Dynamical description of strongly interacting matter

The PHSD-PHQMD Team

PHSD

PHQMD

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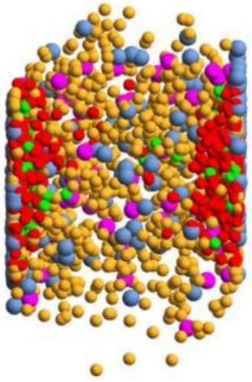
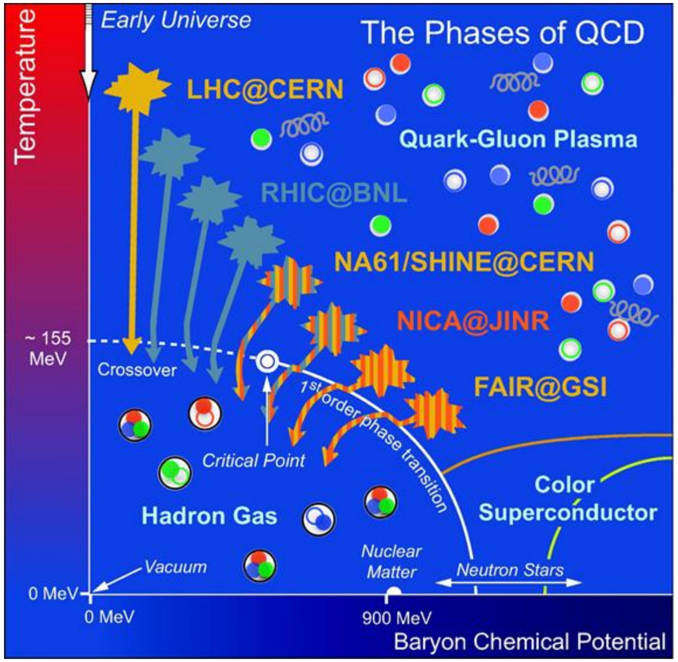
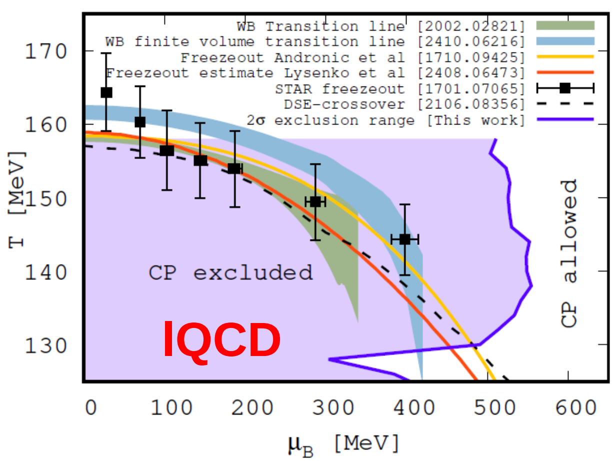


Key questions of HICs at NICA energies:

The phase diagram of QCD





- What are the **properties of the hot and dense matter** created in HICs?
- What are **the degrees-of-freedom**, their properties and interactions?

  QGP: strongly interacting liquid
  → non-perturbative QCD

  Hadronic matter: higly compressed and hot medium
  → chiral symmetry restoration effects

- Origin of the **phase transition**:
  crossover → ? → 1st order?!
- **Strong electromagnetic fields** are created during the HICs
  → polarization phenomena

NICA is located in a very interesting energy range !

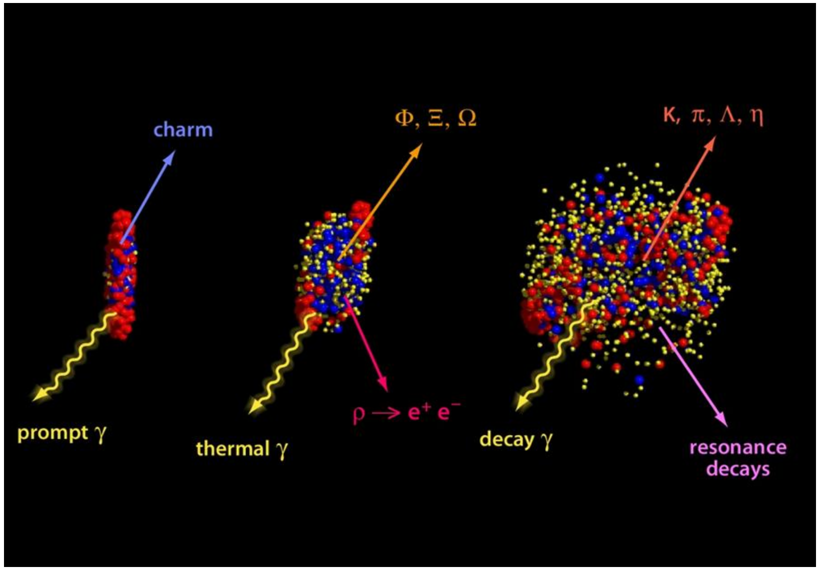
Jana N. Guenther, QM'2025

Experimental observables:

What are the experimental observables ?

- ‚bulk‘ observables - multiplicities,
  y-, p_T - spectra, flow coefficients v_n
- electromagnetic observables –
  dileptons and photons
- clusters and hypernuclei production
- hard probes – open and hidden charm

....



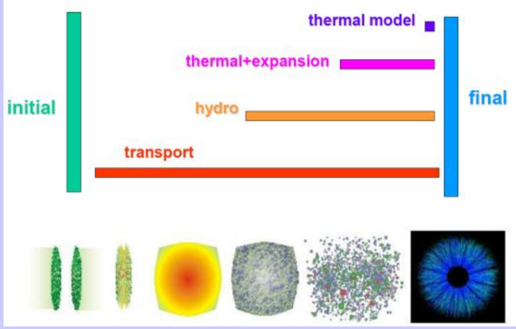
What are the systems to study ?

- **elementary pp and pn reactions:**
  of fundamental interest + provide a ‚reference frame‘ (i.e. input information) for the study of heavy-ion collisions
- **pA (and πA) reaction:** cold nuclear matter effects
- **light AA → heavy AA:** many-body effects, isospin phenonena, EoS, critical point(?), properties of strongly interacting QCD matter

Way to study:

Experimental energy scan of differential observables in comparison with theory

Basic models for heavy-ion collisions

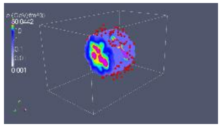


- <u>Statistical models:</u>

basic assumption: system is described by a (grand) canonical ensemble of non-interacting fermions and bosons in **thermal and chemical equilibrium** **= thermal hadron gas at freeze-out** with common T and μ_B

[- : no dynamical information]

- <u>Hydrodynamical models :</u>



basic assumption: conservation laws + equation of state (EoS); assumption of **local thermal and chemical equilibrium**

- Interactions are ‚hidden' in properties of the **fluid** described by **transport coefficients** (shear and bulk viscosity η, ζ, ..), which is **'input'** for the hydro models

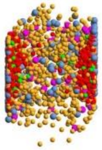
[- : simplified dynamics]

- <u>Microscopic transport models:</u>

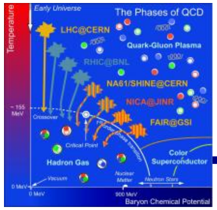
based on transport theory of relativistic quantum many-body systems

- **Explicitly account for the interactions of all degrees of freedom** (hadrons and partons) in terms of cross sections and potentials
- Provide a unique dynamical description of **strongly interaction matter in- and out-off equilibrium**:
- **In-equilibrium:** transport coefficients are calculated in a box – controled by lQCD
- **Nonequilibrium dynamics** – controled by HICs

Actual solutions: Monte Carlo simulations



[+ : full dynamics | - : very complicated]



Dynamical description of strongly interacting matter

Goal: Microscopic modeling of heavy-ion collisions

PHSD & PHQMD

Parton-Hadron-String Dynamics & Parton-Hadron-Quantum-Molecular Dynamics

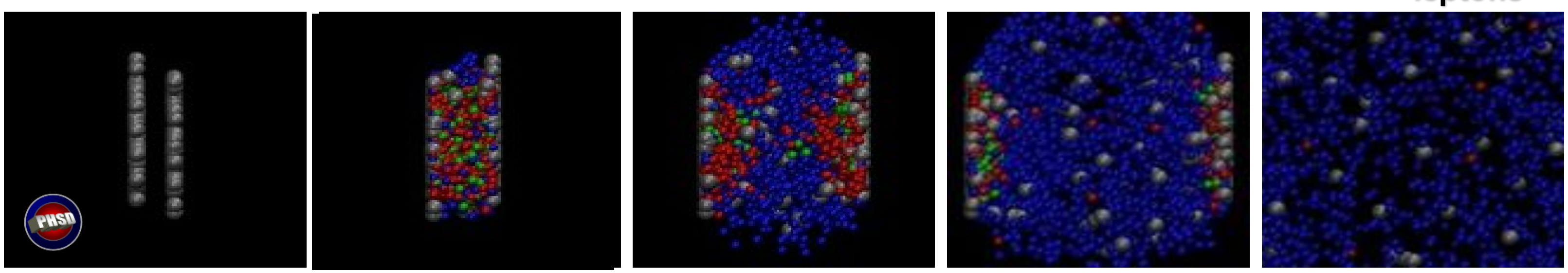
is a unified non-equilibrium microscopic transport approach for the description of the dynamics of strongly-interacting hadronic and partonic matter created in heavy-ion collisions and p+A, p+p, π+A reactions from SIS to LHC energies

Initial state: Au+Au

Quark-Gluon Plasma: lQCD EoS
non-perturbative QCD - quasiaprticles

Dynamical Hadronization

Hadronic interactions
➔final hadrons, clusters + leptons



time

➔ **provides a continuous description of the HIC dynamics:**
– no artificial transition from micro- to macro-description as in hydro-type models, no jump in entropy and energy density

*** PHSD,PHQMD are open source codes, available for experimental collaborations**

PHSD-PHQMD approach

PHSD mode

Initialization A+A
+ propagation of baryons:
Mean Field dynamics
(BUU)

PHQMD mode

Initialization A+A
+ propagation of baryons:
Quantum Molecular dynamics
(QMD) – n-body model

Propagation of partons (quarks, gluons) and mesons:
Mean Field dynamics (Kadanoff-Baym, BUU)

Collision integral = interactions of hadrons and partons (QGP)

Optionally

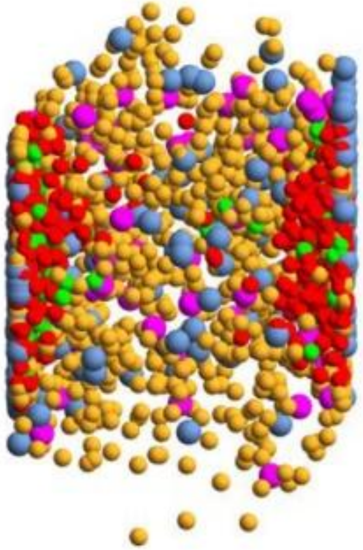
Cluster recognition: MST (Minimum Spanning Tree)
or SACA (Simulated Annealing Clusterization Algorithm)
or coalescence mechanism + kinetic deuterons

Final output – “events” : OSCAR, ROOT, Rivet formats

Realization: parallel ensemble method

Computer language: Fortran

Microscopic transport theory: mean-field (MF) vs. QMD dynamics



Equations-of-Motion

Dynamics of heavy-ion collisions is a many-body problem!

Schrödinger eq. for system of N particles

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) = H(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) \;\; \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t)$$

N-body Hamiltonian:

$$H \approx \sum_{i=1}^{N} T(\vec{r}_i) + \sum_{i<j}^{N} V_{ij}(\vec{r}_1 - \vec{r}_2, t)$$

local two-body potential

N-body wave function:

$$\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) \approx \prod_{i=1}^{N} \psi_i(\vec{r}_i\,, t)$$

Mean field dynamics

Hartree-Fock eq. for a single particle *i*:

$$i\hbar \frac{\partial}{\partial t} \psi_i(\vec{r}, t) = \hat{h} \psi_i(\vec{r}, t)$$

h = T+U **- single particle Hamiltonian**
ψ_i **– single particle wave function**

Self-generated mean-field potential:

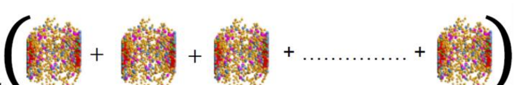
$$U(\vec{r}, t) = \frac{1}{(2\pi\hbar)^3} \int d^3r' \, d^3p V(\vec{r} - \vec{r}\,', t) f(\vec{r}\,', \vec{p}, t)$$

local two-body potential

Testparticle or parallel ensembles method:

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N_t} \sum_{i=1}^{N \cdot N_t} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t))$$

MF- covariant

$\frac{1}{N_t}\Big($  $\Big)$

QMD dynamics

Ritz variational principle:

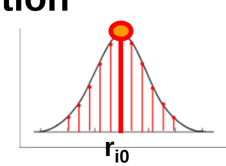
$$\delta \int_{t_1}^{t_2} dt < \psi(t) | i \frac{d}{dt} - H | \psi(t) >= 0$$

H - N-body Hamiltonian
Ψ **- N-body wave function**

Single-particle Wigner density of the nucleon wave function ψ_i:

$$f(\mathbf{r}_i, \mathbf{p}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L}[\mathbf{r}_i - \mathbf{r}_{i0}(t)]^2} e^{-\frac{L}{2\hbar^2}[\mathbf{p}_i - \mathbf{p}_{i0}(t)]^2}$$

Ansatz: Gaussian trial wave function with width *L* centered at r_{i0}, p_{i0}
➔ QMD - non-covariant !

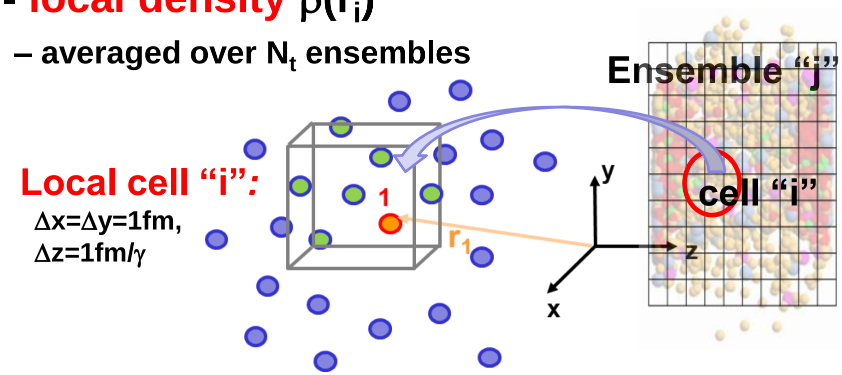


Equations-of-Motion

Mean field dynamics

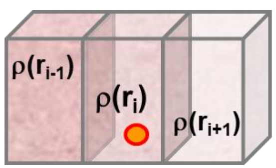
1 event: A+A nucleons
→ N_t ensembles with (A+A)*N_t test particles

Step 1: grid cells
- local density $\rho(r_i)$
– averaged over N_t ensembles



Step 2: forces

$$\vec{F} = -\vec{\nabla} U \sim \vec{\nabla}\rho \, \frac{\partial U}{\partial \rho}$$



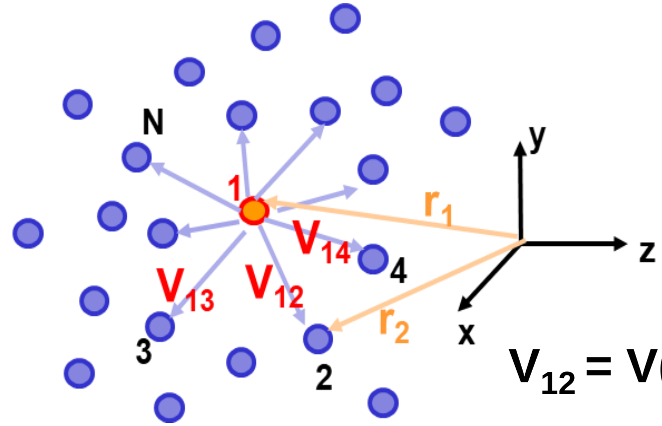
$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_i}$$

$$\dot{\vec{p}}_i = \frac{d\vec{p}_i}{dt} = -\vec{\nabla}_{\vec{r}_i} U(\vec{r}_i, t)$$

$U(\vec{r}, t)$ - mean-field potential

QMD dynamics

1 event: A+A nucleons



$V_{12} = V(r_1 - r_2)$

Expectation value of N-body Hamiltonian:

$$\langle H \rangle = \sum_i \langle H_i \rangle = \sum_i \left(\langle T_i \rangle + \sum_{j \neq i} \langle V_{i,j} \rangle \right)$$

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

$V_{ij} = V(r_i - r_j)$ - local two-body potential

→ QMD: N-body dynamics
via 2-body interaction potential

Where do we see differences in QMD vs MF dynamics?

