\gamma-1} \frac{e^{\omega Y}}{\omega - \bar{\alpha}_s \chi(n,\gamma)}, \quad Y = \ln\left(\frac{s}{s_0}\right),$$

where $s_0 = |\mathbf{l}_{T1}||\mathbf{l}_{T2}|$ (BFKL: $|\mathbf{l}_{T1}| \sim |\mathbf{l}_{T2}|$) and $s_0 = \mathbf{l}_{T1,2}^2$ (DGLAP: $\mathbf{l}_{T1,2}^2 \gg \mathbf{l}_{T2,1}^2$);

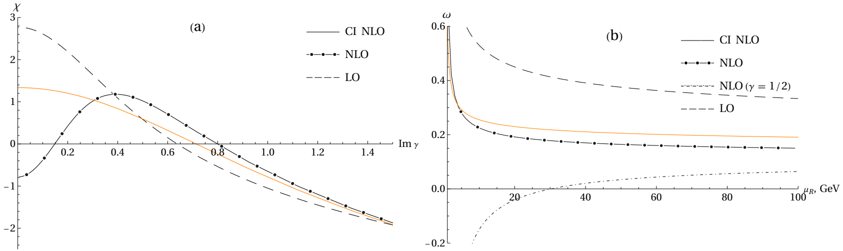
- The ω-shift scheme[G. Salam '98]:

$$d_0^{-k}\left(n, \gamma \pm \frac{1}{2}\bar{\alpha}_s \chi(n,\gamma)\right) \bar{\alpha}_s^l \supset \sum_{l'=l}^{\infty} d_0^{-k-2(l-l')} \bar{\alpha}_s^{l'};$$

- The "all-poles" approximation[A. Sabio-Vera '05; A. Sabio-Vera, F. Schwennsen '07]:

$$\begin{aligned}\bar{\alpha}_s \Delta\chi(n,\gamma) &= \sum_{m=0}^{\infty}\Bigg[\kappa_{1,2}\,\bar{\alpha}_s - d_m \\ &+ \sqrt{2\bar{\alpha}_s\left(\kappa_{0,1} + \kappa_{1,1}\,\bar{\alpha}_s\right) + \left(\kappa_{1,2}\,\bar{\alpha}_s - d_m\right)^2} \\ &- \sum_{l=0}^{1}\sum_{k=1}^{2l+1} \kappa_{l,k}\, d_m^{-k}\, \bar{\alpha}_s^{l+1}\Bigg] + \{\gamma \to 1-\gamma\};\end{aligned}$$

- Doesn't break the diagonalization of the kernel: $\Delta\chi \sim \mathscr{O}(\bar{\alpha}_s^3)$.



The CI NLO:

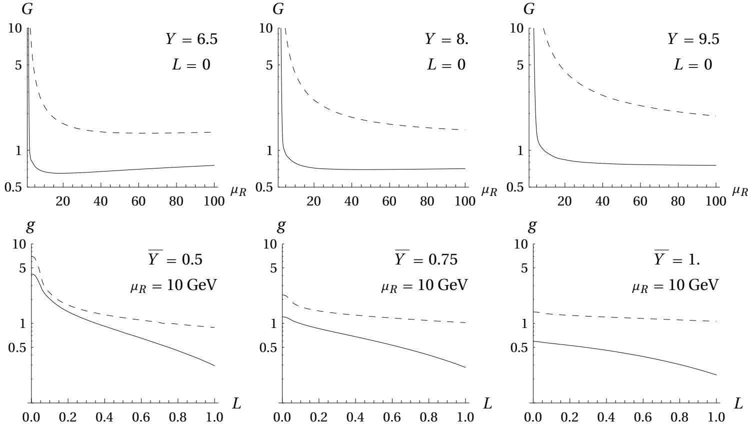
- Only one saddle point located at $\gamma = 1/2$;
- The LO definition of the intercept is preserved at the NLO:

$$\omega = \bar{\alpha}_s(\mu_R^2)\,\chi(0, 1/2)$$

- The ω is approximately scale-independent at relatively large μ_R.

Green's function vs. μ_R & L

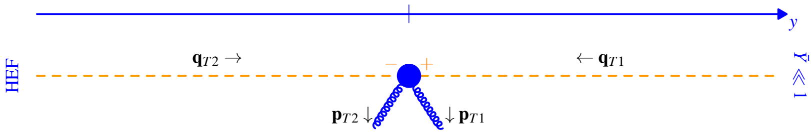
The region of applicability: $\bar{Y} = \bar{\alpha}_s(\mu_R^2)\, Y \sim 1$.



