speaker: **G. Prokhorov** [1,2]

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based on work:

[GP, Oleg V. Teryaev, Valentin I. Zakharov, arXiv: 2304.13151 (2024)]

INFINUM 2025,
BLTP, JINR, Dubna,
May 12-16, 2025

Possible phase transition in accelerated system

Contents

Part 1

Introduction and Motivation

Phase transitions

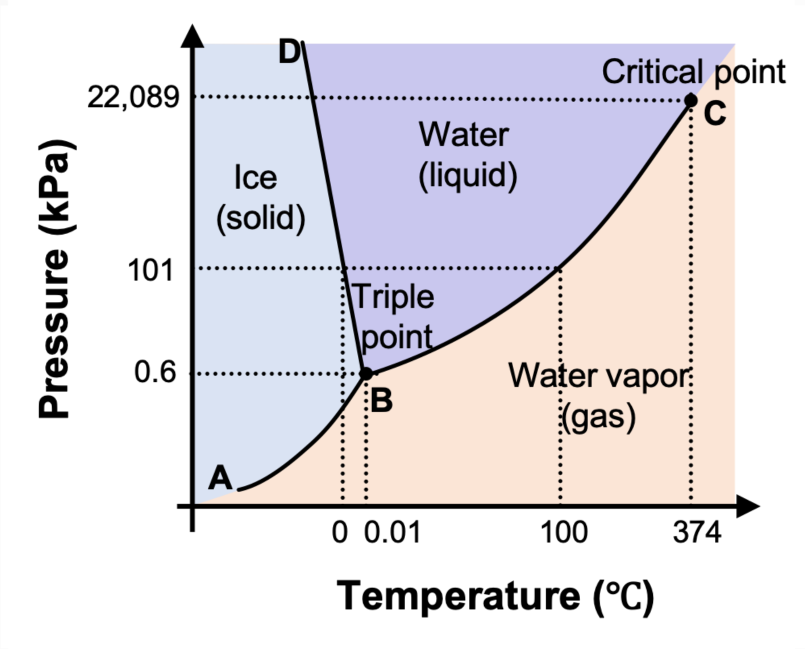
- Phase transitions are a universal phenomenon that play a central role in physics (e.g. the modern **Universe** arose as a result of a series of phase transitions)

grand unification? *electroweak phase transition*

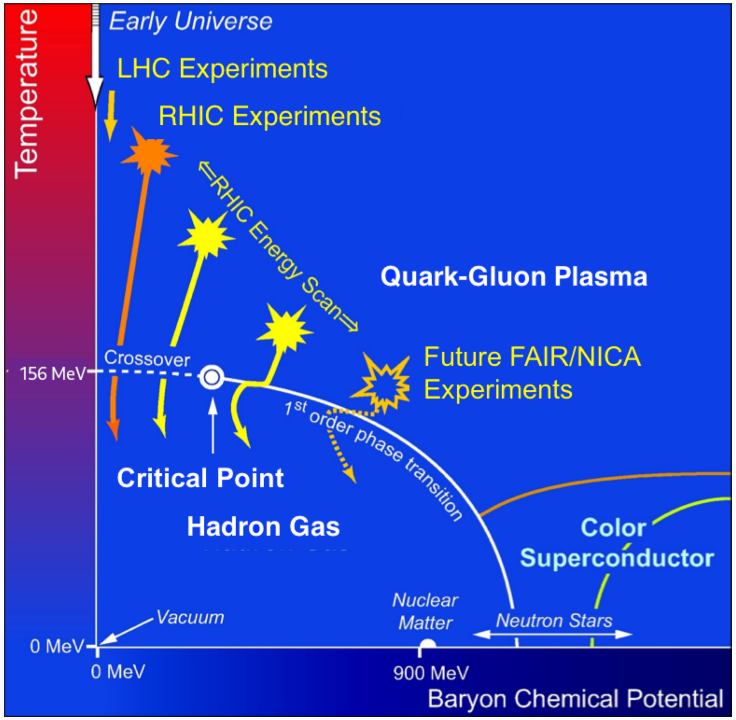
$$SU(5) \to SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1)$$

- The search for a phase transition (or crossover) in **quantum chromodynamics** is one of the main tasks of fundamental high-energy physics

Examples: water-ice-vapor, QCD...



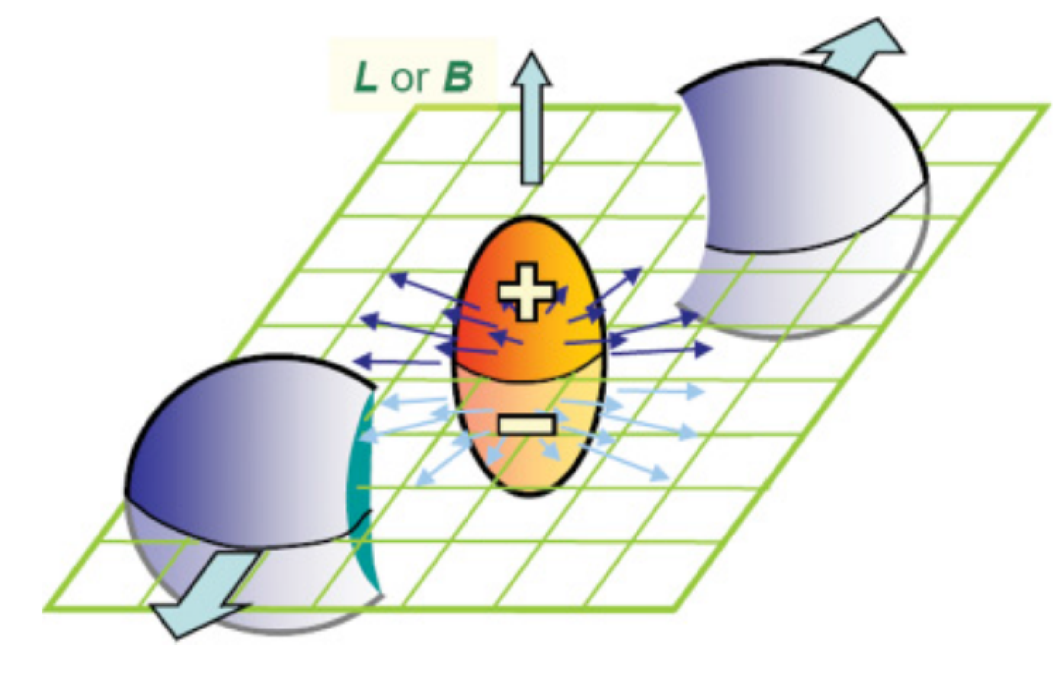
UW-Madison Chemistry 103/104 Resource Book



The Hot QCD White Paper (2015)

Hydrodynamics in heavy ion collisions

- New area: vortical relativistic fluids in external fields
- Off-center collisions of heavy ions produce **huge magnetic fields** and **enormous angular momentum**.



