Transport properties of quark matter under rotation

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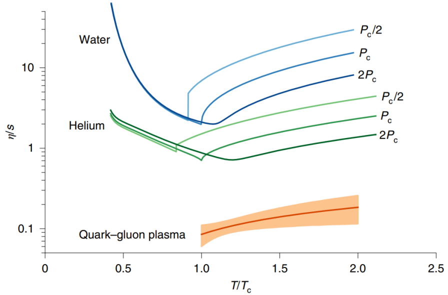
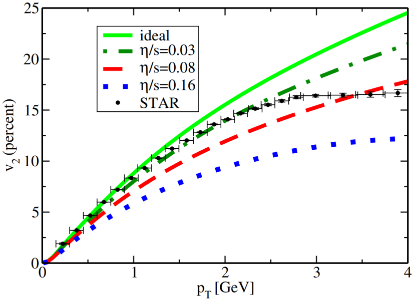
Overview

Motivation: Perfect fluid behavior of QGP



Nature Phys. **15**, 1113–1117 (2019),



PRL **99**, 172301 (2007)

- Specific Viscosity of QGP close to theoretical limit
- Elliptic flow is sensitive to Viscosity
- Transport coefficients are critical input parameters for Hydro evolution

Motivation: Global polarization and vorticity

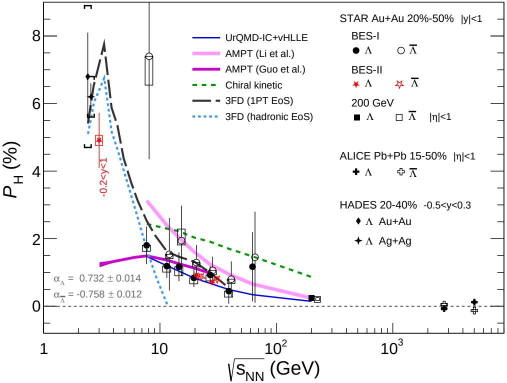


Figure: Λ and $\bar{\Lambda}$ polarization with collision energy. *Becattini et al.*, IJMPE, **33, 06**, 2430006 (2024)

The most vortical fluid, $\Omega \approx 10^{22} s^{-1} \approx 10$ MeV

In inertial frames (non-rotating)

Viscous stress tensor: (Navier-Stokes theory)

$$\tau^{\mu\nu} = \eta \quad \left[(\Delta^{\mu}_{\sigma}\Delta^{\nu}_{\tau} + \Delta^{\nu}_{\sigma}\Delta^{\mu}_{\tau}) - \frac{2}{3}\Delta^{\mu\nu}\Delta_{\sigma\tau} \right] \nabla^{\sigma} u^{\tau} - \ \zeta \ \Delta^{\mu\nu}\nabla_{\sigma} u^{\sigma}$$

In general, $\tau^{\mu\nu} = \eta^{\mu\nu\alpha\beta} V_{\alpha\beta}$,

$V_{\alpha\beta} = \frac{1}{2}(\frac{\partial U_{\alpha}}{\partial x_{\beta}} + \frac{\partial U_{\beta}}{\partial x_{\alpha}})$, $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^{\mu}u^{\nu}$, $\Delta^{\mu\nu}u_{\mu} = 0$.

$\eta^{\mu\nu\alpha\beta} \rightarrow u^{\mu}$ (fluid velocity), $\Delta^{\mu\nu}$, $g^{\mu\nu}$ (metric)

Hydro: Viscous stress (Rotating system)

Rotating System:

- Fluid cells with velocity u^μ, and angular velocity ($\vec{\Omega}$): independent
- Slightly out of equilibrium.
- Slow rotation, $\frac{\omega}{T} \ll 1$
- System under external magnetic field and rotating system posses the same symmetry

$$\eta^{\mu\nu\alpha\beta} \to \eta^{\mu\nu\alpha\beta}\left(u^\mu, \Delta^{\mu\nu}, g^{\mu\nu}, \omega^\mu, \omega^{\mu\nu}\right)$$

Where, $\omega^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\omega_\alpha\omega_\beta$, ω^μ is the direction of angular velocity.

To construct $\eta^{\mu\nu\alpha\beta}$, following condition must satisfy:

- Symmetric stress tensor: η symmetric under $\mu \leftrightarrow \nu$, $\alpha \leftrightarrow \beta$
- Onsager symmetry: $\eta^{\mu\nu\alpha\beta}(\omega^\gamma) = \eta^{\alpha\beta\mu\nu}(-\omega^\gamma)$
- Orthogonality: $u_\mu \tau^{\mu\nu} = 0$.

