Pions in peripheral heavy-ion collisions. On possibility of neutral pion condensation and magnetization

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Plan

- Description of pion self-energy in equilibrium nuclear matter (case of fully overlapped nucleon Fermi spheres).
- Model for π^0 condensation. Spin and magnetic moment and rotation, Gibbs free energy.
- Peripheral HIC, pion fluctuations and π^0 condensation in model of nonoverlapped Fermi spheres

  D.V. Phys.Rev.C in press.

Triple role of mesons in matter:

collective excitations, mediators of *baryon-baryon* interaction and condensates of classical fields

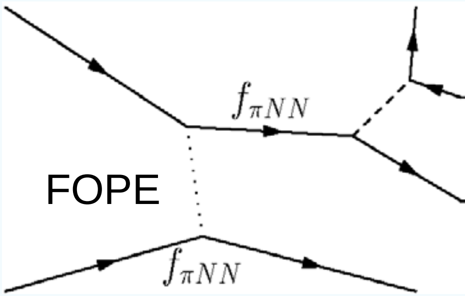
In spite of this, up to now in use are models in which pions in matter are for simplicity treated as obeying **vacuum dispersion law** (ideal pion gas in HIC, minimal cooling paradigm for description of cooling of NS, etc) that may lead to **principal inconsistencies:**

Inconsistencies of FOPE model

P-wave πNN vertex in vacuum (for nonrel. N): $V_{\pi NN}^{n.rel} = -if_{\pi NN}\boxed{\sigma_\alpha}\tau_\beta\partial_\alpha\pi_\beta, \quad \alpha,\beta = 1,2,3,$

for atomic nucleus density is n_0=0.5 m^3_π, for n ~few n_0 typical r~1/$f_{\pi NN}$, $f_{\pi NN}$≈$1/m_\pi$
→ strong coupling

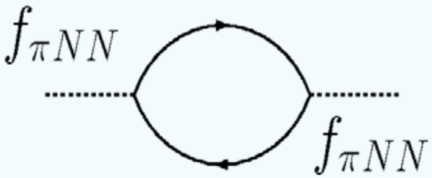
The only diagram in FOPE model which contributes to MU process (key process for NS cooling in minimal cooling scenario) is



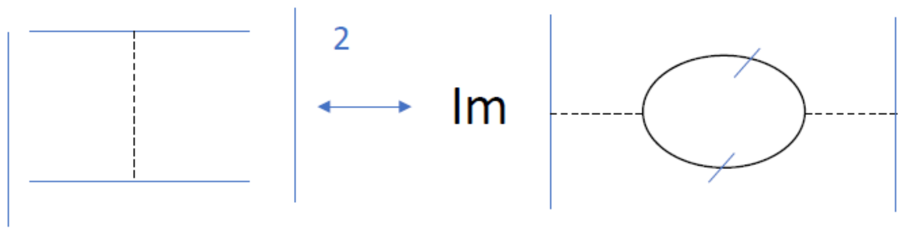
This is Born approximation term

For consistency necessary to include terms of the second-order in $f_{\pi NN}$ in all quantities! Otherwise appear problems with unitarity.

In $f_{\pi NN}^2$ order, pion self-energy is then: Π_0 =

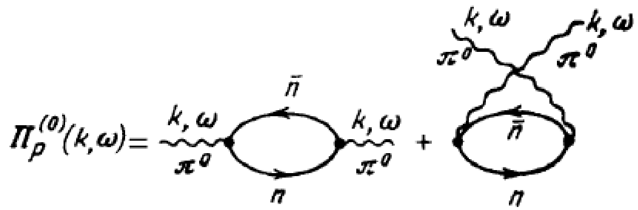


Optical theorem



requirement of unitarity!

Nucleon Fermi distributions



$$=2f^2k^2\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p}+\mathbf{k})-n(p)}{\omega+\varepsilon(\mathbf{p}+\mathbf{k})-\varepsilon(p)}=-2f^2k^2\frac{mp_F^{(n)}}{2\pi^2}\boxed{\Phi(k,\omega)},$$

With equilibrium Fermi distributions n(p) Lindhard function is:

$$\Re\Phi(\omega\ll kv_{\rm F},k,T)\simeq 1-\frac{k^2}{12p_{\rm F}^2}-\frac{\omega^2}{k^2v_{\rm F}^2}-\frac{\pi^2T^2}{12\epsilon_{\rm F}^2}\,,$$

no free parameters!

But Re Π_0 proves to be so attractive that *p*-wave pion condensate in N=Z matter would occur then already at $n>0.3\,n_0$. However experimental data show absence of pion cond. in atomic nuclei.

Nevertheless to describe levels in pion atoms one needs $\omega(k)\simeq\sqrt{m_\pi^2+\boxed{\alpha_0}\vec{k}^2}$

α_0 ~0.2-0.4 *for ω near m_π at $n=n_0$* $k\lesssim m_\pi,$ rather than $\omega^2\simeq m_\pi^2+k^2$