– Rotation is 25 orders of magnitude faster than the rotation of the Earth:
– the vorticity of order 10^{22} sec^{-1}

Annual since 2015:
"International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter"

(Romania 2024, Brazil 2025)

Hydrodynamics in heavy ion collisions

- Plenty results on **vorticity** and **magnetic field** effects:

  -- quantum anomaly transport effects:

  chiral magnetic effect (**CME**), **[Fukushima, Kharzeev, Warringa, PRD (2008), e-Print: 0808.3382]**

  chiral vortical effect (**CVE**), **[Son, Surowka, PRL (2009), e-Print: 0906.5044]**

  kinematical vortical effect (**KVE**), **[Prokhorov, Teryaev, Zakharov, PRL (2022), e-Print: 2207.04449]**

  many other effects…

  -- vortical polarization **[STAR, Nature (2017), arXiv: 1701.06657]** **[Rogachevsky, Sorin, Teryaev, PRC (2010), e-Print: 1006.1331]**

  -- rotation on the lattice **[Braguta, Kotov, Kuznedelev, Roenko, PRC (2021), e-Print: 2102.05084]**

  …

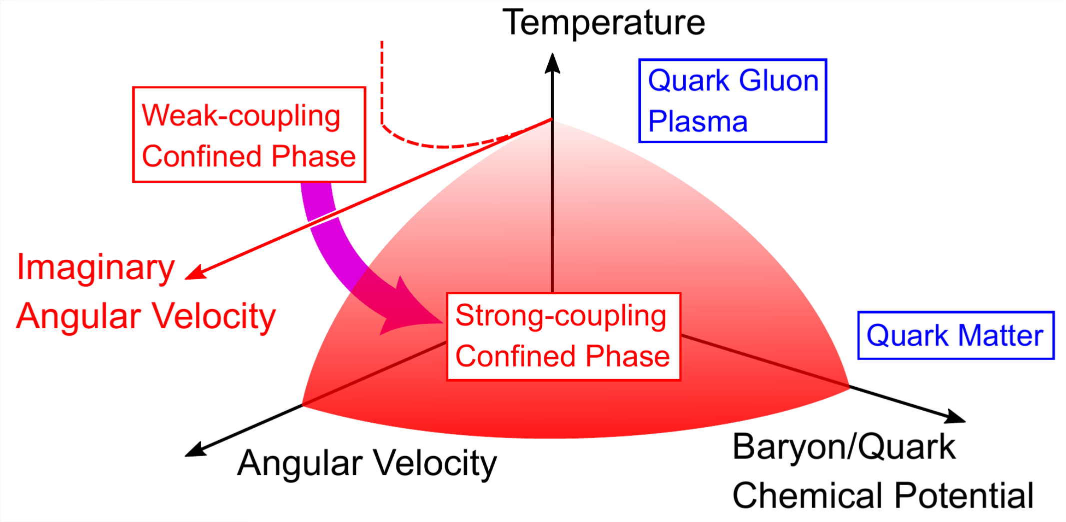
- Modern development: **acceleration** effects

Vorticity	$\leftrightarrow$	**Magnetic field**
Acceleration	$\leftrightarrow$	**Electric field**

It is natural to consider **acceleration** effects in addition to **vorticity**.

Phase diagrams: extra dimensions

- The question of the influence of **magnetic field** and **rotation** on the phase diagram of QCD is currently being actively studied in models and on the lattice:



https://indico.math.cnrs.fr/event/10773/contributions/11970/attachments/5157/8420/fukushima.pdf

- The question of the influence of **acceleration** on the **phase transition** is even less clear.

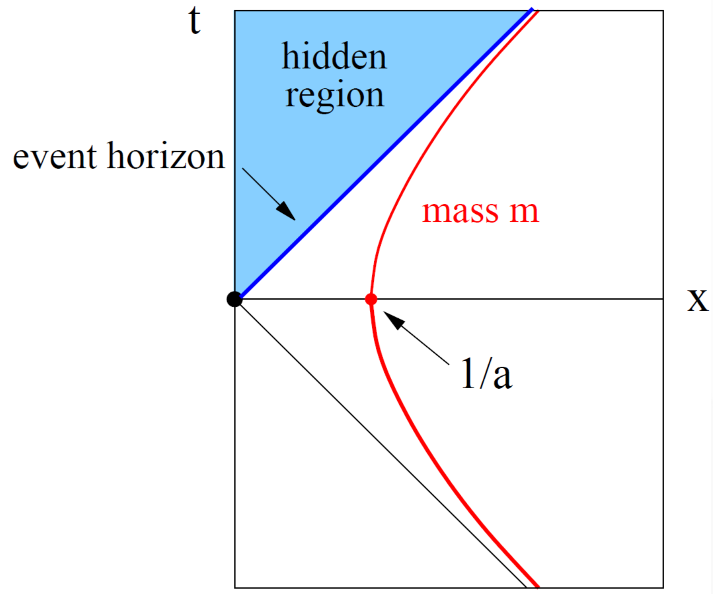
[Chernodub, 2025, e-Print: 2501.16129]
and references therein

Hawking and Unruh effects

- Black hole horizon → thermal Hawking radiation
- Equivalence principle – we expect a similar effect in flat space in an accelerated frame. This is true: the **Unruh effect**.

[W. G. Unruh, Phys. Rev. D 14, 870 (1976)]

In an **accelerated** frame of reference, there is also an event **horizon**.



[Eur.Phys.J. C52 (2007) 187-201]

Formulation

The Minkowski **vacuum** is perceived by an **accelerated** observer as a medium with a finite (Unruh) **temperature**

$$T_U = \frac{a}{2\pi}$$

Toy-like derivation

$$g = GM/R^2 = (R_{black\ hole} = 2GM) = 1/4GM$$

$$T_H = 1/8\pi GM = g/2\pi$$

Equivalence principle: $g \leftrightarrow a$

$$T_H \to \; T_U = \frac{a}{2\pi}$$

Hawking and Unruh effects

Prof. Unruh moving with acceleration feels the temperature

$$T_U = \frac{a}{2\pi}$$

A stationary observer has a temperature

$$T = 0$$

[Blasone, (2018), e-Print: 1911.06002]

Summary of part 1: open questions

- **Unruh effect** in heavy ion **collisions**?

  -- The Unruh effect from a **statistical** mechanics point of view?

  Analogy:

  Huge electric fields are predicted in HIC. They may lead to a (somewhat Unruh-like) Schwinger effect in HIC.

  [Toneev, Rogachevsky, Voronyuk, Eur.Phys.J.A 52, 264 (2016)]

  [Taya, Nishimura, Ohnishi, PRC 110, 014901 (2024)]

- **Critical phenomena** related to **acceleration**?

Part 2

Unruh effect from the quantum-statistical approach

[Prokhorov, Teryaev, Zakharov, JHEP (2020), e-Print: 1911.04545]

The Unruh effect in statistical physics

Consider relativistic fluid of particles with spin 1/2:

Quantities

4-velocity of the fluid	$u_\mu(x)$
Proper temperature	$T(x)$
Inverse temperature vector	$\beta_\mu = u_\mu / T$
Thermal vorticity tensor (analogous to the acceleration tensor)	$\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu \beta_\nu - \nabla_\nu \beta_\mu)$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

$$\alpha_\mu = a_\mu / T$$

acceleration

We consider a medium in a state of
(global) thermodinamic equilibrium

[F. Becattini, L. Bucciantini, E. Grossi, L. Tinti, Eur. Phys. J. C 75, 191 (2015)]

Killing equation

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

Very close to the Tolman-Ehrenfest's criterion and the Luttinger relation

The Unruh effect in statistical physics

The density operator contains the effects of **thermal vorticity**

$$\hat{\rho} = \frac{1}{Z}\exp\Big\{ - \beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \xi\hat{Q}\Big\}$$

The **angular momentum** describes the effects of **vorticity**, while the **boost** generator describes the effects of **acceleration**.

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} \quad = \quad -2\alpha^\rho\hat{K}_\rho - 2w^\rho\hat{J}_\rho$$

The density operator can be used to find the **mean value** of the operator in a medium with a *thermal vorticity*

$$\langle\hat{O}(x)\rangle = \mathrm{tr}\{\hat{\rho}\hat{O}(x)\}_{\mathrm{ren}}$$

The effects of **thermal vorticity** can be calculated in the framework of the **perturbation theory** (a feature is the presence of *non-commuting* operators)

$$\langle\hat{O}(x)\rangle = \langle\hat{O}(0)\rangle_{\beta(x)} + \sum_{N=1}^{\infty}\frac{\varpi^N}{2^N|\beta|^N N!}\int_0^{|\beta|} d\tau_1 d\tau_2 ... d\tau_N \langle T_\tau \hat{J}_{-i\tau_1 u} ... \hat{J}_{-i\tau_N u}\hat{O}(0)\rangle_{\beta(x),c}$$

The Unruh effect in statistical physics

The meaning of the Unruh effect is that the accelerated observer sees the **Minkowski vacuum** as a medium filled with particles with a **Unruh temperature** proportional to the acceleration

$$T_U = \frac{a}{2\pi}$$

Thus, the **mean values** of the thermodynamic quantities normalized to Minkowski vacuum should be **equal to zero** when the proper temperature, measured by comoving observer, equals to the **Unruh temperature**.

