

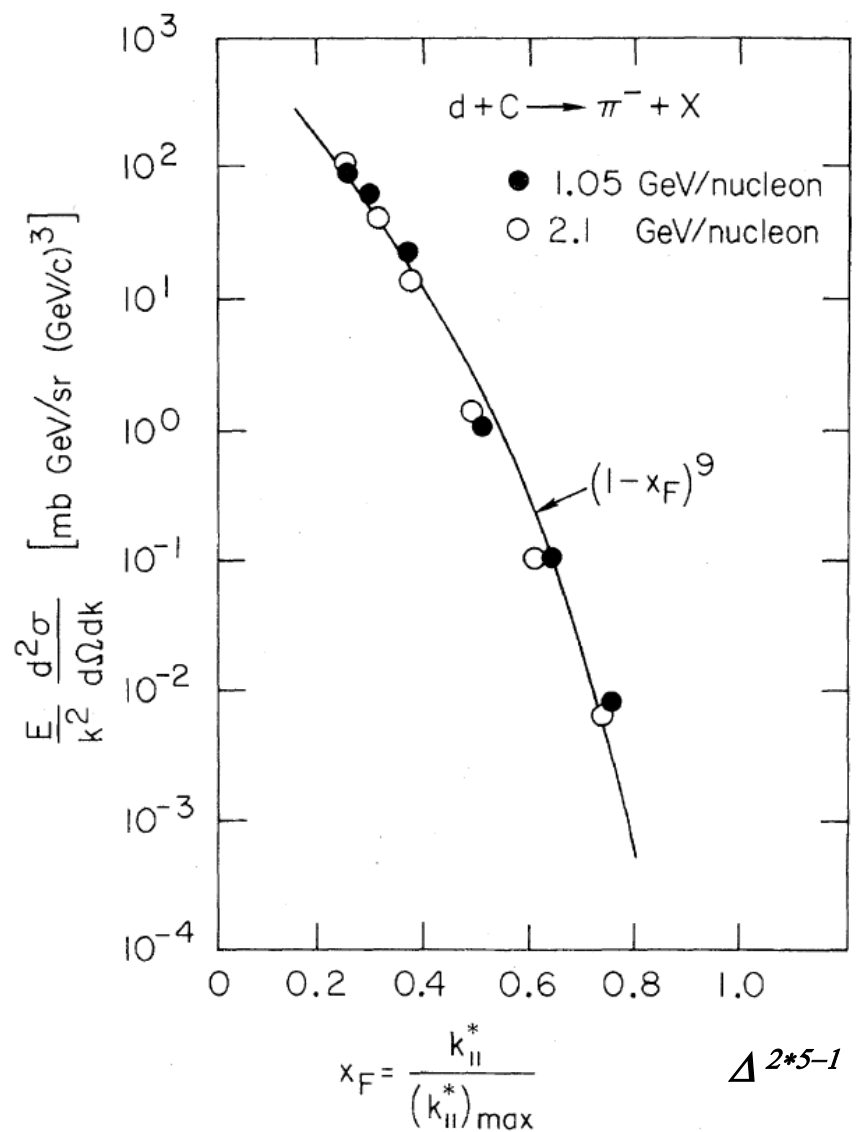
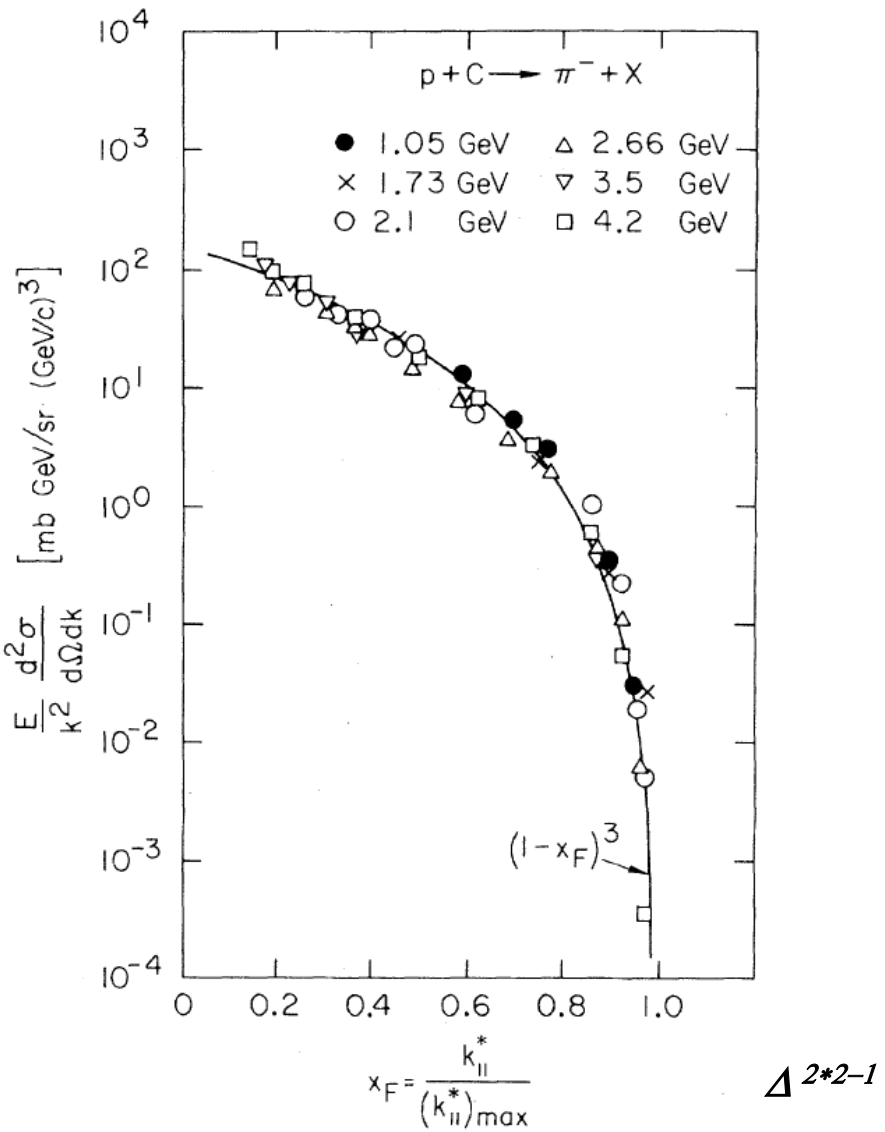
Quark counting rules for the inclusive cross-section for the production of particles with large transverse momenta at midrapidities

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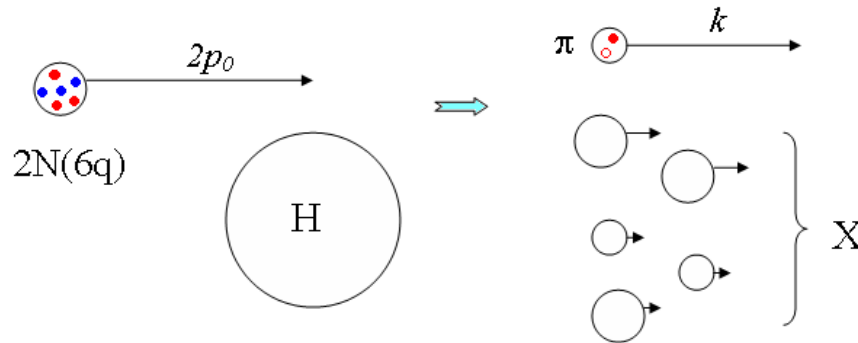
Quark counting rules in fragmentation region

Cumulative production at $|t| \ll s$
due to flucton fragmentation



Threshold behaviour of **inclusive cross sections** (quark counting rules) at $|t| \ll s$.
The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Limiting fragmentation of light nuclei. Quark counting rules.



$1 < x < 2$ - the cumulative region ($1 < x < f$ - for the fN flucton)

Theoretical description near upper threshold: for $2N(6q)$ flucton $k \rightarrow 2p_0$, $x = k/p_0 \rightarrow 2$
(Limiting fragmentation of a nucleus)

Quark counting rules for inclusive cross-section in the fragmentation region: $I \sim \Delta^{2p-1}$

Δ – the deviation of x from its maximal value f , $\Delta = f - x$

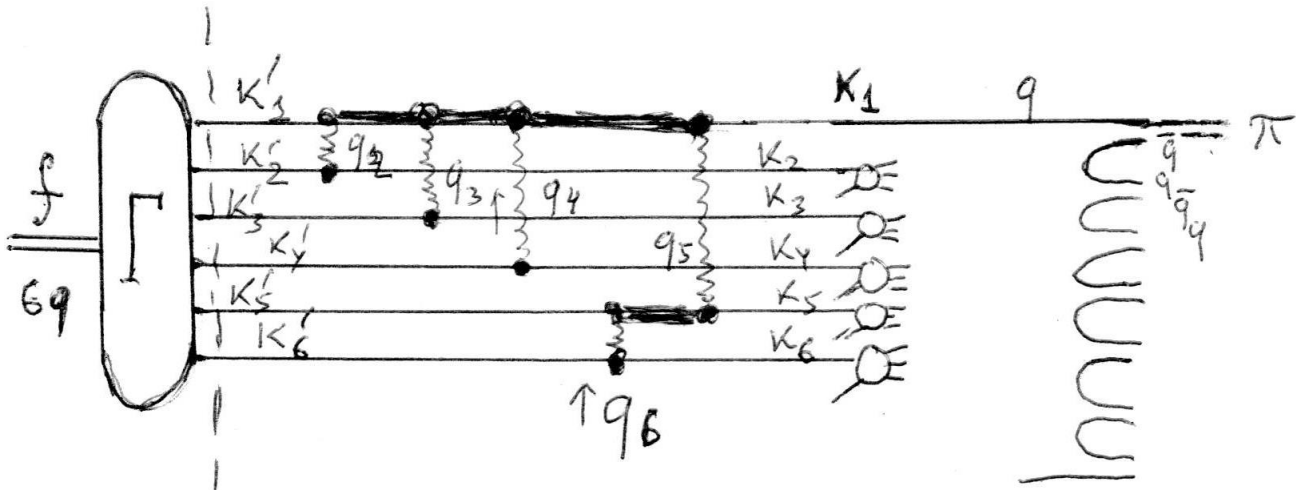
p – the number of “donors”, stopped quarks, $p = n - 1$

n – the number of constituents.

For $2N(6q)$ flucton $f = 2$, $n = 6$, $p = 5$, then $I \sim (2-x)^{2 \cdot 5 - 1} = (2-x)^9 = \Delta^9$

Almost no dependence on collision energy p_0 at $p_0 \gg m_N$.

[G.A. Leksin. Nuclear scaling. 1977]



f – number of nucleons which formed flucton
 n – number of quarks in flucton
 $p=n-1$ – number of "donors", stopped quarks

$\Gamma = \Gamma(k'_{+i}, k'_{\perp i})$ then after integration over all k'_{-i} we get:

$\Gamma(k'_{+i}, k'_{\perp i}) \rightarrow \Psi(k'_{+i}, k'_{\perp i})$ – light cone parton wave function of flucton

In all rest parts of the diagram we can put: $k'_{+i} = \frac{f p_+}{n} = \frac{f}{n} p_+ = \frac{1}{3} p_+$

Then we get: $\int \Psi(k'_{-i}, k'_{\perp i}) \delta(\sum_{i=1}^n k'_{+i} - f p_+) \delta^2(\sum_{i=1}^n k'_{\perp i}) \prod_{i=1}^n \frac{dk'_{+i}}{2k'_{+i}} d^2 k'_{\perp i} \sim \bar{\Psi}_{cms}(\{r_i - r_j = 0\})$

Contribution of $(n-1)$ "Gluon" exchanges and $(n-2)$ internal quark propagators limits to constant, when at $x_1 \Rightarrow f$ all $x_2, \dots, x_n \Rightarrow 0$

The main contribution comes from propagators of stopped quarks k_2, \dots, k_n , which defined the longitudinal and transverse momentum dependence.

