Weakening of the deconfinement phase transition in an external gravitational field

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Outline

- **Motivation**
- **Work plan**
- **The analogy with temperature.**
- **Acceleration on the lattice**
- **Results**
- **Conclusions**

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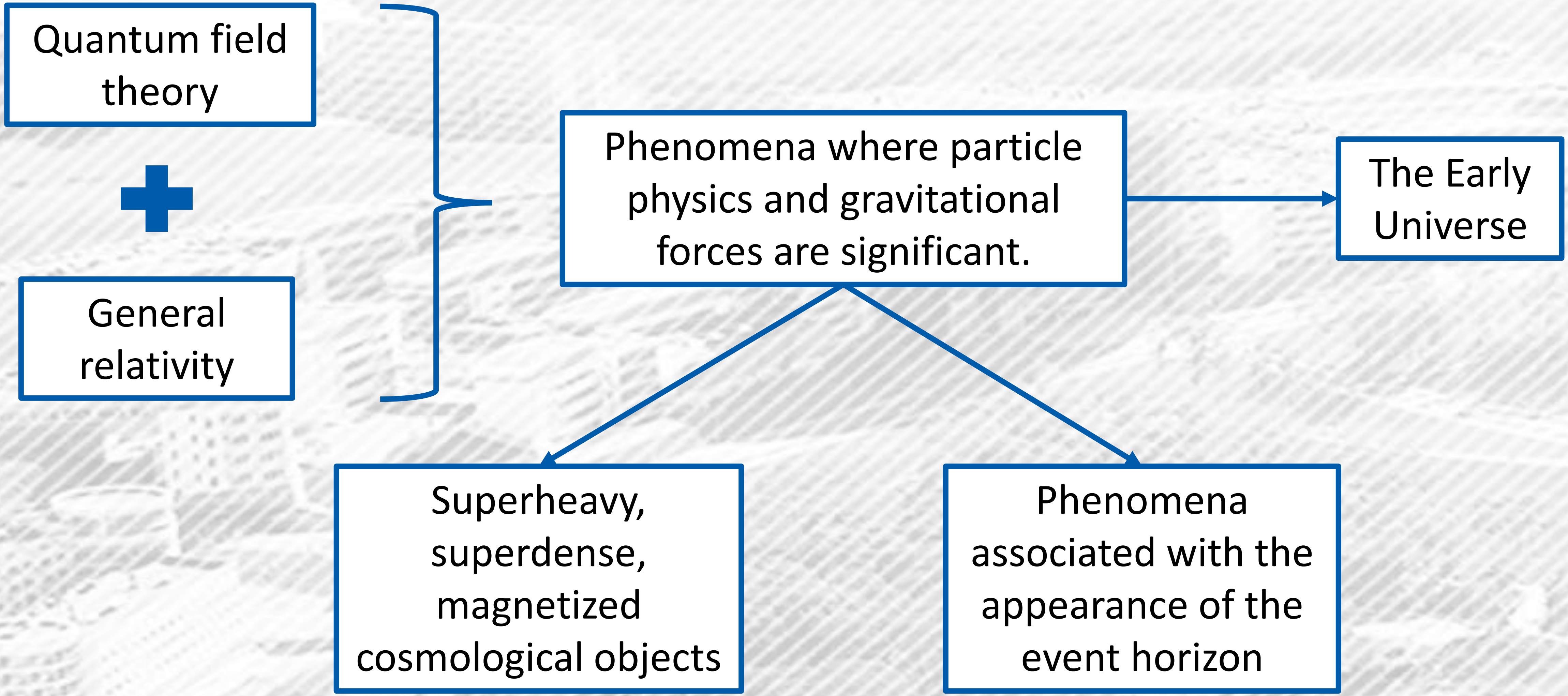
Motivation


Quantum field theory
General relativity
Phenomena where particle physics and gravitational forces are significant.
The Early Universe
Superheavy, superdense, magnetized cosmological objects
Phenomena associated with the appearance of the event horizon

Work plan

Pure gluodynamics

+

Acceleration

Deconfinement phase transition ?

Approach

Lattice quantum field theory

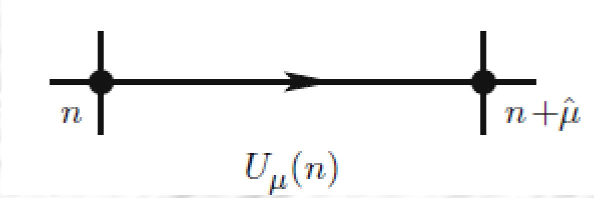
Tasks:

- Obtaining a theoretical description of acceleration within the framework of the lattice approach.
- Data generation for theory with zero acceleration.
- Data generation for theory with non-zero acceleration.
- Data analysis and processing.