- It has been shown for scalar particles.

[F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018)]

For **fermions**, a similar effect was observed based on the Wigner function in the Boltzmann limit at a **double** Unruh temperature $2T_U$

[W. Florkowski, E. Speranza and F. Becattini, Acta Phys. Polon. B 49, 1409 (2018)]

- This is due to the **approximate** nature of the Wigner function used.

Unruh effect from quantum statistical mechanics

$$\langle \hat{T}^{\mu\nu} \rangle = (\rho_0 + A_1 T^2 |a|^2 + A_2 |a|^4) u^\mu u^\nu - (p_0 + A_3 T^2 |a|^2 + A_4 |a|^4) \Delta^{\mu\nu}$$
$$+ (A_5 T^2 + A_6 |a|^2) a^\mu a^\nu + \mathcal{O}(a^6) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \,,$$

The final momentum integral for A_2 has the form ($\tilde{p} = |\mathbf{p}|/T$)

$$A_2 = \int_0^\infty d\tilde{p} e^{\frac{9\tilde{p}}{2}} \tilde{p}^3 \Big(5600\tilde{p}\,(49\tilde{p}^2 - 95)\cosh\left(\frac{\tilde{p}}{2}\right) + 2016\tilde{p}\,(25 - 119\tilde{p}^2)\cosh\left(\frac{3\tilde{p}}{2}\right)$$
$$+53200\left(\sinh\left(\frac{3\tilde{p}}{2}\right) - 11\sinh\left(\frac{\tilde{p}}{2}\right)\right)\cosh^4\left(\frac{\tilde{p}}{2}\right) + \tilde{p}\Big(-224\,(\tilde{p}^2 + 25)\cosh\left(\frac{7\tilde{p}}{2}\right)$$
$$+224\,(119\tilde{p}^2 + 575)\cosh\left(\frac{5\tilde{p}}{2}\right) + 18\tilde{p}\sinh\left(\frac{\tilde{p}}{2}\right)\Big(-5786\tilde{p}^2 + (\tilde{p}^2 + 210)\cosh(3\tilde{p})$$
$$-6\,(41\tilde{p}^2 + 1890)\cosh(2\tilde{p}) + 3\,(1349\tilde{p}^2 + 9450)\cosh(\tilde{p})$$
$$+39900\Big)\Big)\Big)(50400\pi^2\,(e^{\tilde{p}} + 1)^9)^{-1} \,,$$

$$A_2(m = 0) = -\frac{17}{960\pi^2}$$

This integral be found analytically

The Unruh effect in statistical physics

Similarly for the other components of the energy-momentum tensor

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{fermi}} = \left(\frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2} \right) u^\mu u^\nu - \left(\frac{7\pi^2 T^4}{180} + \frac{T^2 |a|^2}{72} - \frac{17|a|^4}{2880\pi^2} \right) \Delta^{\mu\nu}$$

The energy-momentum tensor **vanishes** at the **Unruh temperature**

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

Thus, a consequence of the **Unruh effect** for Dirac fields is **justified**.

- **The same can be shown for other spins and finite mass.**

Part 3

Phase transition at the Unruh temperature

Minimal temperature?

- It is assumed that T_U is minimal, since the energy becomes negative below T_U
  [F. Becattini, Phys. Rev., D97(8):085013, 2018.]

- We know that in heavy ion collisions there is a problem of fast thermalization *[D. Kharzeev, K. Tuchin. Phys. A, 753:316–334, 2005.]*

⇩ ⇩ ⇩

Presumably there are states with $T < T_U$

Negative energies exist in physics (for example, in the ergosphere of rotating black holes).

Goal: construct an analytical continuation to the region $T < T_U$

- We will show that for massless fields with spin 1/2 it is impossible to use the old formulas → a phase transition occurs

Novel phase transition

First signs:

- Instability at the Unruh temperature in the axial current of fermions (thermodynamic approach):

  [Prokhorov, G., Teryaev, O., & Zakharov, V. (2018). Phys. Rev. D, 98(7), 071901]

- Similar instability in energy density (thermodynamic approach):

  [Prokhorov, G. Y., Teryaev, O. V., & Zakharov, V. I. (2019). Phys. Rev. D, 100(12), 125009]

Similar observations:

- Imaginary mass in scalar theory with interaction below the Unruh temperature:

  [Diakonov, D. V., & Bazarov, K. V. (2023), 2301.07478]

- Different formulas for the free energy of massive scalar fields above and below the Unruh temperature:

  [Akhmedov, E. T., & Diakonov, D. V. (2022). Phys. Rev. D, 105(10), 105003]

- “Critical” points with “imaginary” rotation:

  [Chernodub, M. N. (2022), 2210.05651]

- Problems with the limit $T \to T_U$ for higher spins (3/2, 2) in Schwinger-DeWitt coefficients: ***[D. V. Fursaev, G. Miele. Nucl. Phys. B, 484:697–723, 1997]***

Novel phase transition

The effects of acceleration can also be investigated from the point of view of an **accelerated observer**. In this case, the **Rindler coordinates** are to be used:

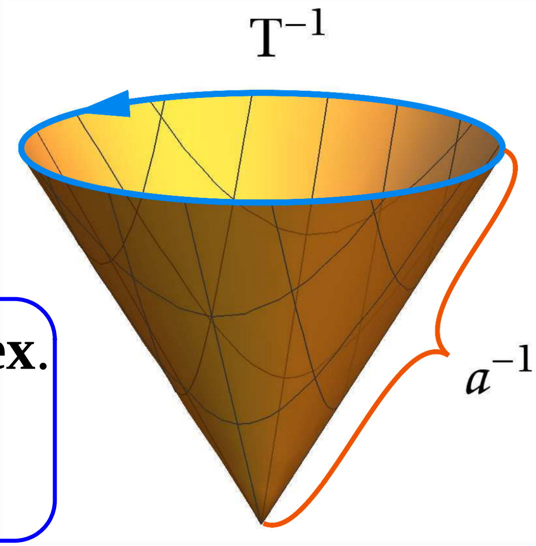
$$ds^2 = -\rho^2 d\theta^2 + dx^2 + dy^2 + d\rho^2$$

Passing to imaginary time:

$$ds^2 = \rho^2 d\theta^2 + d\rho^2 + d\mathbf{x}_\perp^2$$

$\mathcal{M} = \mathbb{R}^2 \otimes \mathcal{C}_\nu^2$

It describes a flat two-dimensional cone with an angular deficit $2\pi - a/T$. This metric contains a **conical singularity** at $\rho = 0$



