Classical and quantum shear

International Workshop "Infinite and Finite Nuclear Matter" (INFINUM-2025)

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Main Topics

- Classical Energy-Momentum Tensor and shear viscosity
- Holographic viscosity bound and Planck constant
- Quantum EMT: Hadronic gravitational FFs ("old" and "new"): EP for spin and its Extension
- Spin-1 in exclusive and inclusive processes: shear and viscosity
- Quantrum shear and vorticity in HICs: handedness
- Conclusions

EMT and shear viscosity

- EMT : $T^{\mu\lambda} = (e+p)\, v^{\mu} v^{\lambda} - p\, g^{\mu\lambda}$
- Viscosity term: T –odd : $\eta\, dv^{\{\mu}/d x_{T\ \lambda\}}$
- Bound $\eta/s > 1/(4\pi)$

- Checked in models (PHSD etc.)

- Dimensionfull $\eta/s > \hbar/(4\pi)$

Sources of Ћ

- η ~ ρ v l, s~ n
- η/s ~ m v l = S: action of particle propagating at mean free path l
- S> Ћ : holds always (but close to saturation for "quantum" l (Cf MB talk:
- Δp Δx ~ 4 Ћ
- Other representation (D.T.Son)
- mvl = Ћ l/ (Ћ/mv) = Ћ $l/l_{deBroglie}$

Rotation as a sources of Ћ

- η ~ ρ v l
- s~ n
- η/s ~ m v l ~ L: Angular momentum of "vortex" of mean free path size (recall Kolmogorov cascade!)
- L > Ћ : holds always (but close to saturation for "quantum" vortex size

Sources of ħ and 3D and 2D

- $\eta/s > \hbar/(4\pi)$
- 3D: ħ comes from η, s "classical"
- But at 2D (surface) entropy ~ 1/ ħ
- Susskind argument for Black Hole: entropy is increased by 1 when photon of BH size R carrying energy ~ ħ is absorbed
- L at 2D is NOT quantized (anyons)

Quantum EMT: Gravitational Formfactors (Pagels'66, Ji'97 : O (Δ)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- No M_{Pl}! May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons Ji's SRs
- Describe interaction with both classical and "equivalent" gravity

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle
- Low-energy theorem

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $\vec{H}_J = \frac{1}{2} rot \vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot \vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on (quantum!) SPIN – known since 1962 (Kobzarev and Okun’; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW
LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 25 September 1991)

- If (CP-odd!) GEDM (new EMT FF, also forbidden by EP: extra γ_5 in B) =0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