Hydro: Viscosity coefficient matrix

Satisfying the symmetry conditions, we can construct 7 independent components

$$
\begin{aligned}
i)\ & \Delta^{\mu\nu}\Delta^{\alpha\beta}, \\
ii)\ & \Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\nu\alpha}\Delta^{\mu\beta}, \\
iii)\ & \Delta^{\mu\nu}\omega^{\alpha}\omega^{\beta} + \Delta^{\alpha\beta}\omega^{\mu}\omega^{\nu}, \\
iv)\ & \omega^{\mu}\omega^{\nu}\omega^{\alpha}\omega^{\beta}, \\
v)\ & \Delta^{\mu\alpha}\omega^{\nu}\omega^{\beta} + \Delta^{\nu\beta}\omega^{\mu}\omega^{\alpha} + \Delta^{\mu\beta}\omega^{\nu}\omega^{\alpha} + \Delta^{\nu\alpha}\omega^{\mu}\omega^{\beta}, \\
vi)\ & \Delta^{\mu\alpha}\omega^{\nu\beta} + \Delta^{\nu\beta}\omega^{\mu\alpha} + \Delta^{\mu\beta}\omega^{\nu\alpha} + \Delta^{\nu\alpha}\omega^{\mu\beta}, \\
vii)\ & \omega^{\mu\alpha}\omega^{\nu}\omega^{\beta} + \omega^{\nu\beta}\omega^{\mu}\omega^{\alpha} + \omega^{\mu\beta}\omega^{\nu}\omega^{\alpha} + \omega^{\nu\alpha}\omega^{\mu}\omega^{\beta}.
\end{aligned}
$$

Hydro: Viscosity coefficient matrix

5 Traceless components → Shear Viscosity (η).
2 Non-zero trace components → Bulk Viscosity (ζ).

Traceless:

$$C^0_{ijkl} = (3\omega_i\omega_j - \delta_{ij})\left(\omega_k\omega_l - \frac{1}{3}\delta_{kl}\right)$$

$$C^1_{ijkl} = \delta_{il}\delta_{jk} + \delta_{jl}\delta_{ik} - \delta_{ij}\delta_{kl} + \delta_{ij}\omega_k\omega_l - \delta_{jl}\omega_i\omega_k - \delta_{jk}\omega_i\omega_l + \delta_{kl}\omega_i\omega_j - \delta_{ik}\omega_j\omega_l - \delta_{il}\omega_j\omega_k + \omega_i\omega_j\omega_k\omega_l$$

$$C^2_{ijkl} = \delta_{ik}\omega_j\omega_l + \delta_{il}\omega_j\omega_k + \delta_{jk}\omega_i\omega_l + \delta_{jl}\omega_i\omega_k - 4\omega_i\omega_j\omega_k\omega_l$$

$$C^3_{ijkl} = \delta_{il}\omega_{jk} + \delta_{jl}\omega_{ik} - \omega_{ik}\omega_j\omega_l - \omega_{jk}\omega_i\omega_l$$

$$C^4_{ijkl} = \omega_{ik}\omega_j\omega_l + \omega_{il}\omega_j\omega_k + \omega_{jk}\omega_i\omega_l + \omega_{jl}\omega_i\omega_k$$

Non-Zero Trace:

$$C^5_{ijkl} = \delta_{ij}\delta_{kl}, \qquad C^6_{ijkl} = \delta_{ij}\omega_k\omega_l + \delta_{kl}\omega_i\omega_j$$

Viscous stress tensor: $\tau_{ij} = \sum_{n=0}^{4} \eta_n C^n_{ijkl} V_{kl} + \zeta_0 C^n_{ijkl} V_{kl} + \zeta_1 C^n_{ijkl} V_{kl}$

Kinetic theory

$$\text{Viscous Stress}: \tau_{ij} = N_f N_c N_s \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p_i p_j}{E} \delta f$$

$$\text{Charge Current}: J_i = q\, N_f N_c N_s \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p_i}{E} \delta f$$

$$\text{Distribution function, } f = f(x^\mu, p^\mu) = f_0 + \delta f$$

$$f_0 = \frac{1}{\exp\left[\frac{p^\mu \cdot u_\mu(x) - \mu(x)}{T(x)}\right] \pm 1}, \quad P^\mu = (E, \vec{p})$$

δf : Solve Boltzmann transport equation.