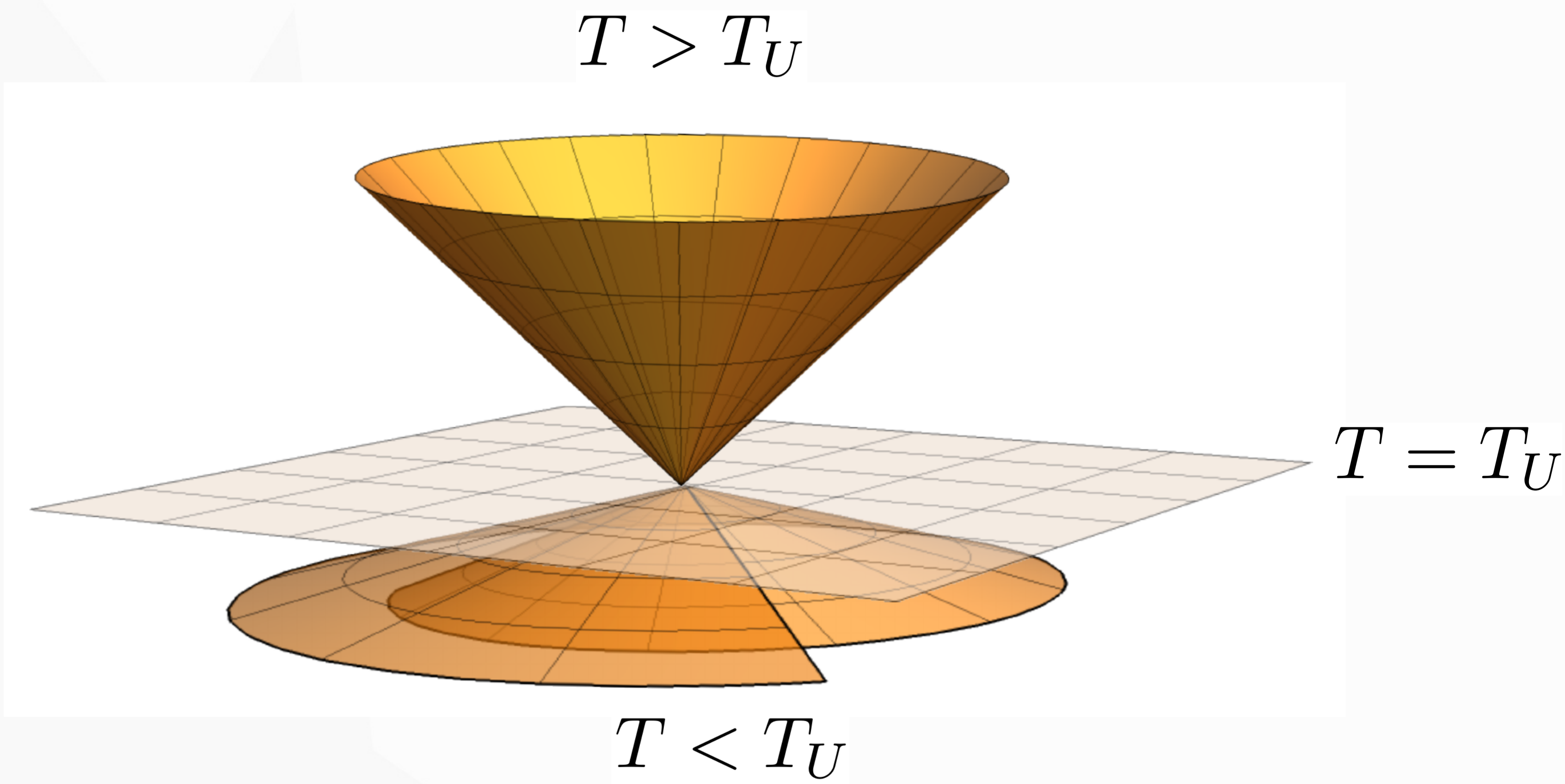
<u>**Dictionary**</u> for translation

Thermodynamic characteristics in *Geometrical*:

Inverse **acceleration** ⟺ **distance** from the **vertex**.

Inverse proper **temperature** ⟺ **circumference**.

Novel phase transition



- The region $T < T_U$ corresponds to a cone with an angle **greater than 360 degrees**.
- **Odd number of full** rotations of 360 degrees: $T_k = T_U/(2k+1)$

Matsubara modes on the horizon

[V. B. Bezerra, N. R. Khusnutdinov. Class. Quant. Grav., 23:3449–3462, 2006]

- Consider the **Green function** of the Dirac fields in the Euclidean Rindler space: $$\not{D}_x S_E(x;x') = -I_4 \frac{\delta^4(x-x')}{\sqrt{g}}$$
- It is more convenient to consider the Green function of the **square** of the Dirac operator: $$\not{D}_x^2 G_E(x;x') = -I_4 \frac{\delta^4(x-x')}{\sqrt{g}}$$
- They are **related** to each other: $$S_E(x;x') = \not{D}_x G_E(x;x')$$
- Dirac operator is defined as: $$\not{D} = \gamma_E^\mu \nabla_\mu$$

We use the tetrad of the form:

$$e^\mu_{(a)} = e^{\mu(a)} = \begin{pmatrix} \frac{\nu}{\rho} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

[B. Linet. J. Math. Phys., 36:3694–3703, 1995]

- Curved Dirac matrices: $$\gamma_E^\mu = e^\mu_{(a)} \gamma_E^{(a)}$$
- Covariant derivative includes spinor connection: $$\nabla_\mu \psi = (\partial_\mu + \Gamma_\mu)\psi$$ $$\Gamma_\mu = \frac{1}{2}\Sigma_{(a)(b)} e^{(a)\lambda} \nabla_\mu e^{(b)}_\lambda$$
- **Squared Dirac operator** (we neglect singularity in the apex):

$$\not{D}_x^2 = I_4 \left(g^{\mu\nu}\partial_\mu\partial_\nu - \frac{1}{4\rho^2} + \frac{1}{\rho}\partial_\rho \right) + i\frac{2\nu}{\rho^2}\Sigma_0$$

Matsubara modes on the horizon

- Consider the **eigenmodes** of the **square** of the Dirac operator

$$\not{D}_x^2 \phi(x) = -\lambda^2 \phi(x)$$

- Tetrad choise fixes **antiperiodic** boundary conditions:

$$\phi(\varphi + 2\pi n) = (-1)^n \phi(\varphi)$$

- Solutions can be classified by eigen values of mutually commuting operators:

$$\widehat{p}_{\rm x}\phi(x) = -i\partial_{\rm x}\phi(x) = p_{\rm x}\phi(x)$$

$$\widehat{p}_{\rm y}\phi(x) = -i\partial_{\rm y}\phi(x) = p_{\rm y}\phi(x)$$

$$\widehat{p}_0\phi(x) = -i\partial_\varphi\phi(x) = \left(n + \frac{1}{2}\right)\phi(x)$$

$$\Sigma_3\phi(x) = s_2\frac{1}{2}\phi(x)\,, \quad s_2 = \pm 1$$

$$\Sigma_0\phi(x) = s_1\frac{1}{2}\phi(x)\,, \quad s_1 = \pm 1$$

-- Matsubara frequency, e.g.

$$i(n + \tfrac{1}{2})\varphi = i\,\pi T(2n+1)\tau$$

- All the eigen values: $q = (p_{\rm x}, p_{\rm y}, n + 1/2, \lambda, is_1/2, s_2/2)$

Matsubara modes on the horizon

The solution to the eigenvalue equation of operator $\not{D}_x^2$ is well known...

- But there are **two solutions!**

$$\phi_q^{\pm}(x) = \frac{\sqrt{\nu}}{4\pi^{3/2}} e^{ip_{\mathrm{x}}\mathrm{x}+ip_{\mathrm{y}}\mathrm{y}+i(n+\frac{1}{2})\varphi} J_{\pm\beta_{s_1}}(\xi\rho)\, w_{(s_1,s_2)}$$

where $\beta_{s_1} = \nu(n+\frac{1}{2}) - \frac{s_1}{2}$

- Only one of them is regular on the horizon $\rho \to 0$

Really, on the horison, asymptotiacally, one has $J_a(x) \sim x^a$

Matsubara modes on the horizon

- At the Unruh temperature **two lowest** Matsubara modes **change** there solutions!