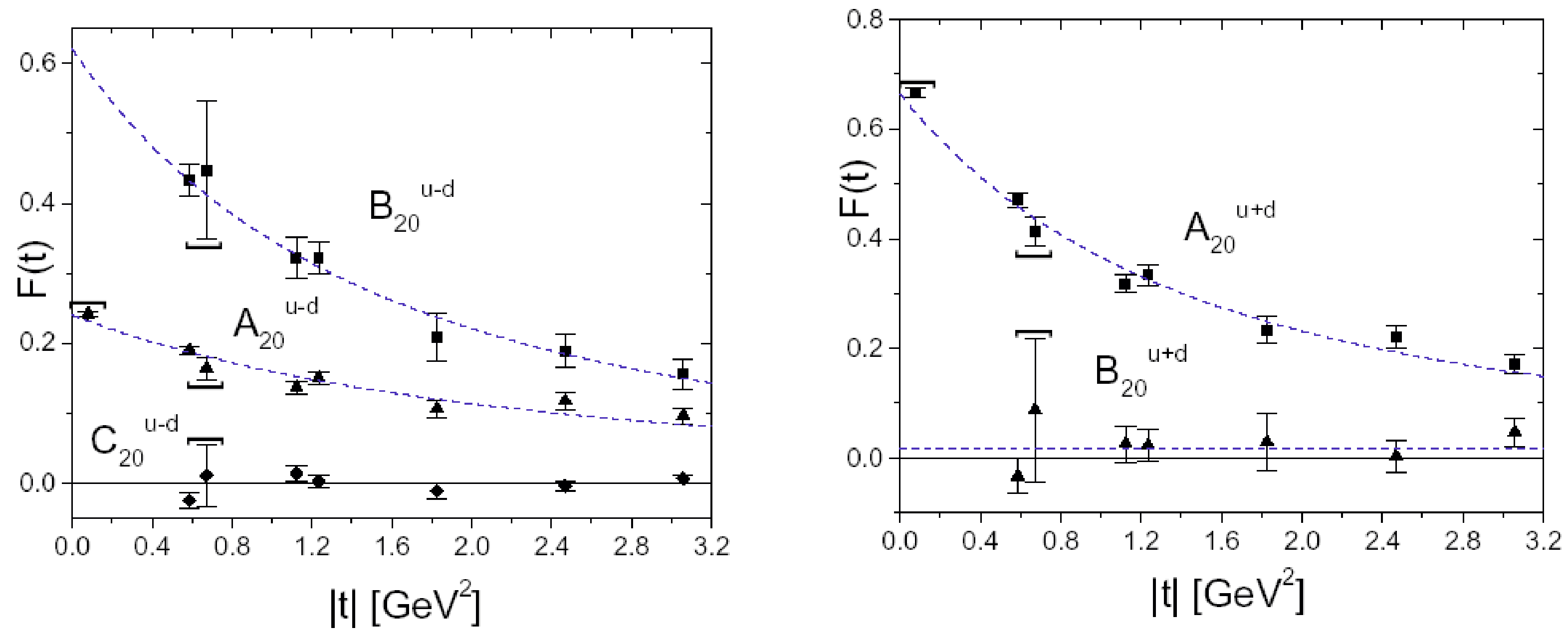
- Gravitational test of Ji's SR (for Hg)!
- New high precision EDM experiments: gravity is essential (NN Nikolaev,Vergeles,Obukhov, Silenko,OT, 2204.00427 and UFN)

EP and quantum measurement

- If spin is just a geometric vector, EP for Earth’s rotation is “trivial”: looking from stars, spin rotates with Earth’s angular velocity like Foucault pendulum
- Non-trivial if quantum measurement (quite practical here) is performed in the rotating frame
- Cf with Unruh effect (talk of G. Prokhorov): measurement in accelerated frame is crusial, medium as an (active) detector