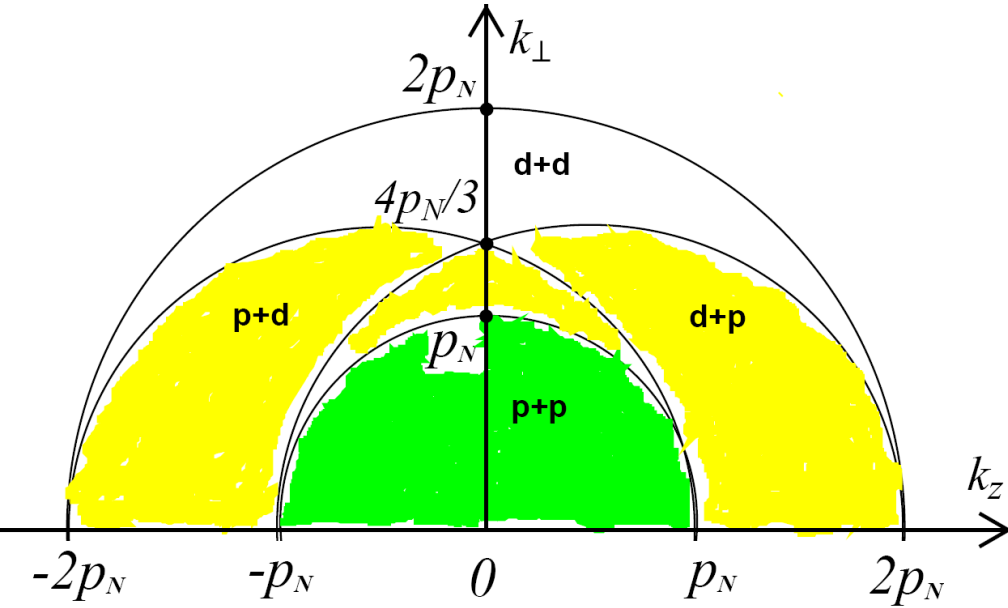
Scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x, k_\perp) \equiv \frac{k_0 d^3 \sigma_\pi}{d^3 \mathbf{k}} = C s^0 (f - x)^{2p-1} \Phi_p \left(\frac{k_\perp}{m_q} \right)$$

Central rapidity region

Cumulative production at $|t| \sim s$
due to flucton-flucton interaction

Cumulative regions in dd and pd collisions



$$p_N \gg m_N \quad p_N \approx \sqrt{s}/2$$

$$\frac{k_{\perp}}{p_N} = \frac{\sqrt{A_1 A_2}}{(A_1 + A_2)/2} \sqrt{\left(A_1 - \frac{k_z}{p_N}\right) \left(A_2 + \frac{k_z}{p_N}\right)}$$

We expect

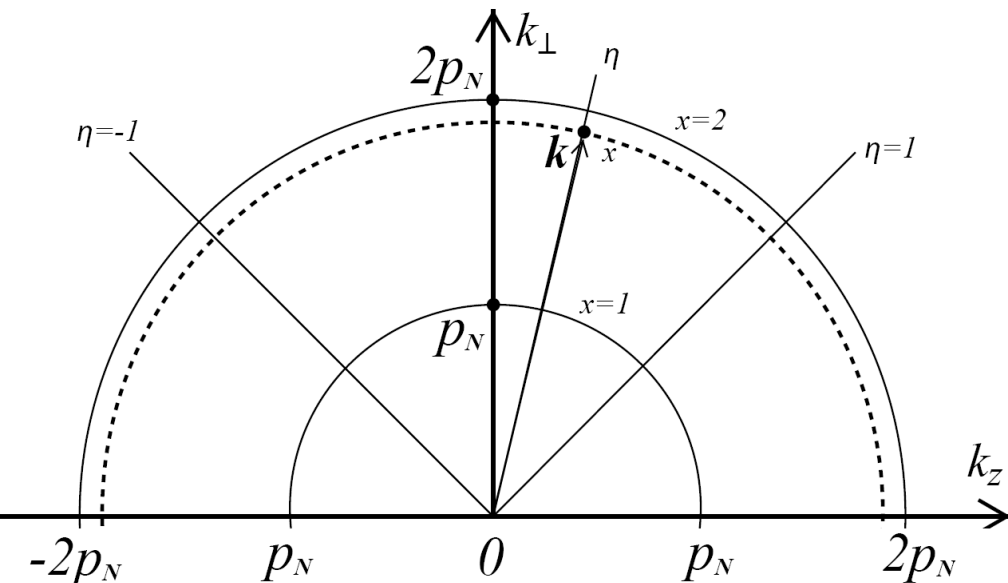
for the inclusive cross-section
of the production of particles with large
transverse momenta at midrapidities:

$$\frac{k_0 d^3 \sigma}{d^3 \mathbf{k}} = f(x, \eta) = C s^{-q_1} (f - x)^{q_2} F(\eta)$$

**Quark Counting Rules with
two asymptotic parameters:**
 $s \gg m^2$ and $(f-x)$

x - cumulative number

$\eta = -\ln \operatorname{tg} \frac{\theta^*}{2}$ - pseudorapidity



Flucton-flucton interaction in dd collisions

- It can be studied **only in new cumulative region of large transverse momenta in mid-rapidity region at NICA** (not in the traditional cumulative region of fragmentation of one of the nuclei).
- There are **no additional interactions in dd collision**, compared with collisions of heavier nuclei, if both deuterons are in flucton configuration at the moment of collision. =>
- The possibility to register, in addition to the cumulative particle, the **particles formed from fragmentation of the flucton residue**.
- **Higher frequency of dd collisions** that can be recorded by the SPD, compared to the slower MPD (important for a registration of rare cumulative events).
- The studies in new cumulative region becomes **possible due to the moderate energy of the NICA** collider and is completely impossible at ultrahigh energies of the RHIC and LHC.

Quark counting rules for *elastic and quasi elastic reactions with nuclei*

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

$s \rightarrow \infty$, t/s fixed

$$(d\sigma/dt)_{\pi p \rightarrow \pi p} \sim s^{-8}, (d\sigma/dt)_{pp \rightarrow pp} \sim s^{-10}, (d\sigma/dt)_{\gamma p \rightarrow \pi p} \sim s^{-7}, (d\sigma/dt)_{\gamma p \rightarrow \gamma p} \sim s^{-6}$$

$$\sim s^{-n} \quad A+B \rightarrow C+D \quad n=n_A+n_B+n_C+n_D-2 \quad n_p=3 \quad n_\pi=2 \quad n_\gamma=1$$

$$\frac{d\sigma}{dt}(A+B \rightarrow C+D) \rightarrow \frac{1}{t^{N-2}} f(t/s) \quad N=n_A+n_B+n_C+n_D$$

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

the deuteron break-up by a photon, $\gamma + D \rightarrow p + n$

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \quad \frac{t}{s} = \text{const}, \quad K-2=1+6+3+3-2=11$$

For light nuclei:

Yu.N. Uzikov, Indication of Asymptotic Scaling in the Reactions
 $dd \rightarrow p^3\text{H}$, $dd \rightarrow n^3\text{He}$ and $pd \rightarrow pd$, JETP Letters 81 (2005) 303.

$$\sim s^{-22} \quad (6+6+3+9-2=22) \quad \text{and} \quad \sim s^{-16} \quad (3+6+3+6-2=16)$$

The same is valid for formfactors:

Brodsky S., Chertok B.T., *Phys.Rev. D14 (1976) 3003*

$$F_n(q^2) \sim \left(\frac{1}{q^2}\right)^{n-1}$$

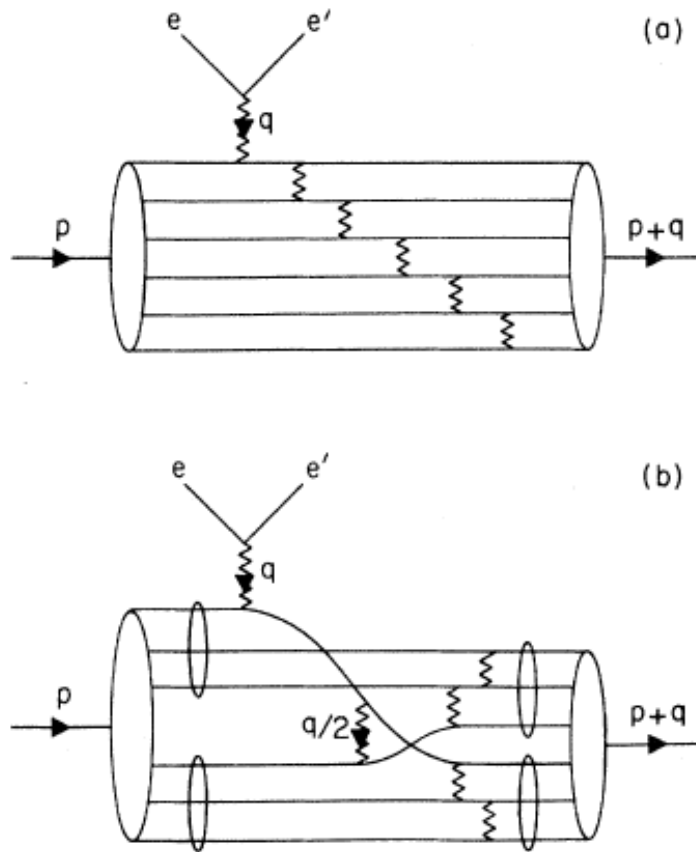


FIG. 2. Two possible quark-constituent views of e - D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

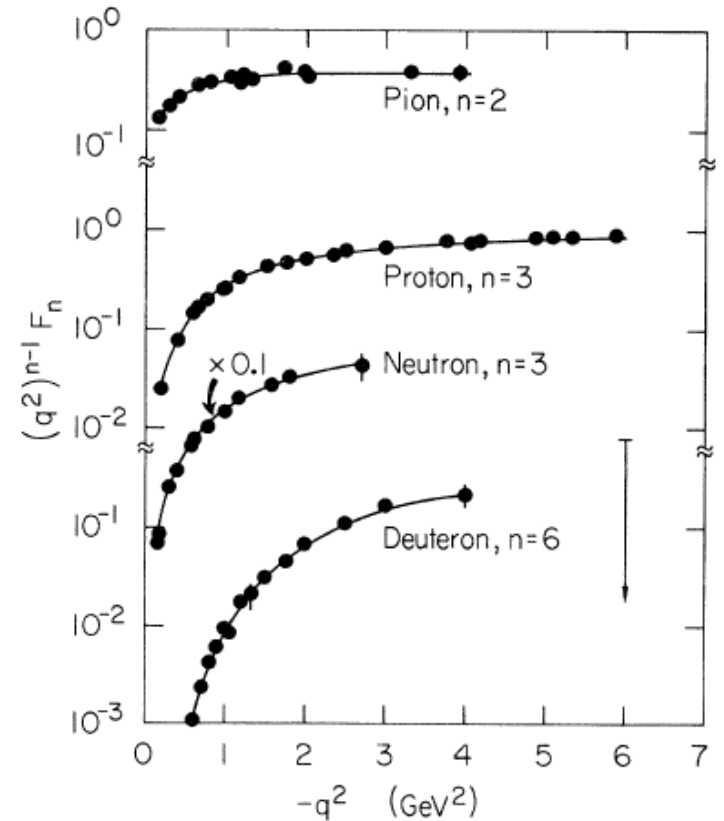


FIG. 1. Elastic electromagnetic form factors of hadrons for large spacelike q^2 in terms of the dimensional-scaling quark model. The curves simply connect the data points. (The neutron data have been multiplied by 0.1.)