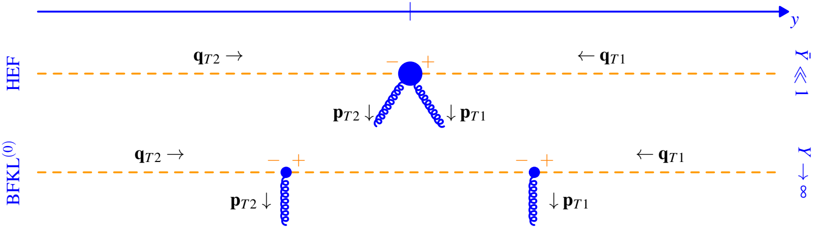
$L = \ln\left(|\mathbf{l}_{T1}|/|\mathbf{l}_{T2}|\right)$

Mueller–Navelet dijet production

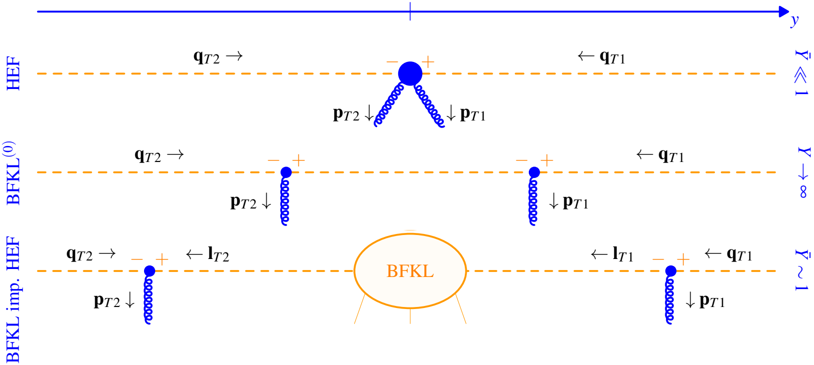
Matching scheme



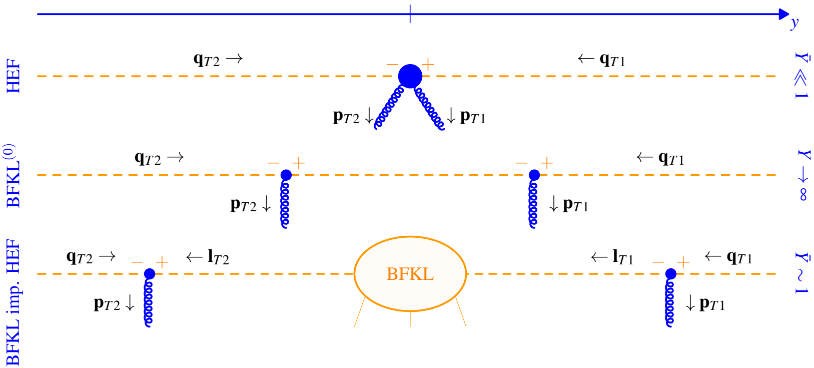
Matching scheme



Matching scheme



Matching scheme



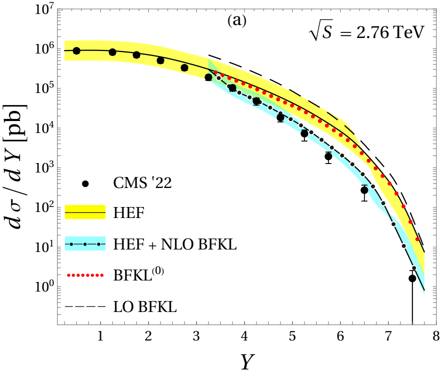
The matching scheme:

$$H_{ij}^{(\text{HEF+BFKL})} = H_{ij}^{(\text{HEF})} - H_{ij}^{(\text{BFKL},0)} + H_{ij}^{(\text{BFKL})},$$

where

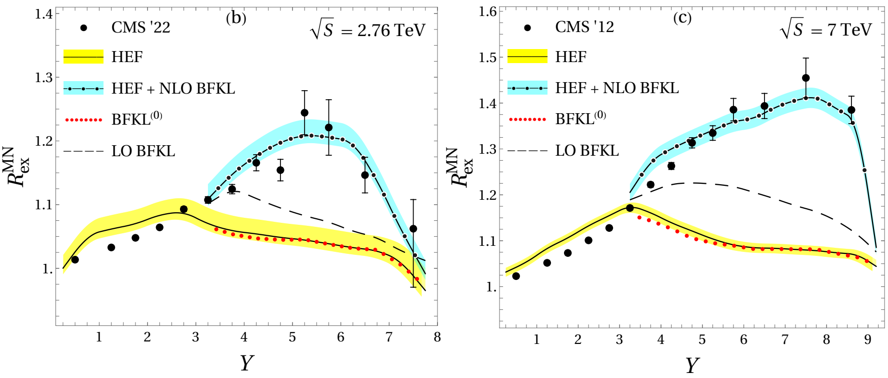
- $H_{ij}^{(\text{HEF})} - H_{ij}^{(\text{BFKL},0)} = 0$ *(w.r.t. NLP in z)* for $Y \to \infty$;
- $H_{ij}^{(\text{BFKL})} - H_{ij}^{(\text{BFKL},0)} = 0$ for $\bar{Y} \ll 1$.