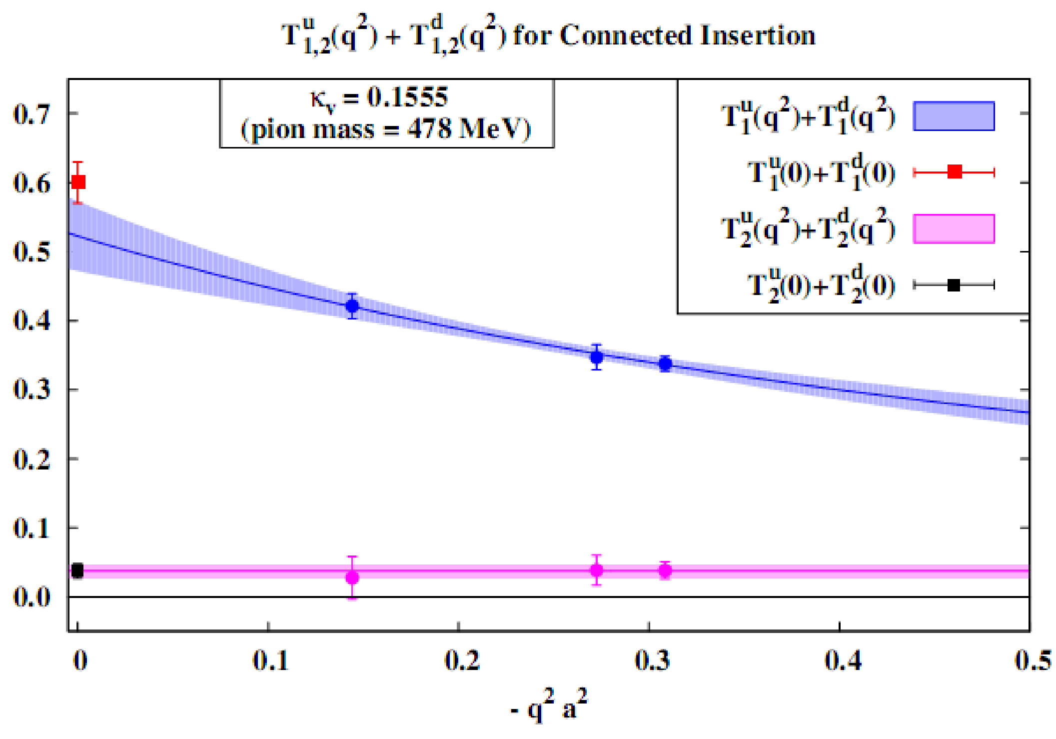
Extension of Equivalence principle

- Various arguments: AGM $\approx$ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



More recent lattice study (M. Deka,...K.-F. Liu et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In NLO pQCD – violated (LF:S.Brodsky et al.)
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 71)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement?! Nucleons are not broken even by black holes?!
- Equivalent gravity effect is absent (limted)?
- Support by recent observation of smallness of (Ex)EP-forbidden "cosmological constant" (talks of C. Roberts, S. Nair) and by separate stability of quarks and gluons (talks of S. Nair, P. Choudhari)

Exact Equipartition and Pivot

- Important notion introduced by C. Lorce to relate transverse spin SR's of Ji&Yuan and Leader et al.
- Naïve interpretation of ExEP: common (approximately, averagely) pivot for quarks and gluons:
- $\langle J_{T(q,G)} \rangle = \langle x_0 \rangle \langle P_{L(q,G)} \rangle$
- Can this be satisfied for some of pivot choices?

Quadrupole formfactor: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Vacuum – Cosmological Constant

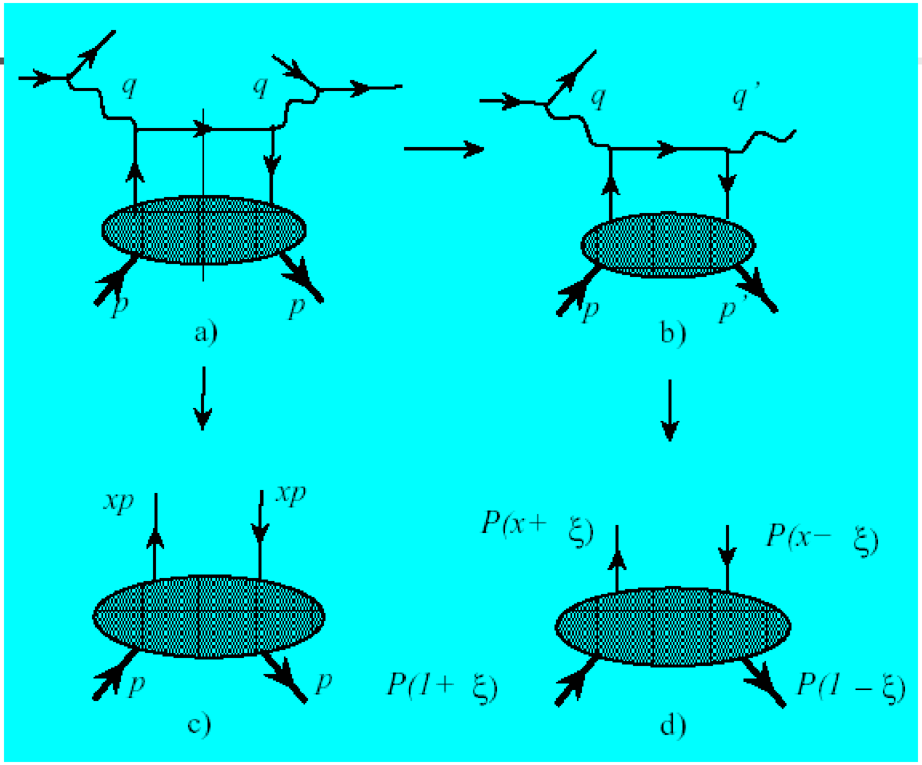
$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe
- Traceless+Trace =
- M_I: (3/4+1/4) X.Ji'96
- M_{Gr}: (3/2-1/2) OT'99 ("Antigravity": seen in trace part of Einstein Eqs)
- Access: D-term in GPDs