- **"Bulk" observables for hadrons are rather similar in QMD and MF!**
  → tested within PHSD/PHQMD framework

- **Cluster formation is sensitive to nucleon dynamics:**
  - QMD – allows to keep over time NN correlations by potential interaction
  - MF – correlations are smeared out
  - Cascade – no correlations by potential interactions

Example: Cluster stability over time:

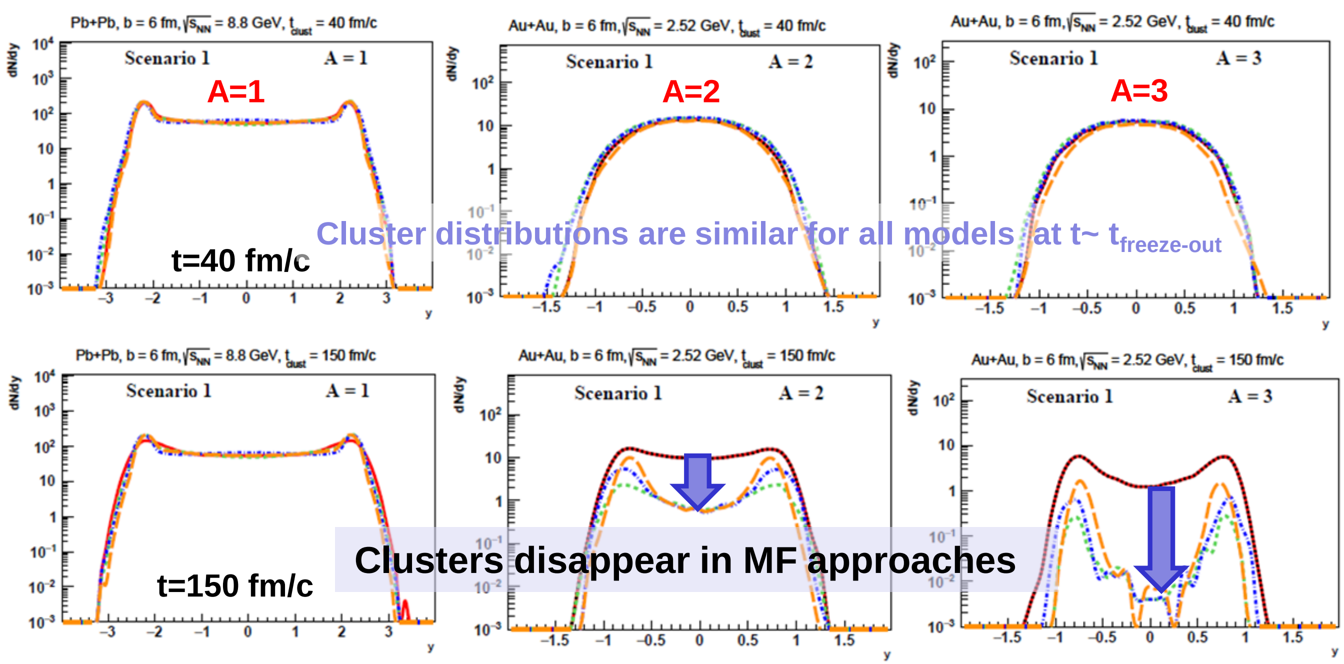
Viktar Kireyeu, Phys.Rev.C 103 (2021) 5

- **Clusters identified by psMST for all models:**

QMD:
PHQMD + psMST

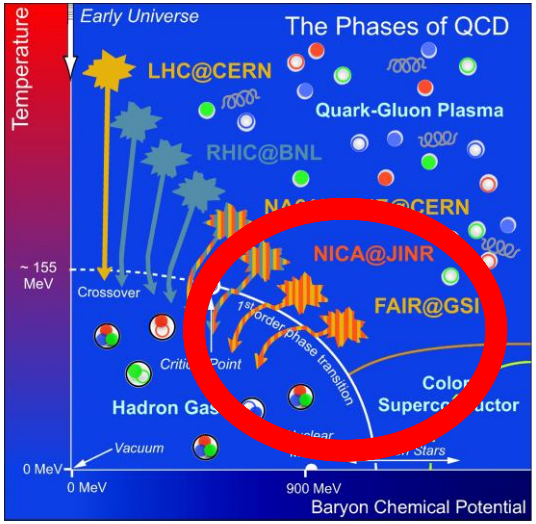
MF:
PHSD + psMST

Cascade:
SMASH + psMST
UrQMD + psMST



Modeling of sQGP in microscopic transport theory:

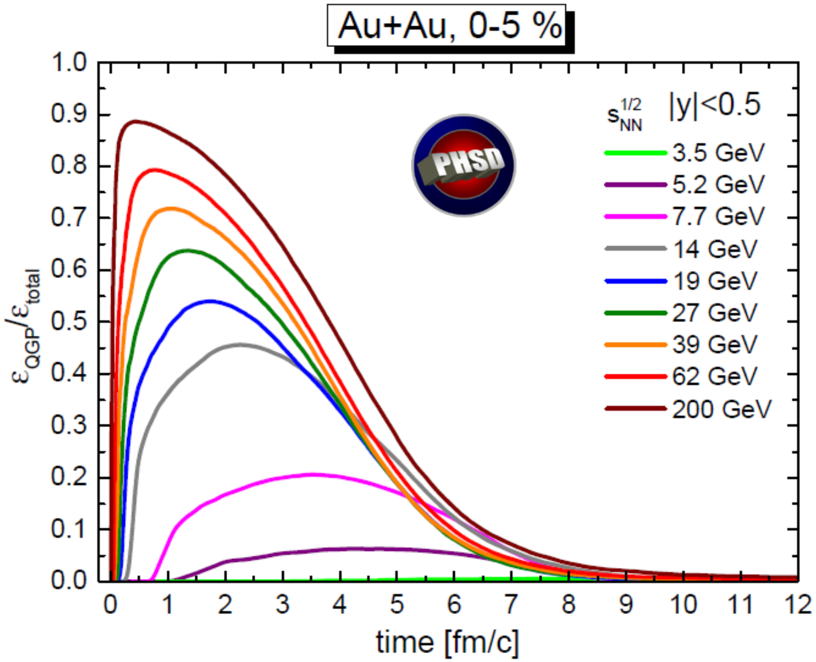
DQPM (T, μ_q) in PHSD/PHQMD



finite μ_q

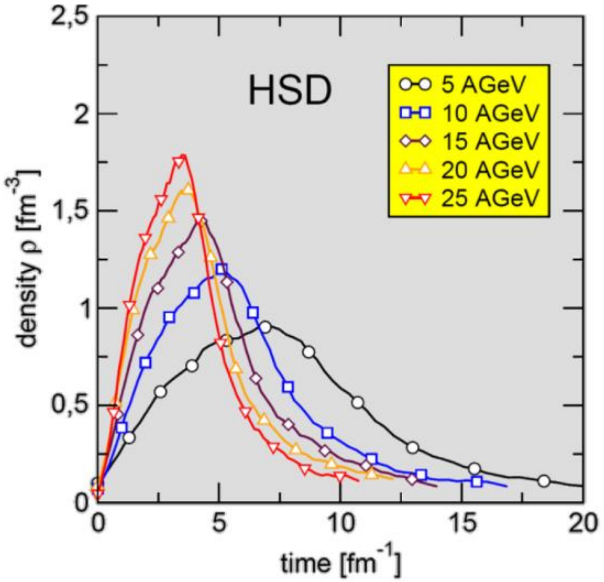
Dense and hot matter created in HICs

Time evolution of the partonic energy fraction



A. W. R. Jorge et al., 2503.05253

Time evolution of the baryon density ρ



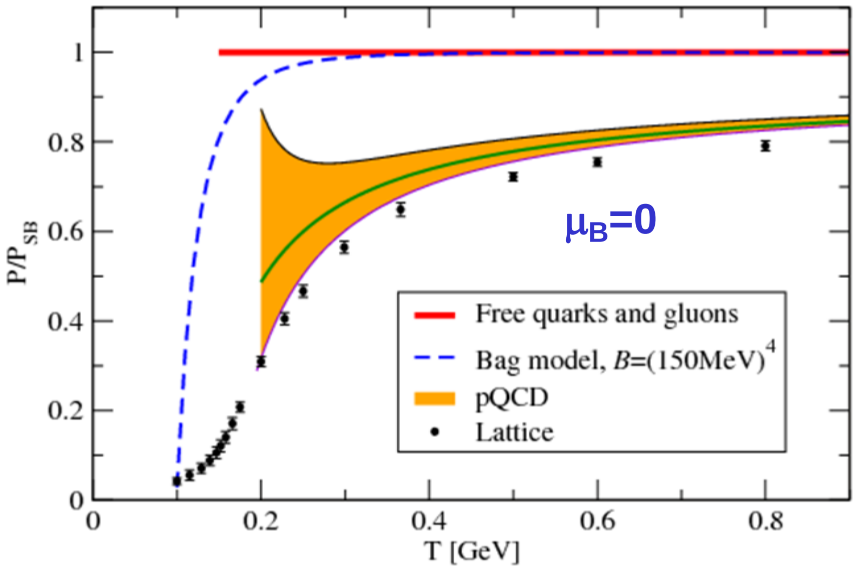
Large energy and baryon densities (above critical $\varepsilon > \varepsilon_{crit} \sim 0.4$ GeV/fm^3) are reachable in central reactions at FAIR/NICA energies*

➔ phase transition form hadronic matter to QGP

*** small volume of QGP (‚droplets‘) at low energies (BM@N)**

Degrees-of-freedom of QGP

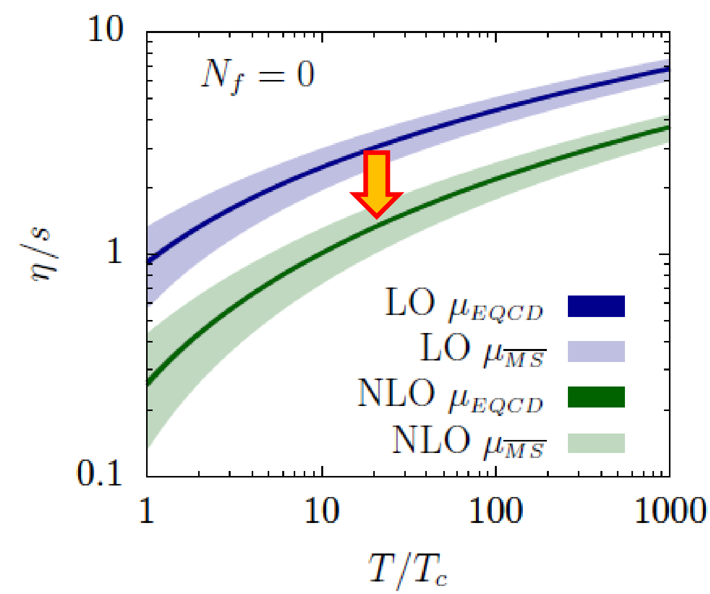
lQCD: QGP EoS at finite μ_B



Non-perturbative QCD ← pQCD

pQCD: (Yang-Mills) shear viscosity η

J. Ghiglieri, G.D. Moore, D. Teaney, JHEP 1803 (2018) 179



pQCD:

- weakly interacting system
- massless quarks and gluons

Thermal (non-perturbative) QCD:

- strongly interacting system
- massive quarks and gluons

➔ **Quasiparticles = effective degrees-of-freedom**

Dynamical QuasiParticle Model (DQPM)

DQPM – effective model for the description of non-perturbative (strongly interacting) QCD based on lQCD EoS

Degrees-of-freedom: strongly interacting dynamical quasiparticles - quarks and gluons

Theoretical basis :

- ‚resummed' single-particle Green's functions → quark (gluon) propagator (2PI) :

gluon propagator: $\Delta^{-1} = P^2 - \Pi$ **&** **quark propagator** $S_q^{-1} = P^2 - \Sigma_q$

gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$ **&** **quark self-energy:** $\Sigma_q = M_q^2 - i2\gamma_q\omega$

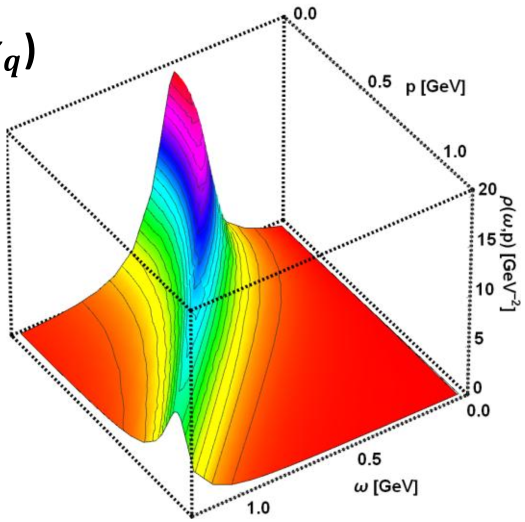
Properties of the quasiparticles are specified by scalar complex self-energies:

$Re\Sigma_q$: **thermal masses** (M_g, M_q); $Im\Sigma_q$: **interaction widths** (γ_g, γ_q)

→ **spectral functions** $\rho_q = -2ImS_q$ → **Lorentzian form:**

$$\rho_j(\omega, \mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2}\right)$$

$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2 - \mathbf{p}^2 - M_j^2\right)^2 + 4\gamma_j^2\omega^2} \qquad \tilde{E}_j^2(\mathbf{p}) = \mathbf{p}^2 + M_j^2 - \gamma_j^2$$



A. Peshier, W. Cassing, PRL 94 (2005) 172301; W. Cassing, NPA 791 (2007) 365: NPA 793 (2007), H. Berrehrah et al, Int.J.Mod.Phys. E25 (2016) 1642003; P. Moreau et al., PRC100 (2019) 014911; O. Soloveva et al., PRC101 (2020) 045203

DQPM: parton properties

Realization concept:

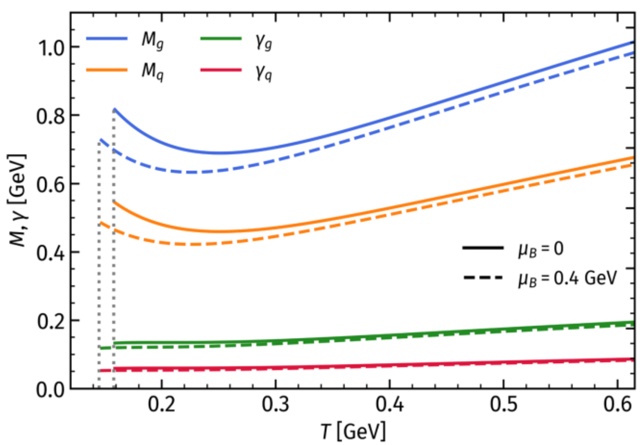
- introduce an ansatz (HTL; with few parameters) for the (T, μ_B) dependence of masses/widths
- evaluate the QGP thermodynamics in equilibrium using the Kadanoff-Baym theory
- fix DQPM parameters by comparison of the DQPM entropy density to lQCD at μ_B=0

- **Masses** and **widths** of quasiparticles depend on T and μ_B

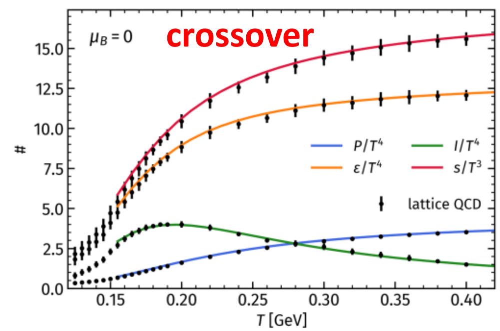
$$m_g^2(T,\mu_\mathrm{B}) = C_g \frac{g^2(T,\mu_\mathrm{B})}{6} T^2 \left(1 + \frac{N_f}{2N_c} + \frac{1}{2}\frac{\sum_q \mu_q^2}{T^2\pi^2}\right)$$

$$m_{q(\bar{q})}^2(T,\mu_\mathrm{B}) = C_q \frac{g^2(T,\mu_\mathrm{B})}{4} T^2 \left(1 + \frac{\mu_q^2}{T^2\pi^2}\right)$$

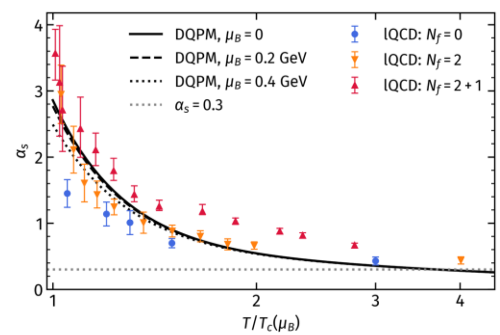
$$\gamma_j(T,\mu_\mathrm{B}) = \frac{1}{3} C_j \frac{g^2(T,\mu_\mathrm{B})T}{8\pi} \ln\left(\frac{2c_m}{g^2(T,\mu_\mathrm{B})} + 1\right)$$



- **Strong coupling** (g) is defined from lQCD entropy density at μ_B=0, using $\frac{\partial}{\partial T}\left(\frac{s_{DQPM}}{T^3}\right) = 0$



$$g^2(s/s_{SB}) = d((s/s_{SB})^e - 1)^f$$
$$s_{SB}^{QCD} = 19/9\,\pi^2 T^3$$



$$\alpha_s = g^2(T,\mu_B)/(4\pi)$$

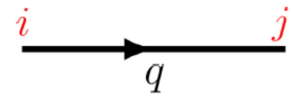
➔ DQPM allows to explore QCD in the non-perturbative regime of the (T, μ_B) phase diagram