Results



- The HEF describes the data well at low $Y < 2 - 3$;
- The HEF gets subtracted by the BFKL$^{(0)}$ starting from $Y > 3 - 3.5$;
- The NLO BFKL-improved HEF is in agreement with the data at large $Y > 3.5$;
- See comparison with [A. Egorov, V. Kim '23].

Results



- The HEF describes the data well at low $Y < 2-3$;
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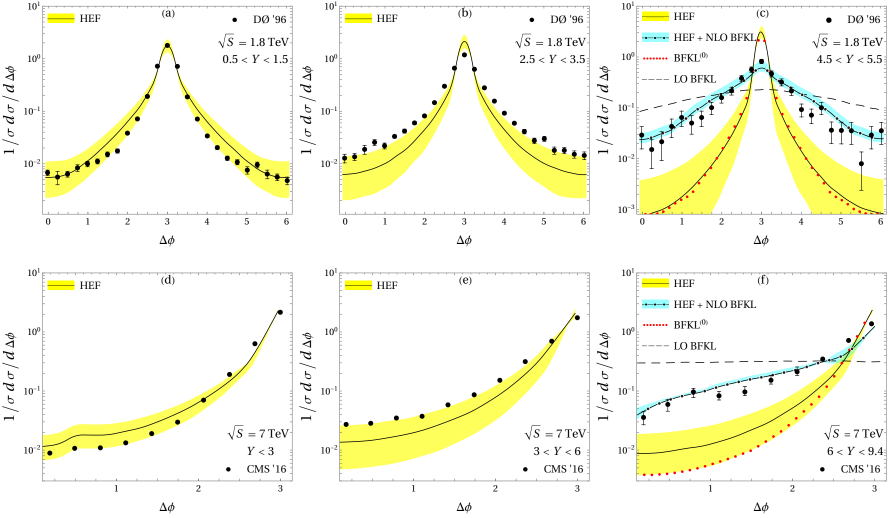
Results



- The HEF describes the data well at low Y **(due to NLL Sudakov resummation)**;
- The NLO BFKL-improved HEF is in agreement with the data at large Y;
- See comparison with [A. Sabio-Vera, F. Schwennsen '07; B. Duclou´ *et.al.* '14].

Conclusions & outlook

- The HEF with the NLL BFKL resummation is pushed to the MN dijets production;
- The basis of the NLO eigenfunctions together with the collinear improvement provides the RG-invariant NLL BFKL Green's function without any pathologies;
- Both, the HEF and NLL BFKL, are crucial for the uniform description of the data across all values of the rapidity difference;
- The matching scheme interpolates predictions between the two descriptions.

Outlook:

- NLO HEF [M. Nefedov, A. Hameren '25];
- Resummation of the transverse logarithms [A. Kovner, M. Lublinsky, V. Skokov, Z. Zhao '23];
- Sub-eikonal corrections [I. Balitsky, A. Tarasov '15; G. Chirilli '21; M. Nefedov '21].

Thank you for your attention!

NLO characteristic function

The NLO characteristic function:

$$\chi^{(1)}(n,\gamma) = \mathscr{S}\chi^{(0)}(n,\gamma) + \frac{3}{2}\zeta(3) - \frac{\beta_0}{8N_c}\left(\chi^{(0)}(n,\gamma)\right)^2$$
$$-\frac{\pi^2\cos(\pi\gamma)}{4(1-2\gamma)\sin^2(\pi\gamma)}\left[\left(3+\left(1+\frac{N_\mathrm{f}}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)}\right)\delta_n^0 - \left(1+\frac{N_\mathrm{f}}{N_c^3}\right)\frac{\gamma(1-\gamma)}{2(1+2\gamma)(3-2\gamma)}\delta_n^2\right]$$
$$+\frac{1}{4}\left[\psi''\left(\gamma+\frac{n}{2}\right)+\psi''\left(1-\gamma+\frac{n}{2}\right)-2\left(\phi(n,\gamma)+\phi(n,1-\gamma)\right)\right],$$

where $\mathscr{S} = (4-\pi^2+5\beta_0/N_c)/12$ and $\beta_0 = (11N_c - 2N_\mathrm{f})/3$. The function ϕ is:

$$\begin{aligned}\phi(n,\gamma) &= \sum_{m=0}^{\infty}\frac{(-1)^{m+1}}{d_m}\Bigg(\psi'(m+n+1)-\psi'(m+1)\\ &+ (-1)^{m+1}\left(\beta'(m+n+1)-\beta'(m+1)\right)+\frac{\psi(m+1)-\psi(m+n+1)}{d_m}\Bigg),\end{aligned}$$

where

$$\beta(z) = \frac{1}{2}\left(\psi\left(\frac{z+1}{2}\right)-\psi\left(\frac{z}{2}\right)\right).$$