Way to D-term: cf QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

=-($\int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$)

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!
- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From quantum D-term to classical pressure

- *Inverse -> 1st moment (model)*
- *Frotm stars by v.Laue in 1912: weighted pressure D~<p r4> (<p r2> =0)*

M.Polyakov'03

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p,S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- *Justification: (Fourier inversed) consistency principle for Born gravitational scatterring*

LETTER

NATURE | www.nature.com/nature

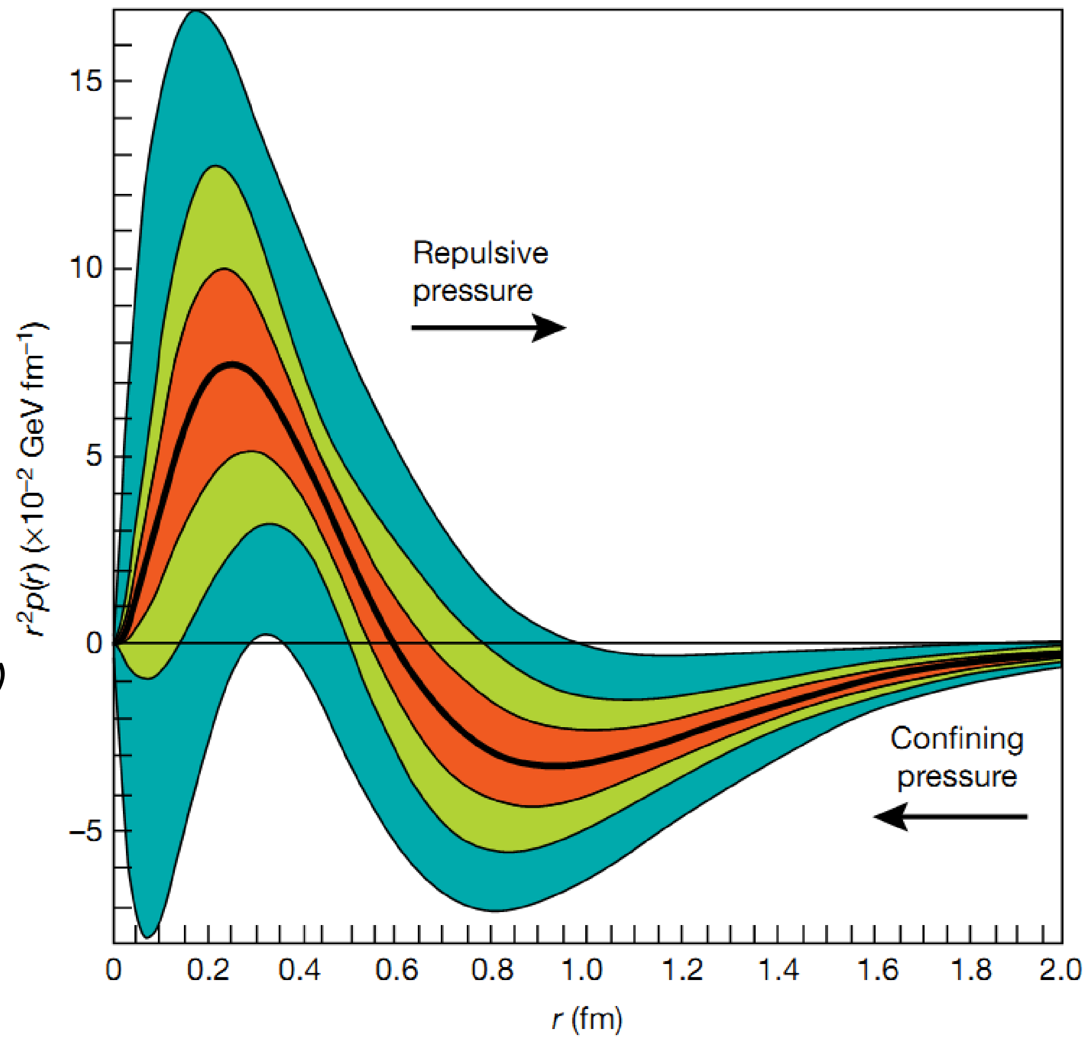
https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