In rotating frame: (about z axis)

$$g_{\mu\nu} = \begin{pmatrix} 1 - \Omega^2 x^2 - \Omega^2 y^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Equation of Motion:

$$\frac{dp^\alpha}{d\tau} + \frac{1}{m} p^\mu p^\nu \Gamma^\alpha_{\mu\nu} = F^\alpha, \quad \Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\frac{\partial g_{\nu\beta}}{\partial x^\mu} + \frac{\partial g_{\beta\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta} \right)$$

$\Gamma^1_{00} = -\Omega^2 x$, $\Gamma^2_{00} = -\Omega^2 y$, $\Gamma^1_{20} = -\Omega$, $\Gamma^2_{10} = \Omega$, Zero otherwise.

For external force $F^\alpha = 0$, $\quad \frac{d\vec{p}}{dt} = \gamma \left[m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} + 2m(\vec{v} \times \vec{\Omega}) \right]$

Force terms: Centrifugal and Coriolis, as in Classical EoM.

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Boltzmann equation and RTA

Modified Boltzmann equation:

$$p \cdot \partial f(x^\mu, p^\nu) = \mathcal{C}[f]$$

$$p^\mu \frac{\partial f}{\partial x^\mu} - p^\mu p^\beta \, \Gamma^\alpha_{\mu\beta} \, \frac{\partial f}{\partial p^\alpha} + g^{\mu\alpha} \, M\partial_\alpha M \, \frac{\partial f}{\partial p^\mu} + qF^{\beta\alpha} p_\alpha \, \frac{\partial f}{\partial p^\beta} = \mathcal{C}[f]$$

Free Streaming term, Rotation, Temperature dependent mass,
Electromagnetic force term

Relaxation time approximation (RTA): $\mathcal{C}(f) = -\frac{p^\mu u_\mu}{E}\frac{\delta f}{\tau_c}$,

ansatz, $\delta f \equiv \delta f \, (\text{relevant gradient forces} : \eta_{\mu\nu\alpha\beta}, \ \sigma_{\mu\nu}, ..)$

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Transport coefficients

Solving RBE we can get:

$$\eta_n = \frac{N_f N_c N_s}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}^2}{E}\right)^2 \tau_c^{(n)} f_0(1 - f_0); \;\; n = 0, ..4$$

$$\sigma_n = \frac{q^2 \; N_f N_c N_s}{9T} \int \frac{d^3 p}{(2\pi)^3} \frac{\tau_c \left(\frac{\tau_c}{\tau_\Omega}\right)^n}{1 + \left(\frac{\tau_c}{\tau_\Omega}\right)^2} \times \frac{p^2}{E^2} f^0(1 - f^0); \;\; n = 0, 1, 2$$

Effective relaxation times: $\tau_c^{(0)} = \tau_c$, $\tau_c^{(1)} = \frac{\tau_c}{1+4(\tau_c/\tau_\Omega)^2}$, $\tau_c^{(2)} = \frac{\tau_c}{1+(\tau_c/\tau_\Omega)^2}$,

$\tau_c^{(3)} = 2\tau_c \frac{\tau_c/\tau_\Omega}{1+4(\tau_c/\tau_\Omega)^2}$, $\tau_c^{(4)} = \tau_c \frac{\tau_c/\tau_\Omega}{1+(\tau_c/\tau_\Omega)^2}$; $\tau_\Omega = \frac{1}{2\Omega} \equiv$ Rotational timescale.

Conductivity Tensor: $\sigma_{ij} = \sigma_0 \delta_{ij} + \sigma_1 \epsilon_{ijk} \omega_k + \sigma_2 \omega_i \omega_j$

Nambu-Jona–Lasinio model for rotating system

Jiang and Liao, **Phys. Rev. Lett. 117, 192302 (2016)**

Two flavor, isospin symmetric NJL:

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m_0]\psi + G_s[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\sigma}\psi)^2],$$

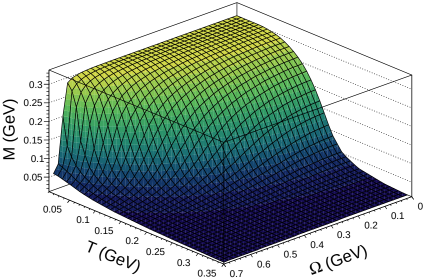
Spinorial affine connection, $\Gamma_\mu = \frac{1}{8}\omega_{\mu ab}[\gamma^a, \gamma^b]$; $\omega_{\mu ab} \equiv$ Spin connection

$$\text{mean field approx. : } \mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - M]\psi - \frac{(M - m_0)^2}{4G_S},$$

$$\text{Constituent quark mass: } M \equiv m_0 - 2G_S\langle\bar{\psi}\psi\rangle$$