Some details of formfactor calculations (compare to our slide 9)

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309

Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

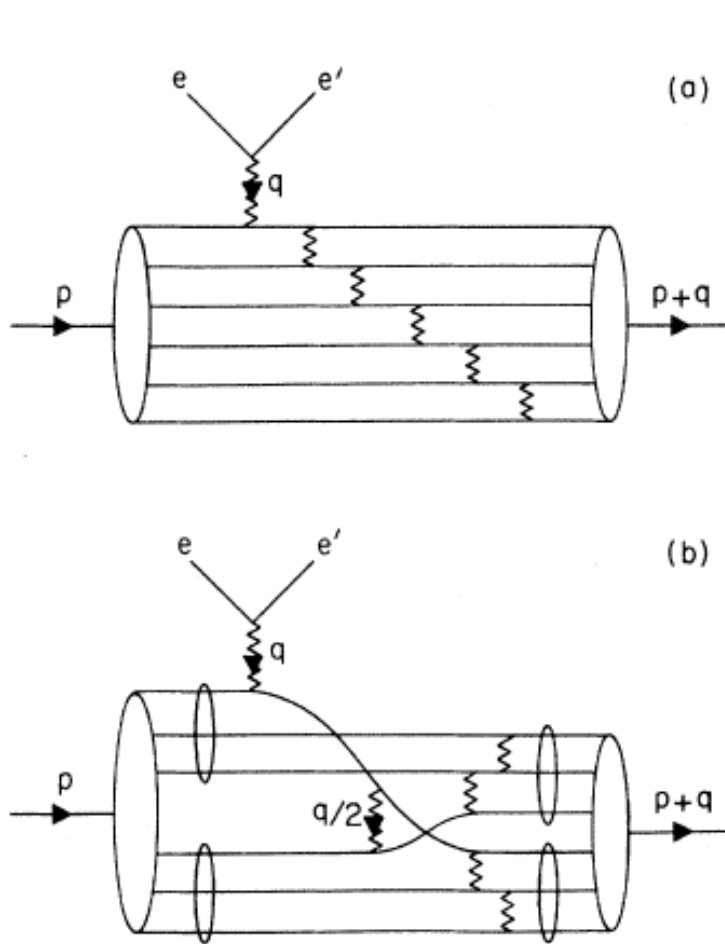


FIG. 2. Two possible quark-constituent views of e - D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

$$\psi_n(0) \equiv \int \prod_{j=1}^{n-1} d^3 \vec{k}_j \psi(\vec{k}_j)$$

Hence, e interacts with d , when d is in the flucton configuration.

$$F_n(\vec{q}^2) \sim \left[\frac{2m}{\vec{q}^2} V(\vec{q}^2) \right]^{n-1} \psi_n^2(0)$$

In the case of quantum electrodynamics, and in fact any renormalizable theory, we have effectively (modulo powers of $\log q^2$ from finite orders in perturbation theory)

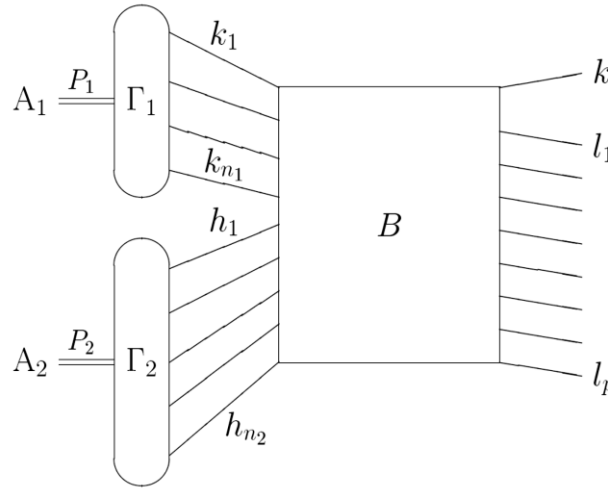
$$V(q^2) \sim \frac{e^2}{q^2} \left[1 + O\left(\frac{q^2}{m^2}\right) \right],$$

i.e., $V(q^2)$ becomes constant in the relativistic domain and

for large q^2 the gluon propagator is always compensated by its couplings to the quark currents

$$F_n(q^2) \sim \left(\frac{1}{q^2} \right)^{n-1}$$

Quark counting rules for the **inclusive cross sections** of particle production in **new cumulative region of large transverse momenta at mid-rapidities** (the decrease with both the initial energy **s** and the cumulative number **x**)

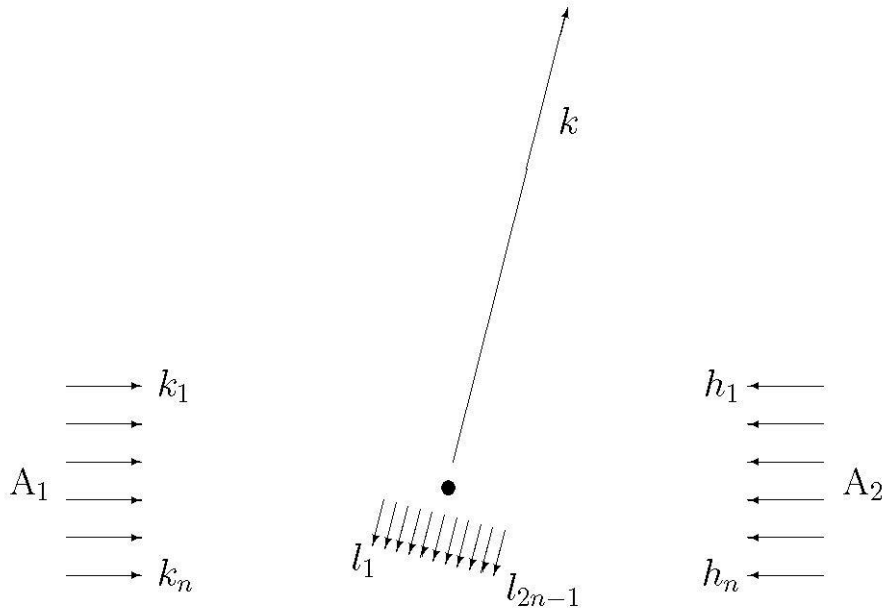


$$p = n_1 + n_2 - 1$$

To evaluate this behavior and find **asymptotes at $s \gg m$ and $(2-x) \ll 1$** we need to generalize the **quark counting rules**, known now only for
a) the **inclusive cross sections** in the fragmentation region ($|t| \ll s$) and
b) the **elastic and quasielastic** cross sections in the high p_T region ($|t| \sim s$),
to the case of **inclusive cross sections** in the high p_T region ($|t| \sim s$).

$$I(x) \equiv (2\pi)^3 2k_0 \frac{d^3 \sigma}{d^3 \mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p - \frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}$$

Kinematics



$d+d \rightarrow \pi+X$ at quark level
($A_1=A_2=A=2, n=6$)

$$p_N = P_A/A \quad p_N \gg m_N$$

Initial state:

$$k_i \sim P_A/n = p_N/3 \quad n = 3A$$

$$h_i \sim -P_A/n = -p_N/3$$

Final state:

$$k \sim P_A = A p_N = n p_N/3$$

$$l_i \sim -P_A/(2n-1) = -\frac{n}{3(2n-1)} p_N$$

First small parameter: $\frac{m_N}{p_N} = \frac{2m_N}{\sqrt{s_{NN}}} \ll 1$

Second small parameter: $1 - \frac{k}{k_{max}} \ll 1$

$$k \rightarrow k_{max} \Rightarrow M_X^2 = (\sum_{i=1}^{2n-1} l_i)^2 \rightarrow M_{Xmin}^2$$

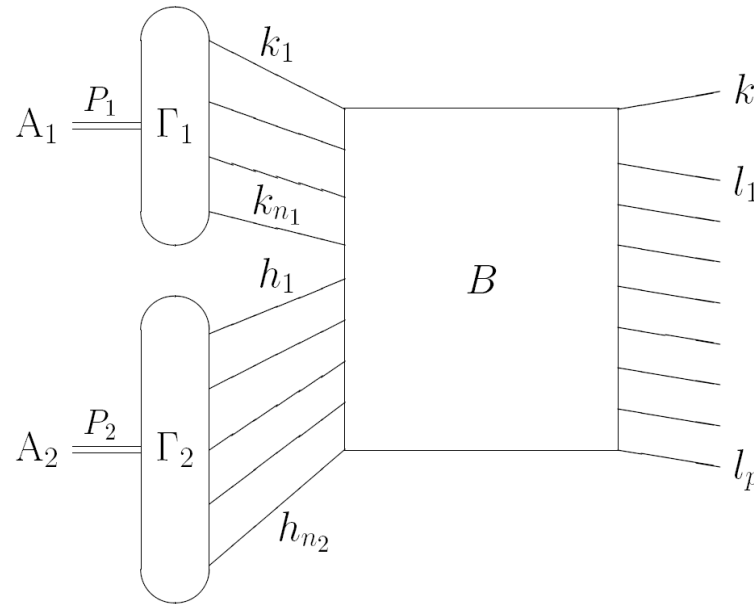
$$l_1 = l_2 = \dots = l_{2n-1} = -\frac{k_{max}}{2n-1}$$

V. Vechernin, S. Yurchenko
Int.J.Mod.Phys. E, 2441022,
DOI: 10.1142/S0218301324410222

$$S = S_{NN}$$

Amplitude (\mathcal{T})

$\mathcal{T} =$



$$p = n_1 + n_2 - 1$$

$$I(\mathbf{k}) \equiv (2\pi)^3 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{1}{J} \int |\mathcal{T}|^2 d\tau_p,$$

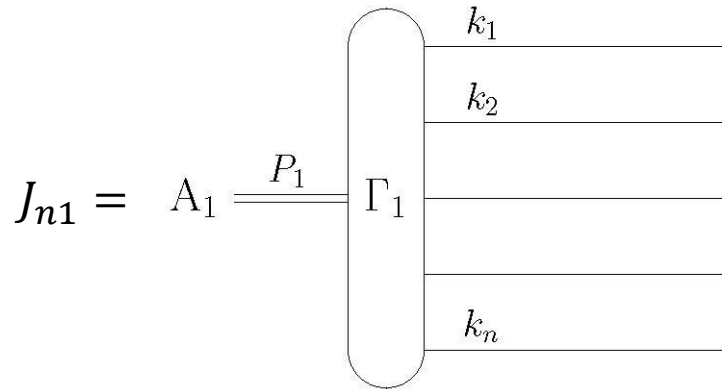
$$J = 2A_1 A_2 \sqrt{s(s - 4m_N^2)}$$

$$d\tau_p \equiv (2\pi)^4 \delta^4(P_1 + P_2 - k - \sum_{i=1}^p l_i) \prod_{i=1}^p \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

$$s = s_{NN}$$

Light-cone partonic wave function

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519



$$x_i \equiv \frac{k_{i+}}{p_{1+}}, \quad k_{i+} \equiv \frac{k_{i0} + k_{iz}}{\sqrt{2}}$$

$$p_1 = P_1/A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2 + \mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

$$J_{n_1} = \int \psi_1(x_i, \mathbf{k}_{i\perp}) 2\delta\left(\sum_{i=1}^{n_1} x_i - A_1\right) (2\pi)^3 \delta^{(2)}\left(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}\right) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2 \mathbf{k}_{i\perp}}{(2\pi)^3}$$

M.A. Braun, V.V. Vechnin,
Nucl.Phys. B427 (1994) 614

$$J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)$$

$$J_{n_1} = \frac{C_1}{m^{(n_1-1)/2} R_1^{3(n_1-1)/2}}$$

m - mass of the constituent quark

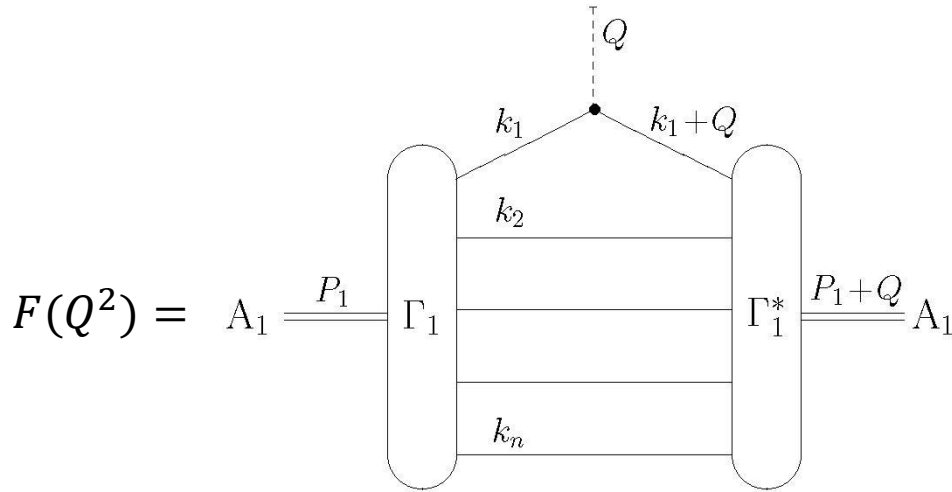
R_1 - size of the system ($R_1 = R_A$ or R_N)