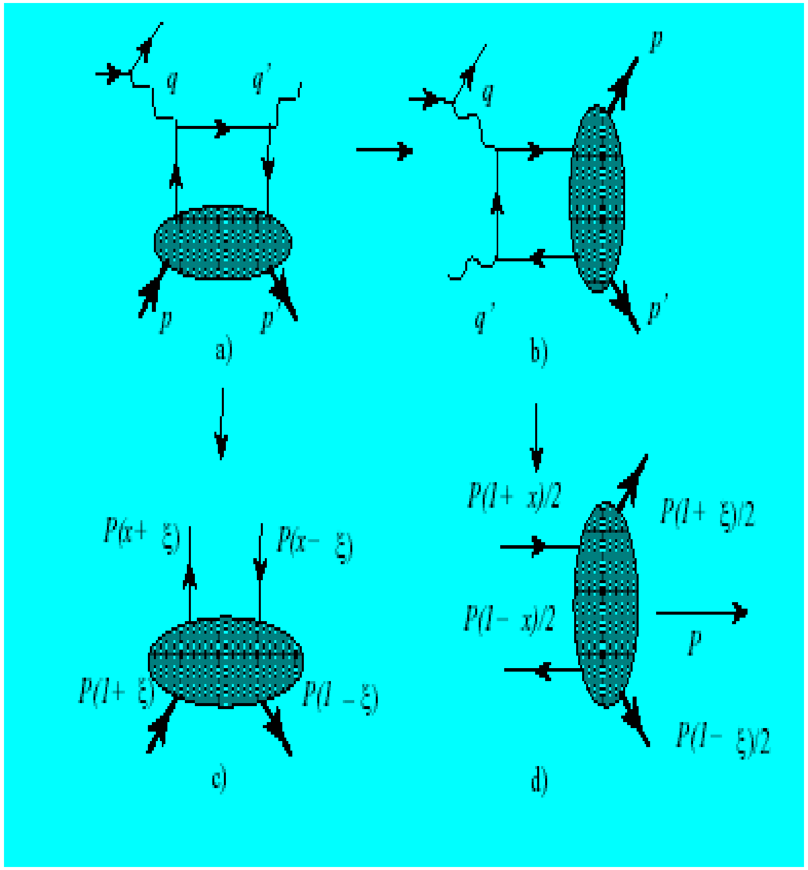
5. Teryaev, O. V. Gravitational form factors and nucleon spin structure. Front. Phys. 11, 111207 (2016)

15. Anikin, I. V. & Teryaev, O. V. Dispersion relations and QCD factorization in hard reactions. Fizika B 17, 151–158 (2008)



Road to timelike GrFFs: Crossing for DVCS and GPD (cf e+e- and HICs: small systems)

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes (Diehl,Gousset, Pire,OT'98)