Partonic interactions: matrix elements

DQPM partonic cross sections → leading order diagrams

- **Propagators for massive bosons and fermions:**

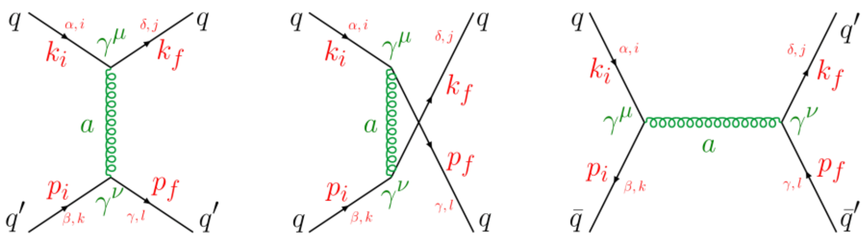
$$= -i\delta_{ab}\frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$



$$= i\delta_{ij}\frac{\not{q} + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

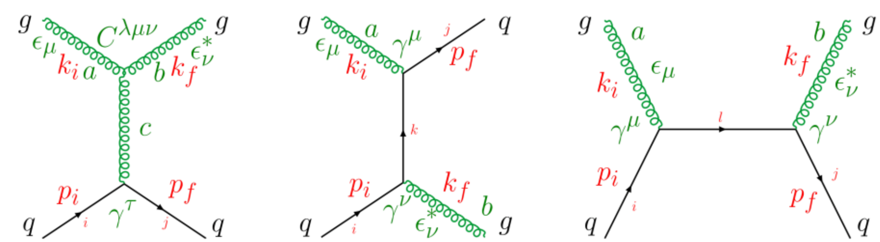
- **(Quasi-) elastic channels:**

qq' → qq' scattering



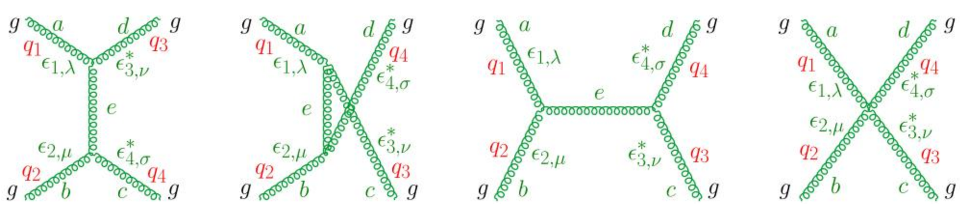
t – channel | u – channel | s – channel

gq → gq scattering



t – channel | u – channel | s – channel

gg→ gg scattering



t – channel | u – channel | s – channel | 4 – point

- **Inelastic channels:**

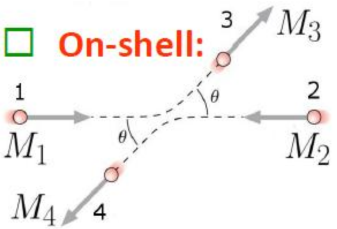
$$q + \bar{q} \to g$$
$$g \to q + \bar{q}$$

H. Berrehrah et al, PRC 93 (2016) 044914,
Int.J.Mod.Phys. E25 (2016) 1642003,

P. Moreau et al., PRC100 (2019) 014911

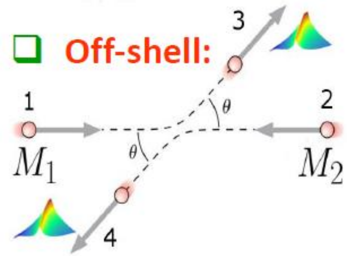
Partonic cross sections

❑ **On-shell:**

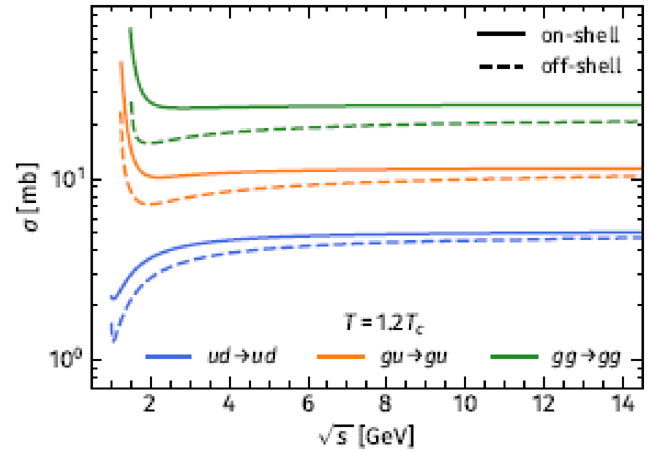


Initial masses: pole masses
Final masses: pole masses

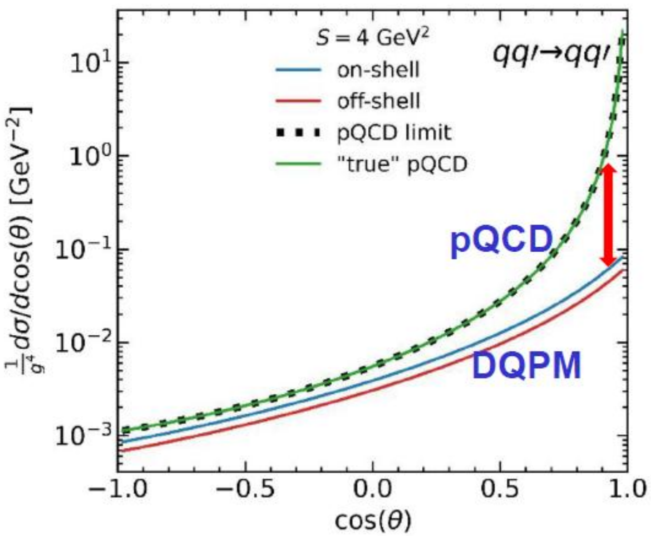
❑ **Off-shell:**

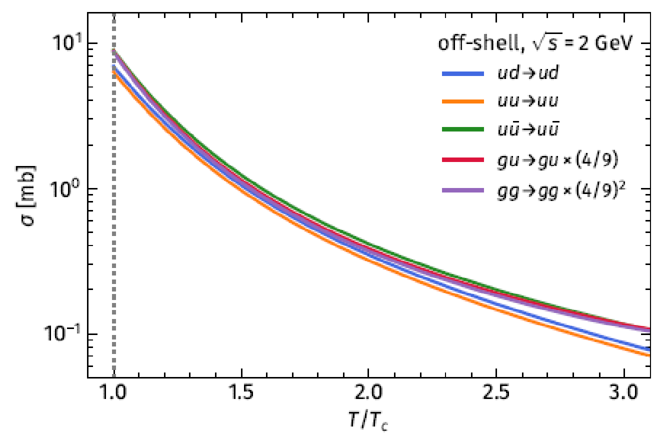


Initial masses: pole masses
Final masses: integration over spectral functions



❑ **off-shell effects are stronger at low $s^{1/2}$**





❑ **strong *T* dependence**
❑ **~ scaling with color ratio**

$$|\bar{\mathcal{M}}_{gg}|^2 \approx \frac{C_g}{C_q} |\bar{\mathcal{M}}_{gq}|^2 \approx \left(\frac{C_g}{C_q}\right)^2 |\bar{\mathcal{M}}_{qq}|^2$$

❑ **DQPM: M→0, γ→0 ➔ reproduces pQCD limits**
❑ **Differences between DQPM and pQCD :**
less forward peaked angular distribution
❑ **leads to more efficient momentum transfer**

P. Moreau et al., PRC100 (2019) 014911

Transport coefficients: shear viscosity η at finite (T, μ_q)

Relaxation-Time Approximation (RTA)

(+ cross-check with Kubo formalism):

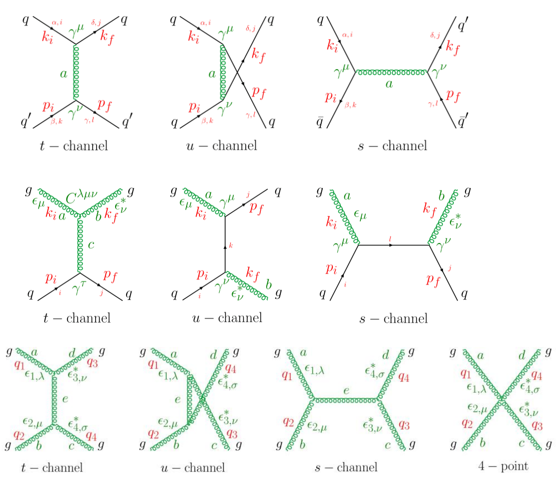
$$\eta^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\boxed{\tau_i(\mathbf{p},T,\mu_B)}\,d_i(1\pm f_i)f_i$$

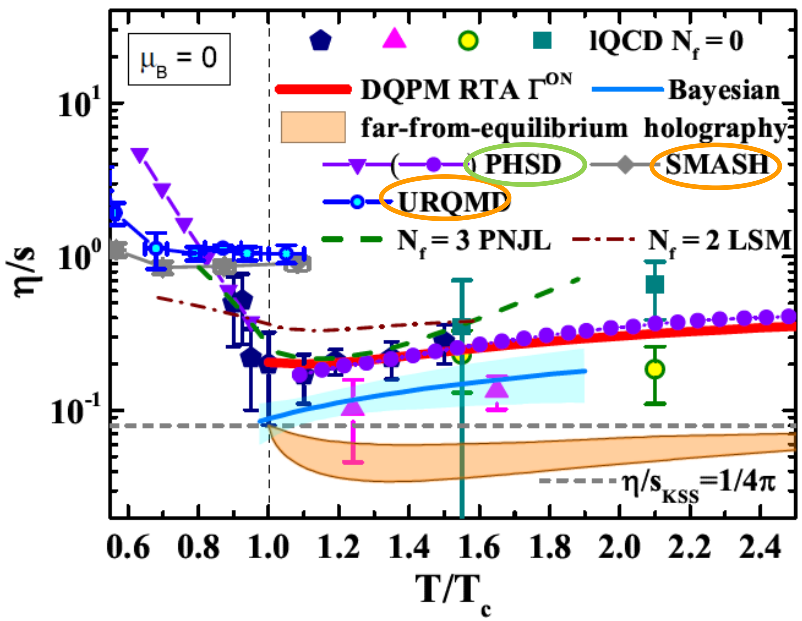
$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

Parton interaction rate:

$$\Gamma_i \propto \sum_j d_j\, f_j\, \sigma_{ij}$$

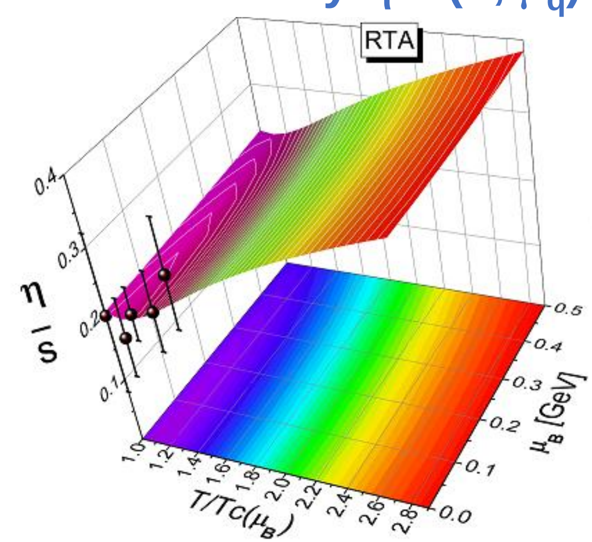
σ_{ij} → **'leading order' diagrams**





η/s from DQPM = η/s from PHSD in a box

Shear viscosity η/s (T, μ_q)



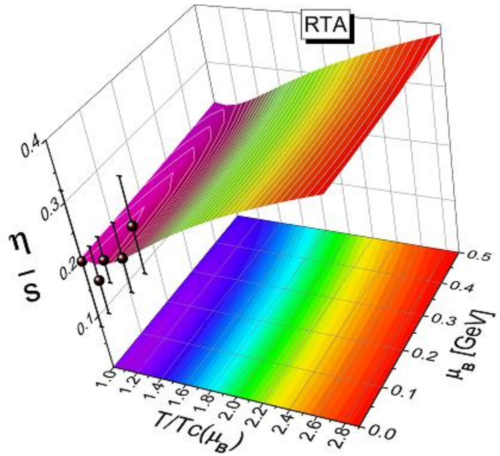
- **Good agreement with lQCD**
- **Light increase of shear viscosity with μ_B**

P. Moreau et al., PRC100 (2019) 014911; O. Soloveva et al., PRC110 (2020) 045203

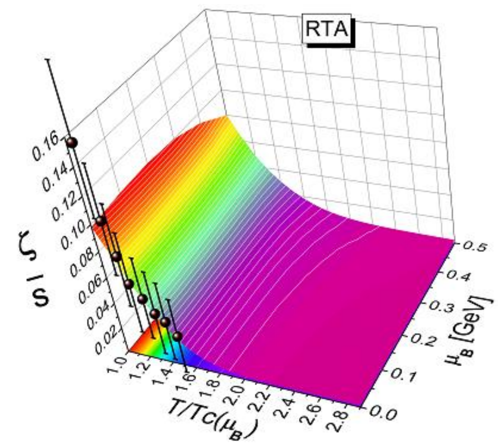
Lattice QCD: N. Astrakhantsev et al, JHEP 1704 (2017) 101

Transport coefficients

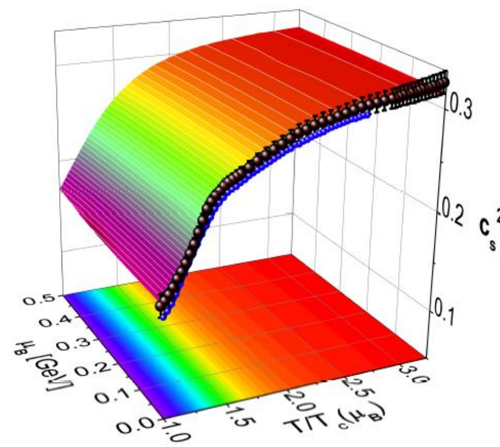
η/s versus (T,μ_B)



Bulk viscosity ζ/s

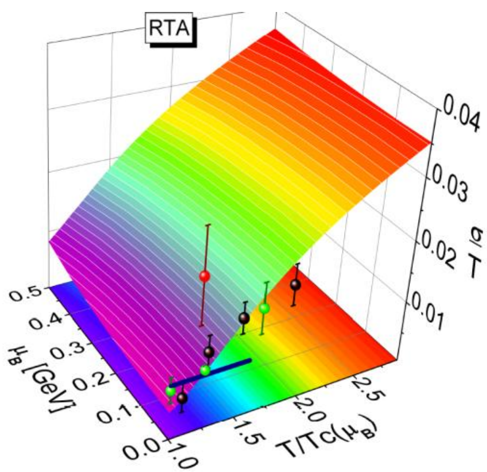


Speed of sound c_s^2

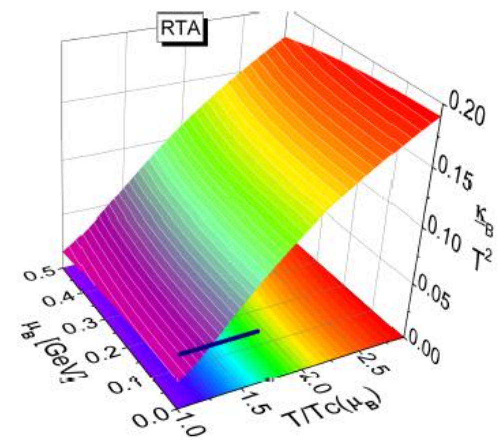


P. Moreau et al., PRC100 (2019) 014911;
O. Soloveva et al., PRC110 (2020) 045203

Electric conductivity σ_e/T



Baryon diffusion coefficient κ_B/T^2



Full diffusion coefficient matrix:

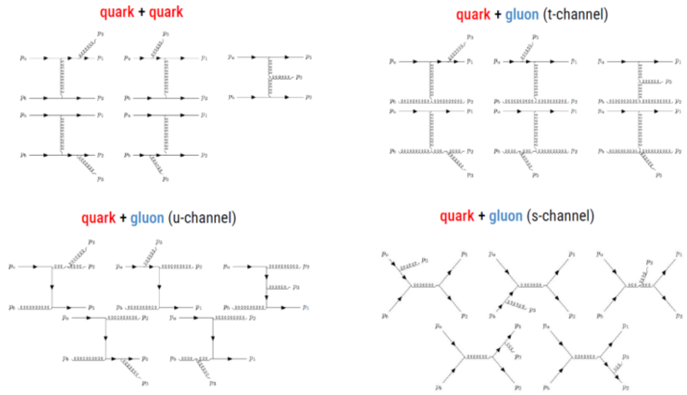
$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

J. A. Fotakis et al., PRD 104 (2021) , 034014

→ **Weak dependence of transport coefficients on μ_B**

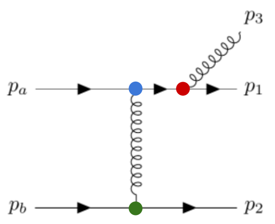
Partonic interaction rates (elastic and inelastic) in equilibrated sQGP