C_1 - dimensionless constant, independent of the dimensional parameters of the model

$$J_n^2 \sim |\psi(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)|^2 \sim \frac{1}{V^{n-1}} \sim \frac{1}{R^{3(n-1)/2}}$$

for a wave function with
one dimensional parameter

Normalization of the wave function (Γ)



$$Q^2 \rightarrow 0 \Rightarrow F(Q^2) \rightarrow 1$$

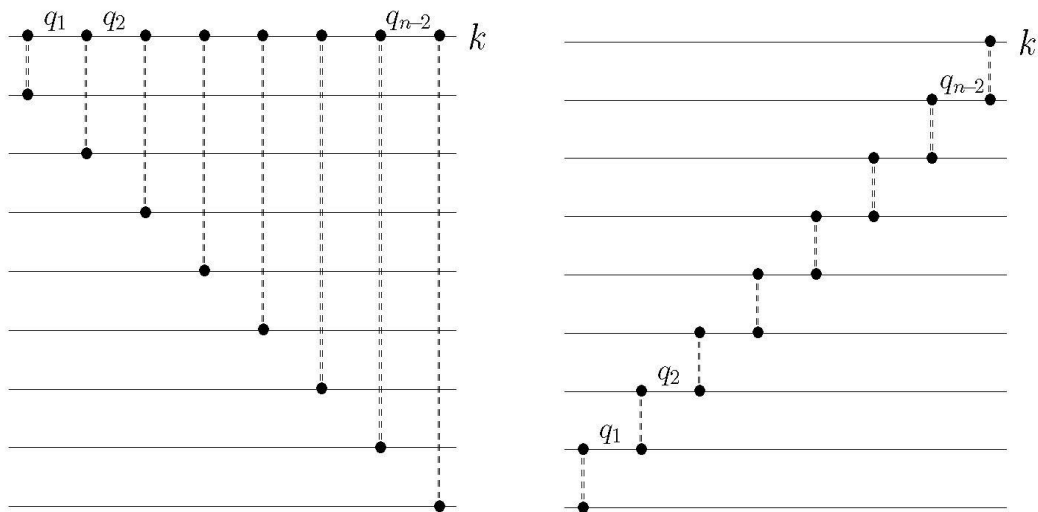
**M.A. Braun, V.V. Vechernin,
Nucl.Phys. B427 (1994) 614**

$$\int |\psi_1(x_i, \mathbf{k}_{i\perp})|^2 2\delta\left(\sum_{i=1}^{n_1} x_i - A_1\right) (2\pi)^3 \delta^{(2)}\left(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}\right) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3} = A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2 + \mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

Similarly for $\psi_2(y_i, \mathbf{h}_{i\perp})$ $y_i \equiv \frac{h_{i-}}{p_{2-}}$, $h_{i-} \equiv \frac{h_{i0} - h_{iz}}{\sqrt{2}}$ $p_2 = P_2/A_2$

Block of hard exchanges (B)



S.J. Brodsky, B.T. Chertok,
Phys.Rev. D 14 (1976) 3003

$$B = \frac{C_B}{s^{n-2}} = \frac{C_B}{s^{n_1+n_2-2}}$$

$$n = n_1 + n_2$$

$$p = n - 1$$

$$B(k_i, h_i; \mathbf{k}, \mathbf{l}_i) \approx B(P_1/n_1, P_2/n_2; \mathbf{k}_{max}, -\mathbf{k}_{max}/p)$$

$$T = \frac{C_1 C_2 C_B}{m^{(n-2)/2} R_1^{3(n_1-1)/2} R_2^{3(n_2-1)/2} s^{n-2}} = \frac{C_1 C_2 C_B}{m^{(n-2)/2} R^{3(n-2)/2} s^{n-2}}$$

$$I(\mathbf{k}) = \frac{1}{J} |T|^2 \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^p l_i) \prod_{i=1}^p \frac{d^{(3)}l_i}{2l_{i0}(2\pi)^3}$$

Calculation of Phase Volume

$$\tau_p = (2\pi)^{4-3p} \int \prod_{i=1}^p \frac{d^3 \mathbf{l}'_i}{2l_{i0}} \delta^{(3)}\left(\sum_{i=1}^p \mathbf{l}'_i\right) \times$$

$$\times \delta\left(\sum_{i=1}^p \left[\sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2} - \sqrt{(\mathbf{k}/p)^2 + m^2}\right] - \Delta\right)$$

$$\mathbf{l}'_i = -\mathbf{k}/p + \mathbf{l}_i$$

$$l_{i0} = \sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2}$$

$$p = n_1 + n_2 - 1$$

$$\Delta = A\sqrt{s} - \sqrt{k^2 + m^2} - \sqrt{k^2 + p^2 m^2}$$

$$k \rightarrow k_{max} \Rightarrow \Delta \rightarrow 0$$

$$\sqrt{k_{max}^2 + m^2} + \sqrt{k_{max}^2 + (pm)^2} = A\sqrt{s}$$

$$\tau_p = \frac{1}{2^p m^{p-1} p^{\frac{3}{2}}} \frac{\left(\frac{E_p}{2\pi} \Delta\right)^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$E_p \equiv \sqrt{k^2/p^2 + m^2}$$

Relation with Cumulative Number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)m]^2}.$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\Delta = (A - x)[\sqrt{s} + O(1/\sqrt{s})]$$

$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{\left[\frac{A}{\pi} s(A-x)\right]^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$p = p(A)$$

$$I(x) \equiv (2\pi)^3 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p - \frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}$$

two (!)

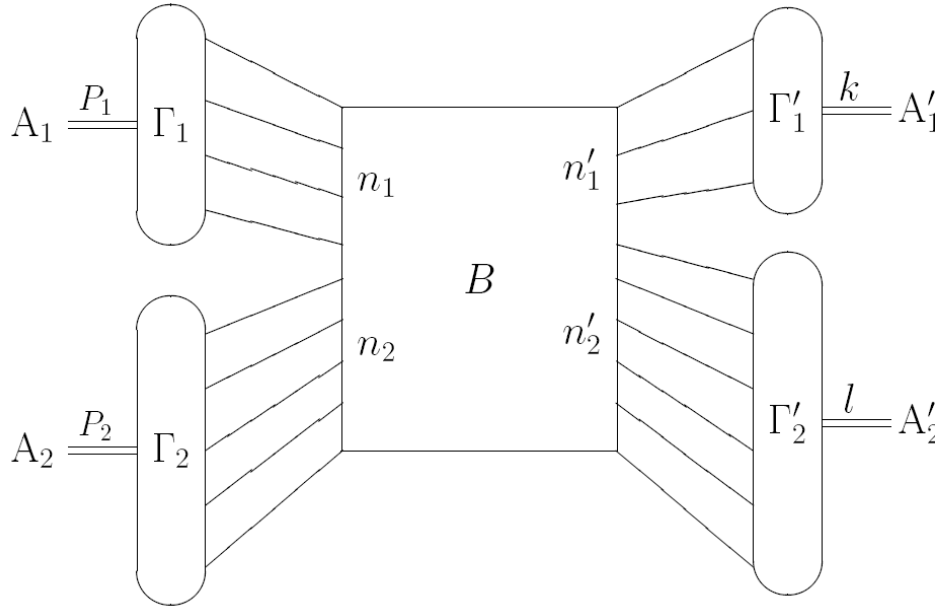
small parameters:

$$m/\sqrt{s} \ll 1$$

$$A - x \ll 1$$

Comparison with Quark Counting Rules for Quasi-Elastic Processes

one
small parameters:
 $m/\sqrt{s} \ll 1$



$$\frac{d\sigma}{dt} \sim \frac{1}{s^{n_1+n_2+n'_1+n'_2-2}}$$

**Matveev V.A., Muradyan R.M., Tavkhelidze A.N.,
Lett. Nuovo Cimento 7 (1973) 719**

**Brodsky S., Farrar G.,
Phys.Rev.Lett. 31 (1973) 1153**

**Brodsky S., Chertok B.T.,
Phys.Rev. D14 (1976) 3003**

Uzikov Yu. N., JETP Lett. 81 (2005) 303

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s_{A_1 A_2}^2} |T_{2 \rightarrow 2}|^2 = \frac{1}{16\pi A_1^2 A_2^2 s^2} |T_{2 \rightarrow 2}|^2$$

$$T_{2 \rightarrow 2} = J_{n_1} J_{n_2} B J_{n'_1} J_{n'_2}$$

$$\frac{d\sigma}{dt} = \frac{C'}{s^{2n-2} m^{2n-4} R_1^{3(n_1-1)} R_2^{3(n_2-1)} R_1'^{3(n'_1-1)} R_2'^{3(n'_2-1)}}$$

$$n = n_1 + n_2 = n'_1 + n'_2$$

Future studies

□ From pion to proton production ($AA \rightarrow \pi X \Rightarrow AA \rightarrow pX$)

1. leading cumulative diquark (including diquarks into scheme),

V. T. Kim, *Modern Phys. Lett. A* 3 (1988) 909

2. coalescence (recombination) of three cumulative quarks

M.A. Braun, V.V. Vechernin, *Nucl.Phys.B*92 (2001) 156;

Theor.Math.Phys 139, 766 (2004);

V.Vechernin, *AIP Conf.Proc.*1701 (2016) 060020

(three point B , can't to neglect all small initial momemta in B)

□ From partonic to hadronic final states

$M_X \sim (A - x)\sqrt{s}$, so at rather small NICA energies \Rightarrow

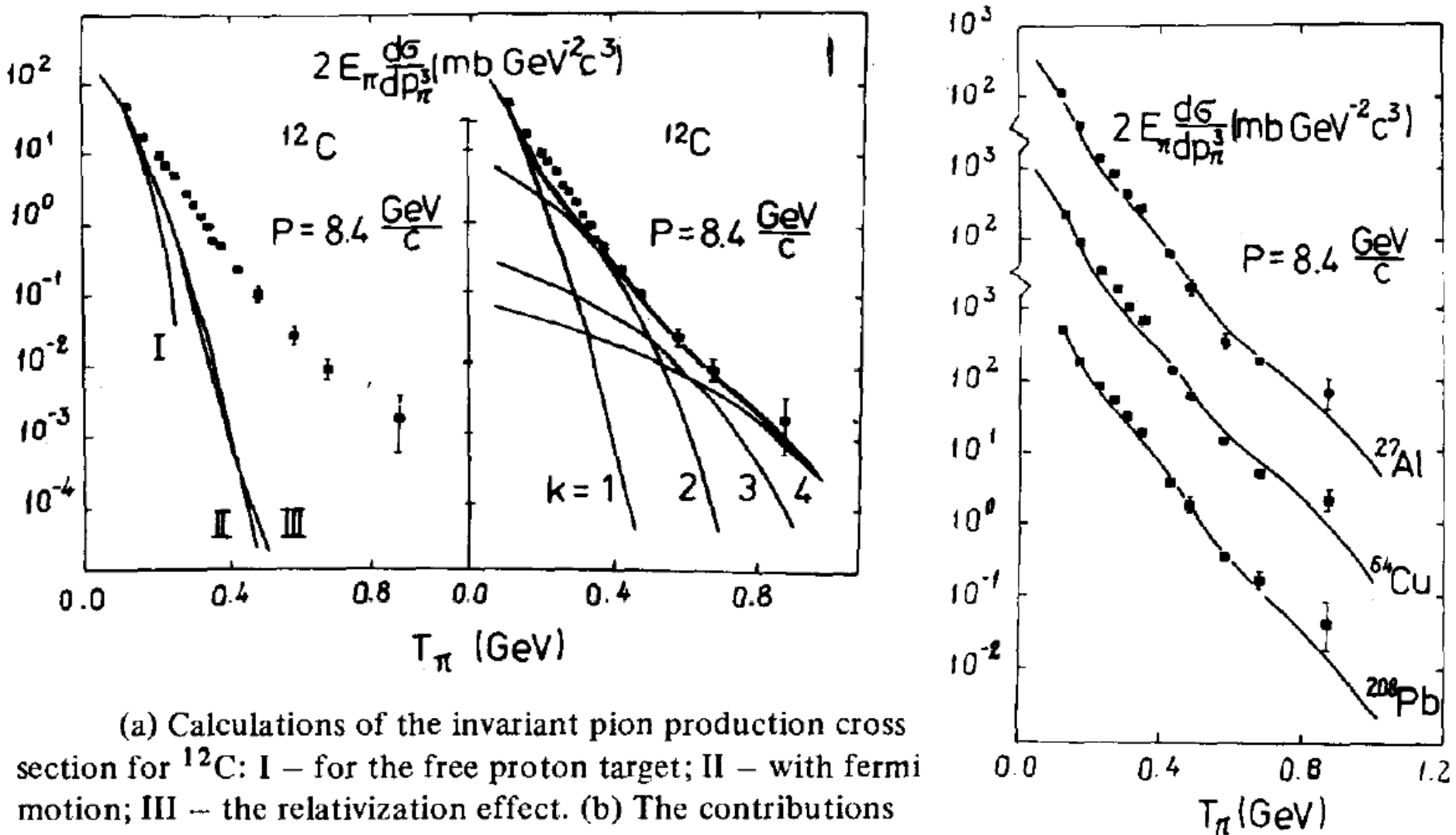
$dd \rightarrow \pi NNNN$ and $dd \rightarrow pNNN$

Summary

- ❖ The contribution from **the flucton-flucton interaction** into the process of pion production in AA collisions in **a new cumulative region of central rapidities and large transverse momenta** is analyzed, supposing that this process takes place through **the production of cumulative quark, which then fragments into a pion.**
- ❖ The **asymptotic behavior of the inclusive cross section at high initial energies near the kinematic boundary** of the process is calculated and **quark counting rules for the inclusive cross section in this region** are formulated.
- ❖ It is shown that according to the obtained new quark counting rules, **the decrease of the inclusive cross section with increasing initial energy turns out to be significantly slower** than for elastic and quasi-elastic processes. **At the same time, it contains an additional small factor depending on the deviation of the cumulative number x from its maximum possible value A .**
- ❖ The found **dependences of the inclusive cross section on the initial energy and cumulative number** can be used to describe the production of **pions with high transverse momentum in dd collisions in the SPD** experiment at the NICA collider.