We consider two modes $(n = 0, s_1 = -1)$ and $(n = -1, s_1 = 1)$

finite solutions at

$T > T_U \quad (\nu > 1)$

finite solutions at

$T < T_U \quad (\nu < 1)$

$\phi^{+}_{n=0,s_1=-1} = (\dots)\ J_{\nu/2-1/2}(\xi\rho)$ ⇒ $\phi^{-}_{n=0,s_1=-1} = (\dots)\ J_{-(\nu/2-1/2)}(\xi\rho)$

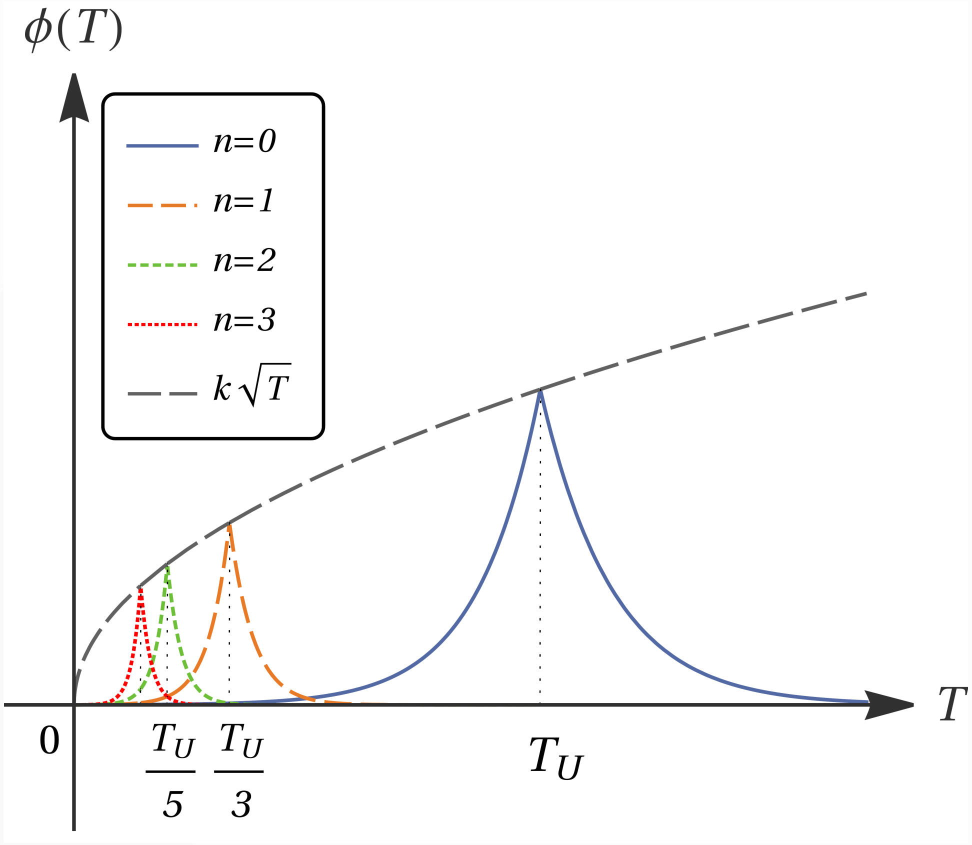
$\phi^{-}_{n=-1,s_1=1} = (\dots)\ J_{-(-\nu/2+1/2)}(\xi\rho)$ ⇒ $\phi^{+}_{n=-1,s_1=1} = (\dots)\ J_{-\nu/2+1/2}(\xi\rho)$

- E.g. when passing through $T = T_U$

$$\phi^{+}_{(n=0,\, s_1=1)} \to \phi^{-}_{(n=0,\, s_1=1)}$$

$$\phi^{-}_{(n=-1,\, s_1=-1)} \to \phi^{+}_{(n=-1,\, s_1=-1)}$$

Novel phase transition



- When geometry changes from cone to plane, **lowest Matsubara modes** become **singular on the horizon** and change solution. This leads to **peaks** in the behavior of the modes.

- The situation repeats for other pairs of higher Matsubara modes at the points

$$T_k = T_U/(2k+1)$$

Novel phase transition

- But **different** mean values below and above $T_k = T_U/(2k+1)$
- Let us consider the **first critical point**:

$$\frac{|a|}{6\pi} < T < \frac{|a|}{2\pi}$$

$$\langle \hat{T}^{\alpha}_{\beta} \rangle = \left(\frac{127\pi^2 T^4}{60} - \frac{11|a|^2 T^2}{24} - \frac{17|a|^4}{960\pi^2} \right) \left(u^{\alpha} u_{\beta} - \frac{1}{3} \Delta^{\alpha}_{\beta} \right) + \left(\pi |a| T^3 - \frac{T|a|^3}{4\pi} \right) \widetilde{\Delta}^{\alpha}_{\beta} \,,$$

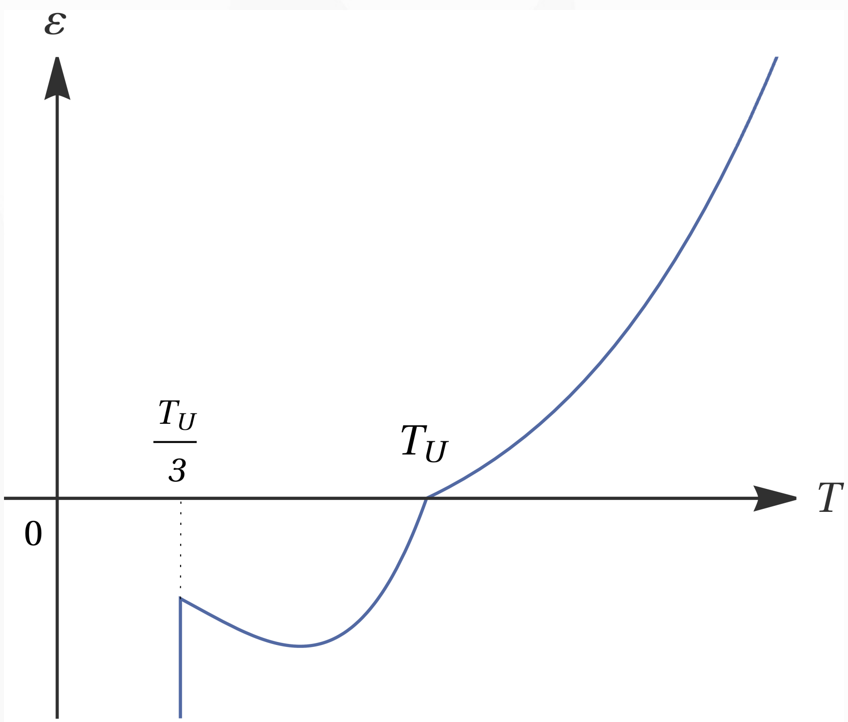
$$\widetilde{\Delta}^{\alpha}_{\beta} = \Delta^{\alpha}_{\beta} + \frac{a^{\alpha} a_{\beta}}{|a|^2}$$