Gravitational FFs from Belle data on GDAs 1711.10086

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

- Gravitational FFs are related to twist-2 GDAs

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\,\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi^q\left(z,\xi,W^2\right)$$

$$\int dz(2z-1)\Phi_q^{+}\left(z,\xi,W^2\right) = \frac{2}{(P^+)^2}\left\langle \pi^+(p_1)\pi^-(p_2)\right|T_q^{++}(0)\left|0\right\rangle$$

$$\left\langle \pi^0(p_1)\pi^0(p_2)\right|T^{\mu\nu}(0)\left|0\right\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^{\mu}P^{\nu}\right)\Theta_1 + \Delta^{\mu}\Delta^{\nu}\Theta_2\right]$$

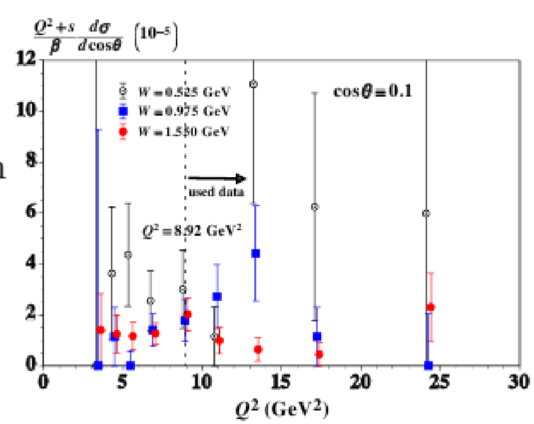
$$P = p_1 + p_2\ ,\ \Delta = p_1 - p_2$$

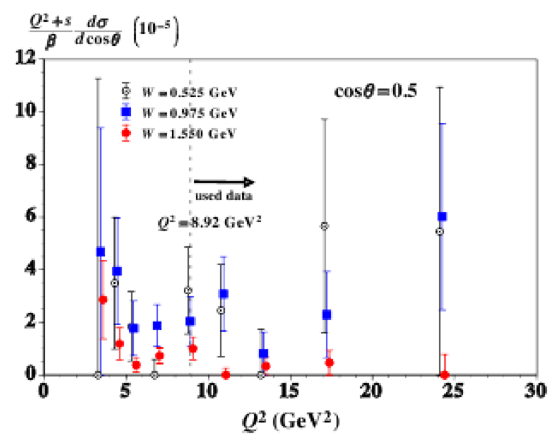
M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003

- Belle data and scaling : W=0.525,0.975, 1.55 GeV

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\cos\theta,W,Q)\right|^2$$

$$\sqrt{\langle r^2\rangle_{\rm mass}} = 0.39\,{\rm fm},\ \ \sqrt{\langle r^2\rangle_{\rm mech}} = 0.82\,{\rm fm}$$





New EMT formfactor : “Shear viscosity” (OT’2020)

- From spherically symmetric object to fluid (EoS!)
- $T^{\mu\lambda} = (e+p)\, v^{\mu} v^{\lambda} - p\, g^{\mu\lambda}$
- $V^{\mu} = P^{\mu}/M$: correct normalization but no coordinate dependence
- Another suggestion (OT’19):
- $V^{\mu} = (P^{\mu} + a(t)\, k_T^{\ \mu}) / (M^2 + a^2(t)\, k_T^{\ 2})^{1/2}$
- Viscosity: $\eta\, dv^{\mu}/d\, x_T^{\ \lambda} \sim E_{\eta}\, p^{[\mu} \Delta^{\lambda]}$
- NO such term in total EMT (but can be for quarks separately)
- Naïve T-oddness: phases in GPD channel from decays in TDA
- Phases <-> dissipation: polarization in pionic superfluidity model (V. I. Zakharov, OT’ 17)

Timelike GrFF: Viscosity in GDA channel

- Viscosity:will correspond to Exotic $J^{PC}=1^{-+}$ meson (studied long ago without mentioning gravity: Anikin, Pire, Szymanowski,OT, Wallon'06)
- Spin: related to structure of matrix element: One index of EMT (0^{th} in rest frame) is carried by momentum and other by polarization vector - just what we need for viscosity
- NO for conserved EM: zero coupling for (G)DA!
- $\pi\eta$ pairs observation instead of $\pi\ \pi$ required
- Smallness of viscosity: related to smallness of exotic GDAs and ExEP violation?!