SU(3) gauge field theory on the lattice

Link:

$$U_\mu(n) = \exp(iaA_\mu(n))$$



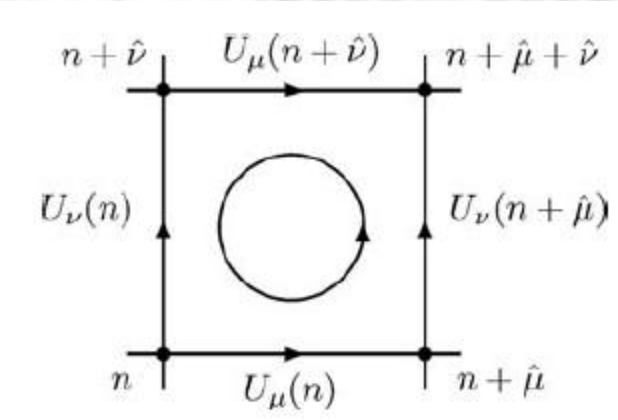
Action:

$$S[U] = \frac{2}{g^2} \sum_{n \in \lambda} \sum_{\mu<\nu} Re\ tr[\mathbb{1} - U_{\mu\nu}(n)]$$

$$S[A] = \frac{a^4}{2g^2} \sum_{n \in \lambda} \sum_{\mu,\nu} tr[F_{\mu\nu}(n)^2]$$

Plaquette:

$$U_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu(n+\hat{\nu})^\dagger U_\nu(n)^\dagger$$



Observables :

Polyakov Loop:

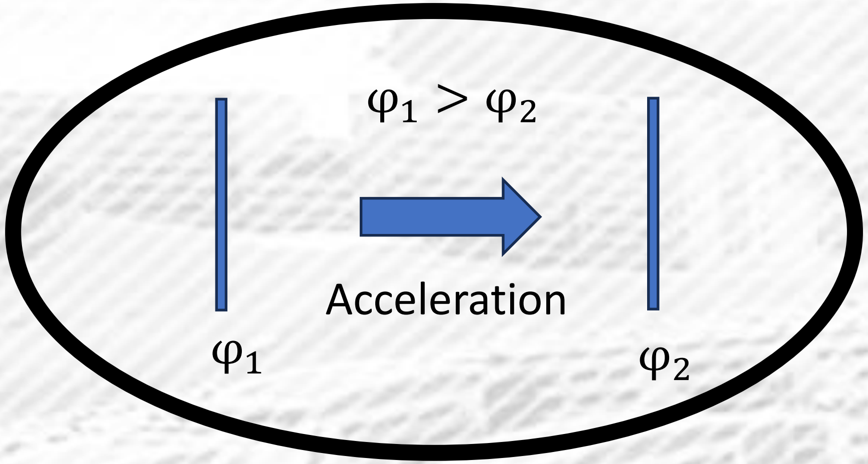
$$L(z) = \langle |L(\boldsymbol{x})| \rangle_{x,y}$$

$$L(\boldsymbol{x}) = \frac{1}{3} Re\ tr \left[\prod_{t=0}^{N_\tau - 1} U_\tau(\boldsymbol{x}, t) \right]$$

Susceptibility of the Polyakov loop:

$$\chi(z) = \langle |L(\boldsymbol{x})|^2 \rangle_{x,y} - \langle |L(\boldsymbol{x})| \rangle^2{}_{x,y}$$

The analogy with temperature.



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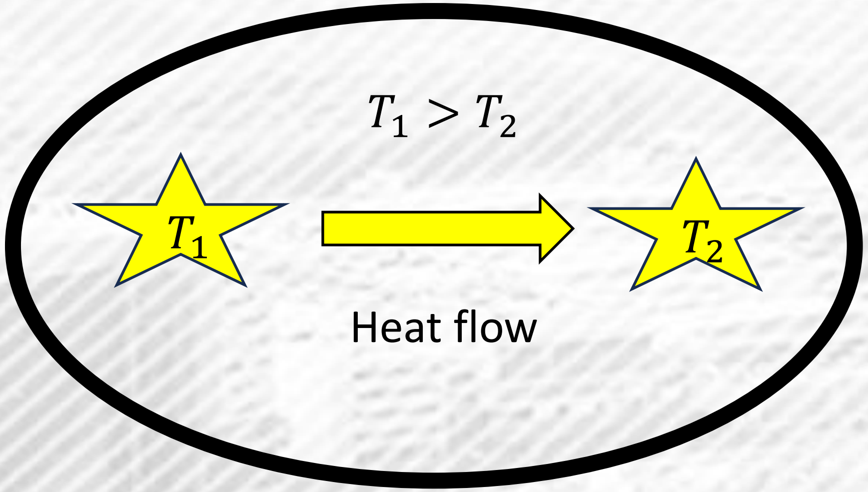


The presence of a gravitational field => thermodynamic equilibrium with inhomogeneous temperature [1,2,3]:

$$T(z)\sqrt{g_{00}} = T_0 = const$$

$$T(z) = \frac{T_0}{1 + a(z - z_0)}$$

[1] Tolman R. C. On the Weight of Heat and Thermal Equilibrium in General Relativity // Phys. Rev. — 1930. — Vol. 35. — P. 904–924

[2] Tolman C., Ehrenfest P. Temperature Equilibrium in a Static Gravitational Field // Phys. Rev. — 1930. — Vol. 36. — P. 1791–1798

[3] Luttinger J. M. Theory of Thermal Transport Coefficients// Phys. Rev. — 1964. — Vol. 135. — P. A1505–A1514