Migdal,Saperstein,Troitsky,D.V. Phys.Rep.192 (1990),

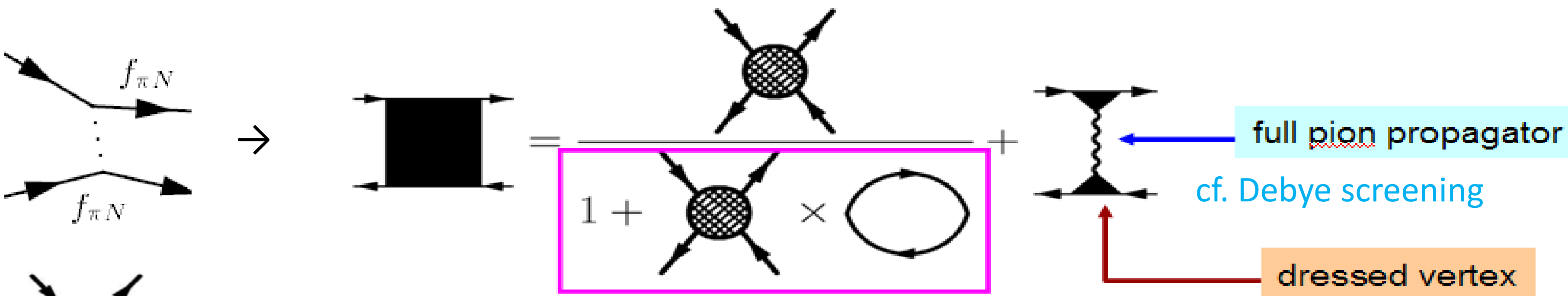
Friedman, Gal *Acta Phys.Polon.B* 51 (2020)

Low-energy excitations in nuclear Fermi liquid (Migdal)

Strong p-wave pion-nucleon interaction: No free pions in matter!

Resummed NN interaction

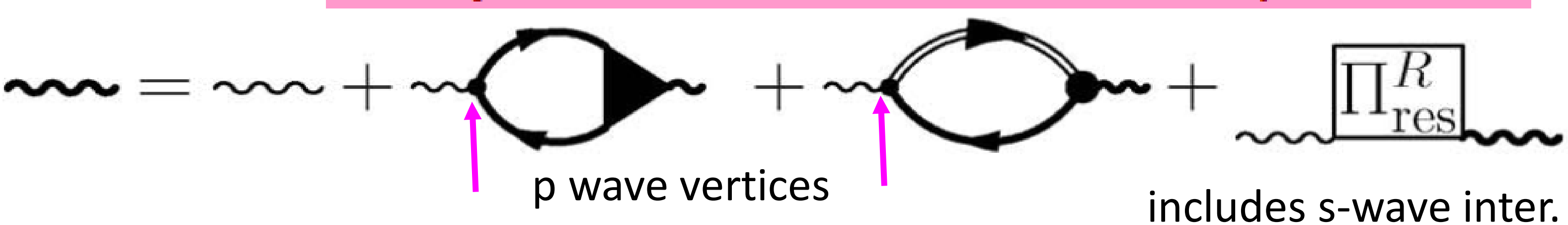
based on a separation of long (pion) and short (NN correlation) scales



$= C_0\,(f_{12} + g_{12}\boldsymbol{\sigma_1}\boldsymbol{\sigma_2})$

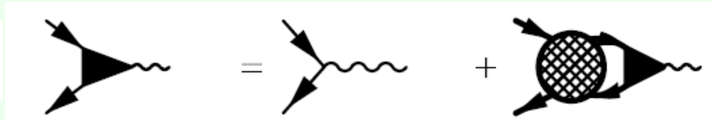
Info. on short-range interact. is extracted from data on atomic nuclei & calculations

Poles yield zero-sound modes in scalar and spin channels



Pole part of the pion self-energy is suppressed by NN correlations

$$\Pi_N = (\tau_N, G_N G_N \mathcal{T}_N)\,,$$

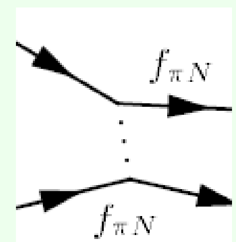


Thus solution of mentioned puzzle is as follows:

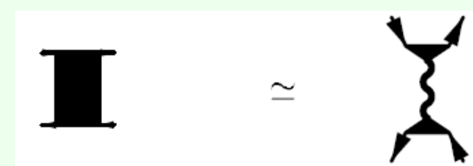
Vertex suppression factor needed to describe various atomic nucleus data at $\omega \ll m_\pi$, $k \sim p_F$ is $\Gamma((g_{12}, n_0) = 0.35\text{-}0.45$ from where one extracts LM parameters:

$g_{12}(n_0)$ –LM parameters of NN interaction in spin channel, 1,2 – n or p.

For $n > n_{c1}$ ($n_{c1} \sim 0.5 n_0$) NN amplitude gets peak at $k \neq 0$.
However for $n < n_0$ and low ω the full NN amplitude is still few times weaker than that approximated by FOPE



But NN amplitude sharply increases with increasing n above n_0, that m.b. results in p-wave pion condensation for $n > n_c > (1.5\text{-}3)\, n_0$, n_c is several times larger than n_{c1} !.



One should replace FOPE by full *NN* interaction, essential part of which is due to MOPE with vertices including *NN* correlations.