Exotic hybrid meson production

On exotic hybrid meson production in $\gamma^*\gamma$ collisions

I.V. Anikin[1], B. Pire[2,a], L. Szymanowski[3,4,5], O.V. Teryaev[1], S. Wallon[5]

Eur. Phys. J. C 47, 71–79 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2006-02533-7

- Possible candidate π_1 (1400)

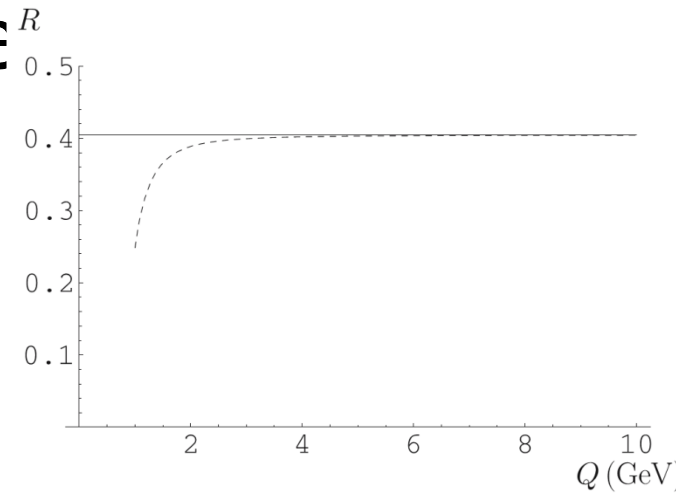


Fig. 2. The ratio $R(Q^2)$ of the squared amplitudes for H and π^0 production in $\gamma^*\gamma$ collisions at leading twist and zero-th order in α_s (*solid line*) and including twist three contributions in the numerator (*dashed line*)

Estimate of viscosity

$(e+p)\, v^{\mu} v^{\lambda} \sim A\, P^{\mu} P^{\lambda}$

- $\eta\, dv^{\mu}/d\, x_T^{\ \lambda} \sim E_{\eta}\, p^{[\mu} \Delta^{\lambda]}$
- TD: e+p -> Ts
- η/s (> $1/(4\pi)$)~ $E_{\eta} T / AM$ (smallness due to ExEP and small coupling to exotics)
- Correct dependence on Planck constant recovered via Δ^{λ} ->- $i\hbar\, d/d\, x_T^{\ \lambda}$
- Song,OT,Yoshida,2503.11316: relation in QCD factorization to structure of pseudoscalar mesons: $\eta/s \sim 0.05$

From time like FFs to HICs:properly averaged momentum correlations (of handedness type): talk of E. Dlin

- Shear : $dv^{\{\mu}/d\, x_{T\ \lambda\}} \sim \langle p_{\{i}\, p_{j\}}\rangle/\hbar$
- Vorticity: $dv^{\{[\mu}/d\, x_{T\ \lambda]} \sim \langle p_{[i}\, p_{j]}\rangle/\hbar$
- Helicity: $\langle v \text{ curl } v\rangle \sim e_{ijk}\langle p_i\, p_j\, p_k\rangle/\hbar$

Shear for deuterons: Spin 1 EMT and inclusive processes

- Forward matrix element -> density matrix
- Contains P-even term: tensor polarization $S^{\alpha\beta}$ New EMT FF
- $< P|T^{\alpha\beta}|P> = A P^{\alpha} P^{\beta} + T S^{\alpha\beta}$
- Symmetric and traceless: correspond to (average) shear forces
- Cf with spin ½ and spin 1 vector polarization : P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT’82 : zero sum rules:
- Current conservation: 1st moment: also in parton model by Close and Kumano (90)
- EMT conservation: 2nd moment (forward analog of Ji’s SR: $\sum B=0 \Leftrightarrow \sum T=0$)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT’09
- No monopole spin-gravity coupling!