Partonic inelastic 2→3 interactions



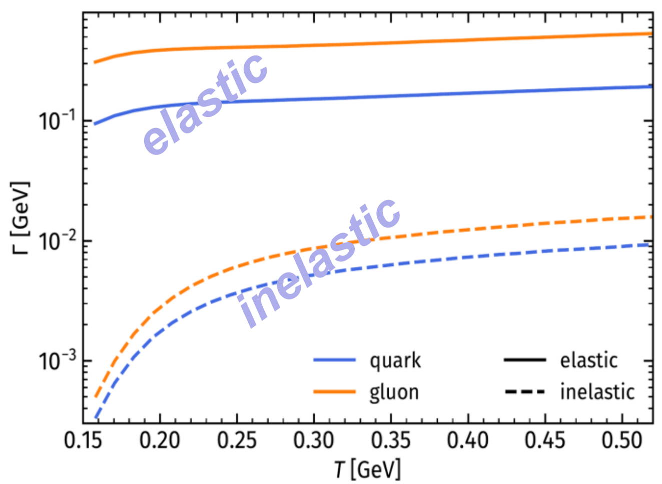
→ **Transport coefficients are sensitive to the choice of the strong coupling**

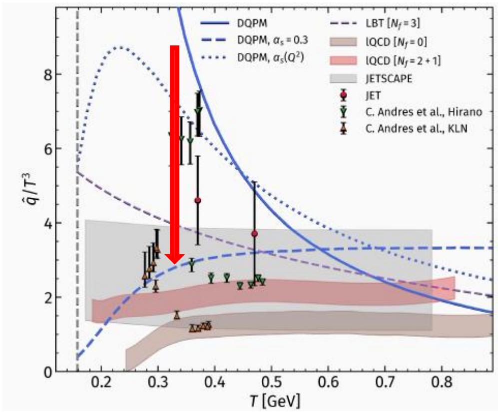
Model	Vertex		
	● thermal	● jet	● emitted gluon
DQPM	$\alpha_s(T)$		
DQPM, $\alpha_s = 0.3$	$\alpha_s = 0.3$		
DQPM, $\alpha_s(Q^2)$	$\alpha_s(T)$	$\alpha_s(Q^2)$	$\alpha_s(k_t^2)$

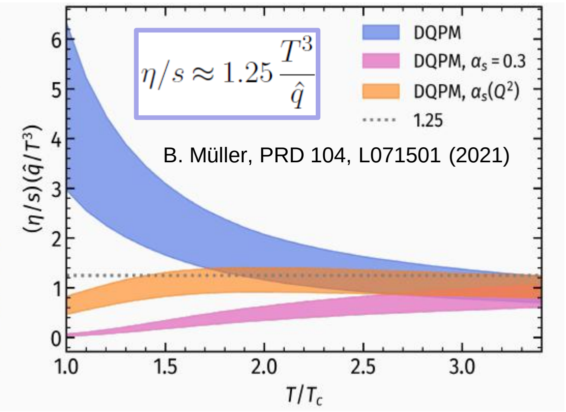


Zakharov model

Interaction rate







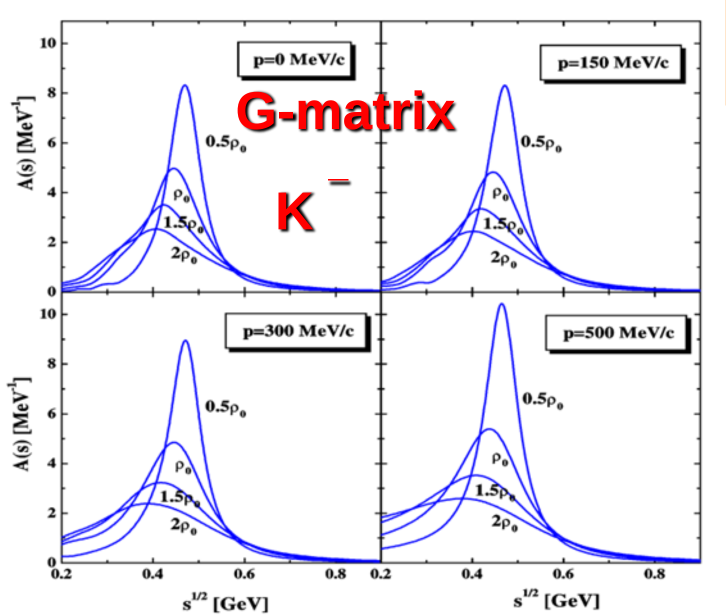
Inelastic interactions are suppressed in a thermalized QGP medium, but are crucial in the context of jet attenuation

★ **Strong dependence on the choice of α_s**
★ **Consistency with the weak-coupling limit at high T**
★ **Strong deviation from the weak-coupling limit at low T**

I. Grishmanovskii et al., PRC 109, 024911 (2024)

Strongly interacting hadrons (vector mesons, strange mesons) in a hot and dense medium:

from BUU to Kadanoff-Baym



Tolos et al., NPA 690 (2001) 547

From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels) = changes of particle properties in a hot and dense medium

Examples: hadronic medium - vector mesons, strange mesons, baryons
QGP – dressing of partons

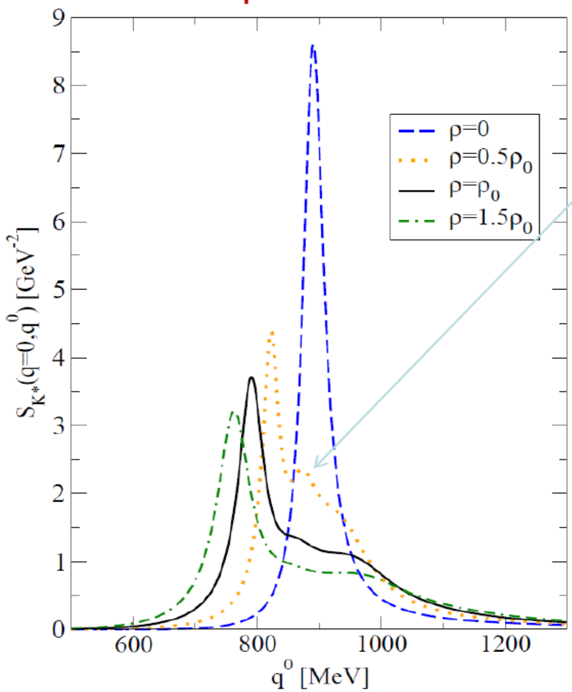
Many-body theory:
Strong interaction ➔ large widths ➔ broad spectral functions ➔ quantum objects

Semi-classical on-shell BUU: applies for weakly interacting systems of particles

- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

It is doable with quantum Kadanoff-Baym equations

Kbar* spectral function



Λ(1783)N^{-1} and Σ(1830)N^{-1} exitations

G-matrix:

Barcelona / Valencia group

Dynamical description of strongly interacting systems

❑ **Quantum field theory ➔**

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

(1962)

$$\boxed{\hat{S}_{0x}^{-1}\, S_{xy}^{<} = \Sigma_{xz}^{ret} \odot S_{zy}^{<} + \Sigma_{xz}^{<} \odot S_{zy}^{adv}}$$

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS_{xy}^{<} = \eta \langle \{ \Phi^{+}(y) \Phi(x) \} \rangle$$

$$iS_{xy}^{>} = \langle \{ \Phi(y) \Phi^{+}(x) \} \rangle$$

$$iS_{xy}^{c} = \langle T^{c} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - causal$$

$$iS_{xy}^{a} = \langle T^{a} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad - anticausal$$

$$S_{xy}^{ret} = S_{xy}^{c} - S_{xy}^{<} = S_{xy}^{>} - S_{xy}^{a} \quad - retarded$$

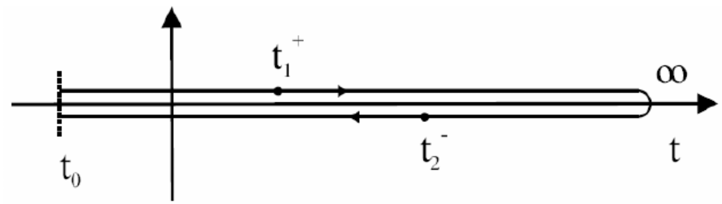
$$S_{xy}^{adv} = S_{xy}^{c} - S_{xy}^{>} = S_{xy}^{<} - S_{xy}^{a} \quad - advanced$$

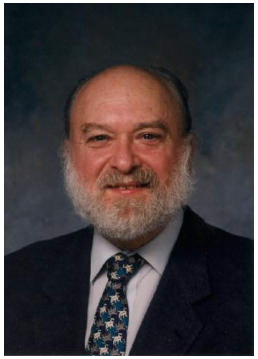
$$\hat{S}_{0x}^{-1} \equiv -(\partial_{x}^{\mu} \partial_{\mu}^{x} + M_{0}^{2})$$

$$\eta = \pm 1 (bosons\ /\ fermions)$$

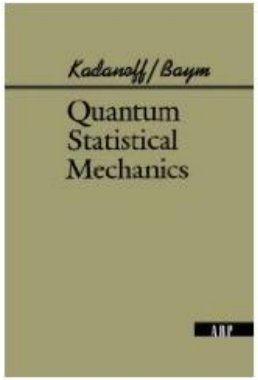
$$T^{a}(T^{c}) - (anti-)time - ordering\ operator$$

Real-time (Keldysh-) Contour





Leo Kadanoff



Gordon Baym

1st application for a spacially homodeneous system with deformed Fermi sphere:
P. Danielewicz, Ann. Phys. 152, 305 (1984); … H.S. Köhler, Phys. Rev. 51, 3232 (1995); …

Off-shell propagation: Kadanoff-Baym

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term | Vlasov term | backflow term | collision term = ‚gain‘ - ‚loss‘ term

$$\diamondsuit \{ P^2 - M_0^2 - Re\Sigma_{XP}^{ret} \} \{ S_{XP}^{<} \} - \boxed{\diamondsuit \{ \Sigma_{XP}^{<} \} \{ Re S_{XP}^{ret} \}} = \frac{i}{2} [\Sigma_{XP}^{>} S_{XP}^{<} - \Sigma_{XP}^{<} S_{XP}^{>}]$$

off-shell behavior

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

- KB propagates 1-body 2-point Green functions $S^<(x,p) \rightarrow A(x,p)*N(x,p)$ in 8 dimensions

- $S^<$ carries information not only on the occupation number N_{XP} (as BUU), but also on the particle properties, interactions and correlations via the spectral function A_{XP}

Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

Reaction rate of particle:

$$\Gamma_{XP} = -Im\,\Sigma_{XP}^{ret} = 2\,p_0 \Gamma$$

On-shell limit of KB:

- $A_{XP} \rightarrow \delta(p^2-M^2)$ or A_{XP} has a constant shape in a medium, i.e. $\nabla_X \Gamma = 0, \quad \nabla_P \Gamma = 0$
- backflow term vanishes: KB → BUU

→ Generalized Cassing-Juchem off-shell equations of motion for testparticles → PHSD

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

Generalalized testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- **Employ testparticle Ansatz for the real valued quantity $i\,S^{<}_{XP}$**

$$\boxed{F_{XP} = A_{XP} N_{XP} = i\,S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))}$$

insert in generalized transport equations and determine equations of motion !

➔ **Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:**

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

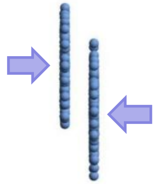
$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Realized in PHSD

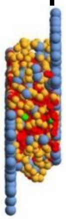
Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime' of particle (i) !

Time evolution in PHSD / PHQMD

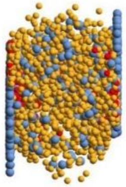
Initial A+A collision



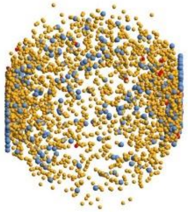
Partonic phase



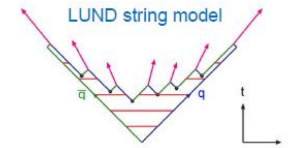
Hadronization



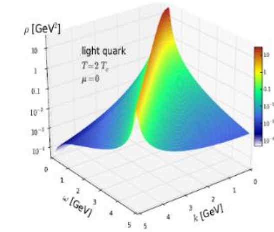
Hadronic phase



- **Initial A+A collisions :**
  N+N → string formation → decay to pre-hadrons + leading hadrons

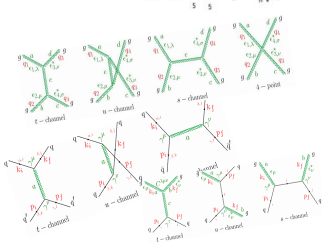


- **Formation of QGP stage if local $\varepsilon > \varepsilon_{critical}$:**
  dissolution of pre-hadrons → partons



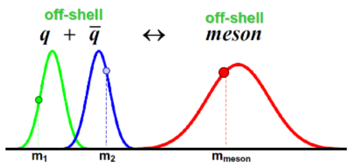
- **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM)
  matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

- Degrees-of-freedom: strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

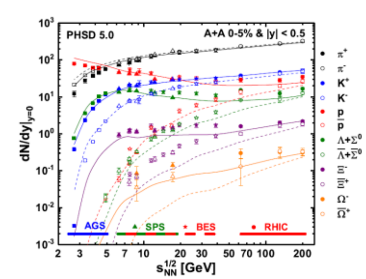


- Interactions: (quasi-)elastic and inelastic collisions of partons
- parton propagation within Kadanoff-Baym equations

- **Hadronization to colorless off-shell mesons and baryons:**
  Strict 4-momentum and quantum number conservation



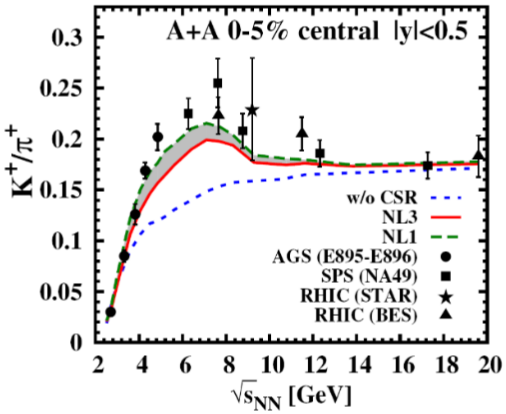
- **Hadronic phase: hadron-hadron interactions – off-shell HSD**
  including *n* ←→ *m* selected reactions
  (for strangeness, anti-baryons, deuteron production)

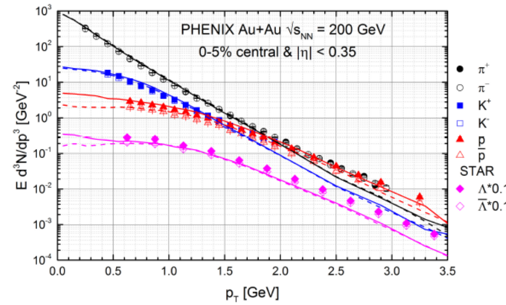


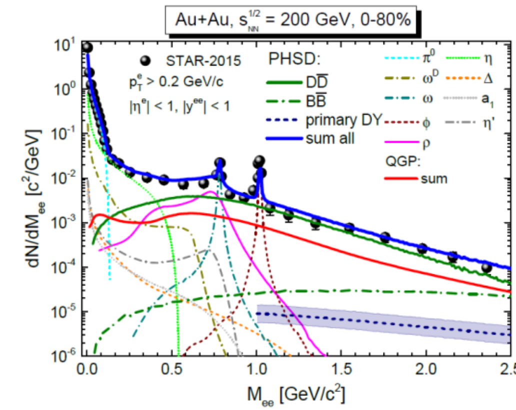
→ PHSD/PHQMD provides a good description of 'bulk' hadronic and electromagnetic observables from SIS to LHC energies

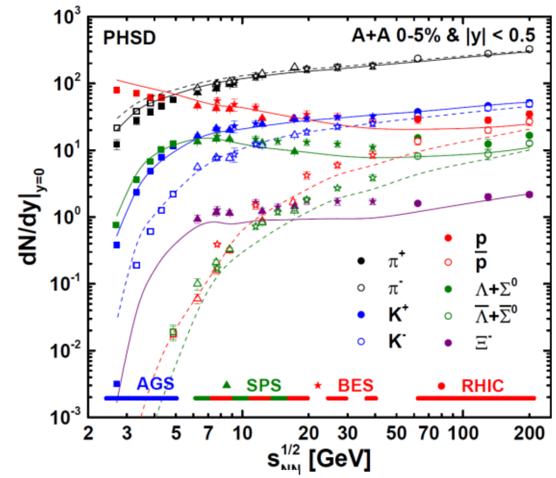
PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; PHQMD: J. Aichelin et al., PRC 101 (2020) 044905

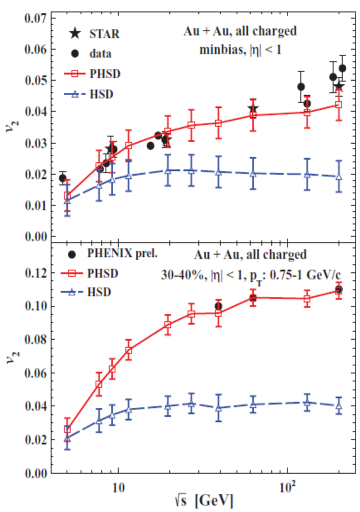
Description of A+A with PHSD (highlights)