Pion spectrum

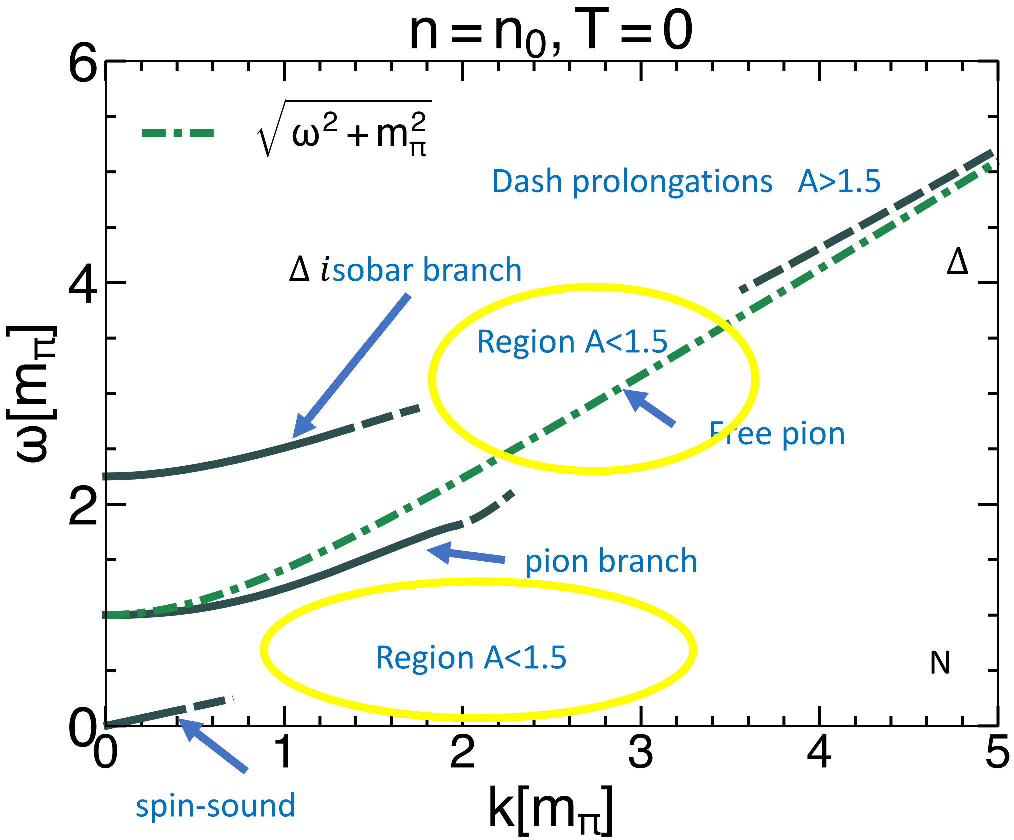
Heavy nucleons are good quasiparticles, except vicinity of π condensation transition, light relativistic π are incorporated explicitly with help of the full spectral function

$$A_\pi = -2\Im D_\pi^R = \frac{\Gamma_\pi}{(\omega^2 - k^2 - m_\pi^2 - \Re\Pi^R)^2 + \Gamma_\pi^2/4}\,, \quad \Gamma_\pi = -2\Im\Pi^R$$

Δ (1232) isobars include width ~100 MeV already in vacuum.

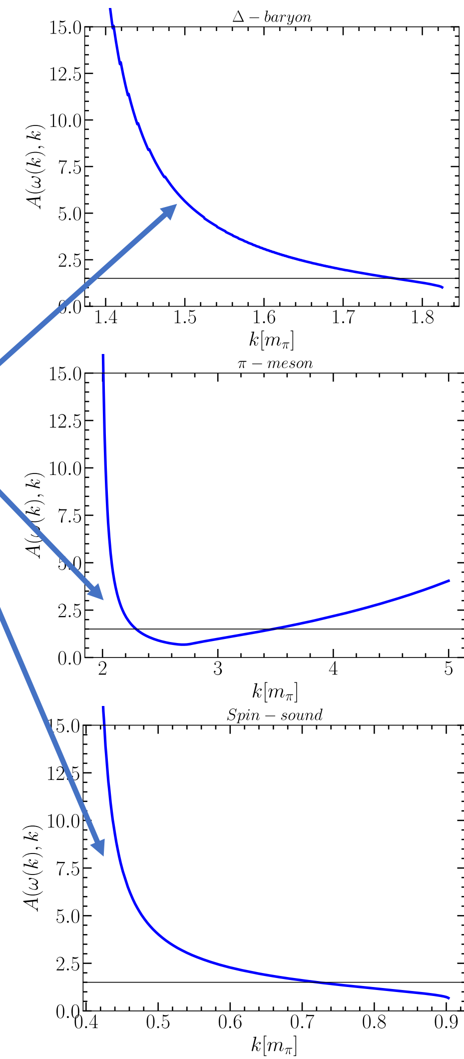
Pion spectrum at N=Z with full spectral function A(ω,k)