The work was supported by the NICA grant (SPD-SPbGU)
“Study of multiquark fluctons in dd scattering at SPD”.

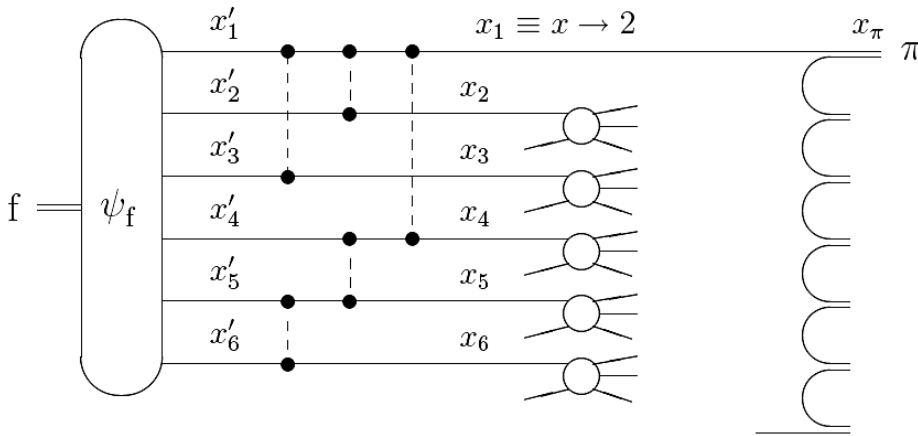
Backup slides



(a) Calculations of the invariant pion production cross section for ^{12}C : I – for the free proton target; II – with fermi motion; III – the relativization effect. (b) The contributions of separate fluctuons with mass $M_k = km_p$ where k is the order of cumulativity.

The experimental points from A.M. Baldin et al., Yad. Fiz. 18 (1973) 79.

Transverse momentum spectra of cumulative pions



- the cumulative pion production

k_{\perp} – dependence:

*M.A. Braun, V.V. Vechernin,
Phys.Atom.Nucl. **63**, 1831 (2000)*

$$\sigma_{pion}(x, k_{\perp}; p) = C(p) (x_{frag} - x)^{2p-1} f_p\left(\frac{k_{\perp}}{m}\right)$$

$$x < x_{frag}(p) = 1/3 + p/3$$

p – the number of “donors”, stopped quarks

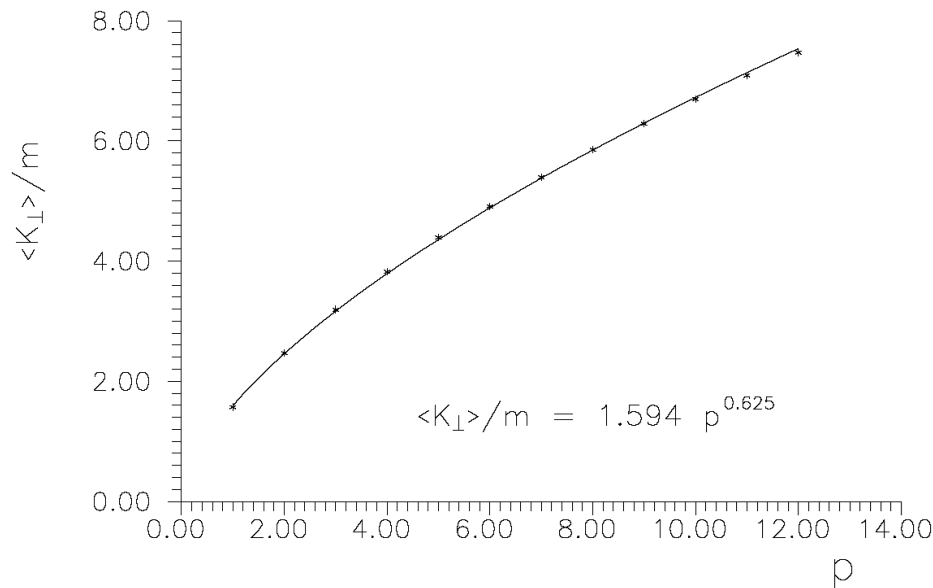
m – the constituent quark mass

$$f_p(t) = \frac{1}{\pi^p} \int \prod_{i=1}^p \frac{d^2 t_i}{(t_i^2 + 1)^2} (2\pi)^2 \delta^{(2)}\left(\sum_{i=1}^p t_i + t\right)$$

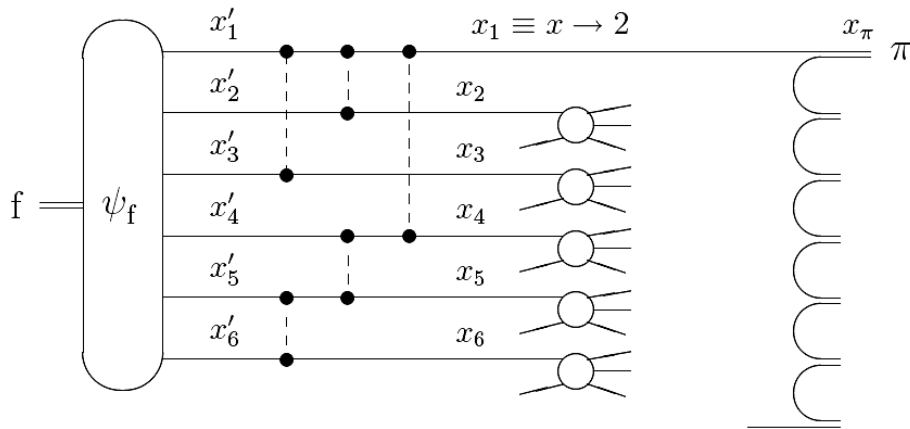
$$t = k_{\perp}/m, \quad t_i = k_{i\perp}/m$$

$$f_p(t) = 2\pi \int_0^{\infty} dz z J_0(tz) [z K_1(z)]^p$$

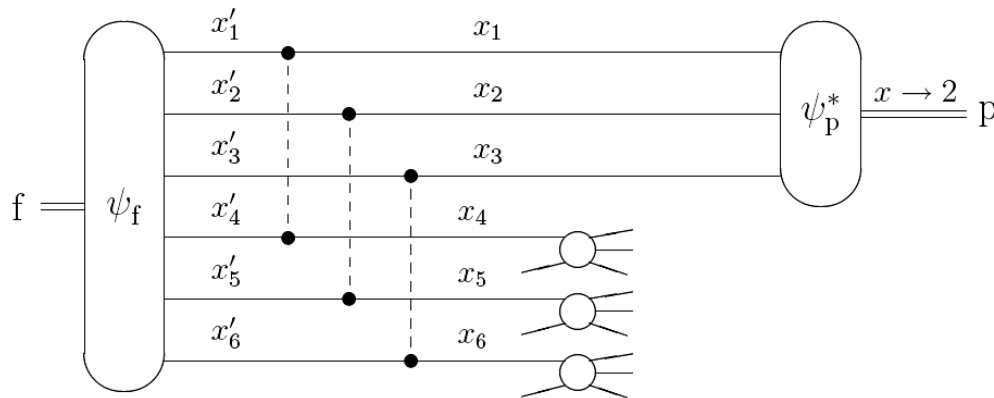
$$\langle |K_{\perp}| \rangle = pm \int_0^{\infty} dz K_0(z) (z K_1(z))^{p-1}$$



Coherent Quark Coalescence and Production of Cumulative Protons



- the cumulative pion production by hadronization of one fast quark
M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427, 614 (1994); Phys.Atom.Nucl. 60, 432 (1997); 63, 1831 (2000)



- the cumulative proton production by **coherent** quark coalescence mechanism:
M.A. Braun, V.V. Vechernin, Nucl.Phys.B 92, 156 (2001); Theor.Math.Phys 139, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.