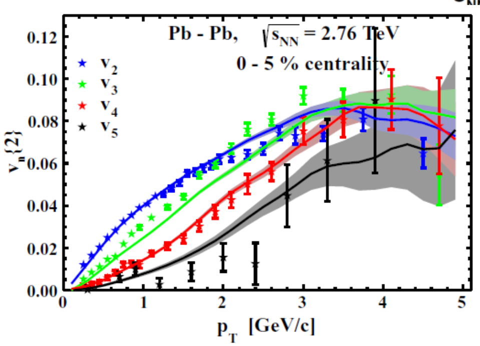


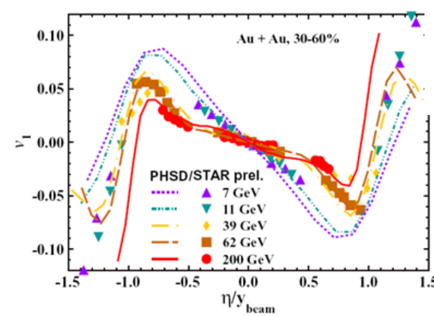


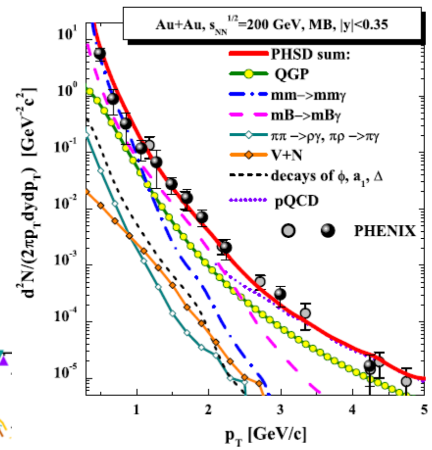












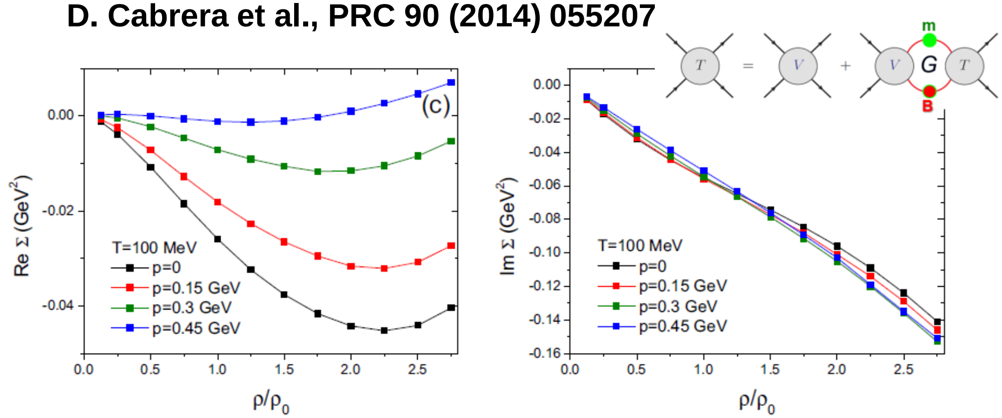
❑ PHSD provides a good description of ‚bulk' and electromagnetic observables (y-, p_T-distributions, flow coefficients v_n, …) from SIS to LHC

Off-shell dynamics for antikaons at SIS energies

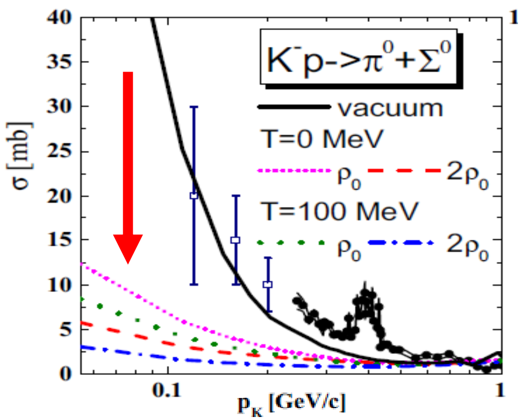
Spectral function of K^- within the **G-matrix** **approach:**

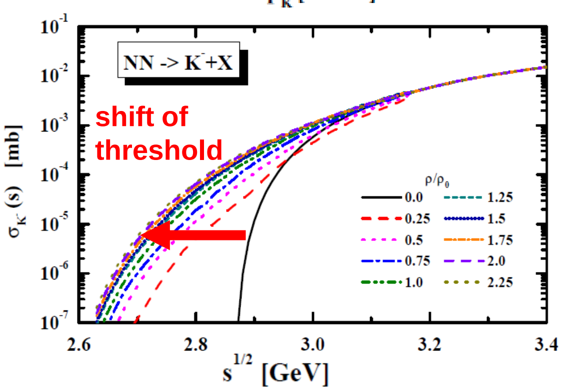
$$S_{\bar K}(k_0,\vec k;T) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma_{\bar K}(k_0,\vec k;T)}{\left|k_0^2-\vec k^{\,2}-m_{\bar K}^2-\Sigma_{\bar K}(k_0,\vec k;T)\right|^2}.$$

D. Cabrera et al., PRC 90 (2014) 055207

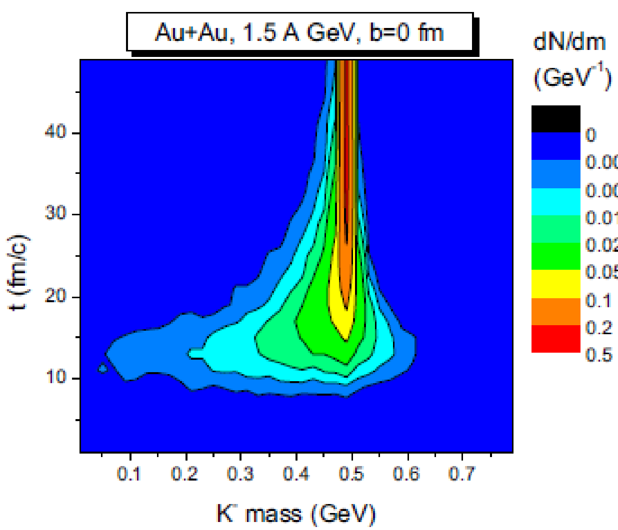


In-medium cross sections for K^- production and absorption are strongly modified in the medium:

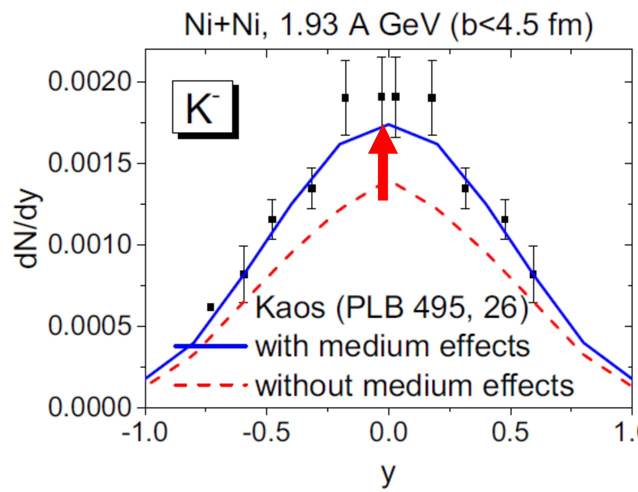




Time evolution of the K^- masses

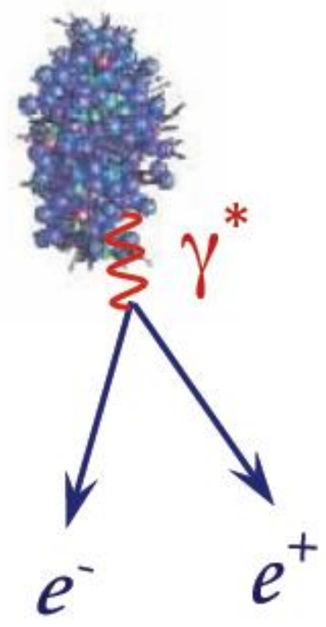


In-medium effects are mandatory for the description of experimental K^- spectra

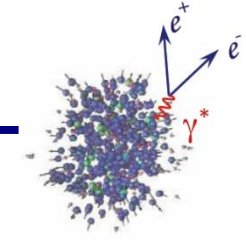


Taesoo Song et al., PRC 103, 044901 (2021)

Electromagnetic probes of the strongly interaction matter: dileptons

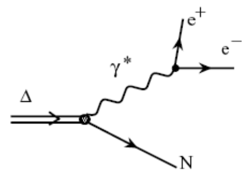


Physics with dileptons

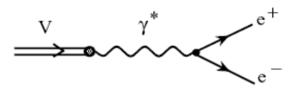


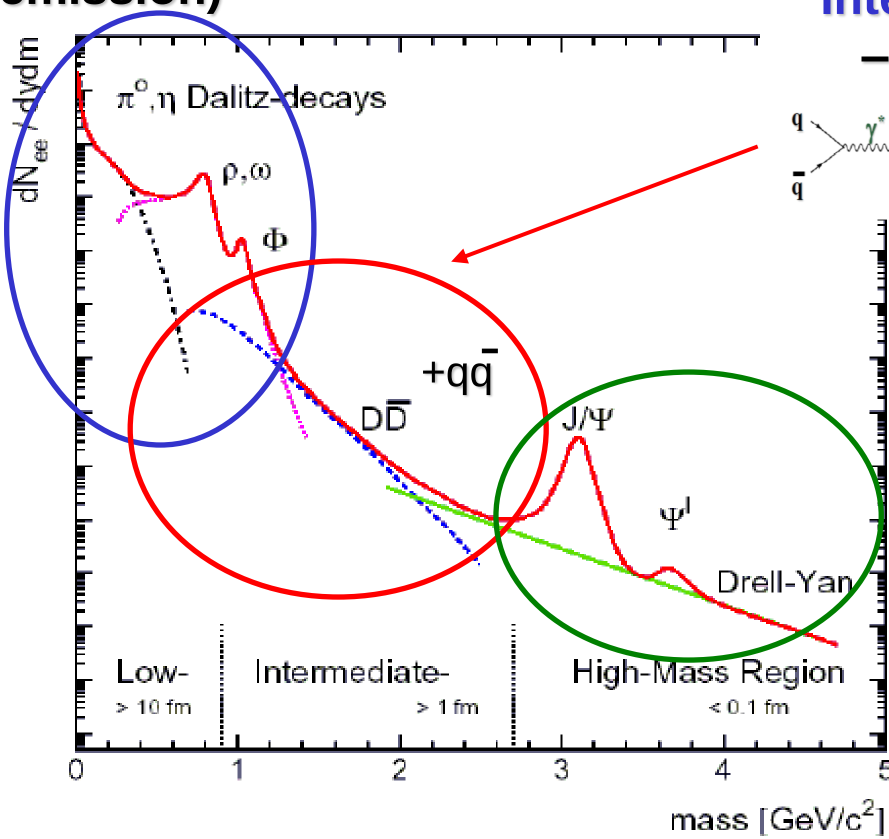
Low mass dileptons
- probe of hadronic in-medium effects – chiral symmetry restoration
(late time emission)

- **Dalitz decay of mesons and baryons (π^0,η, Δ,…)**

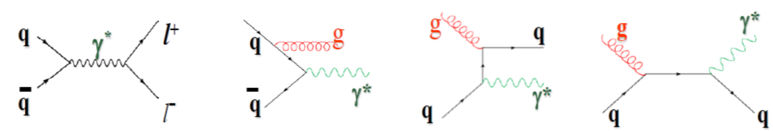


- **direct decay of vector mesons**

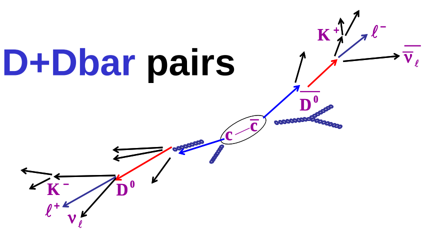




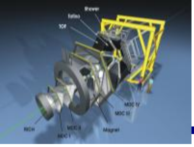
Intermediate mass dileptons – probe of ‚thermal QGP‘



- correlated D+Dbar pairs



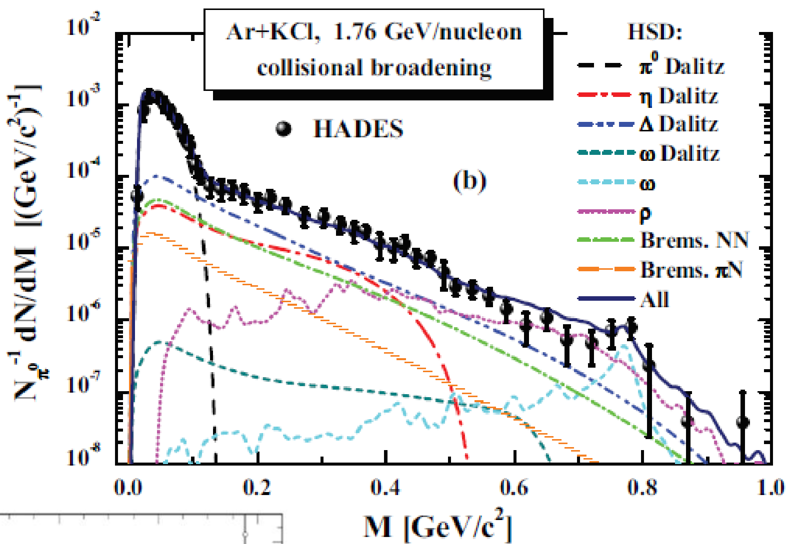
High-mass dileptons – probe of pQGP (DY) and hard probes
(early time emission)

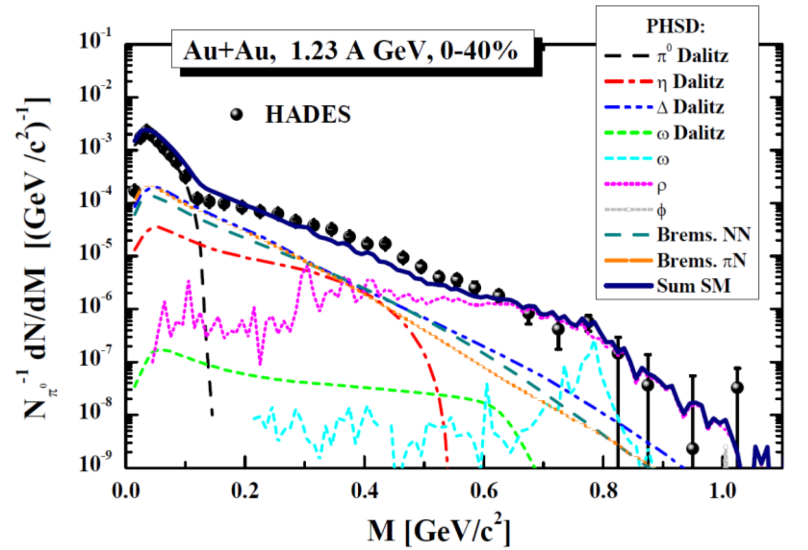


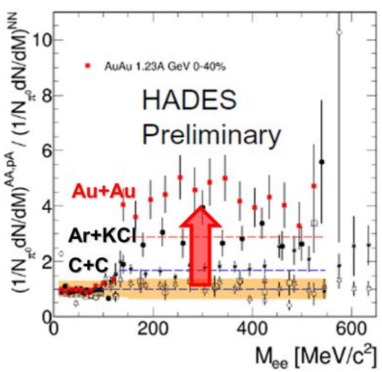
Dileptons at SIS energies - HADES

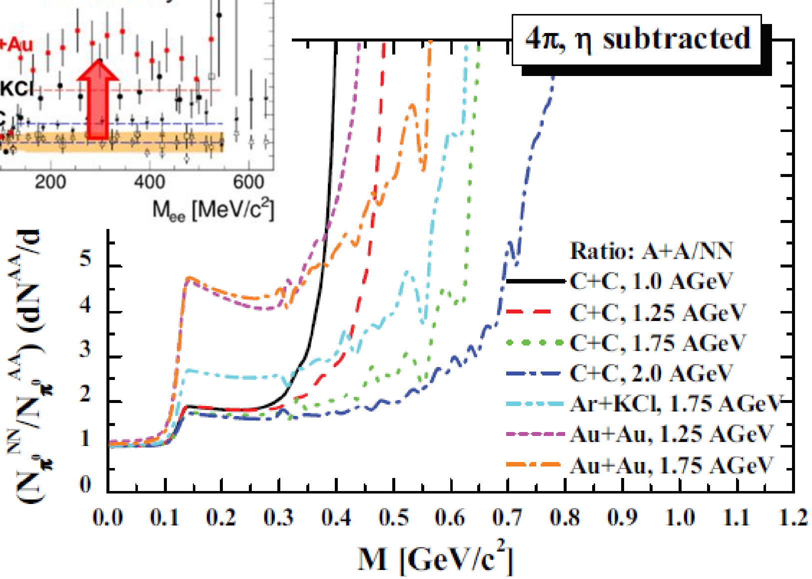
E. B., J. Aichelin, M. Thomere, S. Vogel, M. Bleicher, PRC 87 (2013) 064907

I. Schmidt, E.B., M. Gumberidze,R. Holzmann, PRD 104 (2021), 015008









- **Strong in-medium enhancement** of dilepton yield in Au+Au vs. NN
- **Increases with the system size:**