Acceleration on the lattice

Temperature:

$$T = \frac{1}{N_\tau \cdot a_\tau}$$

We need to realize $\begin{cases} a_\tau = a_\tau(z) \\ a_\sigma \neq a_\sigma(z) \\ a_\sigma = a_\tau(z_0) = a_0 \end{cases}$

Action [4] :

$$S[U] = \sum_{x} \sum_{i>j}^{3} \beta_\sigma(z)\, Re\, tr\left[\mathbb{1} - U_{ij}(\boldsymbol{x})\right] + \sum_{x} \sum_{i=1}^{3} \beta_\tau(z)\, Re\, tr[\mathbb{1} - U_{4i}(\boldsymbol{x})])$$

Anisotropy coefficient:

$$\xi(z) = \frac{a_\sigma}{a_\tau(z)} = \frac{1}{1 + a(z - z_0)}$$

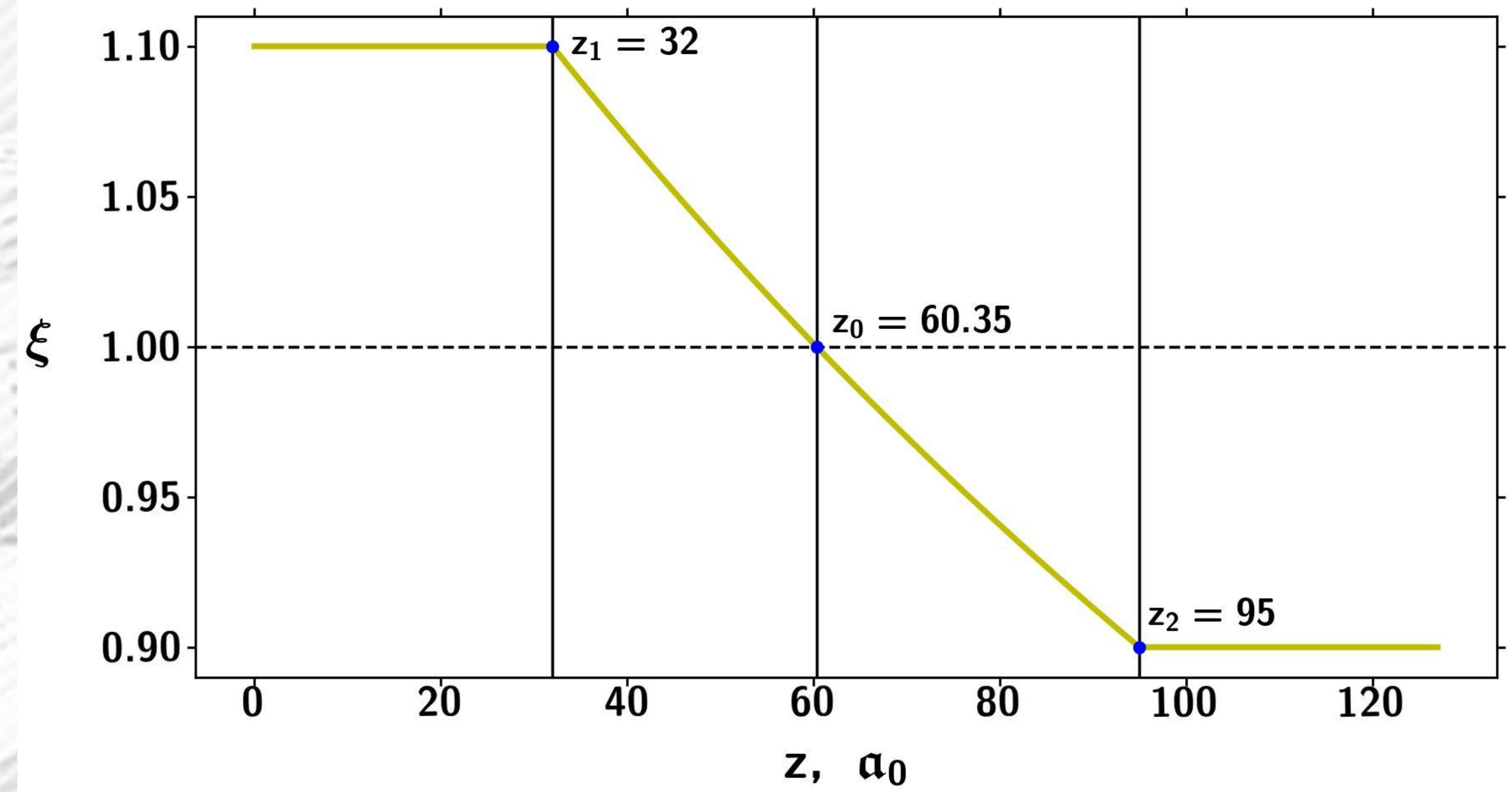
Acceleration:

$$a = \left(\frac{1}{\xi_{min}} - \frac{1}{\xi_{max}}\right) / (z_2 - z_1)$$

[4] Karsch F. SU(N) Gauge Theory Couplings on Asymmetric Lattices // Nucl. Phys. B. — 1982. — Vol. 205. — P. 285–300.