The last **recalls** the few nucleon **short-range correlations** in a nucleus

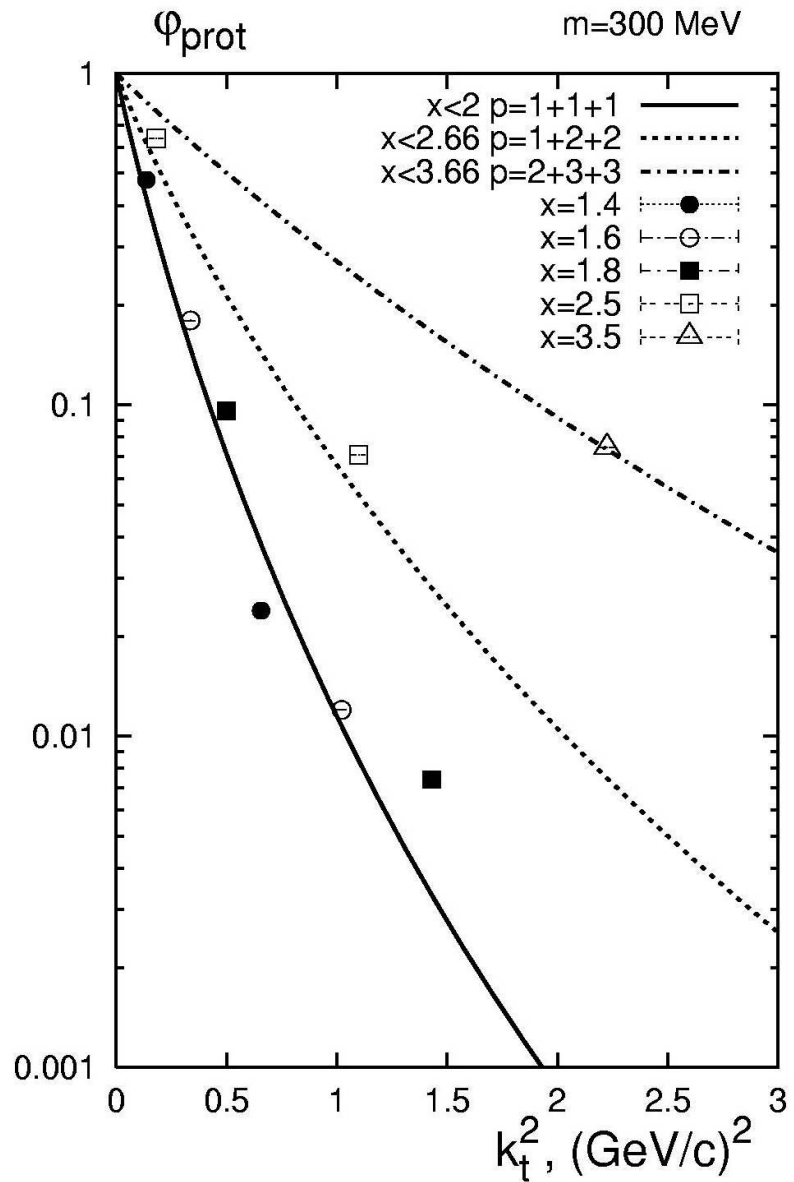
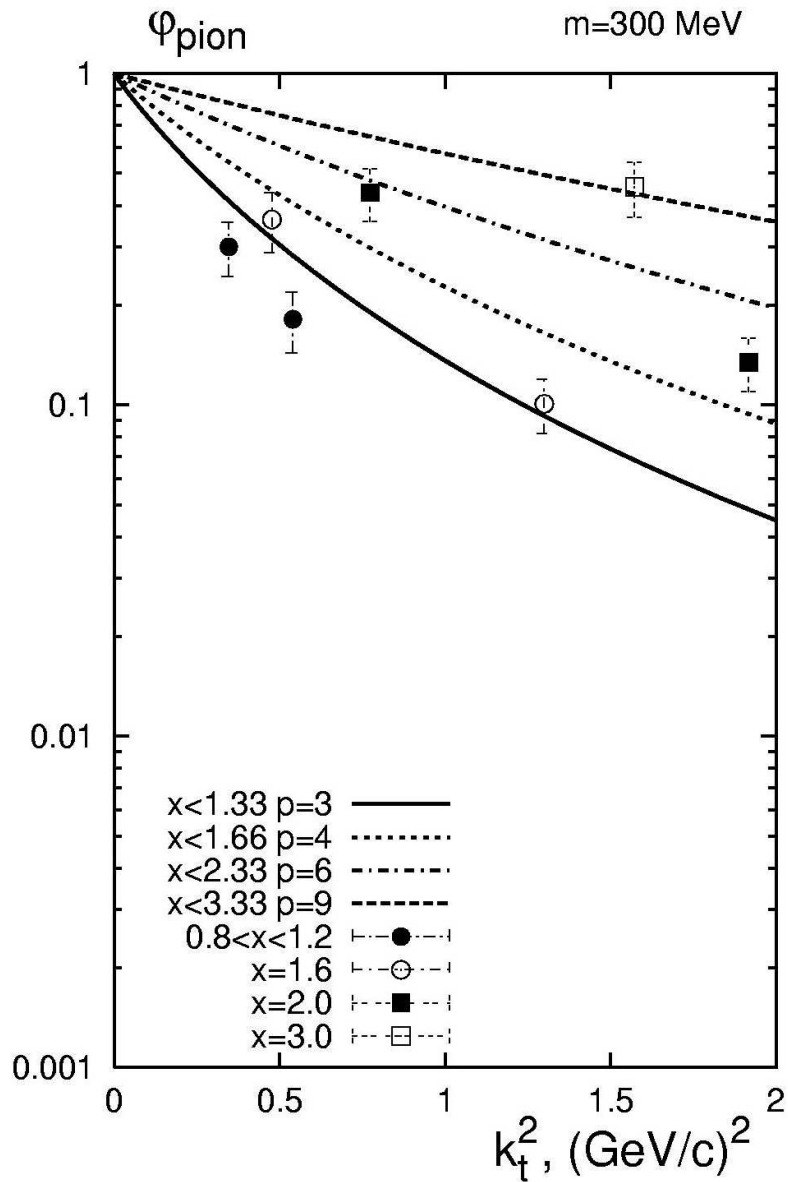
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).

But instead of using the relativistic generalization of non-relativistic NN wave function

the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge

Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. B369 (1992) 519.

was developed and applied.



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **46**, 871 (1987)
S.V. Boyarinov et al., *Physics of Atomic Nuclei* **57**, 1379 (1994)
S.V. Boyarinov et al., *Sov.J.Nucl.Phys.* **55**, 917 (1992)

Application of this old approach for higher p_T

For AA interaction:

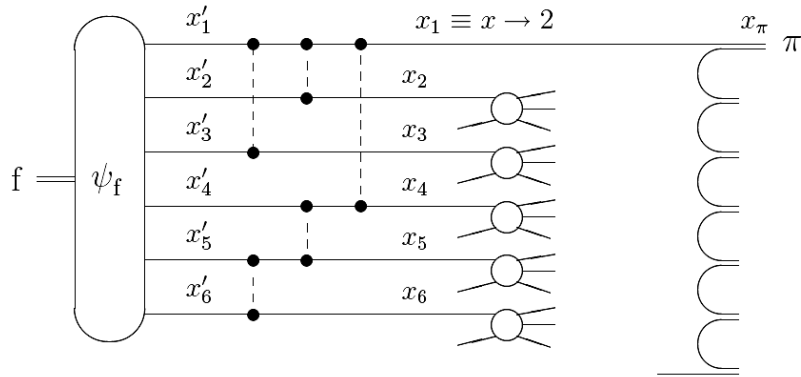
V. Vechernin, S. Belokurova, S. Yurchenko, Dense Cold Quark-Gluon Matter Clusters and Their Studies at the NICA Collider, Symmetry 16 (2024) 79.

For dd interaction:

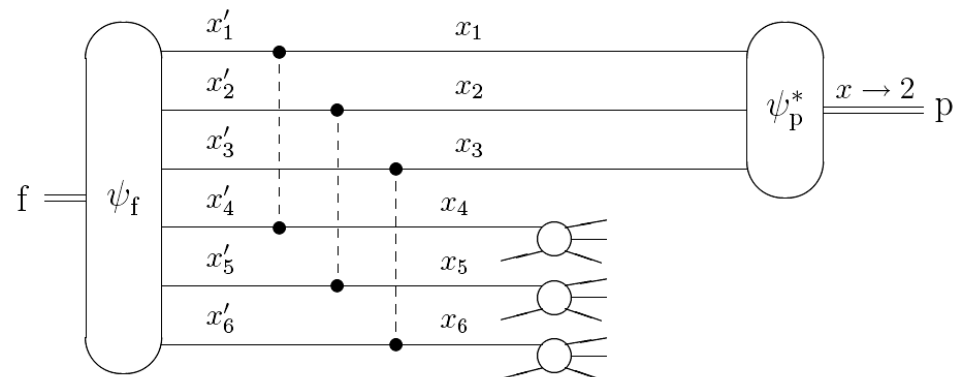
V.V. Vechernin, S.N. Belokurova, S.V. Yurchenko, Cumulative Production in the Region of Central Rapidities and Large Transverse Momenta at the NICA Collider Physics of Particles and Nuclei, 2024, Vol. 55, No. 4, pp. 889-894.

V.V. Vechernin, S.V. Yurchenko, Cumulative production at central rapidities and large transverse momenta in the quark model of flucton fragmentation. Moscow University Physics Bulletin, 2024 (in press).

dd collisions



$$f_\pi(x, k_\perp) \equiv \frac{k_0 d^3 \sigma_\pi}{d^3 \mathbf{k}} = C_\pi (2-x)^9 \Phi_5 \left(\frac{k_\perp}{m_q} \right) / \Phi_5(0)$$



$$f_p(x, k_\perp) \equiv \frac{k_0 d^3 \sigma_p}{d^3 \mathbf{k}} = C_p (2-x)^5 \Phi_1^3 \left(\frac{k_\perp}{3m_q} \right) / \Phi_1^3(0)$$

(1)

$$\Phi_p(t) = 2\pi \int_0^\infty dz z J_0(tz) [z K_1(z)]^p$$

(2)

$$\Phi_1(t) = \frac{4\pi}{(t^2 + 1)^2}$$

$$x \equiv 2x_+$$

$x_+ = 1$
- exact kinematic
boundary for dd reaction

$$x_+ \equiv \frac{k_+}{k_+^{max}},$$

$$k_+ \equiv \frac{k_0 + k_z}{\sqrt{2}}.$$

$x = \frac{k_+}{p_+}$ - light cone variable

$x_F = \frac{k_z}{k_z^{max}}$ - Feynman variable

$M_f^{min} = X m_N$ - cumulative number

$x \approx x_F \approx X$ at $s \rightarrow \infty$

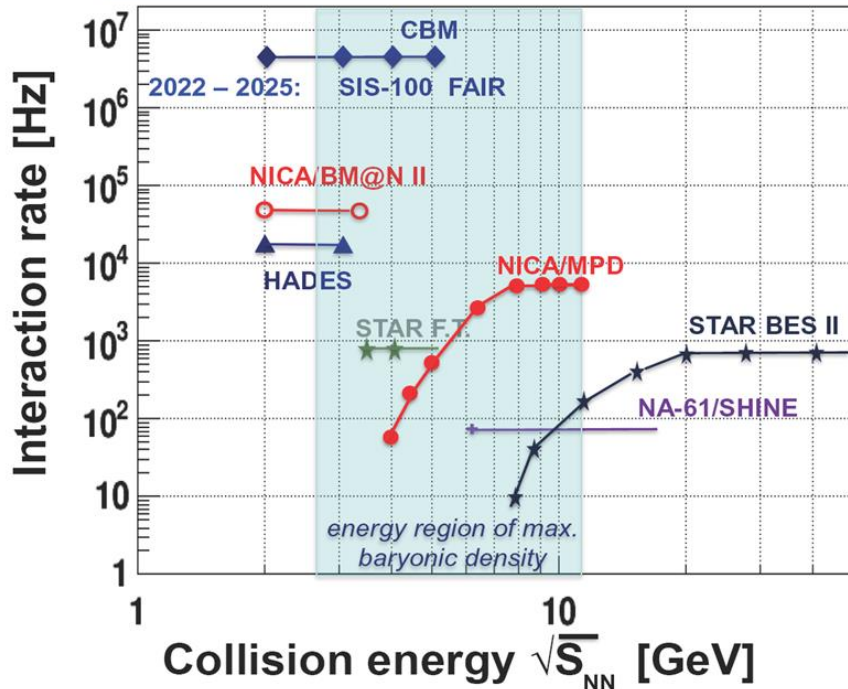
$$\frac{m_N^2}{E^{*2}} = \frac{4m_N^2}{s}$$

Comparison of Interaction Rates in AuAu (BiBi) collisions at MPD and in dd collisions at SPD

$$\text{MPD: } L_{AuAu} = 10^{27} \text{ cm}^{-2} \text{ c}^{-1}$$

$$\sigma_{AuAu}^{tot} \cong 7000 \text{ mb}$$

Present and future HI experiments



$$I_{AuAu} = L_{AuAu} \sigma_{AuAu}^{tot} = 7 \text{ KHz}$$

V. Kekelidze, A. Kovalenko, R. Lednicky, V. Matveev, I. Meshkov, A. Sorin, G. Trubnikov, "Feasibility study of heavy-ion collision physics at NICA", Nuclear Physics A 967 (2017) 884–887.

Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD is important for a registration of rare cumulative events.