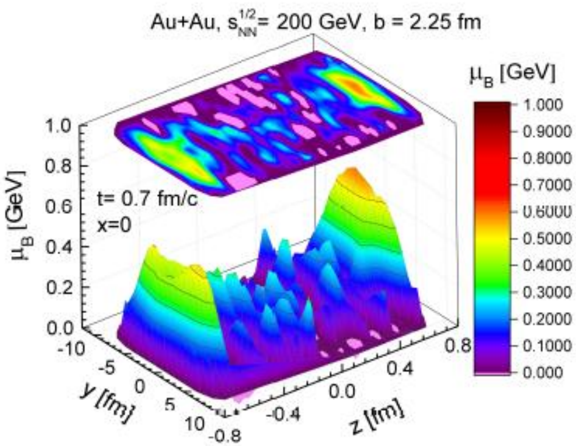
1) **multiple Δ regeneration** – dilepton emission from intermediate Δ's which are part of the reaction cycles
Δ→πN; πN →Δ and NN→NΔ; NΔ→NN

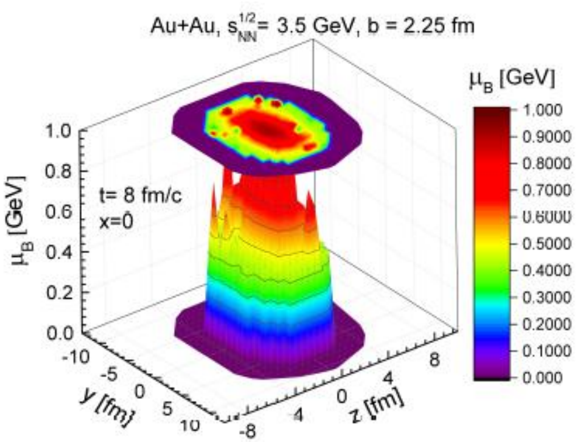
2) **pN bremsstrahlung** which scales with N_{bin} and not with N_{part} , i.e. pions

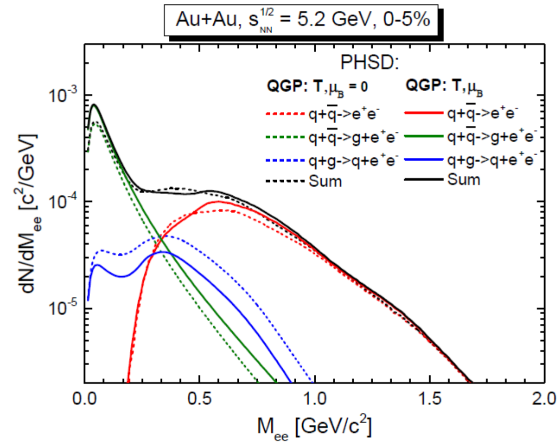
3) **Collisional broadening** of ρ,ω,φ mesons

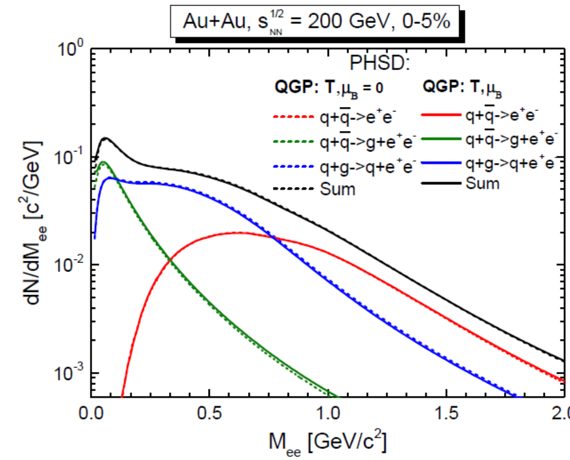
Influence of the (T, μ_B)-dependent EoS on dilepton production

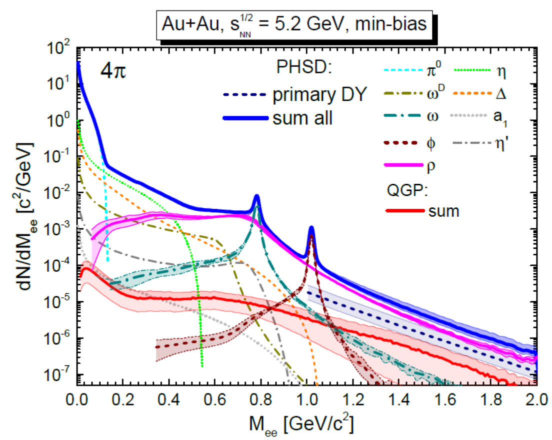
Baryon chemical potential μ_B
in (x=0,y,z) plane in Au+Au

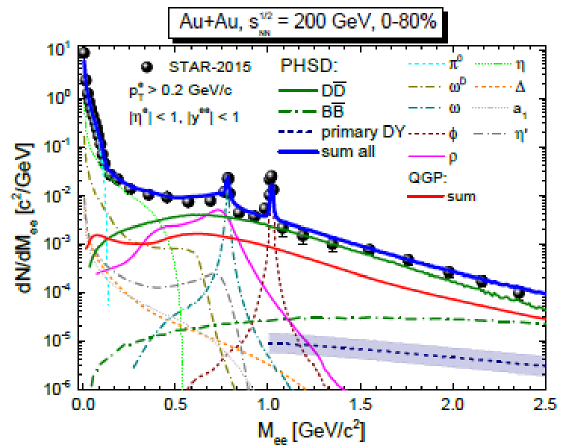












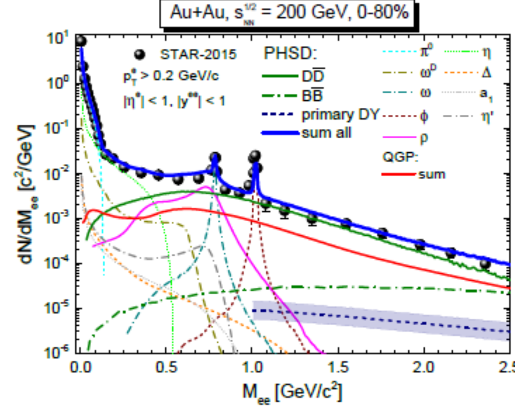
- **Visible influance of μ_B on QGP dilepton spectra at low energies, very small – at high energies**
- **However, small influence of μ_B on the total dilepton yield due to a small volume of QGP at low energies**

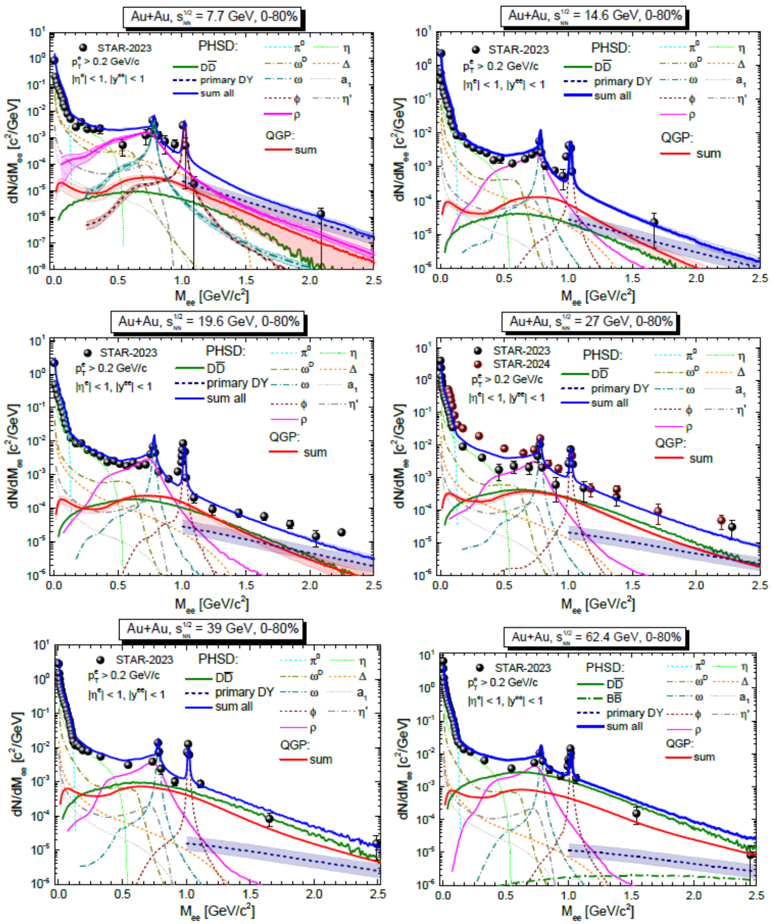
A. R. Jorge et al., 2503.05253, PRC (2025)

Dileptons: excitation function

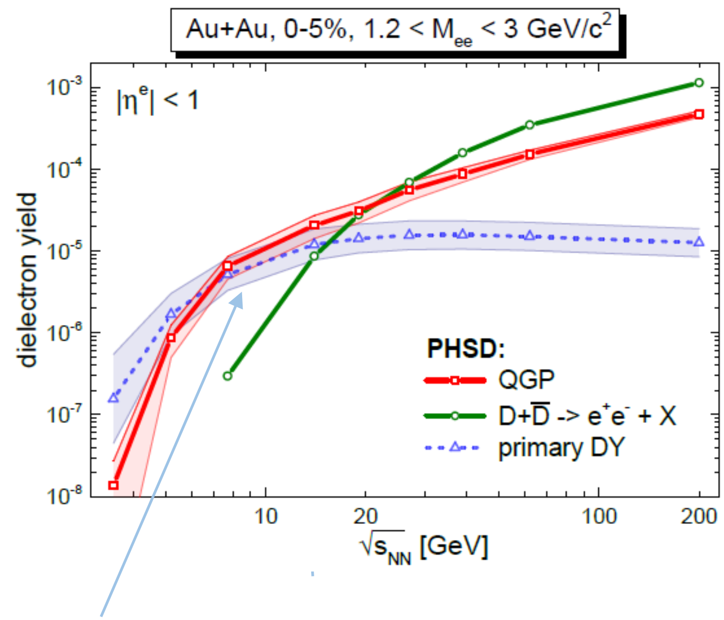
PHSD dilepton spectra including a collisional broadening of the vector meson spectral functions + primary DY + QGP + correlated charm

$$\text{QGP}\begin{cases} q\bar{q} \to e^+e^- \\ q\bar{q} \to ge^+e^- \\ gq(\bar{q}) \to q(\bar{q})e^+e^- \end{cases}$$





Excitation function of dilepton yield integrated for 1.2<M<3GeV



- Dileptons from **QGP overshine charm** dileptons with decreasing beam energy
- **Primary DY** could be "subtracted" from AA dilepton spectra using pp data

➔ **Good perspectives for NICA!**

A. W. R. Jorge et al., 2503.05253, PRC (2025)

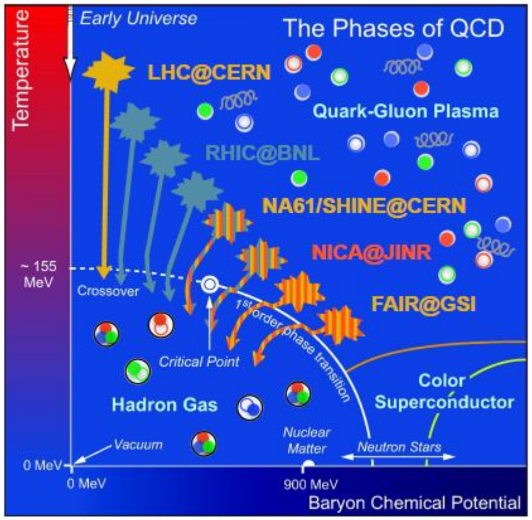
Cf. T. Song et al., Phys. Rev. C97 (2018), 064907

QMD dynamics, cluster production

PHQMD:
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
V. Kireyeu et al., PRC 105 (2022) 044909;
G. Coci et al., PRC 108 (2023) 1, 014902;
V. Kireyeu et al., arXiv:2411.04969

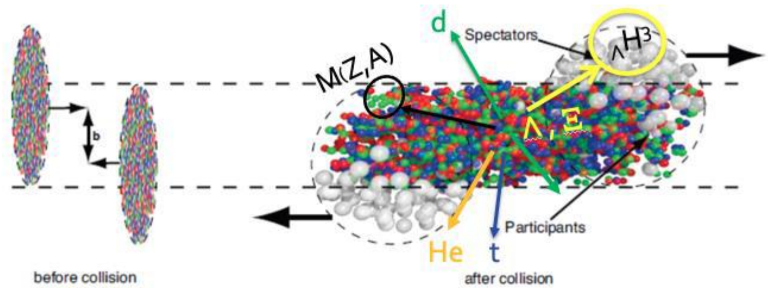
Cluster production in heavy-ion collisions

The phase diagram of QCD

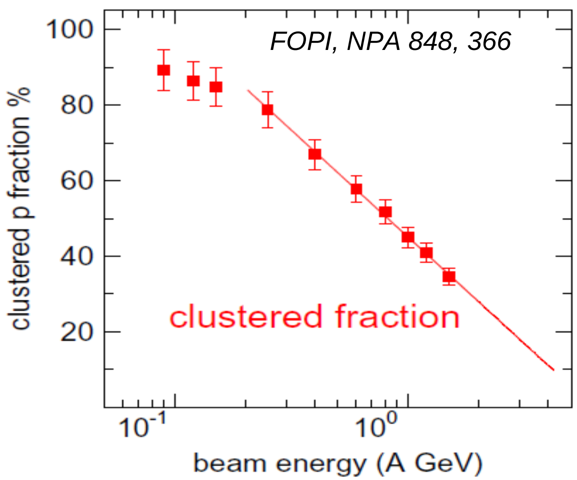


Clusters and (anti-) hypernuclei

are observed experimentally at all energies

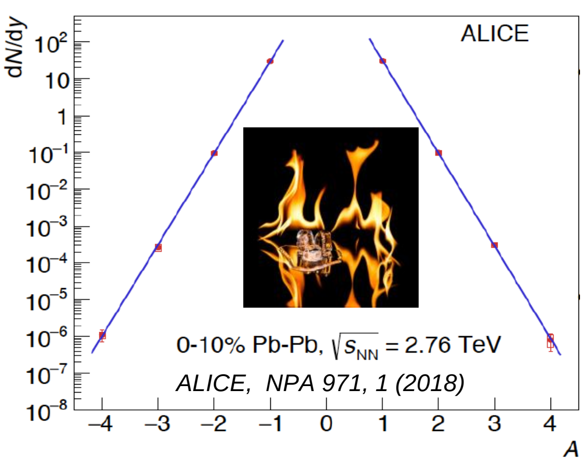


- **Clusters are very abundant at low energy**



- **High energy HIC:** **‚Ice in a fire‘ puzzle:** **how the weakly bound objects can be formed and survive in a hot enviroment?!**

Au+Au, central. midrapidity

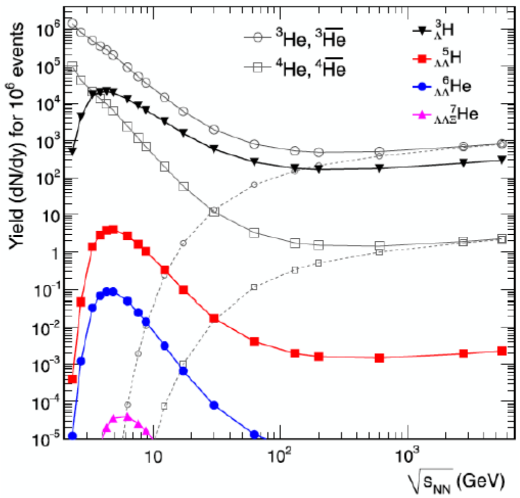


➔ **Mechanisms of cluster formation in strongly interacting matter are not well understood**

Modeling of cluster and hypernuclei formation

Existing models for cluster formation:

A. Andronic et al., PLB 697, 203 (2011)



- ❑ **statistical model:**
  - **assumption of thermal equilibrium**

In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

Dynamical Models:

I. cluster formation by coalescence mechanism
at a freeze-out time by coalescence radii in coordinate and momentum space

II. dynamical modeling of cluster formation based on interactions within microscopic transport models:

– potential‘ mechanism - via potential NN (NY) interactions
(applied during the whole reaction time of HIC)

– ‘kinetic‘ mechanism - by hadronic scattering
(hadronic reactions as NNN→ dN ; NNπ→ dπ, NN→ dπ)

PHQMD

Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

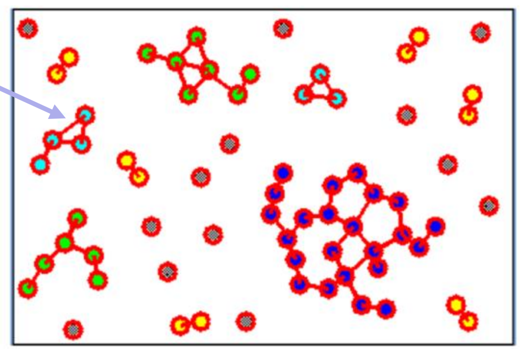
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are 'bound' if their distance in the cluster rest frame fulfills