Sum-rule $\frac{1}{\pi}\int_0^{\infty} d\omega\ [\omega A(\omega,k)] = 1$



Horizontal line A=1.5

rapid decrease

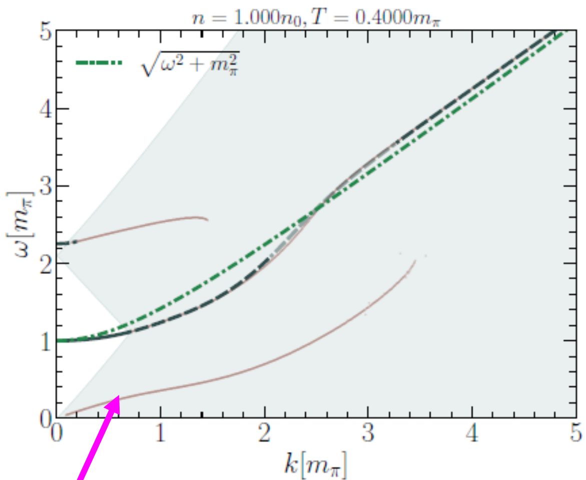


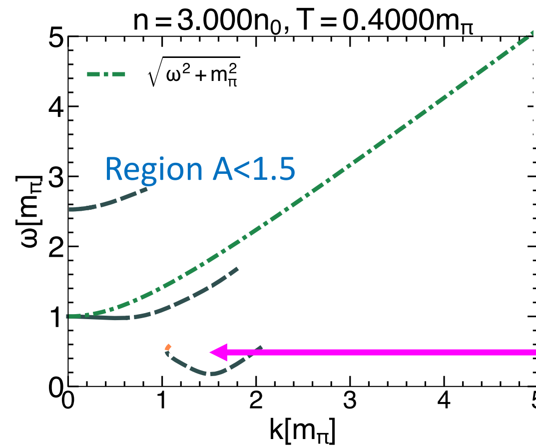
Dash-lines—local maxima of *A*

In the given model pion condensation appears at T=0 for n =2.6 n_0

Pion spectrum at nonzero temperature, *N=Z*

Broader spectral function for T≠0





$$\frac{1}{\pi}\int_0^\infty d\omega\,[\omega A(\omega,k)] = 1$$

Liquid phase of π condensate

Shaded region- is region with Im Π≠0. Thin prolongations show local maxima of A for A<1.5

Spin-sound is fully smeared, but with increasing *n* for n> n_{c1} appears Lorentzian peak at $k \sim p_F$

In this model at T=0.4 m_π pion condensation appears for n =3.1 n_0

L. Grigorian, Master thesis 2024

- **pion softenning**

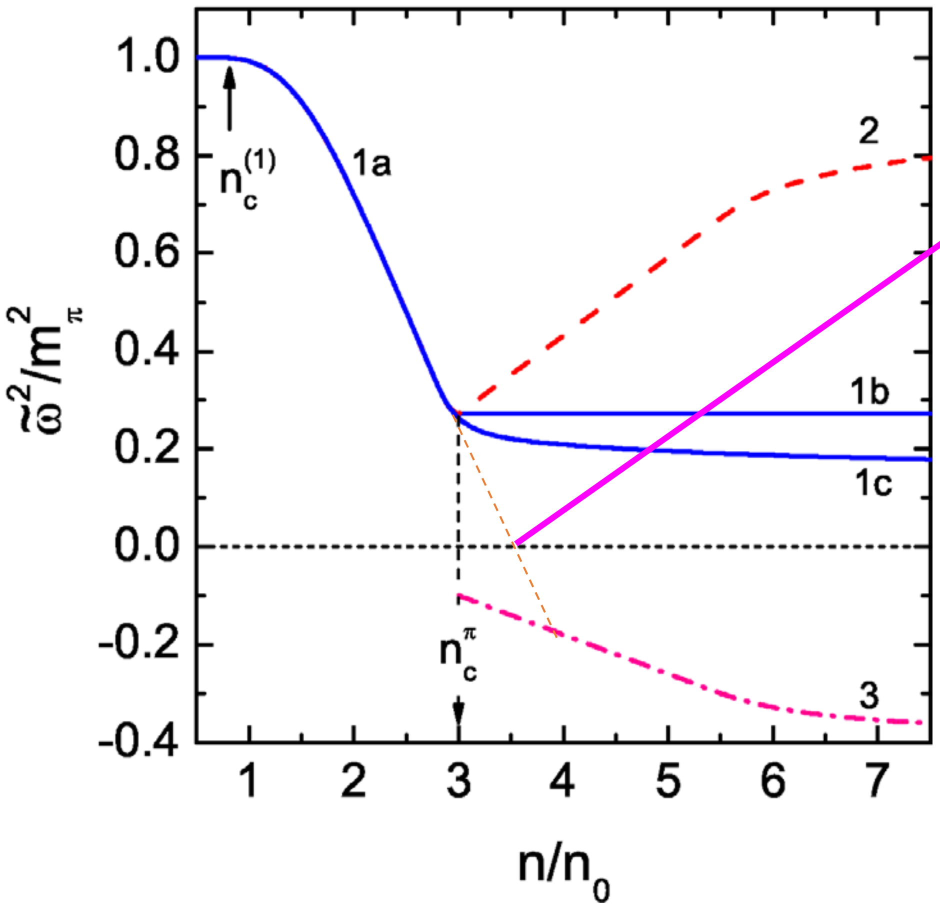
π propagator for **$n_c^\pi > n > n_{c1}$~0.5 n_0**

$\omega \ll k_0 v_F$ gets minimum at $k_0 \neq 0$

$$D_\pi^R(\omega,k) \simeq \frac{1}{\boxed{-\tilde{\omega}^2} - \gamma\,(k-k_0)^2 + i\beta(k)\,\omega}$$

At N=Z, $n_c^\pi > n > n_{c1}$ **liquid phase of pion condensate** e.g. in HIC, cf. D.V. 1989, Nucl. Phys.A555 (1993) , and similarly for Z=0 for neutral pions

Effective pion gap squared



reconstruction of pion spectrum on top of the pion condensate

Crit. point in approx. of II order phase tr.

If LM parameters increase with density -- saturation of pion softening and **no pion condensate**

amplitude of the pion condensate (liquid-crystal or m.b. solid) , e.g.

$\varphi(\boldsymbol{r}) = (1/\sqrt{2})a\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ (traveling wave) ,

$\varphi(\boldsymbol{r}) = a \sin \boldsymbol{k}\cdot\boldsymbol{r}$ (standing wave) .

Next, two models (I and II) were constructed to appropriately treat the region of the pion atom region $\omega \simeq m_\pi, k \ll m_\pi$ as well as low energy domain for $k \gtrsim m_\pi$ at $n \lesssim n_0$,

Models essentially differ in treatment of regular (smooth) s-wave part of pion self-energy Π_{res} =0.