$$\sigma_{dd}^{tot} \cong 120 \text{ mb}$$

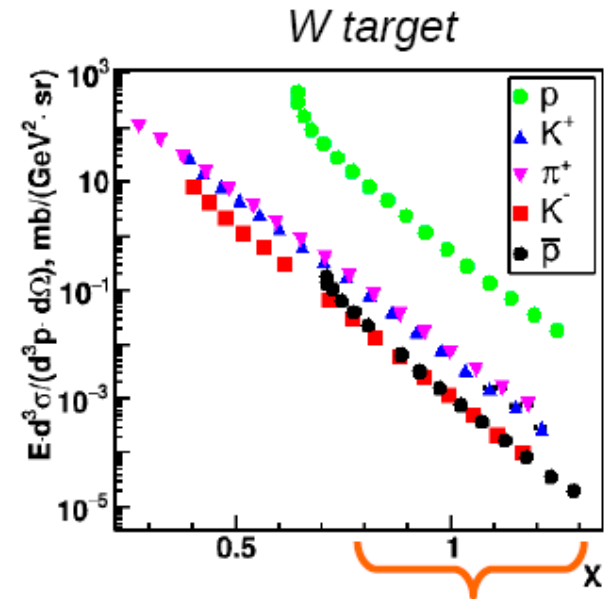
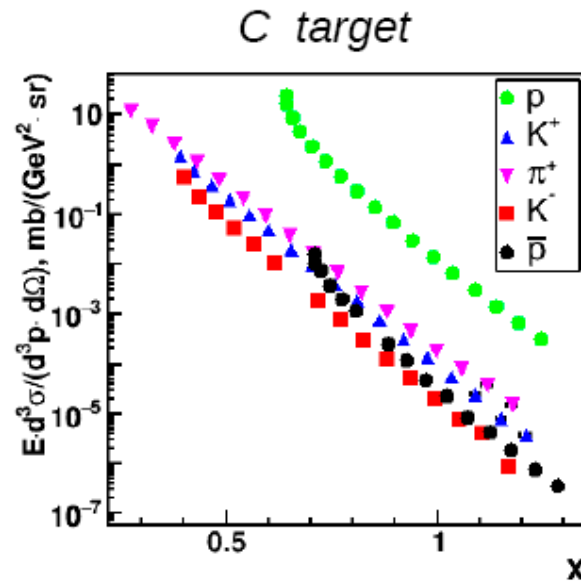
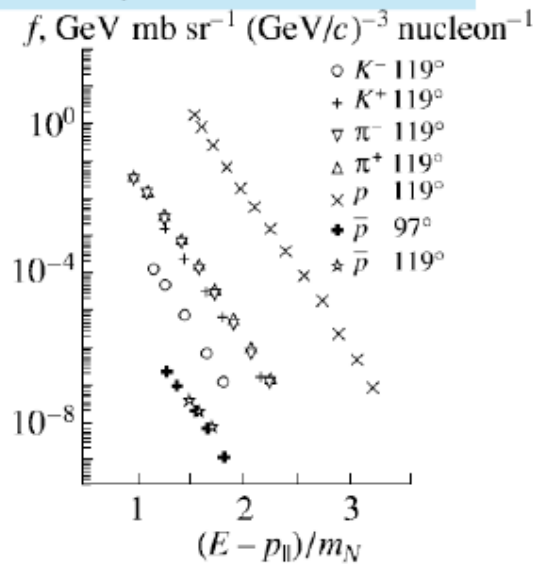
$$\text{SPD: } L_{dd} = 10^{30} \text{ cm}^{-2} \text{ c}^{-1}$$

$$I_{dd} = L_{dd} \sigma_{dd}^{tot} = 120 \text{ KHz}$$

V.M. Abazov, et al. [The SPD collaboration], "Conceptual design of the Spin Physics Detector ArXiv:2102.00442v3 [hep-ex], 2022.

Conclusions (nucleon – flucton interaction)

- We have made **estimates of pion and proton production in a new cumulative region of central rapidities and large transverse momenta in dd collisions** using the theoretical results for the transverse momentum dependence of cumulative particle with given x , obtained in the microscopic (at the quark level) **model of the nucleon – flucton interaction**.
- It is shown that **the observation** of particle yields in this new cumulative region is **accessible for study in dd collisions at the SPD**, due to higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD, which is important for registering of rare cumulative processes.
- The multiplicities of cumulative particles **drop with increase of initial energy** due to general increase of transverse momenta. That in region $\sqrt{s_{NN}}$ **from 4 and 8 GeV** can be **partially compensated by the increase of luminosity**. In this new cumulative region studies are not possible for colliders with large initial energy.
- It is shown also that in this new cumulative region the **yields of pions in comparison with the yields of protons are not suppressed so strongly** as in the nuclear fragmentation region, what can be explained by the different mechanisms of the formation of these cumulative particles. (However, it should be noted that the possible contribution of rescattering processes at large distances to cumulative protons has not been taken into account.)



Large fraction of cumulative processes

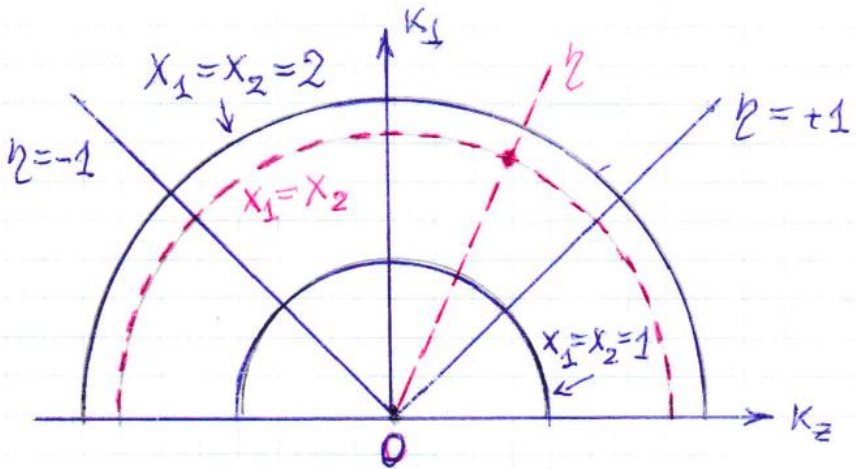
$$\sqrt{s_{NN}} = 9.8 \text{ GeV}$$

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov “High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c” (National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference “NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies”, St Petersburg, October 11-17, 2020.

Flucton-flucton interaction

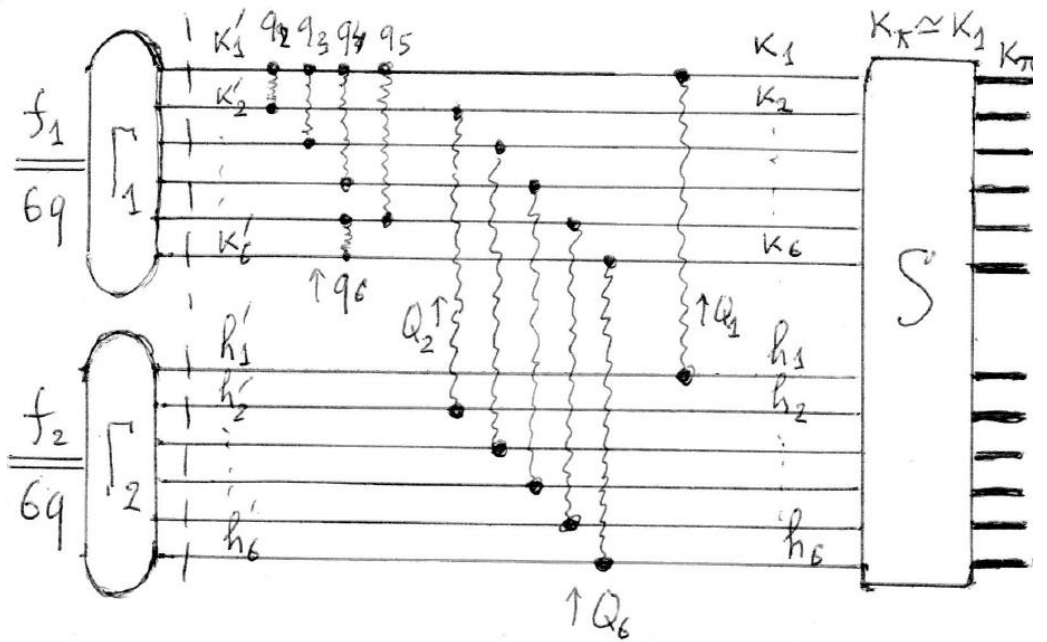
Cumulative production at $|t| \sim s$

Quark counting rules for *inclusive cross sections at* $|t| \sim s$



$$\Gamma_1(k'_{+i}, k'_{\perp i}) \rightarrow \Psi(k'_{+i}, k'_{\perp i})$$

$$\Gamma_2(k'_{-i}, k'_{\perp i}) \rightarrow \Psi(k'_{-i}, k'_{\perp i})$$



Incorporating diquarks

V.T. Kim, Diquarks and Dynamics of Large P(T) Baryon Production, Mod.Phys.Lett.A 3 (1988) 909.

p/π^+ - ratio explanation, using that the diquark distribution function is harder: $(1-x)^1$ vs $(1-x)^3$ for quarks [$(1-x)^{2p-1}$].

Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

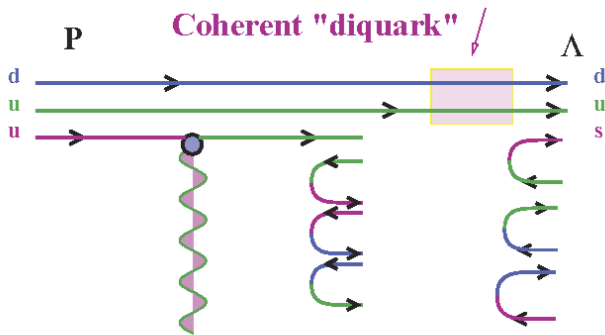


Fig. 4a: Gluon exchange produces a leading baryon.

M.A. Braun, V.V. Vechernin, Nuclear Structure Functions and Particle Production in the Cumulative Region in the Parton Model, Nucl.Phys. B427 (1994) 614

Can string junction carries the baryon number?

L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.

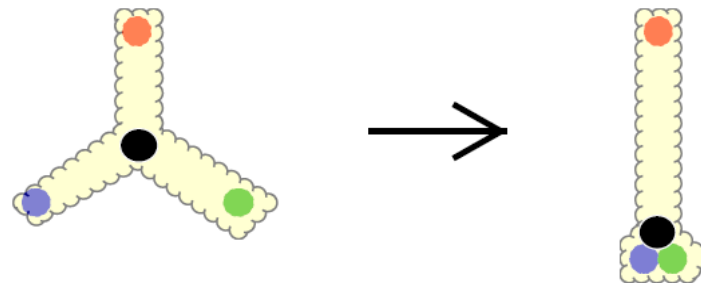
Can be verified experimentally by studying of
baryon stopping in central pp and AA collisions.

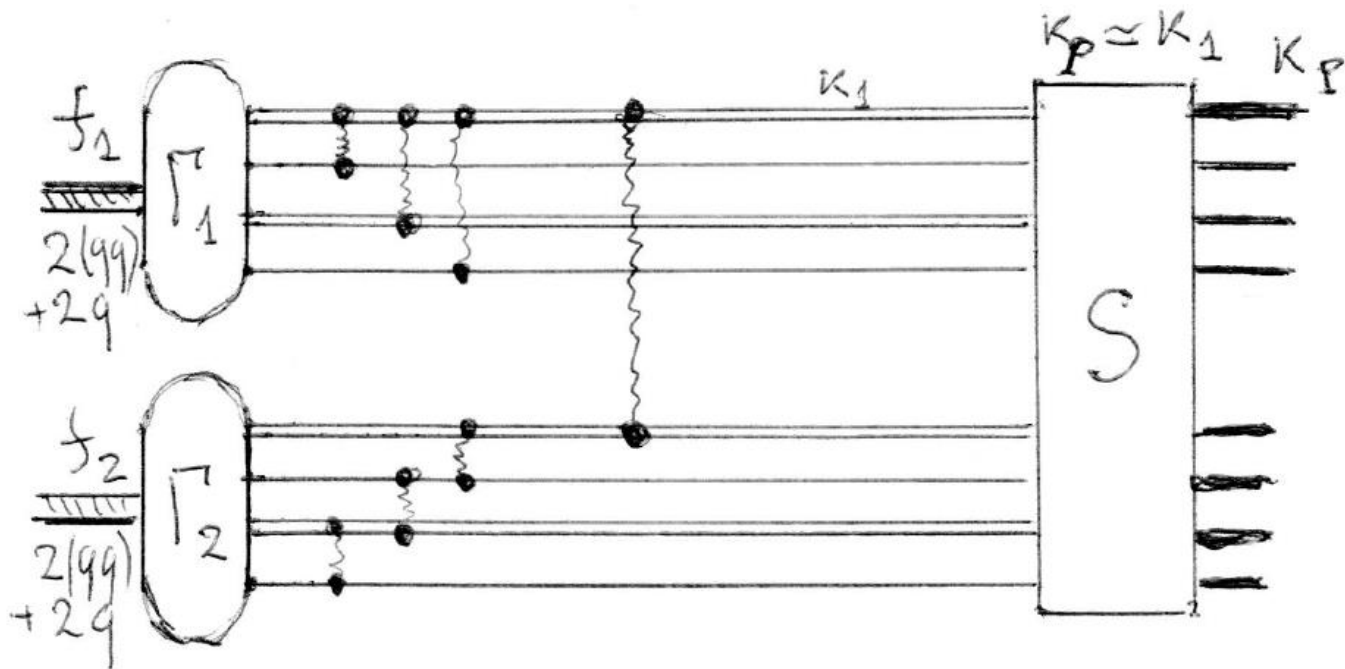
Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).

F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now $B=1$ corresponds to diquark





Possible mechanism for the production of cumulative protons by fragmentation of a diquark into a proton (along with the mechanism of coherent quark coalescence described above).