Temperature profile

$$\xi(z) = \frac{T(z)}{T_0}$$



Polyakov loop

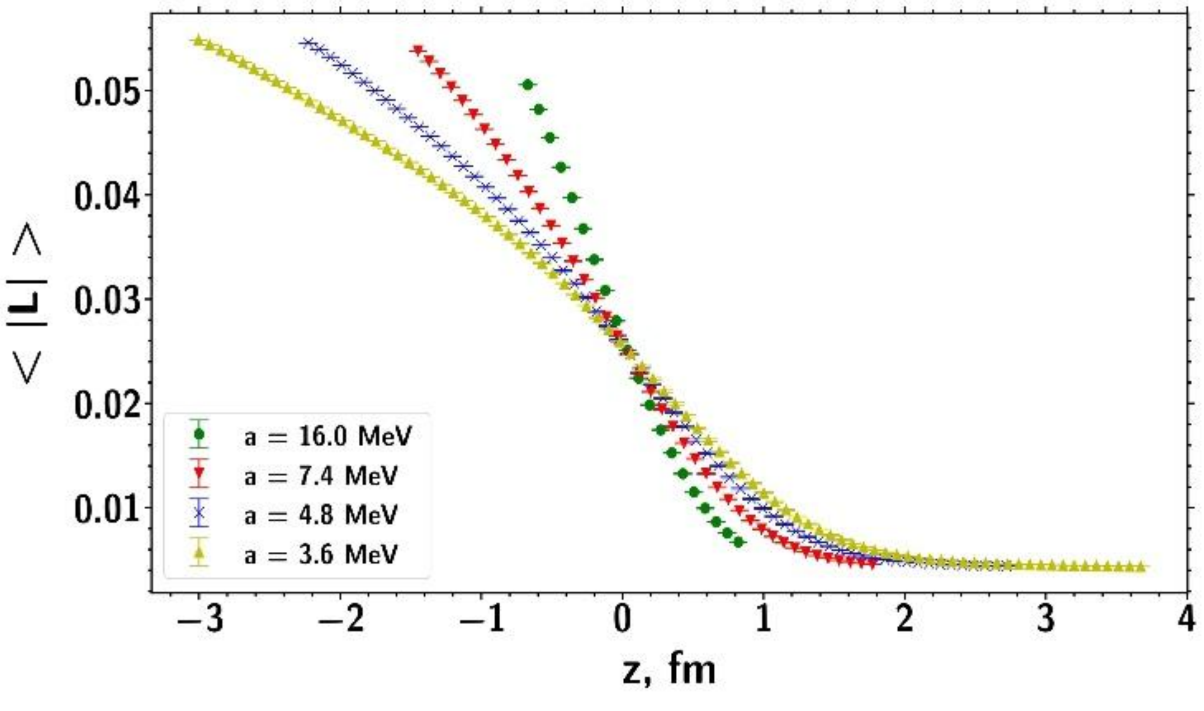


Fig. 1. The average value of the Polyakov loop for different accelerations along the z-direction for lattices with lattice sizes $8 \cdot 84^2 \cdot N_z$. ($N_z = 104 - 170$)