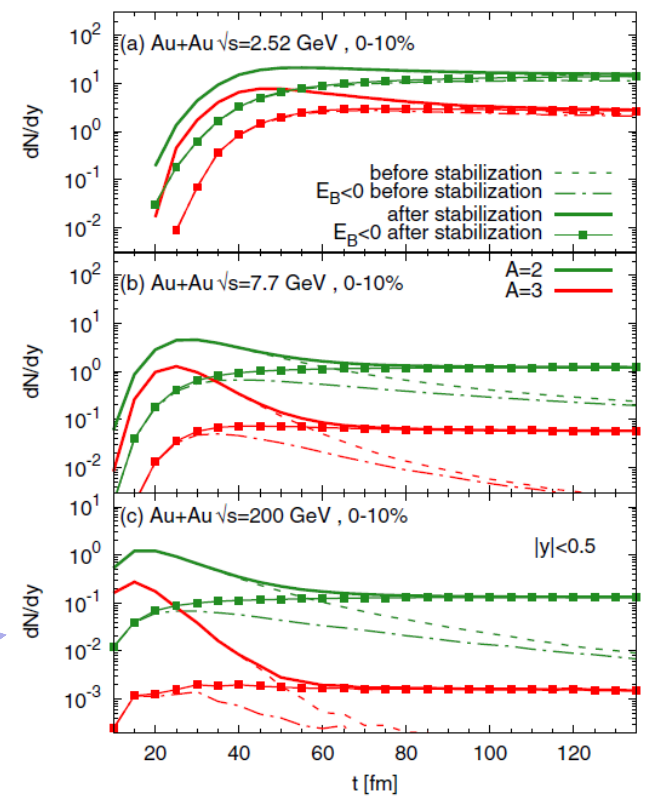
$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm} \quad \text{(range of NN potential)}$$

2. Particle is bound to a cluster if it binds with at least one particle of the cluster



Advanced MST (aMST)

- MST + extra condition: $E_B<0$ negative binding energy for identified clusters
- Stabilization procedure – to correct artifacts of the semi-classical QMD: recombine the final "lost" nucleons back into clusters if they left the cluster without rescattering



Kinetic mechanism for deuteron production in HICs

G. Coci et al., PRC 108 (2023) 014902

"Kinetic mechanism"

1) **hadronic inelastic reactions NN $\leftrightarrow$ dπ , πNN $\leftrightarrow$ dπ , NNN $\leftrightarrow$ dN**
2) **hadronic elastic π+d, N+d reactions**

Transition rate for 3→2 reactions:

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

$$\boxed{P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}}$$

$$\boxed{P_{2,3}(\sqrt{s}) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

W. Cassing, NPA 700 (2002) 618

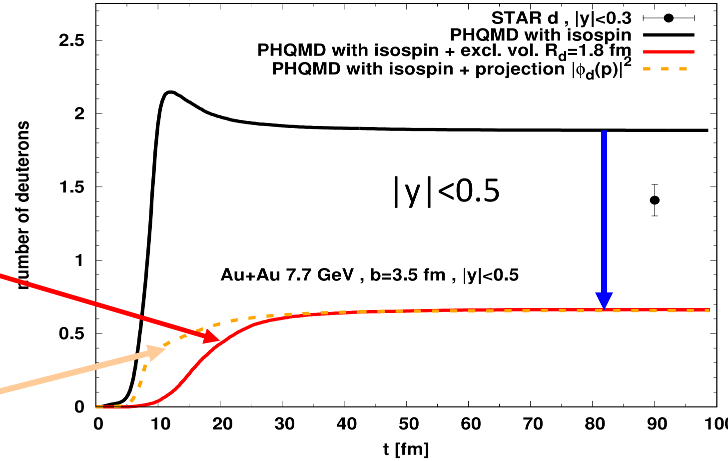
➔ **solved by stochastic method**

+ inclusion of all possible isospin channels which enhance deuteron production

+ accounting of quantum properties of d, modelled by:

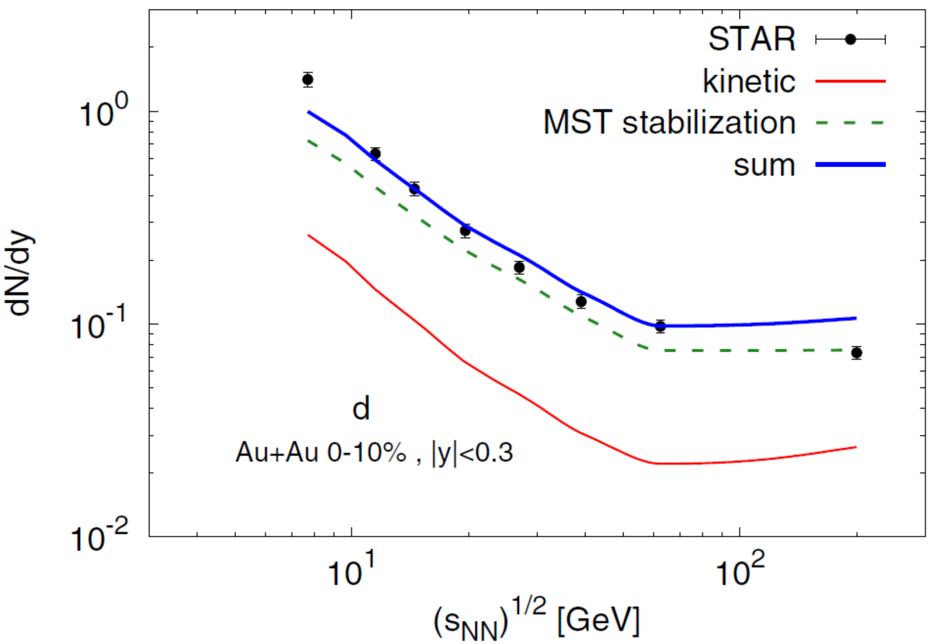
1) the finite-size excluded volume effect in coordinate space $\boxed{|\vec{r}(i)^* - \vec{r}(d)^*| < R_d}$

2) the momentum correlations of p and n inside d by and projection of relative momentum of the p+n pair on the d wave-function in momentum space $|\phi_d(p)|^2$ which lead to a **strong reduction** of d production

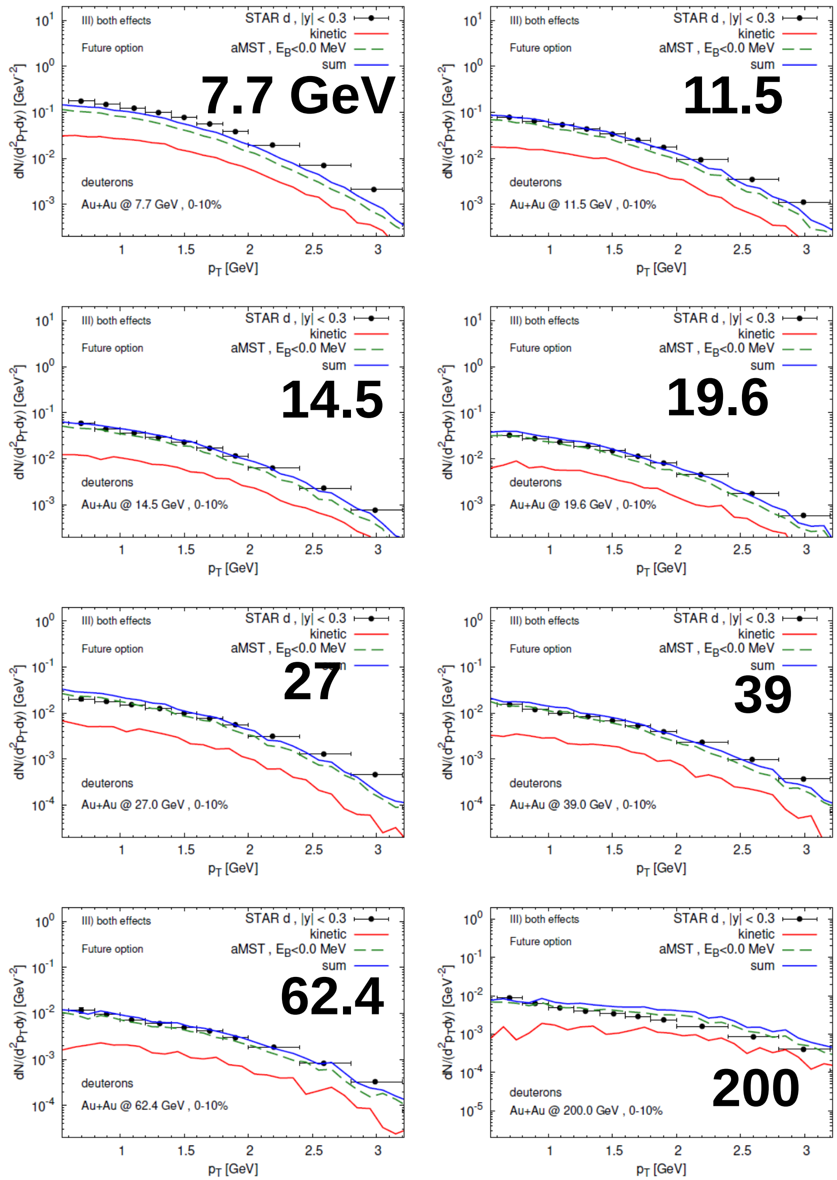


Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



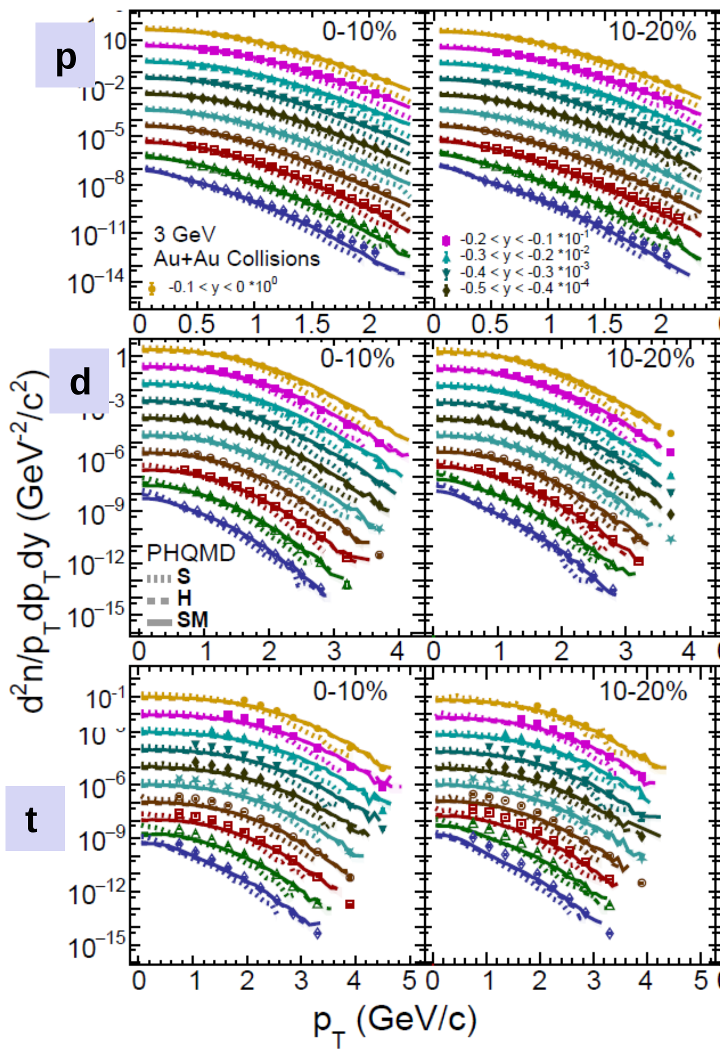
p_T – spectra (BES RHIC)

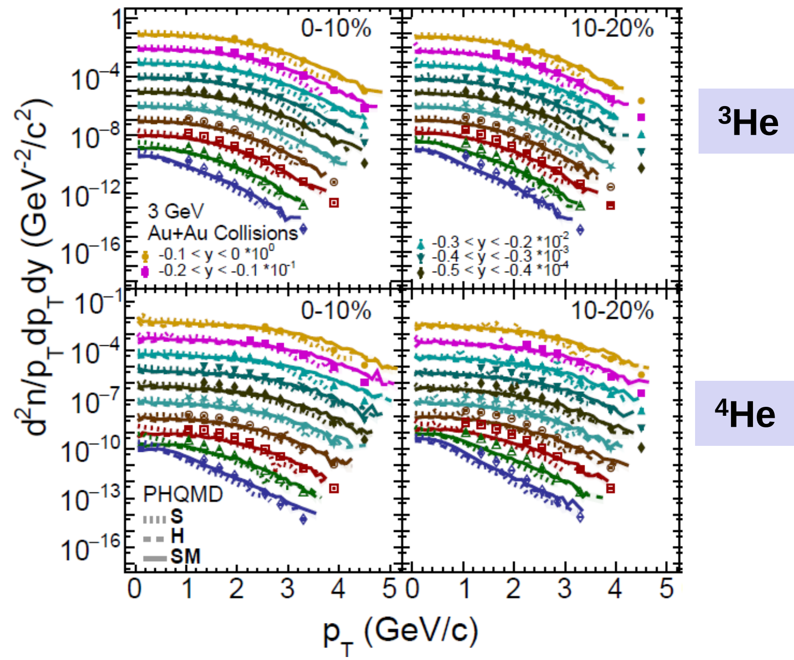


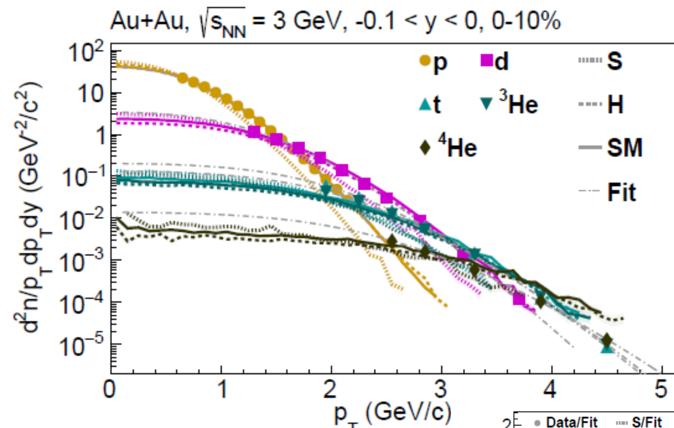
- **PHQMD provides a good description of STAR data**
- **Functional forms of y- and p_T-spectra are slightly different for kinetic and potential deuterons**
- **The potential mechanism is dominant for d production at all energies!**

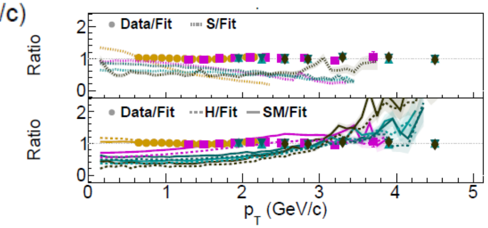
G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

EoS dependence of p_T-spectra at STAR : $s^{1/2}$=3 GeV









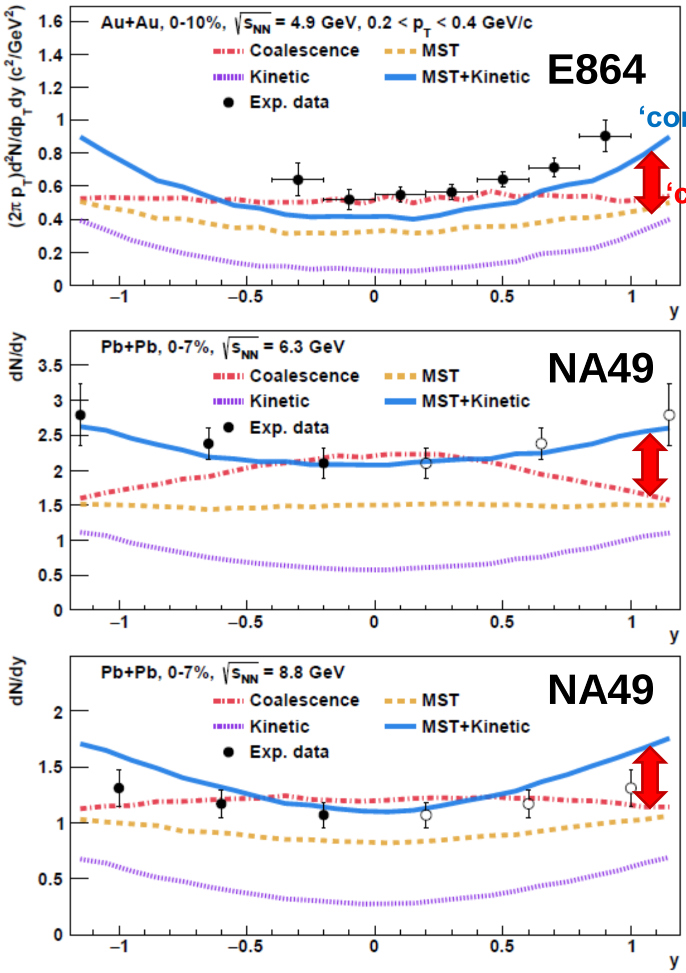
- **STAR p_T data favor a hard or soft-momentum dependent potential (H/SM)**