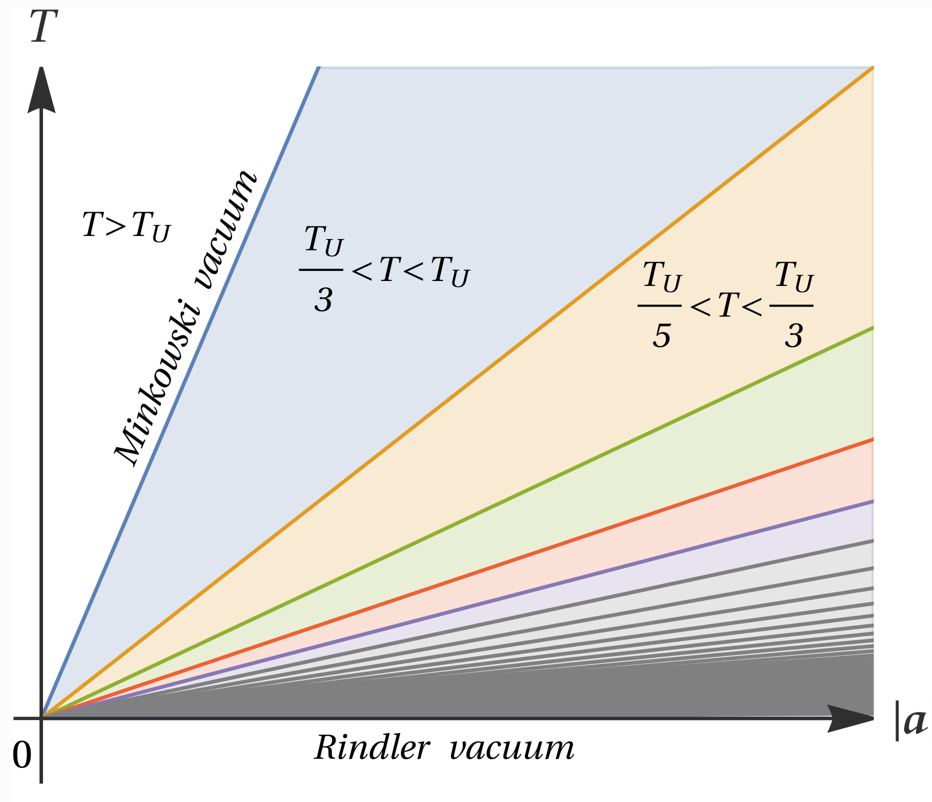
$$\Delta^{\alpha}_{\beta} = \delta^{\alpha}_{\beta} - u^{\alpha} u_{\beta}$$

$$T > T_U$$

$$\langle \hat{T}^{\mu\nu} \rangle = \left(\frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2} \right) u^{\mu} u^{\nu} - \left(\frac{7\pi^2 T^4}{180} + \frac{T^2 |a|^2}{72} - \frac{17|a|^4}{2880\pi^2} \right) \Delta^{\mu\nu}$$

Novel phase transition





We confirm instabilties at the points:

phase diagram

$$T_k = T_U/(2k+1)$$

Novel phase transition

- **Second order** phase transition:

$$\frac{\partial \varepsilon}{\partial T}\Big|_{T\to T_U+0} = \frac{4\pi T_U^3}{5}, \qquad \frac{\partial \varepsilon}{\partial T}\Big|_{T\to T_U-0} = \frac{24\pi T_U^3}{5}$$

$T > T_U$ $\quad \langle \hat{T}^{\beta}_{\beta} \rangle = 0$

$T < T_U$ $\quad \langle \hat{T}^{\beta}_{\beta} \rangle = \frac{\nu(\nu^2-1)}{4\pi^2\rho^4} = 2\pi T|a|\left(T^2 - \frac{|a|^2}{4\pi^2}\right)$

- Trace as an **order parameter**?

- The change in energy is associated with a change in the energy of the two lowest modes:

$$\Delta\varepsilon = \Delta\varepsilon_{(n=0,\, s_1=1)} + \Delta\varepsilon_{(n=-1,\, s_1=-1)} = 2\pi^2 T^4 - \frac{T^2|a|^2}{2}$$

Part 4 Discussion

Novel phase transition

Thus, we found the same critical points as in the (approximate) statistical approach:

[Prokhorov, G., Teryaev, O., & Zakharov, V. (2018). Phys. Rev. D, 98(7), 071901]

$$Critical\ points: \qquad T = T_U/(2k+1) \rightleftarrows \nu = 1/(2k+1) \qquad k = 0, 1, 2...$$

Such mode jumping behaviour will lead to **new Green's functions** and **change** the quantum **mean values**.

Fundamental reason:

”finiteness of the speed of light”

(leads to non-compactness of the Poincaré group → non-unitarity of representations → anti-Hermitianity of the boost generator)

Acceleration as imaginary chemical potential

Wigner function

$$W(x,k)_{AB} = -\frac{1}{(2\pi)^4}\int d^4y\, e^{-ik\cdot y}\langle : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \rangle$$

⇨ Contains **thermodynamic** information about the system

Ansatz:

Acceleration

Boost operator

$$X(x,p) = \left\{\exp\left(\frac{\varepsilon_p I_4}{T} - \frac{|a|\Sigma_0}{T}\right) + I_4\right\}^{-1}$$

[F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338 (2013) 32.]

See also talk of Amaresh Jaiswal

Acceleration as imaginary chemical potential

Energy density of accelerated fermion gas:

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\left(\frac{|\mathbf{p}| + ia}{1 + e^{\frac{|\mathbf{p}|}{T} + \frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1 + e^{\frac{|\mathbf{p}|}{T} - \frac{ia}{2T}}}\right)$$

$$+4\int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}} - 1} \qquad (T > T_U)$$

in red: *modifications* compared to the *Wigner function*

- In the first integral, the **acceleration** enters as an **imaginary chemical potential** $\pm\frac{ia}{2}$
- **Instabilities** (jumps) at the **points:**

$$T_k = T_U/(2k+1)$$

Comparison with the Wigner function

- Can be compared with previous results from the statistical approach and the approximate Wigner function:

Wigner function:

$$X(x,p) = \left\{ \exp\left(\beta_\mu p^\mu I_4 \pm \frac{a_\mu K_s^\mu}{T} \right) + I_4 \right\}^{-1}$$

Energy density

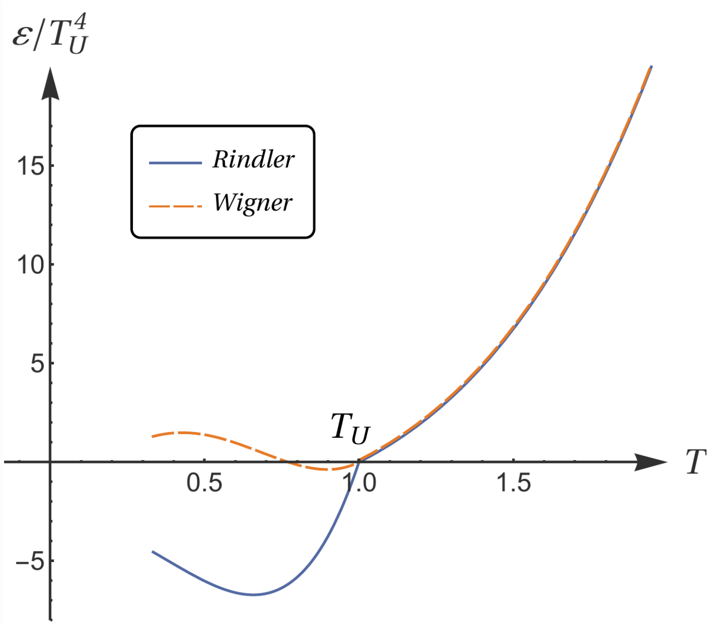
$$\varepsilon = 2 \int \frac{d^3p}{(2\pi)^3} \left(\frac{|\mathbf{p}|+i|a|}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{i|a|}{2T}}} + \frac{|\mathbf{p}|-i|a|}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{i|a|}{2T}}} \right) + 4 \int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{|a|}} - 1}$$

$T < T_U$

$$\varepsilon = \frac{127\pi^2 T^4}{60} - \frac{11 T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2} - \pi T^3 |a| + \frac{T|a|^3}{4\pi}$$

"Rigorous" analytic continuation:

$$\varepsilon = \frac{127\pi^2 T^4}{60} - \frac{11|a|^2 T^2}{24} - \frac{17|a|^4}{960\pi^2}$$