The **gap equation** can also be obtain by minimizing the thermodynamic potential with respect to constituent mass.

$$M = m_0 + \frac{G_S N_c N_f}{4\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\Lambda} dp_\perp^2 \int_{-\sqrt{\Lambda^2 - p_\perp^2}}^{\sqrt{\Lambda^2 - p_\perp^2}} dp_z \left(J_n^2(p_\perp \rho) + J_{n+1}^2(p_\perp \rho) \right) \frac{M}{E}$$
$$- \frac{G_S N_c N_f}{4\pi^2} \sum_{n=-\infty}^{\infty} \int dp_\perp^2 \int dp_z \left(J_n^2(p_\perp \rho) + J_{n+1}^2(p_\perp \rho) \right) \frac{M}{E} \left[\frac{1}{e^{\beta(E-(n+\frac{1}{2})\Omega)} + 1} + \frac{1}{e^{\beta(E+(n+\frac{1}{2})\Omega)} + 1} \right].$$



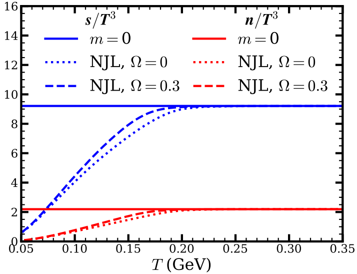


Figure: Left: Constituent quark mass at $\rho = 0.1$ GeV^{-1}, Right: Entropy density and number density with temperature. $\rho\Omega < 1$, $\Lambda = 651$ MeV, $G_S = 5.04$ GeV^{-2}.

Results: Transport coefficients

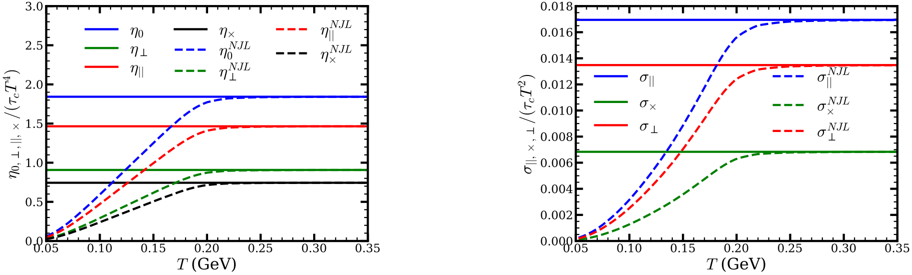


Figure: Left: Shear Viscosity, Right: Electrical conductivity at $\Omega = 0.01$ GeV and $\tau_c = 5$ fm

- Anisotropy is prominent
- At high temperature massless, non-interacting results recovered with chiral restoration
- Substantial contribution from Hall-like component
- Suppressed viscosity/ conductivity: Affect of rotating EoM

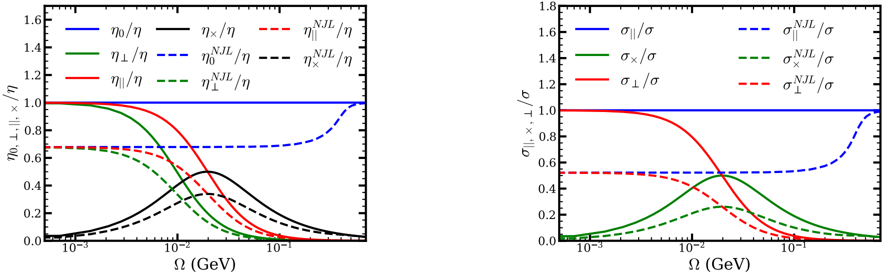


Figure: Left: Shear Viscosity, Right: Electrical conductivity at $T = 0.150$ GeV and $\tau_c = 5$ fm

- Anisotropy vanishes with decreasing rotation.
- At high ω massless results recover due to chiral restoration
- Peak in the Hall-like components arises from term: $\frac{\tau_c/\tau_\Omega}{1+(\tau_c/\tau_\Omega)^2}$.

Summary

Threefold affect of rotation:

1) Anisotropy: Multi-component transports

2) Effective relaxation time, rotational timescale: Reduction in value of transport.

3) Thermodynamic EoS and constituent quark mass: Enhancement in value of transport

Effective relaxation time wins the competition and reduces the net transport value.

Shortcomings and improvements

- Rotating NJL result with **fixed coupling constant** contradicts with lattice estimation.
- Incorporate Ω dependent coupling constant consistent with lattice and study transport.

Thank you for attention!