For equilibrium system (overlapped nucleon Fermi spheres) in Model I crystal-like π condensation appears for N=Z above **3 n_0**, but it occurs at **n_c<2 n_0** in Model II. **Pronounced pion fluctuations with k≠0 for T ≠0** (liquid phase of π condensate) appear in Model I for **$n>n_{c1}\sim 0.5n_0$**, and in Model II already for **$n>0.25\ n_0$**,

Note that at energy of HIC ~ GeV A , pions and nucleons escape fireball at semi-central collisions at ~ 0.5-0.7 n_0

⇒ **Expectation is of enhanced pion fluctuations at k≠0 at freezeout in HIC.**

With increasing n above n_0 results become more model-dependent, but statement about pion softening at k≠0 already for n~n_0 seems to be solid.

About electromagnetic fields and rotation in HIC

Typical magnetic field in non-central HIC at ~ GeV A: $h=h_{VA} \sim (Ze^6)^{1/3} H_\pi$,
$H_\pi=m^2_\pi /e \sim 3.5 \times 10^{18}$ G, D.V., Anisimov, JETP51, 1980,

For ultrarelativistic HIC $h_{SIT} \sim \gamma\, h_{VA}$ Skokov,Illarionov,Toneev,Int.JMPh.A24, 2009

however quantum effects (**rapid fields are not classical**) may result in a decrease of estimate h_{SIT} resulting in that the maximum value of the magnetic field in ultrarelativistic HIC is $h_{max} \sim Ze\ h_{VA}$,D.V. PRD2024

The global polarization of Lambda (1116) hyperon observed by the STAR collaboration in noncentral Au-Au collisions indicated existence of a vorticity with rotation frequency $\Omega \sim (9\mp1)\times 10^{21}$ Hz~ $0.05 m_\pi$, large angular momenta are $L< 10^5\text{-}10^6$

Now assume that we have pion condensation in HIC

Notice that if we dealt with rotated charged pion condensate (superfluid) we could have own mag. field $h \sim h_L \sim 3\times 10^{17}$ G ~ m^2_π for $\Omega \sim 10^{22}$ Hz (``London moment'')

Spin and magnetic moment associated with π^0 classical field

In medium of fully polarized p and n, nucleon mag. moment density would be:

$$\sum_i \vec{\mathcal{M}}_i = \alpha_N \vec{e}^{\,(p)} = \frac{g_p n_p - g_n n_n}{2} M_N \vec{e}^{\,(p)} \qquad M_N = \frac{e_p}{2m_N}, \quad \vec{e}^{\,(p)} = \vec{s}_p / s_p$$

e_p =-e is charge of proton, m_N is vacuum mass of nucleon $\quad n_p \simeq n_n \simeq n/2.$

$$g_n \simeq -3.85, \quad g_p \simeq 5.58$$

p-wave π^0 N interaction produces spin polarization

Averaged n and p densities in presence of *U* potential of π^0 NN p-wave interaction:

$$n_i = \frac{p_{\mathrm{F}i}^3}{3\pi^2} = \frac{(2m_N^*)^{3/2}}{3\pi^2} \frac{1}{2} \mathrm{Tr}_\sigma (\widetilde{\epsilon}_{\mathrm{F}i} - U_{3i})^{3/2}, \qquad \widetilde{\epsilon}_{\mathrm{F}i} \simeq \frac{p_{\mathrm{F}i}^2}{2m_N^*}$$

$$U_{3i} = \mp f_{\pi N} \sigma_j \frac{\partial \phi_3}{\partial x_j}, \qquad f_{\pi N} \simeq m_\pi^{-1}$$

σ-Pauli matrix, *ϕ*- static classical field of π^0

Spin and magnetic moment associated with neutral pion condensate

D.V. Phys.RevC in press

Averaged contribution of pion-nucleon interaction to spin density of n and p is

$$\vec{S}_i^{\pi N} = \frac{(2m_N^*)^{3/2}}{3\pi^2}\frac{1}{2}\mathrm{Tr}_\sigma[\vec{s}_i(\widetilde{\epsilon}_{\mathrm{F}i} - U_{3i})^{3/2}]\,.$$

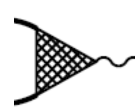
In spite of that for N=Z the total spin polarization vanishes, there exists significant contribution to the net magnetic moment density of nucleons

$$\vec{\mathcal{M}}^{\pi N} = [g_p \vec{S}_p^{\pi N} + g_n \vec{S}_n^{\pi N}] M_N \simeq \alpha_h^{\mathrm{med}} \nabla \phi_3\,,$$

$$\alpha_h^{\mathrm{med}} = \frac{\zeta f_{\pi N} M_N m_N^*(n) p_{\mathrm{F}}(n) \boxed{\Gamma(g'(n))}}{2\pi^2} > 0\,,$$

effective mass of nucleon

$\zeta = g_p - g_n$ =9.43

Γ is dressed πNN vertex  $\Gamma(x) = \left[1 + 2x p_{\mathrm{F,N}} \Phi^R(\omega, k, T)/p_{\mathrm{F,N}}(n_0)\right]^{-1}\,,$

Another (dominant) contribution to magnetic moment of nucleon may arrive from WZW axial anomaly describing anomalous interaction of π^0 field with external electromagnetic field, and the baryon current $A_N^\nu = (\mu_N, \vec{0})$ describing decay $\pi^0 \to 2\gamma$

$$\vec{\mathcal{M}}_N^{\mathrm{WZW}} = \alpha_h^{\mathrm{WZW}} \nabla \phi_3 \,, \quad \alpha_h^{\mathrm{WZW}} = \frac{e_p \boxed{\mu_N}}{2\pi^{3/2} f_\pi}$$

Son, Stephanov, PRD77, 2008

μ_N is nucleon chemical potential, f_π =92 MeV is the pion decay constant.