Cosmic strings: duality

Cosmic string metric is *equivalent* to the **Euclidean Rindler** metric up to a coordinate re-designation *(J. S. Dowker, Class. Quant. Grav. 11, L55 (1994))*

	Euclidean Rindler metric	Cosmic string
Form of metric	$ds^2 = \rho^2 d\theta^2 + d\rho^2 + d\mathbf{x}_\perp^2$	$ds^2 = -dt^2 + dz^2 + \rho^2 d\theta^2 + d\rho^2$
Angular deficit	$2\pi - a/T$	$2\pi - 2\pi/\nu$
The component of the energy-momentum tensor corresponding to the periodic coordinate	T_0^0	T_2^2

⇩

*translation from **cosmic strings** to **Euclidean Rindler** spacetime*

$$\rho_{Rindler} = \langle T_2^2 \rangle_{string} \; , \; \nu = 2\pi T/a$$

Cosmic strings: duality

- Since the metric of the cosmic string has the same form, then similar instabilities appear for the **cosmic string** too:

$$G^E(x;x'|N_0)_{string} = G^E(x;x'|N_0)_{Rindler}\Big|_{\mathrm{x}\to\tau\,,\,\mathrm{y}\to\mathrm{z}\,,\,\Sigma_0\to\,\Sigma_3}$$

cosmic string density becomes negative if we go into the hypothetical region

$$T < T_U \quad \longrightarrow \quad \mu^* = \frac{\nu-1}{4\nu} < 0$$

- Finally we obtain:

$$\nu > 1: \quad \langle T^\alpha_\beta \rangle = \frac{17 - 10\nu^2 - 7\nu^4}{2880\pi^2\rho^4}\mathrm{diag}\,(1,1,-3,1)\ ,$$

$$\frac{1}{3} < \nu < 1: \quad \langle T^\alpha_\beta \rangle = \frac{17 + 110\nu^2 - 127\nu^4}{2880\pi^2\rho^4}\mathrm{diag}\,(1,1,-3,1) + \frac{\nu(\nu^2-1)}{8\pi^2\rho^4}\mathrm{diag}\,(1,0,0,1)\ .$$

Quantum phase transition

Has the features of both **thermal** and **quantum** phase transition.
[Subir Sachdev. Quantum Phase Transitions. Cambridge University Press, 2 edition, 2011]

- On the one hand, it occurs at the finite temperature, which makes it similar to the classical thermal transition.

- On the other hand, the critical temperature, having the Planck scale, is extremely small

$$t_a = c/|a|$$

$$t_r = \xi \frac{\hbar}{k_B T} \quad \leftrightarrows \quad T_U = \frac{\hbar |a|}{2\pi k_B c}$$

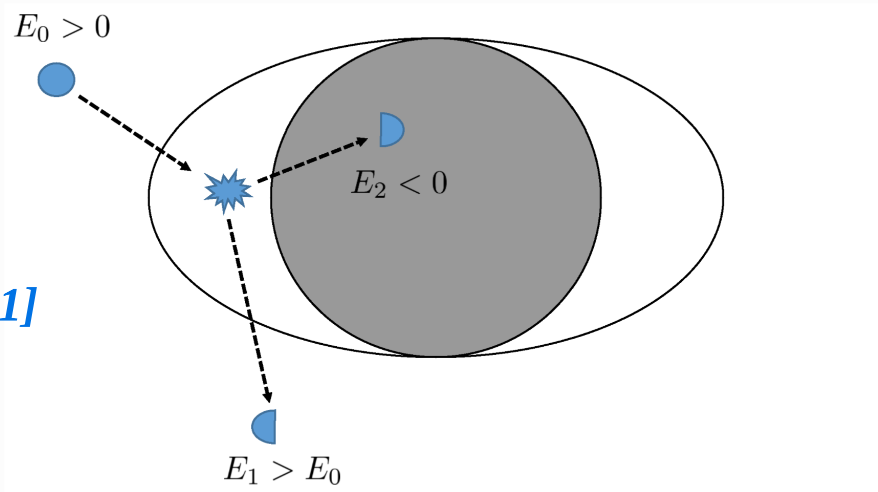
$$\xi = 1/2\pi$$

Inside black hole?

- Do states below the Unruh (Hawking) temperature relate (can be realized or related in some way) to the **interior of a black hole**?

- Simple arguments in favor this:

$$T < T_U \quad \Longrightarrow \quad \varepsilon < 0$$



Inside the ergosphere the energy is negative

[R. Penrose and R. M. Floyd. Nature, 229:177–179, 1971]

[Brito, Richard et al. Lect.Notes Phys. 906 (2015) pp.1-237]

Superradiance: extraction of energy from the black hole

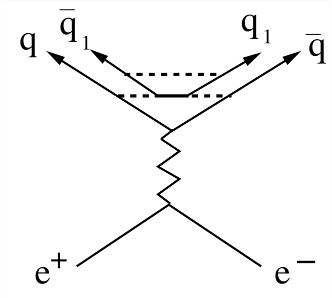
Negative modes inside a black hole (as a source of instability of the classical manifold)

[G. L. Pimentel, A. M. Polyakov, and G. M. Tarnopolsk, Rev. Math. Phys. 30, no. 07, 1840013 (2018)]

$$S_{\text{eff}}(v) = mT\sqrt{1 - V^2} + \dots .$$

Predictions for Phenomenology

- The mechanism of thermalization with the Unruh effect



[P. Castorina, D. Kharzeev, H. Satz, Eur.Phys.J. C 52 (2007) 187-201]

- A quark pair is born in an accelerated state due to string tension: Unruh effect "thermalizes" hadronic spectra

Not everything is clear with this scenario:

-- due to the Unruh effect, particles are born in an accelerated frame of reference, and hadronization is observed in the laboratory, how can this be?

Predictions for Phenomenology

Alternative qualitative description of **thermalisation in HIC**:

-- first the nuclei collide - large acceleration due to stopping forces

-- Thermalization had not yet occurred at the very beginning

It should be expected that at the **initial** moments **after** the **collision**:

$$T < T_U$$

Confirmed by direct collision simulation!

-- if earlier the state $T < T_U$ was "hidden" inside the black hole, now it is hidden by confinement (a region in Hilbert space)?

Predictions for Phenomenology

If there is a **process of type**

$$(\text{black hole} + \text{matter with } T < T_U) \;\rightarrow\; (\text{Minkowskian vacuum}) + (\text{thermal matter})$$

Then in this case it leads to the appearance of a **thermal hadron spectrum** with the **Unruh temperature** (which is of the order of QCD scale), which is what is needed for phenomenology

[T. Morita, Phys. Rev. Lett. 122, 101603 (2019)]

-- thermalization occurs as a sub-barrier transition - an explanation for the **fast thermalization**?

-- is the **phase transition at the Unruh temperature** closely related to the **QCD transition**? Alternative scenario of hadronisation.

Modeling

[Prokhorov, Shohonov, Teryaev, Tsegelnik, Zakharov, (2025) arXiv: 2502.10146]

Details will be given in the next talk of N. Tsegelnik

Statement of the problem and methods

Problems and motivation for modeling:

- The results obtained motivate acceleration modeling in HICs. Little studied, see however:
  [Karpenko, Becattini, Nucl.Phys.A (2019), arXiv:1811.00322]
- Check that extreme accelerations are generated.
- **Verify** the existence of **"exotic" states** with $T < T_U$
- But we obtained a **little more**.

What was done:

- The collision of two gold nuclei Au-Au are considered.
- The parton-hadron-string dynamics (PHSD) model is used: **[Cassing, Bratkovskaya, Nucl.Phys.A (2009)]**
- The space distributions of acceleration and temperature and their time evolution were obtained.