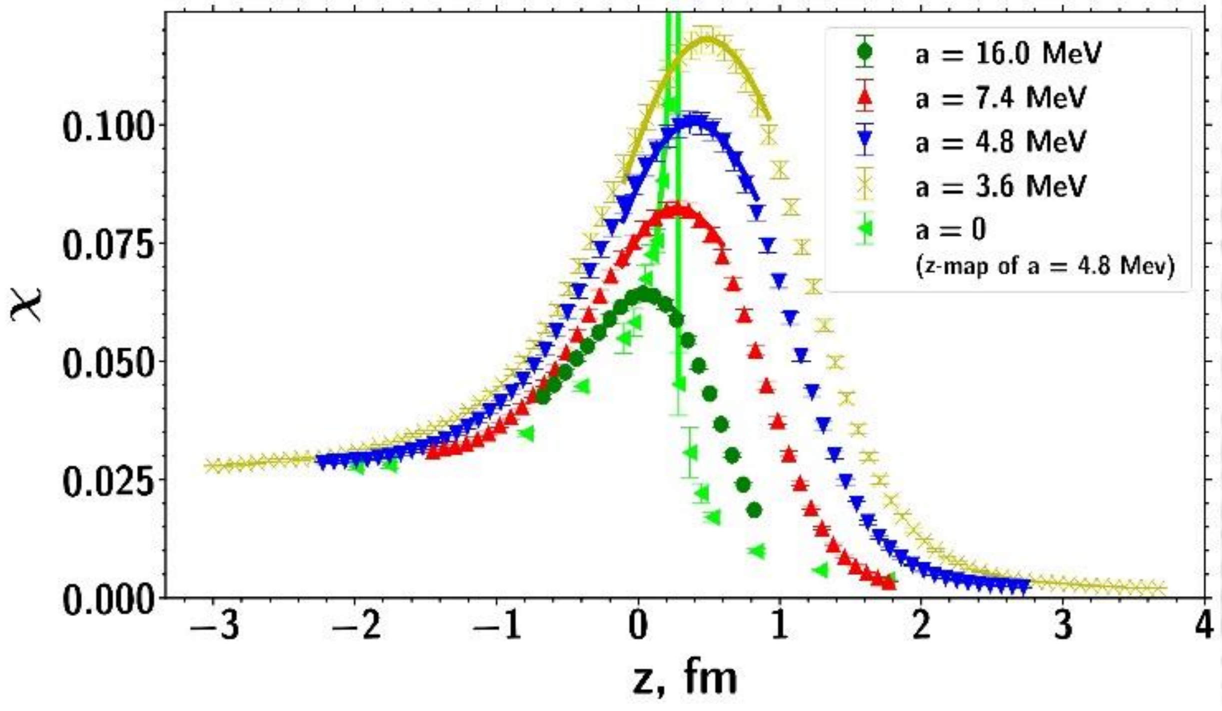
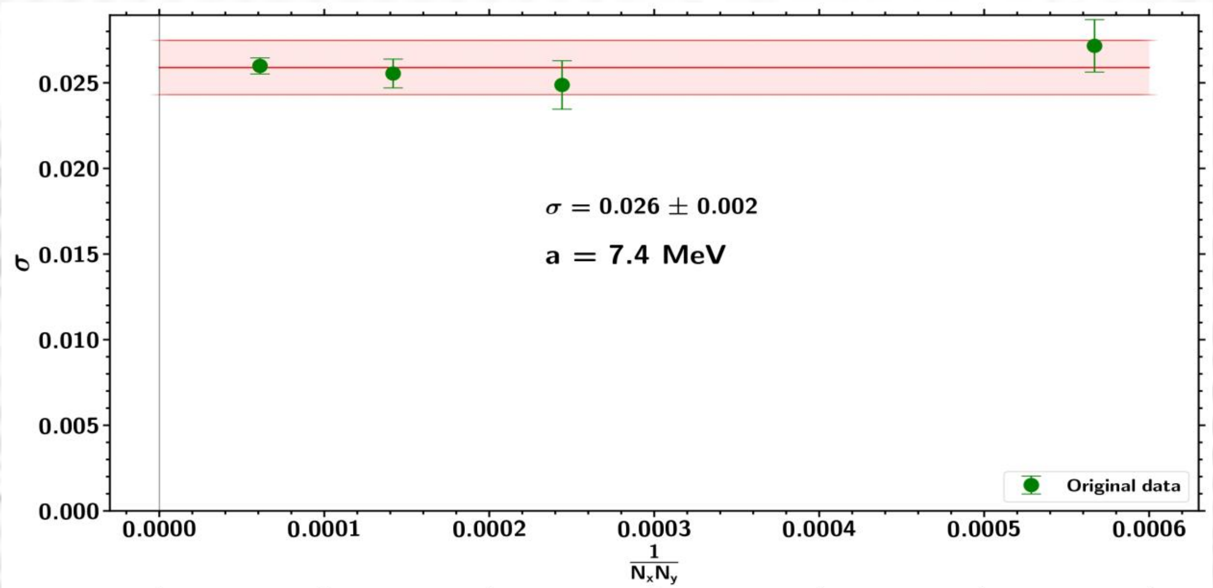
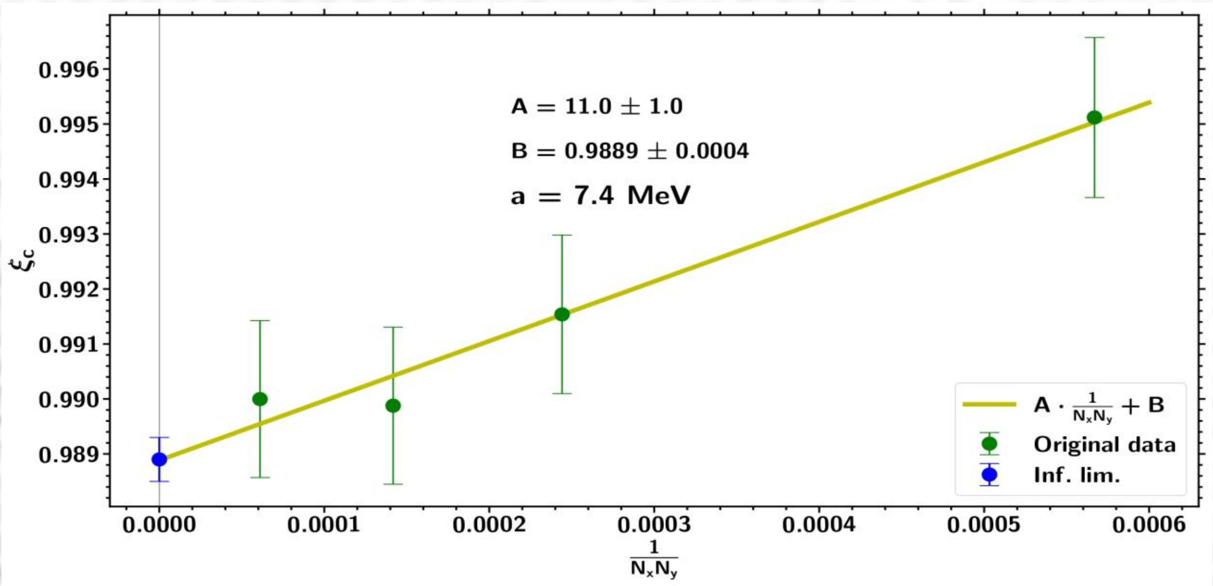
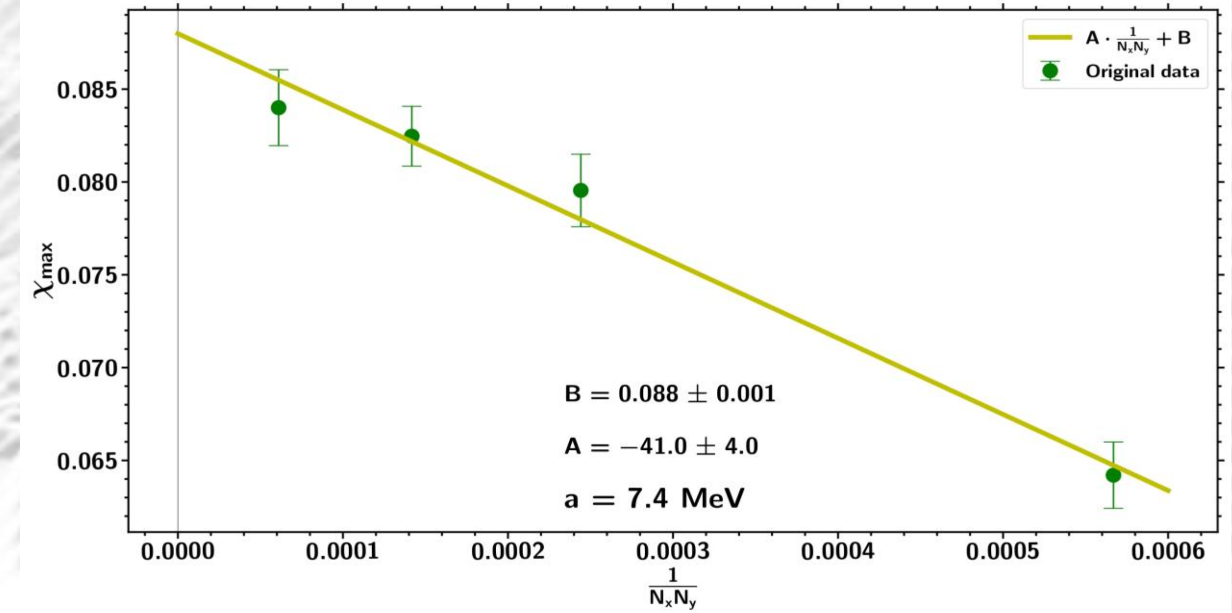