➡ Extra term in Lagrangian $\mathcal{L}_h = \left[(\alpha_h^{\mathrm{med}} + \alpha_h^{\mathrm{WZW}}) \partial_z \phi_3 \right] h_z , \quad \vec{h} = \mathrm{curl} \vec{A}$

It yields contribution not only to mag. moment but also to baryon density $\delta n = \partial \mathcal{L}_h / \partial \mu_N$

In total $\alpha_h \sim 0.1$, ➡ $M_N^{\mathrm{med}} \gg M_N^{\mathrm{vac}}$

Rotation associated with the π^0 condensate

The anomaly also produces the WZW-contribution to the Gibbs free energy density (due to **rotation associated with the π^0 condensate**),

$$\delta E_{\omega}^{\mathrm{WZW}} = -\alpha_\omega m_N \nabla\phi_0 \vec{\omega}, \;\; \alpha_\omega = \frac{\mu_N \boxed{\mu_I}}{2\pi^2 f_\pi m_N},$$ Brauner, Yamamoto, JHEP04, 2017

μ_I is nucleon isospin chemical potential,

$\vec{\omega} = \vec{\Omega} + \vec{\omega}_{\mathrm{own}},$ external rotation Ω and a weak **self-rotation ω.**

For a neutron-rich matter $\mu_I > 0$ and $\delta E_\omega^{WZW} < 0$. This circumstance can result in **appearance of an isospin asymmetry, N≠Z, in rotating nuclear matter, e.g., in peripheral heavy-ion collisions (provided presence of π^0 condensate).**

For $\Omega \sim 0.1\, m_\pi$, $h \sim 0.1\, m^2_\pi$ and N ~ 1.5Z the rotation contribution to the Gibbs energy can become of the order of that from magnetic term.

π^0 condensate contribution to Gibbs free energy

Expand Π^π in low π^0 momenta, $\Pi^{\pi^0}(0, k \ll p_F) \to -\alpha_1 k^2 + \alpha_2 k^4/m_\pi^2$. good for n<1.5$n_0$, k< 1.5$m_\pi$

Pion-mag. field term in Gibbs free-energy density, cf. Ginzburg-Landau model

$$G_{h,\phi} \simeq \frac{(1-\alpha_1)(\nabla\phi_3)^2}{2} + \frac{\alpha_2(\Delta\phi_3)^2}{2} + \frac{m_\pi^{*2}\phi_3^2}{2} + \frac{\lambda\phi_3^4}{4} - \alpha_h h_z \partial_z \phi_3 + \frac{(h_z - H_z)^2}{8\pi} + \frac{H_z^2}{8\pi}. \qquad \vec{h} = \mathrm{curl}\vec{A}$$

$\alpha_1,\ \alpha_2,\ \alpha_h$ are the coefficients of the expansion, $\alpha_h = \alpha_h^{\mathrm{med}} + \alpha_h^{\mathrm{WZW}}$.

π^0 condensate Gibbs free energy

Chiral limit of negligible pion mass (often used e.g. in quark models)

Pion wall solution $\phi = az$ for $\lambda \to 0$, for $a = const.$

$$G_{h,\phi} \simeq \frac{(1-\alpha_1)a^2}{2} - \alpha_h h_z a + \frac{(h_z - H_z)^2}{8\pi} + \frac{H_z^2}{8\pi}\,.$$

Minimizations over $h_z = h$ and then over a yield

$$h = H + 4\pi\alpha_h a\,,\ \ a = \alpha_h H/\xi\,,\ \ \xi = 1 - \alpha_1 - \boxed{4\pi\alpha_h^2}.$$

h (H=0) is **own magnetic field.** For H=0 instability arises for $n>n_{c1}$: $\xi(n_{c1}) = 0\,,$

$n>n_{c1}$, $n_{c1}<n_0$!

effect of mag. field on pion

For external field *H* ≠0 pion condensate appears even for n→ 0 !

$$G_{h,\phi} = \chi\frac{H^2}{8\pi}\,, \quad \chi \simeq 1 - \frac{4\pi\alpha_h^2}{\xi} < 1\,.$$

Case of non-zero pion mass, n>n_c^π >n_0

There is P-wave condensate solution $\quad \phi = \phi_0 \sin(kz)\,,$

$$G_{h,\phi} \simeq \frac{(1-\alpha_1)k^2\phi_0^2\cos^2(kz)}{2} + \frac{(m_\pi^{*2}m_\pi^2+\alpha_2 k^4)\phi_0^2\sin^2(kz)}{2m_\pi^2}$$

$$+\frac{\lambda\phi_0^4\sin^4(kz)}{4} - h_z\alpha_h k\phi_0\cos(kz) + \frac{(h_z-H_z)^2+H_z^2}{8\pi}.$$ (Min in *h*:

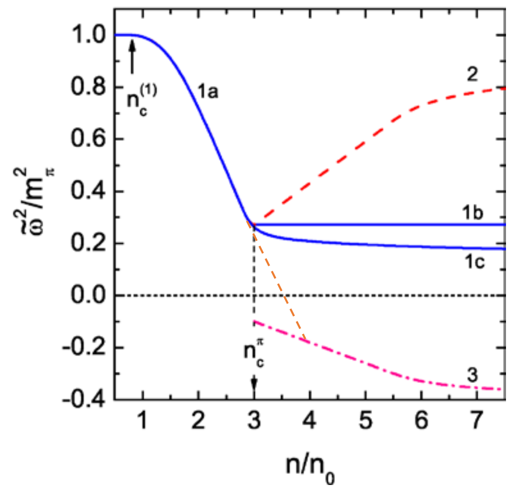
$$h = 4\pi\alpha_h k\phi_0\cos(kz) + H\,,$$

own periodic magnetic field h at H=0 for n>n_c^π

$$\overline{G}_{h,\phi} = -\frac{\widetilde{\omega}_0^4(n)}{6\lambda}\Theta[-\widetilde{\omega}_0^2(n)]\,.\quad \widetilde{\omega}_0^2(n) \propto n_c^\pi - n,$$

for $n > n_c^\pi$,

$$\widetilde{\omega}_0^2 = m_\pi^{*2} - \frac{m_\pi^2\xi^2}{4\alpha_2}\,,\qquad \xi = 1-\alpha_1-\boxed{4\pi\alpha_h^2}.$$



Significant difference compared with case of zero pion mass!

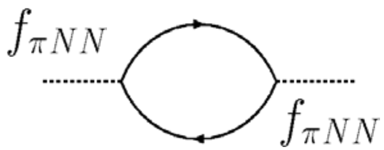
Nonequilibrium model of nonoverlapped Fermi spheres

Gyulassy, Greiner Ann.Phys. 109,1977, Pirner, D.V. Phys.Lett.B343, 1995

in peripheral HIC for E_{lab}>160 MeV for a while Fermi spheres of nucleons belonging to projectile and target nuclei are not overlapped in momentum space

$$f_{\rm tot} = f(\vec{p}) + f\boxed{(\vec{p} + \vec{p}_l + \vec{k})}$$

Factor $f(\vec{p}) f(\vec{p}+\vec{p}_l+\vec{k})$ vanishes for $p_l > 2p_{\rm F}(n/2)$ for $k \perp \vec{p}_l$.

In particle hole term: 

$$\boxed{p_{\rm F}(n)}\Phi(\omega, k, n) \rightarrow \boxed{p_{\rm F}(\tfrac{1}{2}n)}$$
$$\times[\Phi(\omega, k, \tfrac{1}{2}n) + \Phi(\omega - \frac{k p_l \cos\theta}{m_N^*}, k, \tfrac{1}{2}n)]$$

➡ **sharp angular dependence of NN amplitude.**

For $g' = 0$, $m_N^* = m_N$, $\vec{k} \perp \vec{p}_l$, extra πN attraction factor in particle-hole part of pion self-energy would be $2p_{\rm F}(n/2)/p_{\rm F}(n) \simeq 4^{1/3}$ that would correspond to effectively **4 times larger density** **favoring occurrence of the pion condensation already for n< n_0** in the model of the nonoverlapped Fermi spheres.
However Δ (1232) isobar and regular terms of pion self-energy **remain the same as in equilibrium** case, g' ≠0, and thereby effective enhancement is weaker (resulting in ~2 rather than 4- times enhanced effective density).

Nonoverlapped Fermi spheres

Pion mode in extremely dilute nuclear matter   x=**$n/n_0 \ll 1$**

Exactly solvable model

$$n_{c1} = \frac{n_0}{(\alpha^0_{(2)} - 2^{5/3}\boxed{g'_0})^3} \cdot \alpha^0_{(2)} = \frac{\boxed{2^{5/3}} f^2_{\pi N} m_N p_{F0}}{\pi^2}, \; p_F = p_{F0} x^{1/3}, \quad p_{F0}=p_F(n_0),$$

$2^{5/3}$ instead of 2 for equilibrium case

Taking into account NN correlations, $g_0'=1/3$ at $x=n/n_0 \ll 1$, Ericson, Weise book,

$n_{c1} \simeq (0.04 \div 0.05) n_0.$ at which **pion fluctuation effects at k ≠ 0** may start to appear

in peripheral heavy-ion collisions, resulting in observable effects, such as enhancement of pion distributions, especially for $\vec{k} \perp \vec{p}_l$,

In Model I in case of nonoverlapped Fermi spheres $n_c^\pi \sim 1.5 n_0$, in Model II, $n_c^\pi \sim 0.5\, n_0$

Model dependence is associated with uncertainty in the knowledge of off-shell behavior of s-wave πN interaction.

However already for $n>n_{c1}$ ~0.04-0.05 n_0 in case of nonoverlapped Fermi spheres the NN scattering amplitude shows **strong angular dependence**.

For $\vec{k}\vec{p}_l \neq 0$ there appears extra factor in NN amplitude

$$\gamma_2 = \frac{1 + p_l^2 p_{\mathrm{F}}^{-2} \cos^2\theta}{2} \,.$$

Such a dependence could be manifested in experimental distributions of pions and other particles undergoing NN collisions.

Conclusions

Models (I and II) were fitted to correctly treat at n ~ n_0 the region of the pion momenta $\omega \simeq m_\pi, k \ll m_\pi$ and the low pion energy domain $k \gtrsim m_\pi$ at $n \lesssim n_0$,
For equilibrium system (fully overlapped nucleon Fermi spheres) crystal-like π^0 condensation at T=0 occurs above 3 n_0 in Model I, and it occurs below 2 n_0 in Model II. **Model dependence!**

Pronounced pion fluctuations with k≠0 for T ≠0 (liquid phase of π condensate) appear in Model I for **$n>n_{c1}$~$0.5n_0$**, and in Model II already for **$n>0.25n_0$**, **$n_{c1} > n_{fo}$ at freezeout in HIC.**

Spin, magnetic moment and vorticity effects slightly decrease values **n_{c1}** and **n_c^π**
Main terms are **WZW anomaly** and ~ $f_{\pi NN}$ terms in-medium π N interaction term.