In this case for the number of recoil quarks and diquarks we have: $p=7$

Conclusions (flucton – flucton interaction)

- The study of **multiquark fluctons in dd collisions at SPD** has a number of advantages compared with MPD (see slide 2).
- The **inclusive cross sections** for particle production in the new cumulative region of large transverse momenta at mid-rapidities **will decrease with both the initial energy s and the cumulative number $x=x_1=x_2$.**
- To evaluate this behaviour and find **asymptotes at $s \gg m$ and $(2-x) \ll 1$** we need to generalize the **quark counting rules**, known now only for
 - a) the **inclusive cross sections** in the fragmentation region ($|t| \ll s$) and
 - b) the **elastic and quasielastic** cross sections in the high p_T region ($|t| \sim s$),to the case of **inclusive cross sections** in the high p_T region ($|t| \sim s$).

Modeling of the dd scattering within the framework of the Glauber approach

Both analytical and MC modeling without fluctons (Belokurova S.N.)

$$T_A(a_1, \dots, a_A) = \prod_{j=1}^A T_A(a_j). \quad \Rightarrow \quad T_{d_1}(a_1, a_2) = T_{d_1}(a_1)T_{d_1}(a_2)\delta(a_1 - a_2).$$

$$T(a) = \int |\Psi(a, z)|^2 dz \quad \Psi(r) = C(e^{-\gamma r} - e^{-\mu r})/r, \quad C^2 = \frac{\gamma(\gamma + \mu)\mu}{2\pi(\mu - \gamma)^2},$$

$\gamma = 45,8 \text{ M}\text{\AA}\text{B}, \quad \mu = 140 \text{ M}\text{\AA}\text{B}.$

$$\sigma(a) = \exp\left(-\frac{a^2}{r_N^2}\right) \quad \sigma_{NN} \equiv \int db \sigma(b), \quad \sigma_{NN} = \pi r_N^2.$$

$$\langle N_{coll}(\beta) \rangle = 4\chi(\beta) \quad \chi(\beta) \equiv c^{-1} \int \sigma(a - b + \beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db,$$

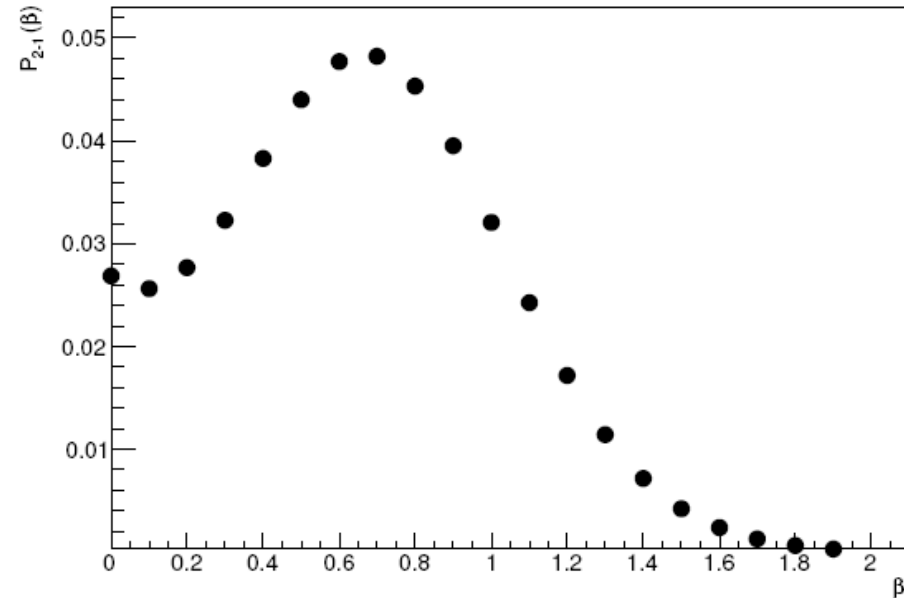
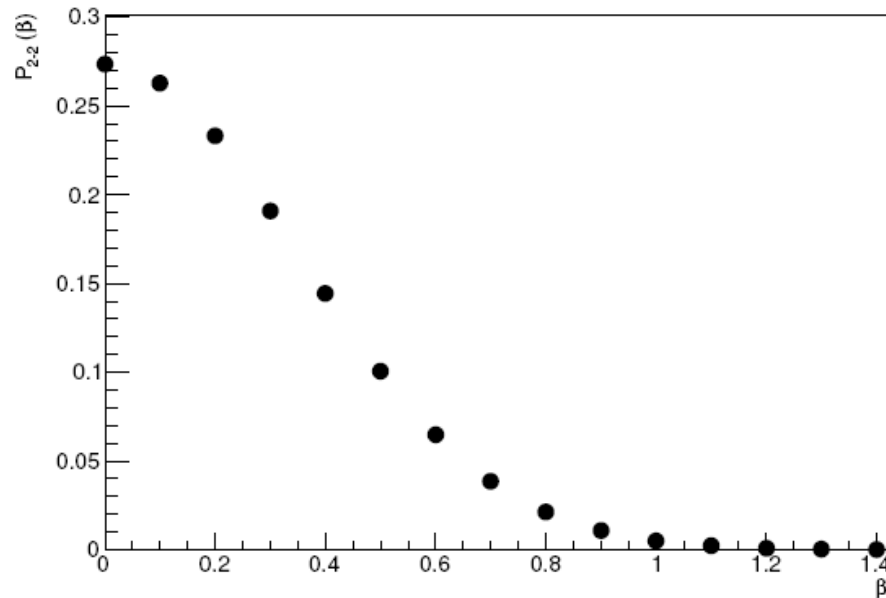
$$V [N_{coll}(\beta)] = \dots$$

$$\langle N_w^{d_1}(\beta) \rangle + \langle N_w^{d_2}(\beta) \rangle = \dots$$

$$V [N_w^{d_1}(\beta) + N_w^{d_2}(\beta)] = \dots$$

$$P_{2-2}(\beta) = c^{-1} \int \sigma(a-b+\beta)\sigma(a+b+\beta)\sigma(-a+b+\beta)\sigma(-a-b+\beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db$$

$$P_{2-1}(\beta) = 2c^{-1} \int \sigma(a-b+\beta) [1 - \sigma(a+b+\beta)] [1 - \sigma(-a+b+\beta)] \sigma(-a-b+\beta) (T_{d_1}(a))^2 da (T_{d_2}(b))^2 db$$



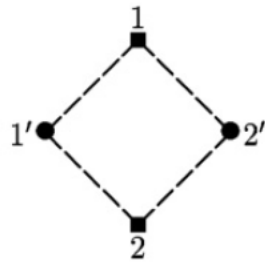
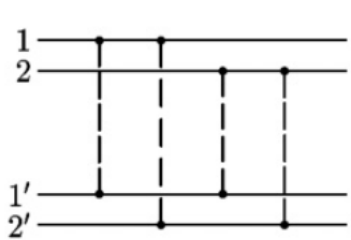
min.bias:

$$P_{2-1}^{min.bias} = 0.0067$$

$$P_{2-2}^{min.bias} = 0.0046$$

Variation of the number of participant and NN collisions in AA (dd) interactions

Vechernin, V.V. and Nguyen, H.S. Phys. Rev. C 84 (2011) 054909



$$\chi(\beta) \equiv \int da db T_A(a) T_B(b) \sigma(a - b + \beta)$$

$$\approx \sigma_{NN} \int da T_A(a) T_B(a + \beta)$$

$$\mathcal{P}_{\text{opt}}(N_{\text{coll}}) = C_{AB}^{N_{\text{coll}}} \chi(\beta)^{N_{\text{coll}}} [1 - \chi(\beta)]^{AB - N_{\text{coll}}}$$

$$\sigma_{NN} \equiv \int db \sigma(b),$$

\Rightarrow

$$I(a) \equiv \int db \sigma(b) \sigma(b + a).$$

!!!

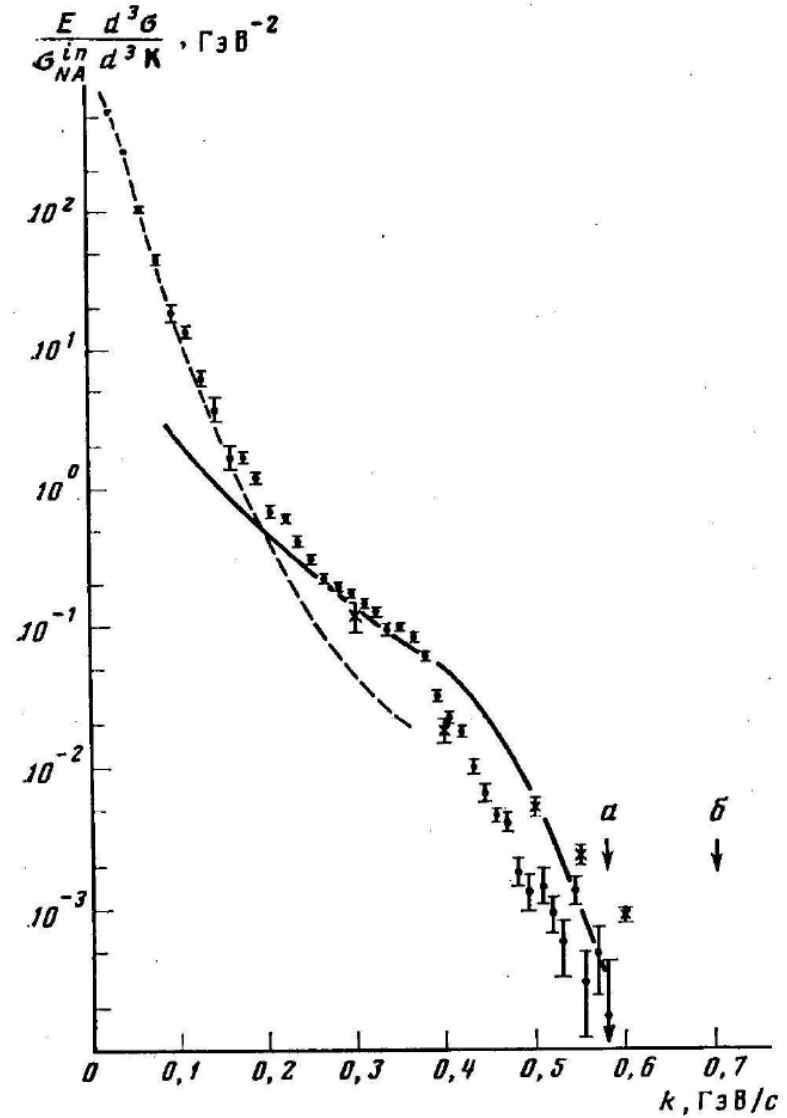
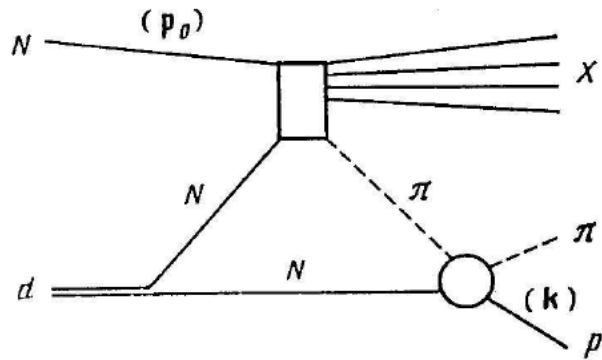
So for variation of the number of participant and NN collisions in AA (dd) interactions the general analytical formulas from textbooks (“the optical approximation”):

C.-Y. Wong, Introduction to High-Energy Heavy-Ion Collisions (World Scientific, Singapore, 1994).

R. Vogt, Ultrarelativistic Heavy-Ion Collisions (Elsevier, Amsterdam, 2007).

are not correct and are not supported by MC simulations.

Contribution of pion rescattering to cumulative proton production from deuteron (long distance contribution !)



Prediction:

Braun M.A., Vechernin V.V., *Yad.Fiz.* 28 (1978) 1466.

Experiment:

Ableev V.G. et al., *Nucl.Phys.A* 393 (1983) 491.

Preprint JINR EI-82-377, Dubna, 1982.

Confirmation:

Braun M.A., Vechernin V.V., *Yad.Fiz.* 40 (1984) 1588.

Braun M.A., Vechernin V.V., *Yad.Fiz.* 43 (1986) 1579.

The shoulder in the spectrum is due to the contribution of the Δ -resonance to elastic πN scattering amplitude