Fig. 2. Average value of the susceptibility of the Polyakov loop for different accelerations along the z-direction for lattices with lattice sizes $8 \cdot 84^2 \cdot N_z$. ($N_z = 104 - 170$)

Infinite volume limit







Fit of susceptibility of the Polyakov loop:

$$\chi(z) = \chi_{max} \cdot exp\left(-\frac{(\xi - \xi_c)^2}{2\sigma^2}\right)$$

- ξ_c, χ_{max} – Linear with volume^{-1}, finite
- σ – const.

Phase structure

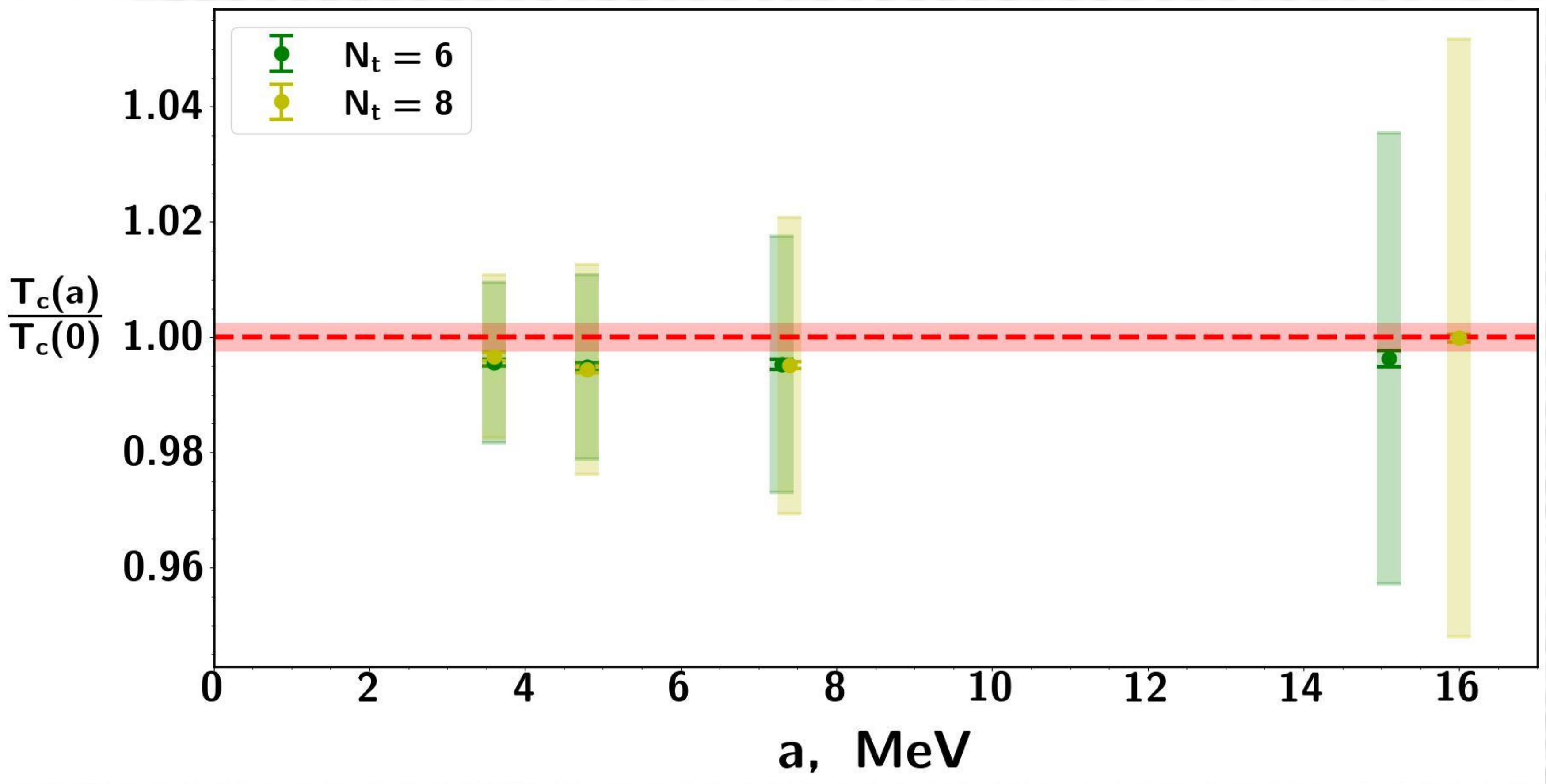
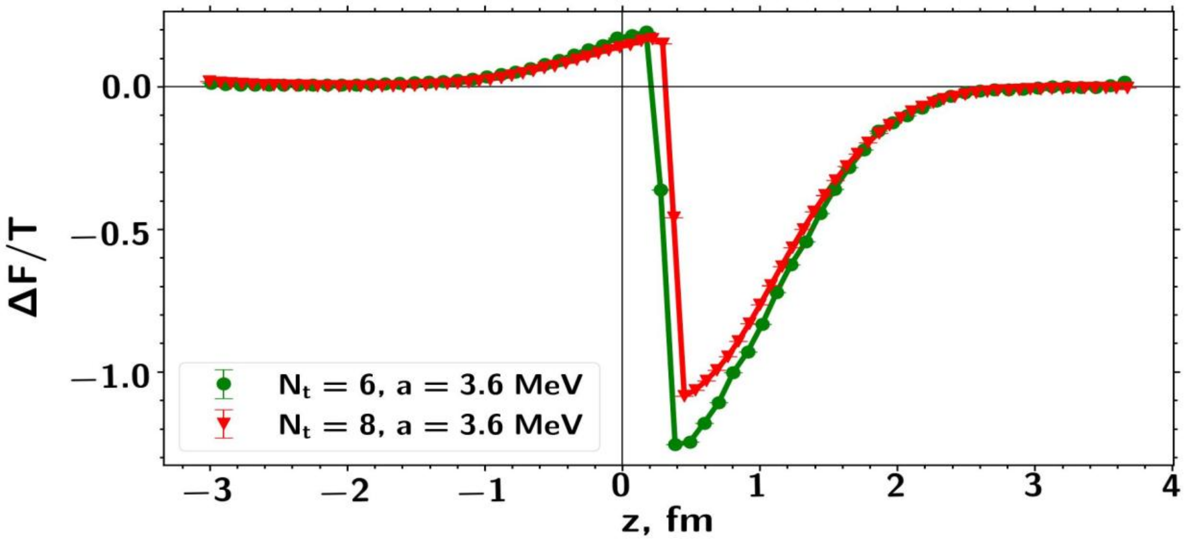
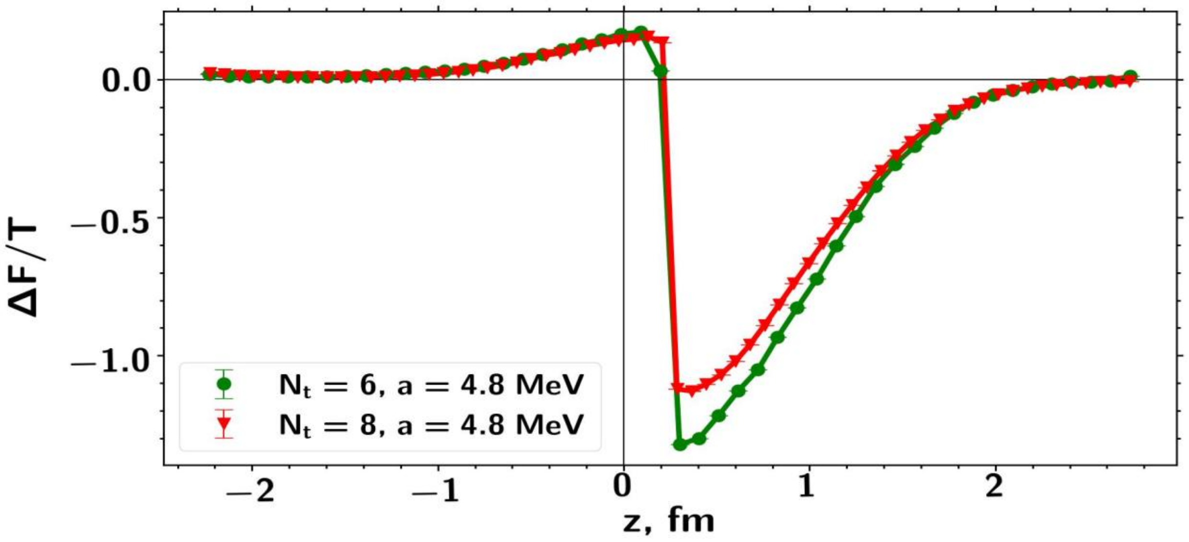
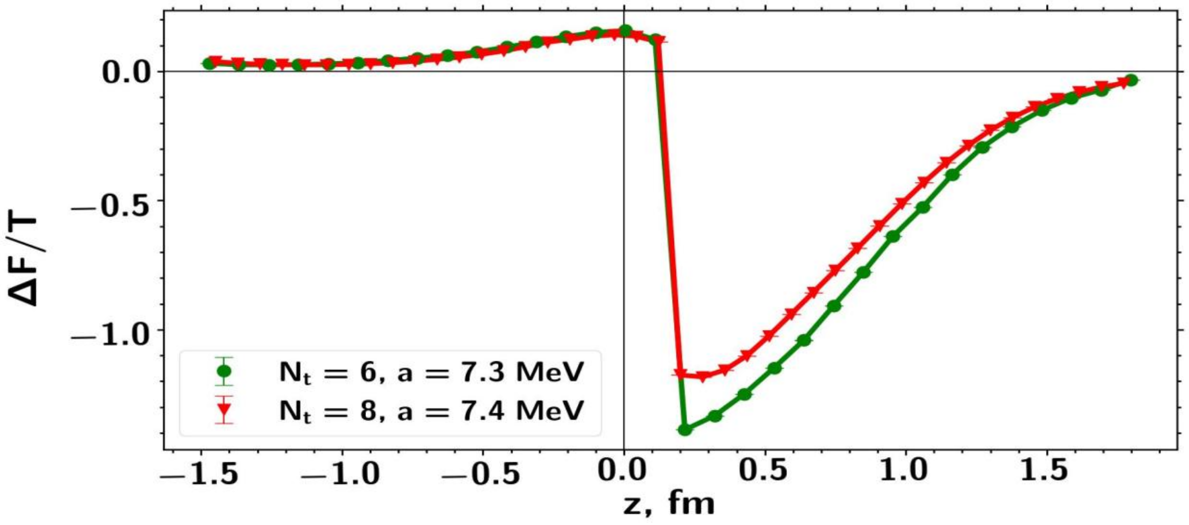