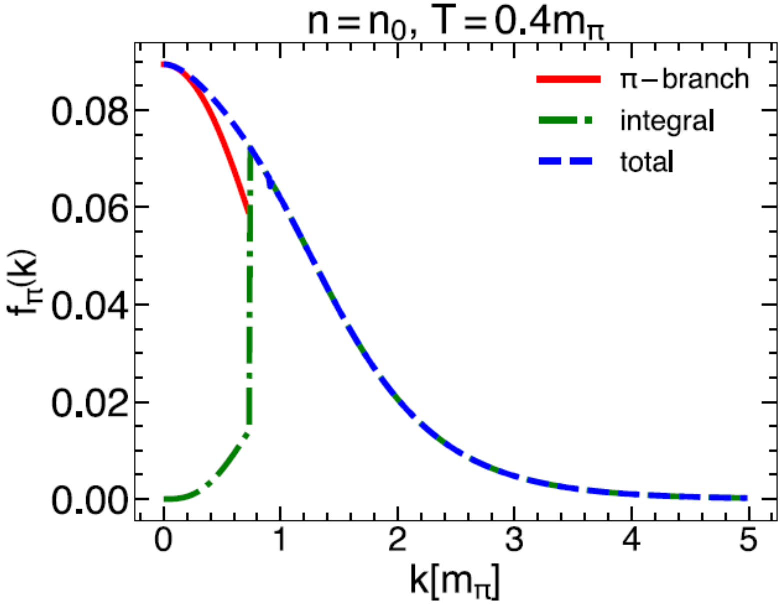
Then focus was made on **the possibility of manifestation of the π^0 condensation and magnetization in peripheral heavy-ion collisions for case of nonoverlapped** (only **for a while)** **Fermi spheres of nucleons belonging to projectile and target nuclei (for E_{lab}>160 MeV A).**

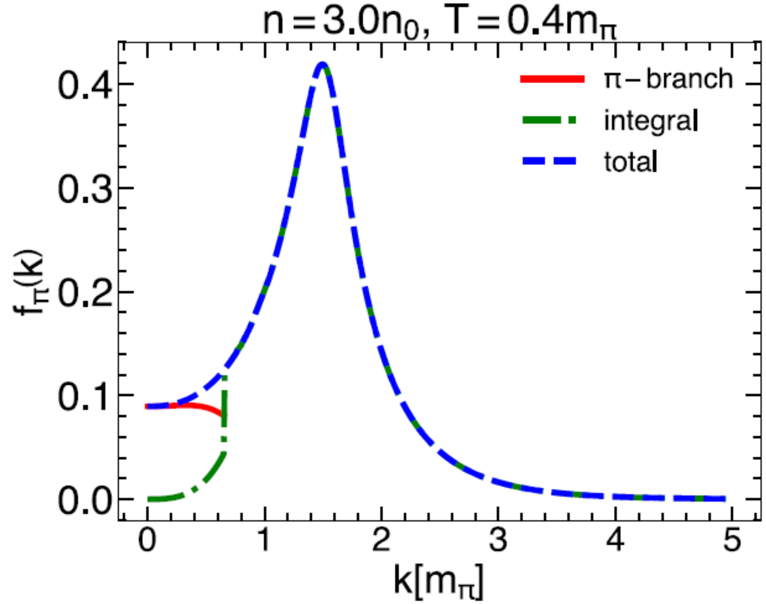
π^0 condensation occurs now at <1.5 n_0 in Model I, and at ~0.5 n_0 in Model II. **Self-rotation in isospin-asymmetrical case.**

In the model of nonoverlapped Fermi spheres of nucleons, n_{c1} ~0.04-0.05 n_0 **(almost model independent result!).** It may stimulate experimental search of effects of enhanced pion fluctuations at $k \neq 0, \vec{k} \perp \vec{p}_l,$ in peripheral heavy-ion collisions.
Especially it would be interesting to check the experimental data on presence or absence of the specific anisotropy in pion distributions for k≠0 as function of the angle $(\vec{k}\vec{p}_l)$

Pion distribution in matter

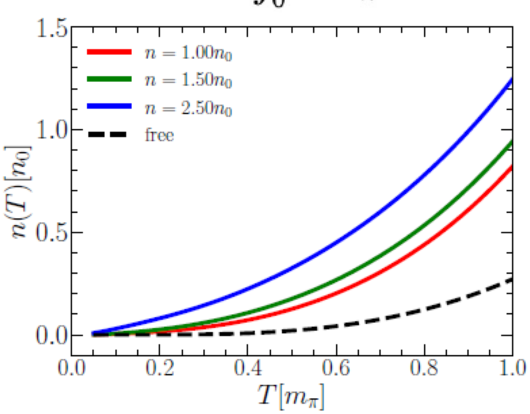
$$f_\pi(k) = \frac{1}{\pi}\int_0^\omega d\omega \, \frac{\omega}{e^{\frac{\omega}{T}}-1} A(\omega, k)$$





Although at $n \sim n_0$ and $T \neq 0$ spectrum visually looks rather similar to that for free pion, at $k>m_\pi$ main contribution to pion distribution comes from spectral function at low ω

$$n(T) = 3\int_0^\infty \frac{dk k^2}{2\pi^2} f_\pi(k) \, .$$



The higher n and T, the larger is difference with free pions