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

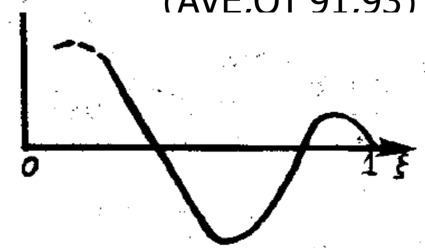
- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)



$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

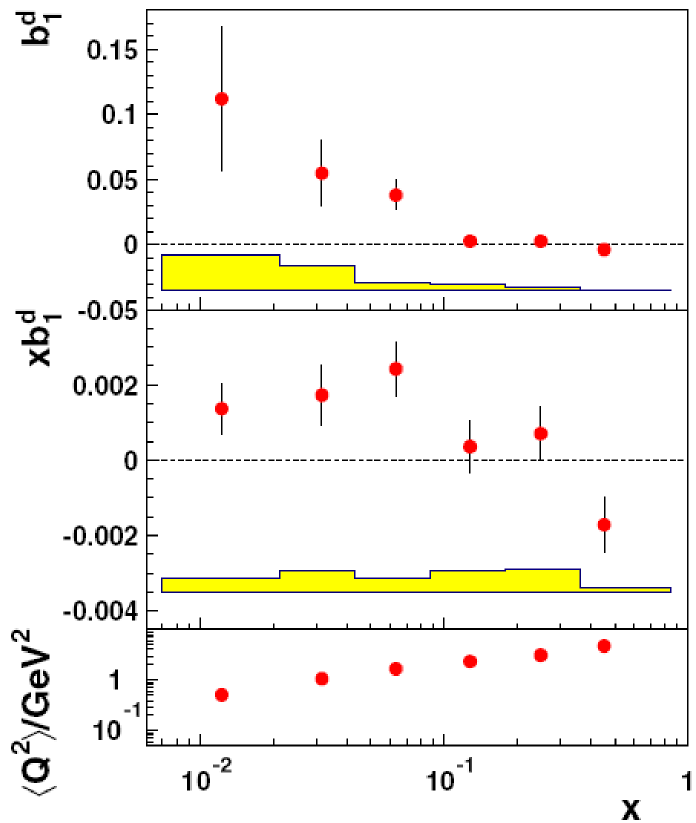
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$ =0 for ExEP

HERMES – data on tensor spin structure function

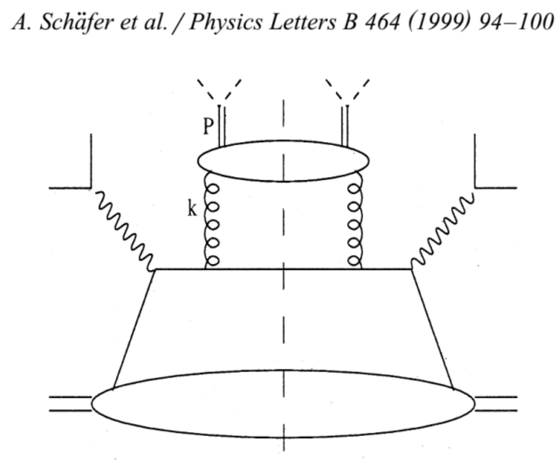
PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective tensor polarized glue << sea)



Fragmentation functions

- Tensor polarized fragmentation functions: (Szymanowski, Schaefer, OT'99)



- Suggestion'21: zero SRs (analogous to momentum SR) may probe the (Ex)EP for hadrons inside partons (EIC: gluons)

Conclusions

- Viscosity bound is non-trivial in "quantum" domain: L,S ~ $\hbar$
- Analog of viscosity may be considered for matrix elements of EMP quantum operator
- HICs: Momentum correlations