Fig. 3. Phase diagram of the hot gluon matter under acceleration in the (a,T) plane for lattices with a temporal lattice size $N_t = 6, 8$. The data are presented for the infinite volume limit $N_{x,y} \to \infty$.

Free energy







$$L(z) \sim e^{-\frac{F(z)}{T}}$$

$$\Delta F(z) = -T_{bulk} \, ln \frac{L_{acc}(z,a)}{L_{bulk}(T_{bulk})}\Bigg|_{T_{bulk}=T(z,a)}$$

Fig. 4. The difference between the free energy of an accelerated and a static heavy quark for different accelerations for lattices with $N_\tau = 6, 8$.

Conclusions:

1. The presence of acceleration "smooths" the phase transition, turning it from a first-order phase transition into a smooth crossover.
2. As the acceleration value increases, so the width of the phase transition does.
3. The critical temperature value does not depend on the acceleration value and remains constant.

An article was published based on the results of this work:

Chernodub M. N., Goy V. A., Molochkov A. V., Stepanov D. V., Pochinok A. S. Extreme Softening of QCD Phase Transition under Weak Acceleration: First-Principles Monte Carlo Results for Gluon Plasma// Phys. Rev. Lett. – 2025. – Vol. 134 – p. 111904

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Some notes:

Inverse temperature four-vector:

$$\beta^\mu(x) \equiv u^\mu(x)/T(x)$$

Local fluid velocity

Killing equation:

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$$

Appropriate solution:

$$\beta^\mu(x)\partial_\mu = (1/T_0)[(1 + a_0 z)\partial_t + a_0 t \partial_z]$$

Local Temperature:

$$T(t,z) = \frac{T_0}{\sqrt{(1 + a_0 z)^2 - (a_0 t)^2}}$$