Results: central Au-Au collisions

- The acceleration is maximum at the **initial time moments** and has the order of:

$$a \sim 1\,GeV \sim 10^{32}\,m/sec^2$$

-- acceleration is **extremely large in nature**

[Вергелес,Николаев, Обухов, Силенко, Теряев, УФН 2023, e-Print: 2204.00427]

- Indeed, **states with** $T < T_U$ **are formed.**

- The region $T < T_U$ corresponds predominantly to the hadron phase,
and region $T > T_U$ to the quark-gluon phase.

- The prediction about the connection between **hadronization**
and **phase transition at Unruh temperature** is qualitatively confirmed

Conclusion

Conclusion

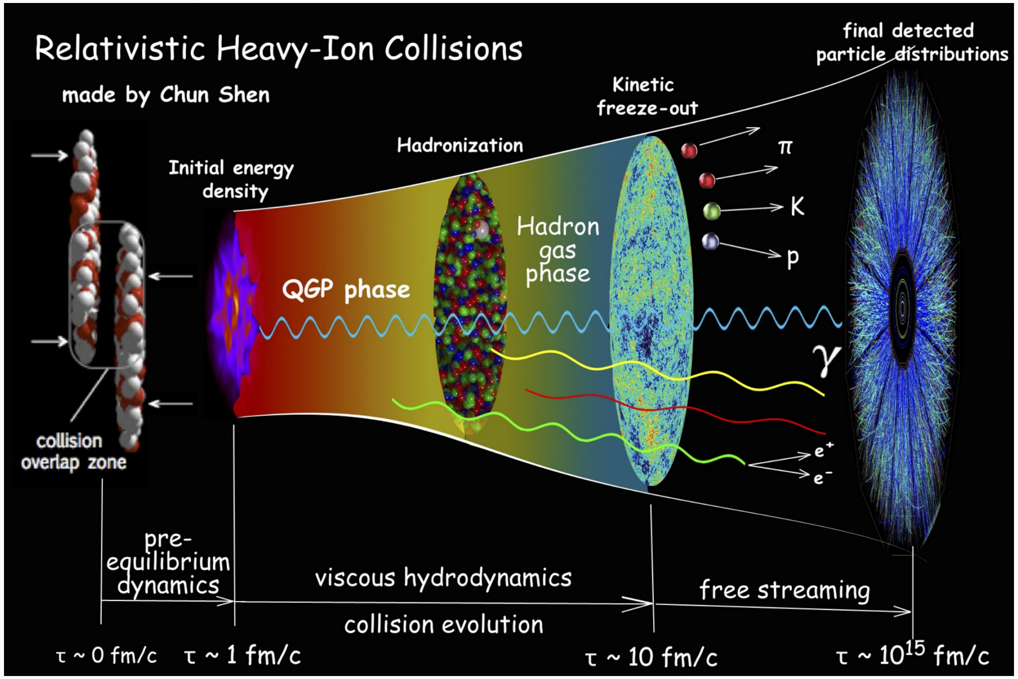
- The Unruh effect, from the point of view of statistical quantum mechanics, leads to **vanishing of averages** (e.g., the energy-momentum tensor) at $T = T_U$
  Averages can be found in ordinary flat space from effective **statistical** interaction.
- We have **shown Unruh effect in statistics** explicitly for massless and massive free fields with spins 0 and ½.
- We have constructed an analytical continuation to the temperature region below the Unruh temperature. At $T = T_U$ a **quantum phase transition** occurs, associated physics near horizon.
- The obtained corrections correspond to the corrections in the field of the **cosmic string**.
- Interpretation of **hadronization** as an effect associated with a phase transition was proposed.
- The **modeling confirms** the formation of states with temperature $T < a/2\pi$ at the early stages of HICs, and demonstrates **phase separation** with respect to $T = a/2\pi$ (QGP at $T > a/2\pi$ and hadrons at $T < a/2\pi$) (*N. Tsegelnik talk*).
  This confirms the suggested hadronization scenario.

Hydrodynamics in heavy ion collisions

- In the early stages of heavy ion collisions, the system exhibits **hydrodynamic** properties.
- There are indications that the QGP is a fluid with almost **minimal viscosity** close to the famous KSS-bound predicted from black hole physics.

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

[Kovtun, Son, Starinets, PRL, 2005, arXiv:hep-th/0405231]



https://u.osu.edu/vishnu/2014/08/06/sketch-of-relativistic-heavy-ion-collisions/

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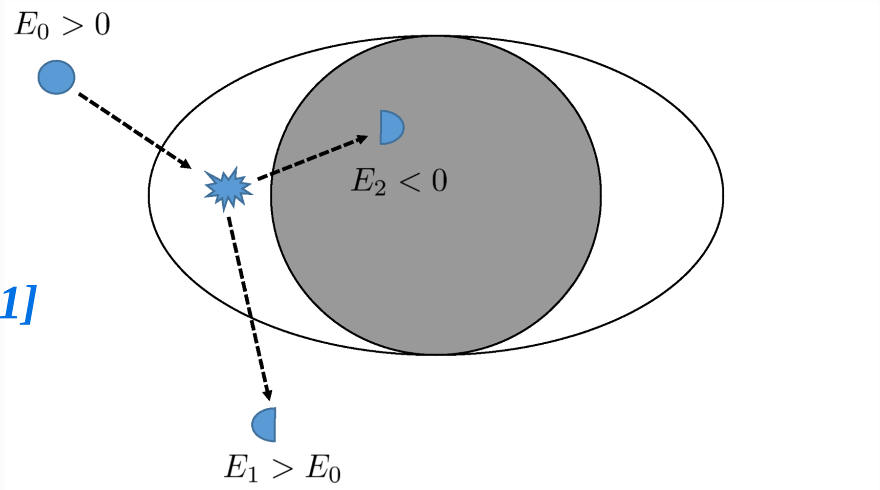

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$$S_{\text{eff}}(v) = mT\sqrt{1 - V^2} + \ldots .$$

Inside black hole?

- Previously, it was hypothesized that the Unruh temperature is the minimal one

  [F. Becattini. Phys. Rev., D97(8):085013, 2018.]

Possible explanation:

Minkowski vacuum is a stable and therefore lower energy state.

[Jin Jia and Pin Yu. Remark on the nonlinear stability of Minkowski spacetime: a rigidity theorem. 4 2023]

Then if we normalize to the Minkowski vacuum

$$T^{\alpha}_{\beta}(\text{Minkowski vacuum}) = 0$$ → Energy cannot be negative

According to the Unruh effect Minkowski vacuum corresponds to Unruh temperature, so in any accelerated frame choice of $T_M = |a_M|/(2\pi)$ leads to zero energy

(selecting a specific acceleration value (or temperature) ~ "selecting gauge")

e.g. $$\epsilon_{s=1/2} \equiv \langle \hat{T}^0_0 \rangle = \left(\frac{7\pi^2 T^4}{60} + \frac{|a|^2 T^2}{24} - \frac{17|a|^4}{960\pi^2} \right) \quad (T \geq T_U)$$

Since at $T < T_U$ the energy is negative → the existence of such states **would contradict the stability of the Minkowski vacuum?**

Inside black hole?

- Thus, the states below $T < T_U$ seem to lead to the instability of the Minkowski vacuum

⇓

Possible way out: states $T < T_U$ refer to the region beyond the horizon (inside the black hole).

Outlook

- Systems with **other spins** and **other horizons**?

-- the relationship with the non-unitarity of the Lorentz group representations allows us to expect a similar effect for any spins except 0. The space, e.g. near the horizon of a black hole is described by the Rindler metric.

- “Tabletop experiment”: **analogy** with the **Casimir effect** – similar effects for different configurations of conducting plates?

-- for example, the Casimir wedge is (almost) mathematically identical to the conical space.