STAR: M. Abdulhamid et al., PRC 110 (2024) 5, 054911

Yingjie Zhou et al., in progress

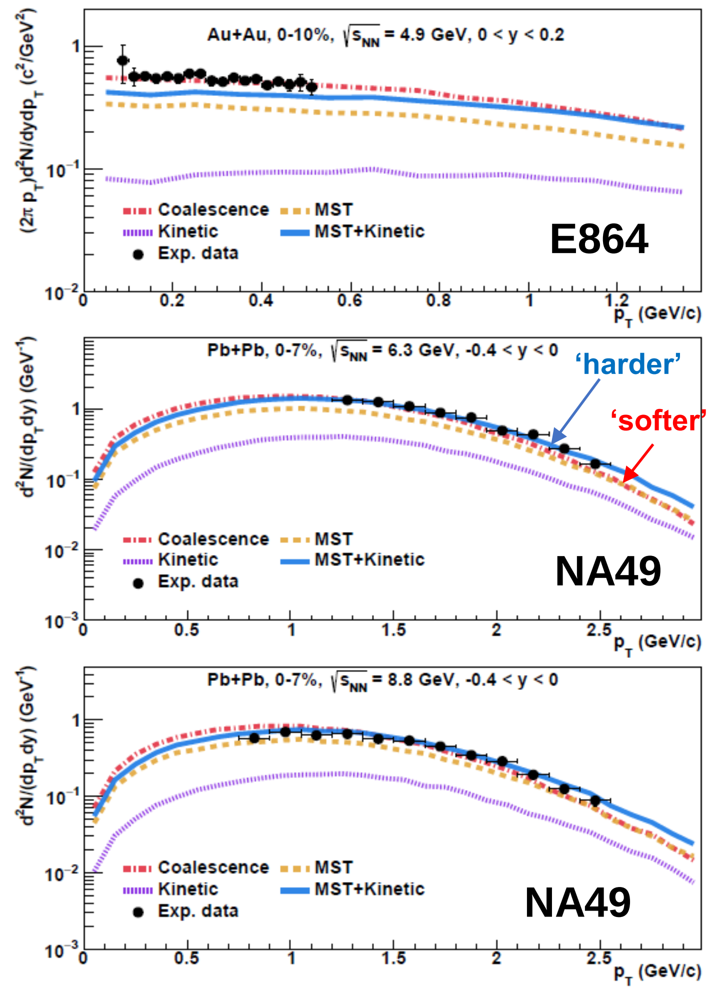
Mechanism for deuteron production: coalescence and MST+kinetic ⟷ experimental data

PHQMD



p_T -distributions have a different slope for coalescence/ MST+kinetic mechanisms

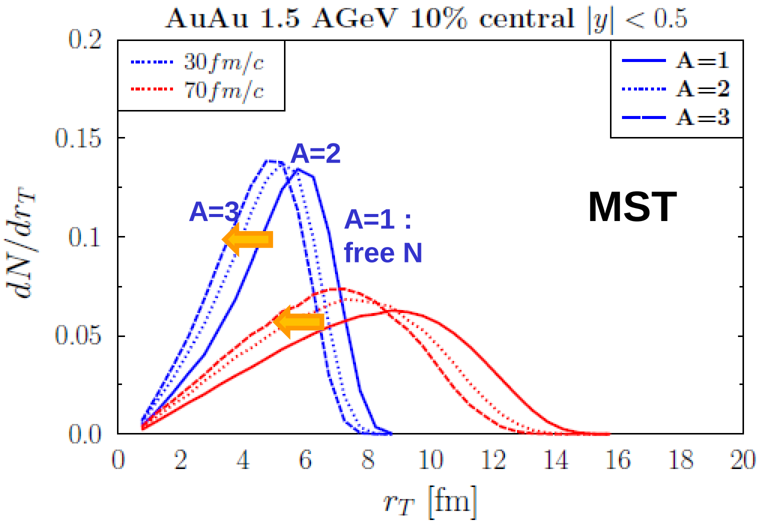
y- distributions show differences

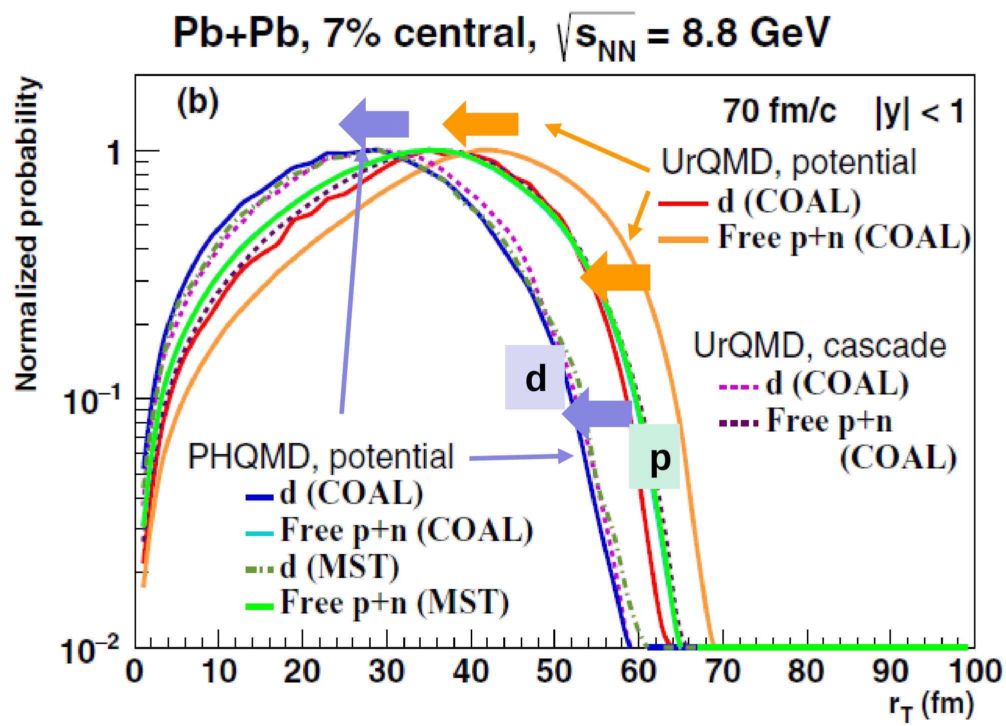


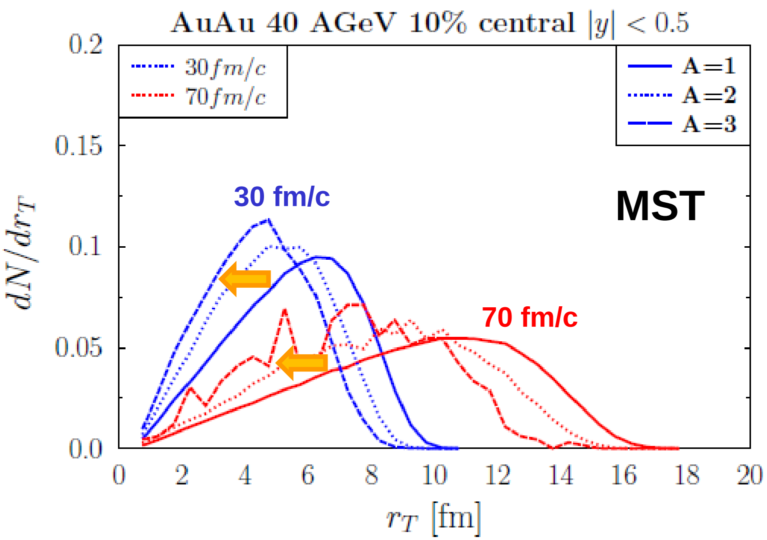
➔ The analysis of the presently available data points tentatively to the MST + kinetic scenario but further experimental data are necessary to establish the cluster production mechanism.

V. Kireyeu et al., PRC109, 044906

PHQMD and UrQMD: Where clusters are formed?







➔ **Stable clusters are formed shortly after elastic and inelastic collisions have ended and behind the front of the expanding energetic hadrons (similar results within PHQMD and UrQMD)**

➔ **since the 'fire' is not at the same place as the 'ice', cluster can survive**

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

Summary

- **NICA & BM@N is an excellent facility to study the properties of the strongly interacting matter at high density and finite temperature**

- **Transport theory is the general basis for an understanding of nuclear dynamics on a microscopic level**

Perspectives with NICA & BM@N:

- **study of EoS by clusters** ***(cf. talk by Viktar Kireyeu),***
- **properties of QGP at high μ_B (in small volume – "droplets") by dileptons,**
- **possible 1st order phase transition,**
- **chiral symmetry restorations by dileptons and strangeness,**
- **subthreshold charm production,**
- **elementary reactions p+p, π+p and p+A**

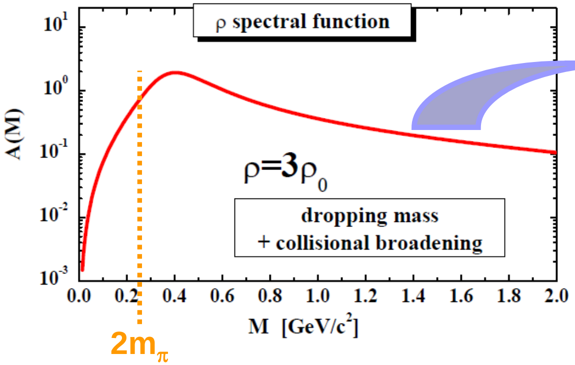
Thanks to the PHSD-PHQMD Team!

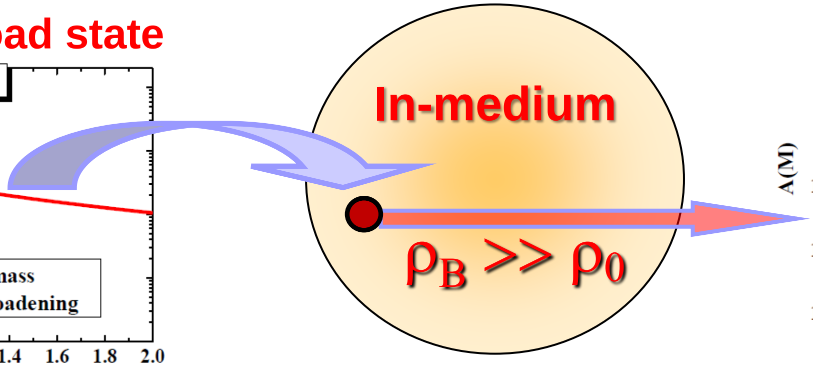
Thanks to Organisers!

Thank you for your attention!

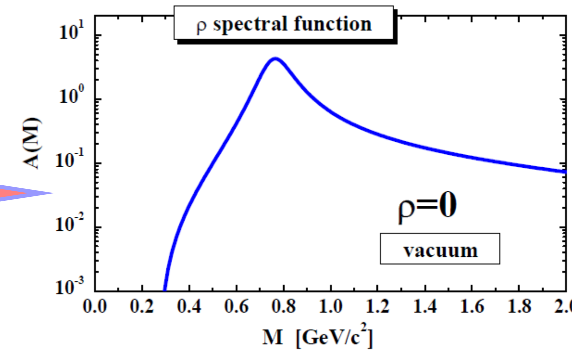
Off-shell vs. on-shell transport dynamics

In-medium:
production of broad state

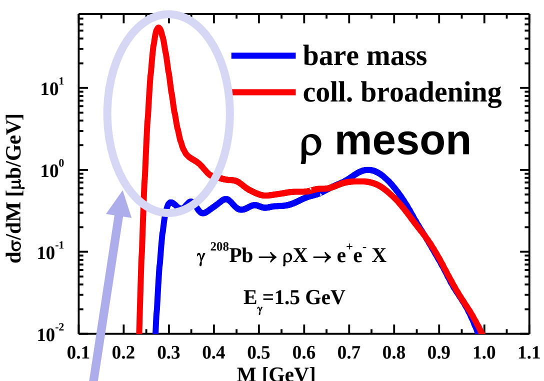




Vacuum (ρ =0) state



1999: dilepton spectra - if one propagates in-medium ρ with on-shell BUU:

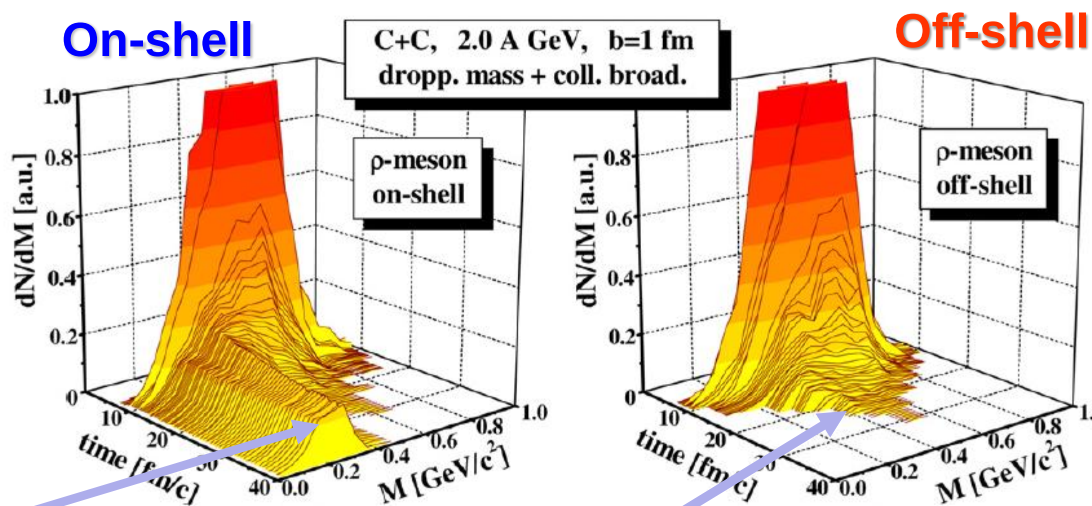


On-shell BUU:

low mass ρ and ω mesons live forever (and shine ‚fake' dileptons)!

(e)GiBUU: M. Effenberger et al, PRC60 (1999) 027601

Mass distribution of ρ mesons vs time



EB & Cassing, NPA 807 (2008) 214

The off-shell spectral function comes to the vacuum shape dynamically by propagation through the medium!

DQPM: Mean-field potential for quasiparticles

Space-like part of energy-momentum tensor $T_{\mu\nu}$ defines the potential energy density:

$$V_p(T,\mu_q) = T^{00}_{g-}(T,\mu_q) + T^{00}_{q-}(T,\mu_q) + T^{00}_{\bar{q}-}(T,\mu_q)$$

space-like gluons + space-like quarks+antiquarks

$$\tilde{\mathrm{Tr}}^{\pm}_{\mathbf{g}} \cdots = \mathbf{d_g} \int \frac{\mathbf{d}\omega}{\mathbf{2}\pi} \frac{\mathbf{d^3 p}}{(\mathbf{2}\pi)^3} \, \mathbf{2}\omega \, \rho_{\mathbf{g}}(\omega) \, \Theta(\omega) \, \mathbf{n_B}(\omega/\mathbf{T}) \, \Theta(\pm \mathbf{P}^2) \cdots$$

$$\tilde{\mathrm{Tr}}^{\pm}_{q} \cdots = d_q \int \frac{d\omega}{2\pi} \frac{d^3 p}{(2\pi)^3} \, 2\omega \, \rho_q(\omega) \, \Theta(\omega) \, n_F((\omega - \mu_q)/T) \, \Theta(\pm P^2) \cdots$$

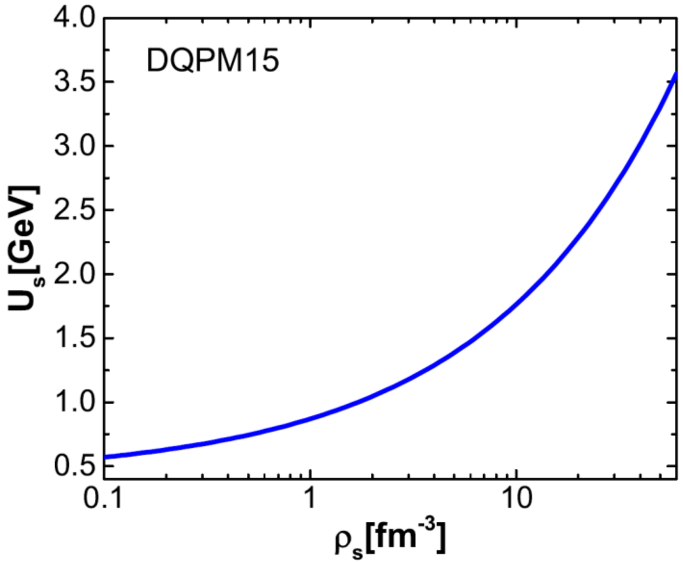
$$\tilde{\mathrm{Tr}}^{\pm}_{\bar{q}} \cdots = d_{\bar{q}} \int \frac{d\omega}{2\pi} \frac{d^3 p}{(2\pi)^3} \, 2\omega \, \rho_{\bar{q}}(\omega) \, \Theta(\omega) \, n_F((\omega + \mu_q)/T) \, \Theta(\pm P^2) \cdots$$

➔ **Mean-field scalar potential (1PI) for quarks and gluons (U_q, U_g) vs parton scalar density ρ_S:**

$$U_s(\rho_s) = \frac{dV_p(\rho_s)}{d\rho_s} \qquad \rho_S = N_g^+ + N_q^+ + N_{\bar{q}}^+$$

$$Uq = Us, \;\; Ug \sim 2Us$$

Quasiparticle potentials (U_q, U_g) are repulsive !



PHSD: ➔ the force acting on a quasiparticle j:

$$F \sim M_j/E_j \nabla U_s(x) = M_j/E_j \; dU_s/d\rho_s \; \nabla\rho_s(x)$$

$$j = g, q, \bar{q}$$

➔ **accelerates particles**

Cassing, NPA 791 (2007) 365: NPA 793 (2007)